# Spatial Spectrum Nulling for Wideband OFDM-DFRC System With Hybrid Beamforming Architecture

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# Why DFRC?

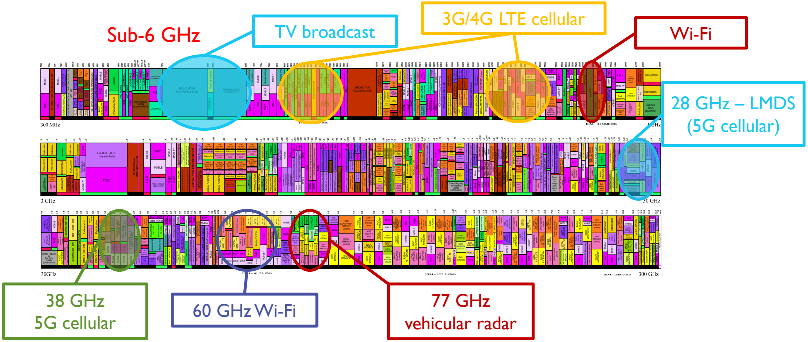
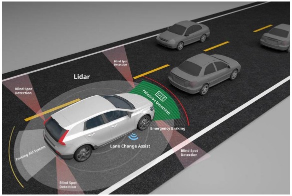


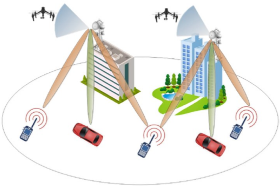

Figure: U.S. Frequency Allocation Chart as of October 2011

How to make efficient use of spectrum resources?

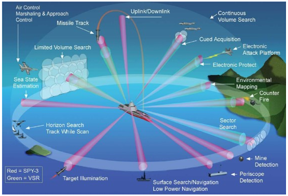
# Why DFRC?




(a)



(b)




(c)

Figure: Practical application of DFRC technology

DFRC has wide application scenarios!

# Motivation

- Existing works on DFRC
    1. Communication-Centric
    2. Radar-Centric
    3. **Joint Waveform Design**
- Drawback
    1. Fully digital beamforming: Requires one high-resolution RF chain per antenna element.
    2. Signal-dependent interference scenarios has not been considered.
- Motivation
    1. Hybrid beamforming provides a trade-off between performance and hardware cost.
    2. Spatial spectrum nulling suppresses the unwanted reflect caused by strong interference.

# System Model

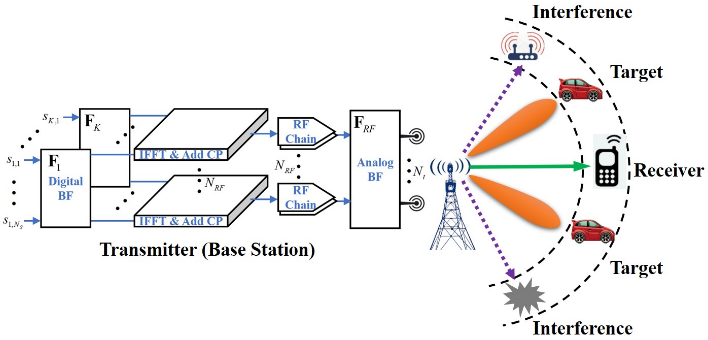



Figure: Overview of a wideband OFDM-DFRC system with hybrid analog and digital beamforming architecture at the transmitter.

# System Model

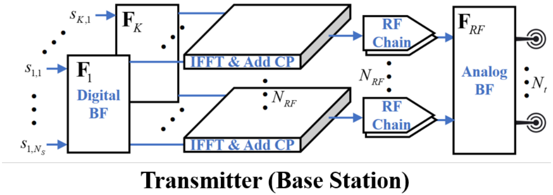



Figure: Overview of a wideband OFDM-DFRC system with hybrid analog and digital beamforming architecture at the transmitter.

