Hybrid Beamforming Design for OFDM Dual-Function Radar-Communication System with Double-Phase-Shifter Structure

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Why DFRC?

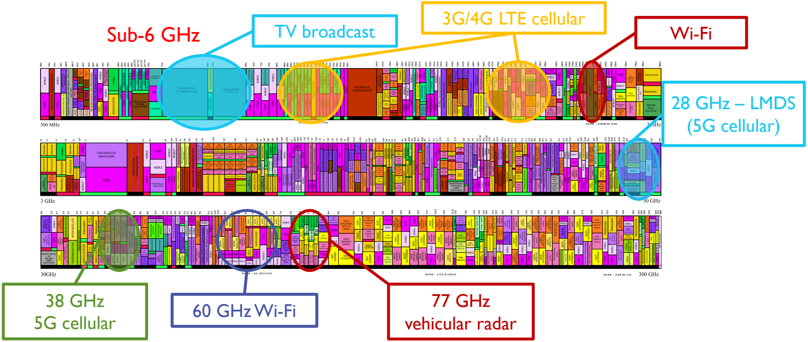
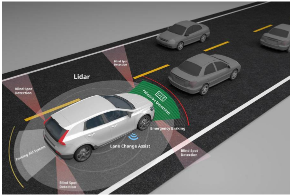


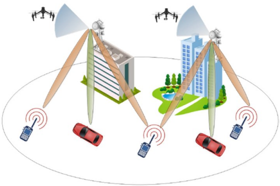
Figure: U.S. Frequency Allocation Chart as of October 2011

How to make efficient use of spectrum resources?

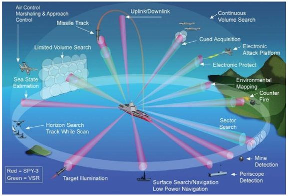
Why DFRC?



(a)



(b)



(c)

Figure: Practical application of DFRC technology

DFRC has wide application scenarios!

Motivation

- Existing works on DFRC
  1. Communication-Centric
  2. Radar-Centric
  3. **Joint Radar-Communication Waveform Design**
- Drawback
  1. Fully digital beamforming: Requires one high-resolution RF chain per antenna element.
  2. There is **no** comprehensive HBF structure to efficiently achieve a satisfactory trade-off between performance and hardware cost.
     - Fully-connected HBF provides satisfactory performance with a large number of phase shifters.
     - Partially-connected HBF saves the usage of phase shifters but leads to performance loss.
- Motivation
  1. Double-phase-shifter based HBF provides a trade-off between performance and hardware cost.

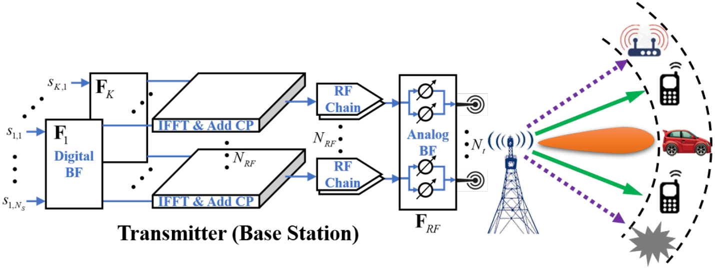


Figure: A mmWave wideband OFDM-DFRC system equipped with a DPS-based sub-connected HBF structure serves multiple downlink users and detects a target of interest in the presence of interference.

System Model

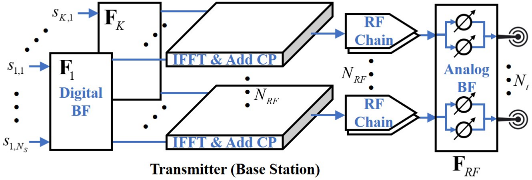


Figure: Overview of a wideband OFDM-DFRC system with DPS based hybrid beamforming architecture at the transmitter.