In the OFDM hybrid beamforming structure, the transmit signal $\mathbf{x}(t) \in \mathbb{C}^{N_t}$ at time instant $t$ can be expressed as

$$\mathbf{x}(t) = \mathbf{F}_{RF} \sum_{k=1}^{K} \mathbf{F}_k \mathbf{s}_k e^{j2\pi f_k t} \text{rect}(t), \quad (1)$$

# Comm. Model

- Communication Precoding Design **Main Idea**:
  1. High Quality of Service (QoS) communication.
- **Downlink Communication Model**:
  1. Received signal at downlink user is given by

$$\hat{\mathbf{s}}_k = \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_k \mathbf{s}_k + \mathbf{W}_k^H \mathbf{n}_k, \forall k, \quad (2)$$

  2. Minimum Mean Square Error (MMSE)

$$\begin{aligned} \mathbf{E}_k\left(\mathbf{W}_k, \mathbf{F}_{RF}, \mathbf{F}_k\right) &= \mathbb{E}\left[\left(\hat{\mathbf{s}}_k - \mathbf{s}_k\right)\left(\hat{\mathbf{s}}_k - \mathbf{s}_k\right)^H\right] \\ &= \left(\mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_k - \mathbf{I}_{N_s}\right)\left(\mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_k - \mathbf{I}_{N_s}\right)^H + \sigma_n^2 \mathbf{W}_k^H \mathbf{W}_k. \end{aligned}$$

- **Communication Performance**: Spectral efficiency achieved at downlink user is given by

$$\mathrm{R}_k = \log\left(\left|\mathbf{I}_{N_s} + \mathbf{C}_k^{-1} \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_k \, \mathbf{F}_k^H \mathbf{F}_{RF}^H \mathbf{H}_k^H \mathbf{W}_k\right|\right), \quad (3)$$

where $\mathbf{C}_k = \sigma_n^2 \mathbf{W}_k^H \mathbf{W}_k$ is the noise covariance matrix at $k$-th subcarrier.

- Radar Waveform Design **Main Idea**:
  1. Effectively control the energy at main-lobe and side-lobe.
  2. Avoid the unwanted return causing by strong interference.
- **Integrated Side-lobe to Main-lobe Ratio** (ISMR) [1] is defined as

$$\begin{aligned} \mathrm{ISMR}(k) &= \frac{\int_{\Delta t}\int_{\Theta_s}\left|\mathbf{a}^H\left(f_k,\theta\right)\mathbf{x}\left(t\right)\right|^2 \mathrm{d}\theta \mathrm{d}t}{\int_{\Delta t}\int_{\Theta_m}\left|\mathbf{a}^H\left(f_k,\theta\right)\mathbf{x}\left(t\right)\right|^2 \mathrm{d}\theta \mathrm{d}t} \\ &= \frac{\mathrm{tr}\left\{\mathbf{F}_{RF}\mathbf{F}_k\mathbf{F}_k^H\mathbf{F}_{RF}^H\mathbf{\Omega}_{s,k}\right\}}{\mathrm{tr}\left\{\mathbf{F}_{RF}\mathbf{F}_k\mathbf{F}_k^H\mathbf{F}_{RF}^H\mathbf{\Omega}_{m,k}\right\}}, \end{aligned} \tag{4}$$

- **Spatial Spectrum Nulling (SNN)** is defined as

$$\left\|\mathbf{a}^H\left(f_k,\vartheta\right)\mathbf{F}_{RF}\mathbf{F}_k\right\|_F^2 \leqslant \Gamma_k, \vartheta \in \Theta_k, \tag{5}$$

[1] H. Xu, R. S. Blum, J. Wang and J. Yuan, "Colocated MIMO radar waveform design for transmit beampattern formation," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 1558-1568, April 2015.

# Problem Formulation

**Problem of interest**:

$$\max_{\substack{\mathbf{F}_{RF},\{\mathbf{F}_k\},\\ \{\mathbf{W}_k\}}} \frac{1}{K}\sum_{k=1}^{K} \mathsf{R}_k \tag{6a}$$

$$\text{s.t.} \quad \mathsf{ISMR}\,(k) \leqslant \gamma_k, \forall k \tag{6b}$$

$$\left\| \mathbf{a}^H \left(f_k, \vartheta\right) \mathbf{F}_{RF}\mathbf{F}_k \right\|_F^2 \leqslant \Gamma_k, \vartheta \in \Theta_k, \forall k \tag{6c}$$

$$\left\| \mathbf{F}_{RF}\mathbf{F}_k \right\|_F^2 = \mathsf{P}_k, \forall k \tag{6d}$$

$$\left| \mathbf{F}_{RF}\left[i,j\right] \right| = 1, \forall i,j, \tag{6e}$$

- Joint waveform design criteria.
- High dimension and non-convex probelm.

# Problem Formulation

- Problem Reformulation based on WMMSE [1]

$$\begin{aligned} \min_{\substack{\mathbf{F}_{RF},\{\mathbf{F}_k\} \\ \{\boldsymbol{\Xi}_k\},\{\mathbf{W}_k\}}} \ & \frac{1}{K}\sum_{k=1}^{K}\left(\mathrm{tr}\left\{\boldsymbol{\Xi}_k\mathbf{E}_k\left(\mathbf{W}_k,\mathbf{F}_{RF},\mathbf{F}_k\right)\right\}-\log\left|\boldsymbol{\Xi}_k\right|\right) \\ \text{s.t.} \quad & (5b)-(5e). \end{aligned} \tag{7}$$

The problem w.r.t $\{\boldsymbol{\Xi}_k\}$ is convex, and the first-order optimality condition yields

$$\boldsymbol{\Xi}_k=\mathbf{E}_k^{-1}\left(\mathbf{W}_k,\mathbf{F}_{RF},\mathbf{F}_k\right),\forall k, \tag{8}$$

- Origin problem can be simplified as

$$\begin{aligned} \min_{\mathbf{F}_{RF},\{\mathbf{F}_k\},\{\mathbf{W}_k\}} \ & \frac{1}{K}\sum_{k=1}^{K}\mathrm{tr}\left\{\boldsymbol{\Xi}_k^t\mathbf{E}_k\left(\mathbf{W}_k,\mathbf{F}_{RF},\mathbf{F}_k\right)\right\} \\ \text{s.t.} \quad & (5b)-(5e), \end{aligned} \tag{9}$$

**Problem (9) is still hard to tackle!**

[1] Q. Shi, M. Razaviyayn, Z. Luo and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," in *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331-4340, Sept. 2011.

# Proposed CADMM-Based Approach

- By introducing auxiliary $\{\mathbf{T}_k\}$ ,$\{\mathbf{V}_k\}$ and $\{\mathbf{Y}_k\}$, the problem (6) can be reformulated as the following consensus problem

$$\min_{\substack{\mathbf{F}_{RF},\{\mathbf{F}_k\},\{\mathbf{W}_k\}\\ \{\mathbf{T}_k\},\{\mathbf{V}_k\},\{\mathbf{Y}_k\}}} \frac{1}{K}\sum_{k=1}^{K} \mathrm{tr}\left\{\boldsymbol{\Xi}^t \mathbf{E}_k\left(\mathbf{W}_k, \mathbf{T}_k\right)\right\} \tag{10a}$$

$$\text{s.t.} \quad \mathbf{T}_k = \mathbf{V}_k = \mathbf{Y}_k = \mathbf{F}_{RF}\mathbf{F}_k, \forall k \tag{10b}$$

$$\mathrm{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\boldsymbol{\Omega}_k\right\} \leqslant 0, \forall k \tag{10c}$$

$$\left\|\mathbf{A}_k^H\mathbf{Y}_k\right\|_F^2 \leq \bar{\Gamma}_k, \forall k \tag{10d}$$

$$\left\|\mathbf{T}_k\right\|_F^2 = \mathsf{P}, \forall k \tag{10e}$$

$$\left|\mathbf{F}_{RF}\left[i,j\right]\right| = 1, \forall i, j, \tag{10f}$$

where $\mathbf{A}_k = \left[\mathbf{a}\left(f_k, \vartheta_1\right), \cdots, \mathbf{a}\left(f_k, \vartheta_{k,I_k}\right)\right]$, $\boldsymbol{\Omega}_k = \boldsymbol{\Omega}_{s,k} - \gamma_k\boldsymbol{\Omega}_{m,k}$ and $\bar{\Gamma}_k = I_k\Gamma_k$.

# Proposed CADMM-Based Approach

- Placing the bi-linear equality constraint into the augmented Lagrangian function of yields

$$\mathcal{L} = \sum_{k=1}^{K} \mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k\right), \tag{11}$$