The transmit signal $\mathbf{x}_k \in \mathbb{C}^{N_t}$ at k-th subcarrier can be expressed as

$$\mathbf{x}_k = \mathbf{F}_{RF}\mathbf{F}_k\mathbf{s}_k = \mathbf{F}_{RF}\sum_{u=1}^{N_u}\mathbf{f}_{u,k}s_{u,k}, \tag{1}$$

System Model

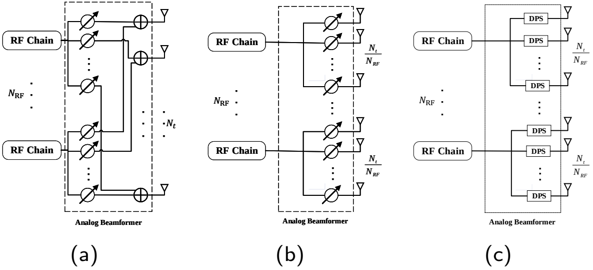


Figure: Different structures of Analog beamformer (a) Fully-connected structure, (b) Partially-connected structure and (c) DPS based Partially-connected structure.

$$\mathbf{F}_{RF} = \mathrm{Bdiag}\left(\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{N_{RF}}\right) \tag{2}$$

where $\mathbf{p}_i \in \mathbb{C}^{N_t/N_{RF}}, \forall i$ with $\mathbf{p}_i\left[j\right] = e^{\jmath\varphi_{i,j}^1} + e^{\jmath\varphi_{i,j}^2} = A_{i,j}e^{\jmath\varphi_{i,j}}, \forall i, j$, and $A_{i,j} \in \left[0, 2\right], \varphi_{i,j} \in \left[0, 2\pi\right)$. With $A_{i,j}$ and $\varphi_{i,j}$, the $\varphi_{i,j}^1$ and $\varphi_{i,j}^2$ are calculated by

$$\varphi_{i,j}^1 = \varphi_{i,j} + \arccos\left(A_{i,j}/2\right), \quad \varphi_{i,j}^2 = \varphi_{i,j} - \arccos\left(A_{i,j}/2\right). \tag{3}$$

Comm. Model

- Communication Precoding Design **Main Idea**:
  1. High Quality of Service (QoS) communication.
- **Downlink Communication Model**:
  1. Received signal at k-th subcarrier for u-th user is given by

$$\hat{s}_{u,k} = \mathbf{w}_{u,k}^H \mathbf{H}_{u,k} \mathbf{F}_{RF} \mathbf{f}_{u,k} s_{u,k} + \mathbf{w}_{u,k}^H \mathbf{H}_{u,k} \mathbf{F}_{RF} \sum_{\ell \neq u}^{N_u} \mathbf{f}_{\ell,k} s_{\ell,k} + \mathbf{w}_{u,k}^H \mathbf{n}_{u,k}, \quad (4)$$

  2. Signal-to-interference-plus-noise-ratio (SINR) at k-th subcarrier for user u is

$$\mathrm{SINR}_{u,k} = \frac{\left| \mathbf{w}_{u,k}^H \mathbf{H}_{u,k} \mathbf{F}_{RF} \mathbf{f}_{u,k} \right|^2}{\sum_{\ell \neq u} \left| \mathbf{w}_{u,k}^H \mathbf{H}_{u,k} \mathbf{F}_{RF} \mathbf{f}_{\ell,k} \right|^2 + \sigma_n^2 \mathbf{w}_{u,k}^H \mathbf{w}_{u,k}}, \quad (5)$$

- **Communication Performance**: Spectral efficiency for user u at k-th sub-carrier can be written as

$$\mathrm{R}_{u,k} = \log\left(1 + \mathrm{SINR}_{u,k}\right). \quad (6)$$

Radar Model

- Radar Waveform Design **Main Idea**:
  1. Effectively control the energy at main-lobe and side-lobe.
  2. Avoid the unwanted return causing by strong interference.
- **Integrated Side-lobe to Main-lobe Ratio** (ISMR) [1] is defined as

$$\begin{aligned}\mathsf{ISMR}(k) &= \frac{\int_{\Delta t}\int_{\Theta_s}\left|\mathbf{a}^H(f_k,\theta)\,\mathbf{x}(t)\right|^2 \mathrm{d}\theta \mathrm{d}t}{\int_{\Delta t}\int_{\Theta_m}\left|\mathbf{a}^H(f_k,\theta)\,\mathbf{x}(t)\right|^2 \mathrm{d}\theta \mathrm{d}t} \\ &= \frac{\mathrm{tr}\left\{\mathbf{F}_{RF}\mathbf{F}_k\mathbf{F}_k^H\mathbf{F}_{RF}^H\mathbf{\Omega}_{s,k}\right\}}{\mathrm{tr}\left\{\mathbf{F}_{RF}\mathbf{F}_k\mathbf{F}_k^H\mathbf{F}_{RF}^H\mathbf{\Omega}_{m,k}\right\}},\end{aligned} \tag{7}$$

where $\mathbf{\Omega}_{s,k} = \int_{\Theta_s}\mathbf{a}(f_k,\theta)\,\mathbf{a}^H(f_k,\theta)\mathrm{d}\theta$ and $\mathbf{\Omega}_{m,k} = \int_{\Theta_m}\mathbf{a}(f_k,\theta)\,\mathbf{a}^H(f_k,\theta)\mathrm{d}\theta$ with Θ_s and Θ_m being side-lobe and main-lobe region, respectively.

[1] H. Xu, R. S. Blum, J. Wang and J. Yuan, "Colocated MIMO radar waveform design for transmit beampattern formation," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 1558-1568, April 2015.

Radar Model

- Radar Waveform Design **Main Idea**:
  1. Effectively control the energy at main-lobe and side-lobe.
  2. Avoid the unwanted return causing by strong interference.
- **Space Frequency Nulling (SFN)** is defined as

$$\mathsf{SFN}\left(k\right)=\sum_{i=1}^{I_k}\int_{\Delta t}\left|\mathbf{a}\left(f_k,\vartheta_{k,i}\right)\mathbf{x}\left(t\right)\right|\mathrm{d}t\leqslant I_k\Gamma_k \tag{8}$$

- **Robust SFN** is defined as

$$\begin{aligned}\overline{\mathsf{SFN}}\left(k\right)&=\sum_{i=1}^{I_k}\int_{\Delta t}\int_{\Theta_{N_{k,i}}}\left|\mathbf{a}\left(f_k,\theta\right)\mathbf{x}\left(t\right)\right|\mathrm{d}\theta\mathrm{d}t\\&=\mathrm{tr}\left\{\mathbf{F}_{RF}\mathbf{F}_k\mathbf{F}_k^H\mathbf{F}_{RF}^H\mathbf{A}_k\right\}\leqslant\overline{\Gamma}_k\end{aligned} \tag{9}$$

Problem Formulation

Problem of interest:

$$\max_{\substack{\mathbf{F}_{RF},\{\mathbf{F}_k\},\\ \{\mathbf{w}_{u,k}\}}} \frac{1}{K}\sum_{k=1}^{K}\sum_{u=1}^{N_u} \mathsf{R}_{u,k} \tag{10a}$$

$$\text{s.t.} \quad \mathsf{ISMR}\,(k) \leqslant \gamma_k, \forall k \tag{10b}$$

$$\overline{\mathsf{SFN}}\,(k) \leqslant \overline{\Gamma}_k, \forall k \tag{10c}$$

$$\|\mathbf{F}_{RF}\mathbf{F}_k\|_F^2 = \mathsf{P}_k, \forall k \tag{10d}$$

$$\mathbf{F}_{RF} \in \mathcal{F}_{RF}, \tag{10e}$$

- Joint waveform design criteria.
- High dimension and non-convex probelm.