  - $\mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k\right)$ is defined as

$$\begin{aligned}
&\mathcal{L}_k \left(\mathbf{F}_{RF}, \mathbf{F}_k, \mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k\right) \\
&= \frac{1}{K}\mathrm{tr}\left\{\boldsymbol{\Xi}^t \mathbf{E}_k \left(\mathbf{W}_k, \mathbf{T}_k\right)\right\} + \frac{\rho_{1,k}}{2}\left\|\mathbf{T}_k - \mathbf{F}_{RF}\mathbf{F}_k + \frac{\boldsymbol{\Lambda}_{1,k}}{\rho_{1,k}}\right\|_F^2 \\
&+ \frac{\rho_{2,k}}{2}\left\|\mathbf{T}_k - \mathbf{V}_k + \frac{\boldsymbol{\Lambda}_{2,k}}{\rho_{2,k}}\right\|_F^2 + \frac{\rho_{3,k}}{2}\left\|\mathbf{T}_k - \mathbf{Y}_k + \frac{\boldsymbol{\Lambda}_{3,k}}{\rho_{3,k}}\right\|_F^2,
\end{aligned}$$

  - $\boldsymbol{\Lambda}_{1,k}, \boldsymbol{\Lambda}_{2,k}, \boldsymbol{\Lambda}_{3,k} \in \mathbb{C}^{N_t \times N_s}, \forall k$ denote the dual variables,
  - $\rho_{1,k}, \rho_{2,k}, \rho_{3,k} > 0$ stand for corresponding penalty parameters.

# Proposed CADMM-Based Approach

- As a result, at the $I$-th iteration, the corresponding CADMM framework takes the following iterations:

$$\left(\mathbf{W}_k^{(I+1)}, \mathbf{T}_k^{(I+1)}, \mathbf{V}_k^{(I+1)}, \mathbf{Y}_k^{(I+1)}\right) \leftarrow \underset{\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k}{\arg\min} \mathcal{L}_k \left(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k, \mathbf{F}_{RF}^{(I)}, \mathbf{F}_k^{(I)}\right) \tag{12a}$$

$$\left(\mathbf{F}_{RF}^{(I+1)}, \mathbf{F}_k^{(I+1)}\right) \leftarrow \underset{\mathbf{F}_{RF}, \mathbf{F}_k}{\arg\min} \mathcal{L}_k \left(\mathbf{W}_k^{(I+1)}, \mathbf{T}_k^{(I+1)}, \mathbf{V}_k^{(I+1)}, \mathbf{Y}_k^{(I+1)}, \mathbf{F}_{RF}, \mathbf{F}_k\right) \tag{12b}$$

# Optimization of $(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

- With fixed $(\mathbf{F}_{RF}, \mathbf{F}_k)$ and $(\mathbf{\Lambda}_{1,k}, \mathbf{\Lambda}_{2,k}, \mathbf{\Lambda}_{3,k})$, the $(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$ are updated by solving the following problem

$$\min_{\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k} \mathcal{L}_k\left(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k\right) \tag{13a}$$

$$\text{s.t.} \quad \mathrm{tr}\left\{\mathbf{V}_k \mathbf{V}_k^H \mathbf{\Omega}_k\right\} \leqslant 0 \tag{13b}$$

$$\left\|\mathbf{A}_k^H \mathbf{Y}_k\right\|_F^2 \leqslant \bar{\Gamma}_k \tag{13c}$$

$$\left\|\mathbf{T}_k\right\|_F^2 = \mathsf{P}, \forall k. \tag{13d}$$

# Optimization of $(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

**STEP1:** Update of $\mathbf{T}_k$

1. By leveraging the Karush-Kuhn-Tucker (KKT) conditions, the close-form solution to $\mathbf{T}_k$ can be calculated by

$$\mathbf{T}_k^\star(\mu_k) = \{\mathbf{\Phi}_k + 2\mu_k \mathbf{I}_{N_t}\}^{-1} \mathbf{\Upsilon}_k, \tag{14}$$

   - $\mathbf{\Phi}_k = \frac{2}{K}\mathbf{H}_k^H \mathbf{W}_k \mathbf{\Xi}_k^t \mathbf{W}_k^H \mathbf{H}_k + (\rho_{1,k} + \rho_{2,k} + \rho_{3,k}) \mathbf{I}_{N_t}$
   - $\mathbf{\Upsilon}_k = \frac{2}{K}\mathbf{H}_k^H \mathbf{W}_k \mathbf{\Xi}_k^t + \rho_{1,k}\mathbf{F}_{RF}\mathbf{F}_k + \rho_{2,k}\mathbf{T}_k + \rho_{3,k}\mathbf{Y}_k - \mathbf{\Lambda}_{1,k} - \mathbf{\Lambda}_{2,k} - \mathbf{\Lambda}_{3,k}$.

2. Defining eigenvalue decomposition (EIG) $\mathbf{\Phi}_k = \mathbf{Q}_k \mathbf{D}_k \mathbf{Q}_k^H$ and substituting (14) into the power constraint, one gets

$$\|\mathbf{T}_k^\star(\mu_k)\|_F^2 = \sum_{n=1}^{N_t} \frac{\left(\mathbf{Q}_k^H \mathbf{\Upsilon}_k \mathbf{\Upsilon}_k^H \mathbf{Q}_k\right)[n,n]}{\left(\mathbf{D}_k[n,n] + 2\mu_k\right)^2} = \mathsf{P}. \tag{15}$$

3. The optimal multiplier $\mu_k$ can be easily obtained via using golden-section search or Newton's method.
4. Plugging the $\mu_k$ into (14), we can obtain the solution of $\mathbf{T}_k$.

# Optimization of $(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k, \mathbf{Y}_k)$

**STEP2:** Update of $\mathbf{V}_k$

1. By leveraging the KKT conditions, we can obtain

$$\mathbf{V}_k^\star = \left\{\rho_{2,k}\mathbf{I}_{N_t} + 2\chi_1\mathbf{\Omega}^H\right\}^{-1}\mathbf{\Psi}_k \tag{16a}$$

$$\chi_1 \text{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\mathbf{\Omega}_k\right\} = 0, \ \chi_1 \geqslant 0 \tag{16b}$$

$$\text{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\mathbf{\Omega}_k\right\} \leqslant 0 \tag{16c}$$

   where $\mathbf{\Psi}_k = \mathbf{\Lambda}_{2,k} + \rho_{2,k}\mathbf{T}_k$.

2. *Case 1:* For $\chi_1 = 0$, form the (16a) we have $\mathbf{V}_k^\star = \{\rho_{2,k}\mathbf{I}_{N_t}\}^{-1}\mathbf{\Psi}_k$, which must satisfy the condition (16c), otherwise
3. *Case 2:* For $\chi_1 \neq 0$, form the (16a) we have

$$\mathbf{V}_k^\star(\chi_1) = \left\{\rho_{2,k}\mathbf{I}_{N_t} + 2\chi_1\mathbf{\Omega}^H\right\}^{-1}\mathbf{\Psi}_k \tag{17}$$

   - Plugging (17) into equality constraint $\text{tr}\left\{\mathbf{V}_k\mathbf{V}_k^H\mathbf{\Omega}_k\right\} = 0$, we obtain

$$\text{tr}\left\{\mathbf{V}_k^\star(\chi_1)\left(\mathbf{V}_k^\star(\chi_1)\right)^H\mathbf{\Omega}_k\right\} = \sum_{n=1}^{N_t}\frac{\mathbf{M}_{1,k}[n,n]\left(\mathbf{Z}_{1,k}^H\mathbf{\Psi}_k\mathbf{\Psi}_k^H\mathbf{Z}_{1,k}\right)[n,n]}{\left(\rho_{2,k} + 2\chi_1\mathbf{M}_{1,k}[n,n]\right)^2} = 0. \tag{18}$$

**STEP3:** Update of $\mathbf{Y}_k$

- Following the same procedure for updating $\mathbf{V}_k$, we can obtain

$$\mathbf{Y}_k^\star = \left\{\rho_{3,k}\mathbf{I}_{N_t} + 2\chi_2\mathbf{A}_k\mathbf{A}_k^H\right\}^{-1}\mathbf{\Pi} \tag{19a}$$

$$\left\|\mathbf{A}_k^H\mathbf{Y}_k^\star\right\|_F^2 \leqslant \bar{\Gamma}_k \tag{19b}$$

$$\chi_2\left(\left\|\mathbf{A}_k^H\mathbf{Y}_k^\star\right\|_F^2 - \bar{\Gamma}_k\right) = 0,\ \chi_2 \geqslant 0, \tag{19c}$$

**STEP4:** Update of $\mathbf{W}_k$

- According to MMSE receiver, its optimal solution of $\mathbf{W}_k$ can be calculated as

$$\mathbf{W}_k = \left(\mathbf{H}_k\mathbf{T}_k\mathbf{T}_k^H\mathbf{H}_k^H - \mathbf{I}\right)^{-1}\mathbf{H}_k\mathbf{T}_k. \tag{20}$$