Problem Formulation

- Problem Reformulation based on WMMSE [1]

$$\begin{aligned} \min_{\mathbf{F}_{RF},\{\mathbf{F}_k\},\{\mathbf{w}_{u,k}\}} \ & \frac{1}{K}\sum_{u=1}^{N_u} \mathrm{tr}\left\{\aleph_{u,k}^{t}\mathcal{E}_{u,k}\left(\mathbf{F}_{RF},\mathbf{F}_k,\mathbf{w}_{u,k}\right)\right\} \\ \text{s.t.} \quad & \text{(10b)} - \text{(10e)}, \end{aligned} \tag{11}$$

where $\aleph_{u,k}^{t} = \mathcal{E}_{u,k}^{-1}\left(\mathbf{T}_k^t, \mathbf{w}_{u,k}^t\right)$.

- MMSE $\mathcal{E}_{u,k}\left(\mathbf{F}_{RF},\mathbf{F}_k,\mathbf{w}_{u,k}\right)$ is defined as

$$\begin{aligned} &\mathcal{E}_{u,k}\left(\mathbf{F}_{RF},\mathbf{F}_k,\mathbf{w}_{u,k}\right) = \mathbb{E}\left[\left(\hat{s}_{u,k}-s_{u,k}\right)\left(\hat{s}_{u,k}-s_{u,k}\right)^H\right] \\ &= \left|\mathbf{w}_{u,k}^H\mathbf{H}_{u,k}\mathbf{F}_{RF}\mathbf{f}_{u,k}-1\right|^2 + \sum_{\ell\neq u}^{N_u}\left|\mathbf{w}_{u,k}^H\mathbf{H}_{u,k}\mathbf{F}_{RF}\mathbf{f}_{\ell,k}\right|^2 + \sigma_n^2\mathbf{w}_{u,k}^H\mathbf{w}_{u,k}. \end{aligned} \tag{12}$$

Problem (11) is still hard to tackle!

[1] Q. Shi, M. Razaviyayn, Z. Luo and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," in *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331-4340, Sept. 2011.

- By introducing auxiliary $\{\mathbf{T}_k\}$,$\{\mathbf{V}_k\}$ and $\{\mathbf{Y}_k\}$, the problem (11) can be reformulated as the following consensus problem

$$
\begin{aligned}
\min_{\substack{\mathbf{F}_{RF},\{\mathbf{F}_k\},\{\mathbf{w}_{u,k}\}\\ \{\mathbf{T}_k\},\{\mathbf{V}_k\},\{\mathbf{Y}_k\}}} \ & \frac{1}{K}\sum_{u=1}^{N_u} \mathrm{tr}\left\{\aleph_{u,k}^{t}\mathcal{E}_{u,k}\left(\mathbf{T}_k,\mathbf{w}_{u,k}\right)\right\} && \text{(13a)}\\
\text{s.t.} \quad & \mathbf{T}_k=\mathbf{V}_k=\mathbf{Y}_k=\mathbf{F}_{RF}\mathbf{F}_k, \forall k && \text{(13b)}\\
& \mathrm{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\mathbf{\Omega}_k\right\}\leqslant 0, \forall k && \text{(13c)}\\
& \mathrm{tr}\left\{\mathbf{Y}_k\mathbf{Y}_k^H\mathbf{A}_k\right\}\leqslant \bar{\Gamma}_k, \forall k && \text{(13d)}\\
& \|\mathbf{T}_k\|_F^2=\mathsf{P}, \forall k && \text{(13e)}\\
& \mathbf{F}_{RF}\in\mathcal{F}_{RF}, && \text{(13f)}
\end{aligned}
$$

where $\mathbf{\Omega}_k=\mathbf{\Omega}_{s,k}-\gamma_k\mathbf{\Omega}_{m,k}$.

Proposed CADMM-Based Approach

- Placing the bi-linear equality constraint into the augmented Lagrangian function of yields

$$\mathcal{L} = \sum_{k=1}^{K} \mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k \right) \tag{14}$$

  - $\mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k \right)$ is defined as

$$\begin{aligned} \mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k \right) &= \frac{\rho_{1,k}}{2} \left\| \mathbf{T}_k - \mathbf{F}_{RF}\mathbf{F}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}} \right\|_F^2 \\ &+ \frac{\rho_{2,k}}{2} \left\| \mathbf{T}_k - \mathbf{V}_k + \frac{\mathbf{\Lambda}_{2,k}}{\rho_{2,k}} \right\|_F^2 + \frac{\rho_{3,k}}{2} \left\| \mathbf{T}_k - \mathbf{Y}_k + \frac{\mathbf{\Lambda}_{3,k}}{\rho_{3,k}} \right\|_F^2 \\ &+ \frac{1}{K} \sum_{u=1}^{N_u} \mathrm{tr} \left\{ \aleph_{u,k}^t \mathcal{E}_{u,k} \left(\mathbf{T}_k, \mathbf{w}_{u,k} \right) \right\} \end{aligned}$$

  - $\mathbf{\Lambda}_{1,k}, \mathbf{\Lambda}_{2,k}, \mathbf{\Lambda}_{3,k} \in \mathbb{C}^{N_t \times N_s}, \forall k$ denote the dual variables,
  - $\rho_{1,k}, \rho_{2,k}, \rho_{3,k} > 0$ stand for corresponding penalty parameters.

Proposed CADMM-Based Approach

- As a result, at the I-th iteration, the corresponding CADMM framework takes the following iterations:

$$\left(\mathbf{w}_{u,k}^{(I+1)}, \mathbf{T}_k^{(I+1)}, \mathbf{V}_k^{(I+1)}, \mathbf{Y}_k^{(I+1)}\right) \tag{15a}$$
$$\leftarrow \underset{\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k}{\arg\min} \; \mathcal{L}_k\left(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k, \mathbf{F}_{RF}^{(I)}, \mathbf{F}_k^{(I)}\right)$$
$$\left(\mathbf{F}_{RF}^{(I+1)}, \mathbf{F}_k^{(I+1)}\right) \tag{15b}$$
$$\leftarrow \underset{\mathbf{F}_{RF}, \mathbf{F}_k}{\arg\min} \, \mathcal{L}_k\left(\mathbf{w}_{u,k}^{(I+1)}, \mathbf{T}_k^{(I+1)}, \mathbf{V}_k^{(I+1)}, \mathbf{Y}_k^{(I+1)}, \mathbf{F}_{RF}, \mathbf{F}_k\right)$$

Optimization of $(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

- With fixed $(\mathbf{F}_{RF}, \mathbf{F}_k)$ and $(\mathbf{\Lambda}_{1,k}, \mathbf{\Lambda}_{2,k}, \mathbf{\Lambda}_{3,k})$, the $(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$ are updated by solving the following problem

$$\min_{\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k} \mathcal{L}\left(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k\right) \tag{16a}$$

$$\text{s.t.} \quad \text{tr}\left\{\mathbf{V}_k \mathbf{V}_k^H \mathbf{\Omega}_k\right\} \leqslant 0 \tag{16b}$$

$$\text{tr}\left\{\mathbf{Y}_k \mathbf{Y}_k^H \mathbf{A}_k\right\} \leqslant \bar{\Gamma}_k \tag{16c}$$

$$\|\mathbf{T}_k\|_F^2 = \mathsf{P}, \forall k. \tag{16d}$$

STEP1: Update of $\mathbf{T}_k$

1. By leveraging the Karush-Kuhn-Tucker (KKT) conditions, the close-form solution to $\mathbf{T}_k$ can be calculated by

$$\mathbf{T}_k^{\star}(\mu_k) = \left\{\mathbf{\Phi}_k + 2\mu_k \mathbf{I}_{N_t}\right\}^{-1} \mathbf{\Upsilon}_k, \tag{17}$$

- $\mathbf{\Phi}_k = \frac{2}{K}\sum_{u=1}^{N_u} \aleph_{u,k}^t \mathbf{H}_{u,k}^H \mathbf{w}_{u,k} \mathbf{w}_{u,k}^H \mathbf{H}_{u,k} \mathbf{T}_k + (\rho_{1,k} + \rho_{2,k} + \rho_{3,k}) \mathbf{I}_{N_t}$
- $\mathbf{\Upsilon}_{\boldsymbol{k}} =$ $\frac{2}{K}\sum_{u=1}^{N_u} \aleph_{u,k}^t \mathbf{H}_{u,k}^H \mathbf{w}_{u,k} \mathbf{e}_u^H + \rho_{1,k}\mathbf{F}_{RF}\mathbf{F}_k + \rho_{2,k}\mathbf{T}_k + \rho_{3,k}\mathbf{Y}_k - \mathbf{\Lambda}_{1,k} - \mathbf{\Lambda}_{2,k} - \mathbf{\Lambda}_{3,k}.$

2. Defining eigenvalue decomposition (EIG) $\mathbf{\Phi}_k = \mathbf{Q}_k \mathbf{D}_k \mathbf{Q}_k^H$ and substituting (17) into the power constraint, one gets

$$\left\|\mathbf{T}_k^{\star}(\mu_k)\right\|_F^2 = \sum_{n=1}^{N_t} \frac{\left(\mathbf{Q}_k^H \mathbf{\Upsilon}_k \mathbf{\Upsilon}_k^H \mathbf{Q}_k\right)[n,n]}{\left(\mathbf{D}_k[n,n] + 2\mu_k\right)^2} = \mathrm{P}. \tag{18}$$

3. The optimal multiplier μ_k can be easily obtained via using golden-section search or Newton's method.
4. Plugging the μ_k into (17), we can obtain the solution of $\mathbf{T}_k$.

Optimization of $(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

STEP2: Update of $\mathbf{V}_k$

1. By leveraging the KKT conditions, we can obtain

$$\mathbf{V}_k^\star = \left\{\rho_{2,k}\mathbf{I}_{N_t} + 2\chi_1\boldsymbol{\Omega}^H\right\}^{-1}\boldsymbol{\Psi}_k \tag{19a}$$

$$\chi_1 \mathrm{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\boldsymbol{\Omega}_k\right\} = 0,\ \chi_1 \geqslant 0 \tag{19b}$$

$$\mathrm{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\boldsymbol{\Omega}_k\right\} \leqslant 0, \tag{19c}$$

   where $\boldsymbol{\Psi}_k = \boldsymbol{\Lambda}_{2,k} + \rho_{2,k}\mathbf{T}_k$.

2. *Case 1:* For $\chi_1 = 0$, form the (19a) we have $\mathbf{V}_k^\star = \boldsymbol{\Psi}_k/\rho_{2,k}$, which must satisfy the condition (19c), otherwise
3. *Case 2:* For $\chi_1 \neq 0$, form the (19a) we have

$$\mathbf{V}_k^\star(\chi_1) = \left\{\rho_{2,k}\mathbf{I}_{N_t} + 2\chi_1\boldsymbol{\Omega}^H\right\}^{-1}\boldsymbol{\Psi}_k, \tag{20}$$

   - Plugging (20) into equality constraint $\mathrm{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\boldsymbol{\Omega}_k\right\} = 0$, we obtain

$$\mathrm{tr}\left\{\mathbf{V}_k^\star(\chi_1)\left(\mathbf{V}_k^\star(\chi_1)\right)^H\boldsymbol{\Omega}_k\right\} = \sum_{n=1}^{N_t}\frac{\left(\mathbf{Z}_k^H\boldsymbol{\Omega}_k\mathbf{Z}_k\right)[n,n]\left(\mathbf{Z}_k^H\boldsymbol{\Psi}_k\boldsymbol{\Psi}_k^H\mathbf{Z}_k\right)[n,n]}{\left(\rho_{2,k} + 2\chi_1\mathbf{M}_k[n,n]\right)^2} = 0. \tag{21}$$

Optimization of $(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

STEP3: Update of $\mathbf{Y}_k$

- Following the same procedure for updating $\mathbf{V}_k$, we can obtain

$$\mathbf{Y}_k^{\star} = \left\{\rho_{3,k}\mathbf{I}_{N_t} + 2\chi_2\mathbf{A}_k\mathbf{A}_k^H\right\}^{-1}\mathbf{\Pi} \tag{22a}$$

$$\left\|\mathbf{A}_k^H\mathbf{Y}_k^{\star}\right\|_F^2 \leqslant \bar{\Gamma}_k \tag{22b}$$

$$\chi_2\left(\left\|\mathbf{A}_k^H\mathbf{Y}_k^{\star}\right\|_F^2 - \bar{\Gamma}_k\right) = 0,\ \chi_2 \geqslant 0, \tag{22c}$$

STEP4: Update of $\mathbf{w}_{u,k}$

- According to MMSE receiver, its optimal solution of $\mathbf{w}_{u,k}$ can be calculated as

$$\mathbf{w}_{u,k} = \left(\mathbf{H}_k\mathbf{T}_k\mathbf{T}_k^H\mathbf{H}_k^H - \mathbf{I}_{M_r}\right)^{-1}\mathbf{H}_k\mathbf{T}_k\left(:, n\right). \tag{23}$$

Optimization of $(\mathbf{F}_{RF}, \mathbf{F}_k)$

- With fixed $(\mathbf{w}_{u,k}, \mathbf{T}_k, \mathbf{V}_k \;\; \mathbf{Y}_k)$ and $(\mathbf{\Lambda}_{1,k}, \mathbf{\Lambda}_{2,k}, \mathbf{\Lambda}_{3,k})$, the $(\mathbf{F}_{RF}, \mathbf{F}_k)$ are updated by solving the following problem

$$\begin{aligned} \min_{\mathbf{F}_{RF}, \{\mathbf{F}_k\}} & \sum_{k=1}^{K} \frac{\rho_{1,k}}{2} \left\| \mathbf{T}_k - \mathbf{F}_{RF}\mathbf{F}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}} \right\|_F^2 \\ \text{s.t.} \quad & \mathbf{F}_{RF} \in \mathcal{F}_{RF}. \end{aligned} \tag{24}$$

STEP1: Update of $\mathbf{F}_k$

- The close-form solution of $\mathbf{F}_k$ is given by

$$\mathbf{F}_k = \left(\mathbf{F}_{RF}^H \mathbf{F}_{RF}\right)^{-1} \mathbf{F}_{RF}^H \left(\mathbf{T}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}} \right), \forall k. \tag{25}$$

Optimization of $(\mathbf{F}_{RF}, \mathbf{F}_k)$

STEP2: Update of $\mathbf{F}_{RF}$

1. According to the block diagonal property of the analog beamformer, the corresponding problem of $\mathbf{F}_{RF}$ can be reformulated as

$$\min_{A_{i,j},\varphi_{i,j}} \sum_{k=1}^{K} \frac{\rho_{1,k}}{2} \left\| \widetilde{\mathbf{T}}_k[i,:] - A_{i,j} e^{\jmath\varphi_{i,j}} \mathbf{F}_k[j,:] \right\|_F^2, \forall i, \tag{26}$$

   - $\widetilde{\mathbf{T}}_k = \mathbf{T}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}}$ and $j = \left\lceil i \frac{N_{RF}}{N_t} \right\rceil$.

2. It is obvious that the optimal solution of $\mathbf{F}_{RF}[i,j]$ is

$$A_{i,j} = \begin{cases} \frac{\sum_{k=1}^{K} \rho_{1,k} \left|\mathbf{F}_k[j,:]\mathbf{T}_k^H[i,:]\right|}{\sum_{k=1}^{K} \rho_{1,k} \|\mathbf{F}_k[j,:]\|_F^2}, & \frac{\sum_{k=1}^{K} \rho_{1,k} \left|\mathbf{F}_k[j,:]\mathbf{T}_k^H[i,:]\right|}{\sum_{k=1}^{K} \rho_{1,k} \|\mathbf{F}_k[j,:]\|_F^2} \leqslant 2 \\ 2, & \text{otherwise}, \end{cases}$$

and

$$\varphi_{i,j} = \text{angle}\left(\sum_{k=1}^{K} \mathbf{F}_k[j,:] \mathbf{T}_k^H[i,:] \right).$$

Computational Complexity

1. The main computational complexity is caused by updating $(\mathbf{F}_{RF}, \{\mathbf{F}_k\})$ and $(\{\mathbf{w}_{u,k}\}, \{\mathbf{T}_k\}, \{\mathbf{V}_k\}, \{\mathbf{Y}_k\})$ in each iteration.
2. Updating $(\mathbf{F}_{RF}, \{\mathbf{F}_k\})$ requires $\mathcal{O}\left(K\left(N_t N_{RF} N_s + N_t N_{RF}^2\right)\right)$ operations.
3. Updating $(\{\mathbf{w}_{u,k}\}, \{\mathbf{T}_k\}, \{\mathbf{V}_k\}, \{\mathbf{Y}_k\})$ needs $\mathcal{O}\left(K N_t^2 \left(\log\left(2\right) + N_s\right)\right)$ operations,.
4. As a result, the overall complexity of the proposed algorithm is given by $\mathcal{O}\left(T_{max} K\left(N_t N_{RF} N_s + N_t N_{RF}^2 + N_t^2 N_s + N_t^2 \log\left(2\right)\right)\right)$.

Parameters Setting

Parameter	Value	Parameter	Value
N_t	32	N_s	4
N_{RF}	8	M_r	4
f_c	10GHz	B	2.56GHz
K	128	P_k	1

Table: Collection of parameters values

- The main-lobe region $\Theta_m = [-10^\circ, 10^\circ]$,
- The side-lobe region $\Theta_s = [-90^\circ, -10^\circ) \cup (10^\circ, 90^\circ]$,
- The SFN azimuths ϑ_k are assumed as $\vartheta_k = \{-22^\circ, 22^\circ\}, \forall k$ with uncertainty $\delta = 4^\circ$,
- The threshold Γ_k of the SFN on k-th subcarrier is set as 40dB,

Comm. Performance

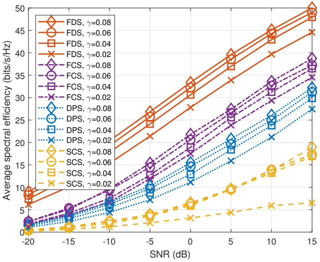


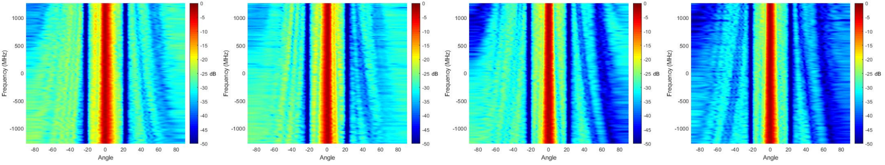
Figure: Achievable SE versus SNR with different ISMR γ.

Table: Comparison between different structures.

Structure	FCS	DPS	SCS
No. of PS	$N_t N_{RF} = 256$	$2N_t = 64$	$N_t = 32$

- The attained spectral efficiencies increase along with the γ for all algorithms,
- With a slight performance loss, DPS based DFRC can significantly reduce the number of PSs than the FCS,
- DPS based DFRC obtains a remarkable improvement in performance with only a moderate increase of PS compared to the SPS.

Radar Performance



(a) (b) (c) (d)

Figure: The space-frequency spectral behaviors of the DPS based OFDM-DFRC with SNR=0dB and different ISMR value (1) $\gamma = 0.08$, (2) $\gamma = 0.06$, (3) $\gamma = 0.04$, (4) $\gamma = 0.02$.

- As the ISMR level γ decreases, the side-lobe energy becomes lower and lower,
- The beampattern has desired SFN at interference azimuth for all γ,
- There exist the coupling effect between the space and frequency.

Conclusions

- In this paper, we have addressed the problem of the HBF design for the OFDM-DFRC system with the DPS structure.
- The corresponding problem is formulated to maximize the spectral efficiency subjecting to the power budget, SFN and the radar ISMR constraint.
- The simulation results shows that:
  - We propose an alternating optimization-based algorithm to tackle this non-convex problem.
  - The simulation results show that the proposed method and novel structure can achieve satisfactory performance for radar and communication.

Thank you !!

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