- With fixed $(\mathbf{W}_k, \mathbf{T}_k, \mathbf{V}_k \;\; \mathbf{Y}_k)$ and $(\mathbf{\Lambda}_{1,k}, \mathbf{\Lambda}_{2,k}, \mathbf{\Lambda}_{3,k})$, the $(\mathbf{F}_{RF}, \mathbf{F}_k)$ are updated by solving the following problem

$$\begin{aligned} \min_{\mathbf{F}_{RF}, \{\mathbf{F}_k\}} & \sum_{k=1}^{K} \frac{\rho_{1,k}}{2} \left\| \mathbf{T}_k - \mathbf{F}_{RF}\mathbf{F}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}} \right\|_F^2 \\ \text{s.t.} \quad & \left| \mathbf{F}_{RF}[i,j] \right| = 1, \forall i, j. \end{aligned} \tag{21}$$

**STEP1:** Update of $\mathbf{F}_k$

- The close-form solution of $\mathbf{F}_k$ is given by

$$\mathbf{F}_k = \left(\mathbf{F}_{RF}^H \mathbf{F}_{RF}\right)^{-1} \mathbf{F}_{RF}^H \left(\mathbf{T}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}}\right). \tag{22}$$

# Optimization of $(\mathbf{F}_{RF}, \mathbf{F}_k)$

**STEP2:** Update of $\mathbf{F}_{RF}$

1. Block Coordinate Descent (BCD) method is implemented to update $\mathbf{F}_{RF}$
2. The optimization problem w.r.t. $\mathbf{F}_{RF}[i,j]$ can be written as

$$\begin{aligned} \min_{\mathbf{F}_{RF}[i,j]} \quad & -\Re\left\{\mathbf{F}_{RF}[i,j]\,\zeta_{i,j}\right\} \\ \text{s.t.} \quad & \mathbf{F}_{RF}[i,j] = e^{-j\varpi_{i,j}}, \end{aligned} \tag{23}$$

   - $\zeta_{i,j} = \sum_{k=1}^{K} \rho_{1,k}\left(\mathbf{F}_k \bar{\mathbf{T}}_k^H - \mathbf{F}_k\left(\bar{\mathbf{F}}_{RF}^{i,j}\mathbf{F}_k\right)^H\right)[j,i]$,
   - $\bar{\mathbf{T}}_k = \mathbf{T}_k + \frac{\mathbf{\Lambda}_{1,k}}{\rho_{1,k}}$,
   - $\bar{\mathbf{F}}_{RF}^{i,j}$ denotes the matrix $\mathbf{F}_{RF}$ with $(i,j)$ element is zero.
3. It is obvious that the optimal solution of $\mathbf{F}_{RF}[i,j]$ is

$$\varpi_{i,j} = \text{angle}\left(\zeta_{i,j}\right) \tag{24}$$

# Computational Complexity

1. The main computational complexity is caused by updating $(\mathbf{F}_{RF}, \{\mathbf{F}_k\})$ and $(\{\mathbf{W}_k\}, \{\mathbf{T}_k\}, \{\mathbf{V}_k\}, \{\mathbf{Y}_k\})$ in each iteration.
2. Updating $(\mathbf{F}_{RF}, \{\mathbf{F}_k\})$ requires $\mathcal{O}\left(K\left(N_t N_{RF} N_s + N_t N_{RF}^2\right)\right)$ operations.
3. Updating $(\{\mathbf{W}_k\}, \{\mathbf{T}_k\}, \{\mathbf{V}_k\}, \{\mathbf{Y}_k\})$ needs $\mathcal{O}\left(K N_t^2 \left(\log(2) + N_s\right)\right)$ operations,.
4. As a result, the overall complexity of the proposed algorithm is given by $\mathcal{O}\left(T_{max} K \left(N_t N_{RF} N_s + N_t N_{RF}^2 + N_t^2 N_s + N_t^2 \log(2)\right)\right)$.

# Parameters Setting

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $M_t$ | 32 | $N_s$ | 4 |
| $M_{RF}$ | 4 | $N_r$ | 2 |
| $f_c$ | 10GHz | $\Delta f$ | 20MHz |
| $K$ | 128 | $P_k$ | 1 |

Table: Collection of parameters values

- The main-lobe region $\Theta_m = [-10^\circ, 10^\circ]$,
- The side-lobe region $\Theta_s = [-90^\circ, -10^\circ) \cup (10^\circ, 90^\circ]$,
- The SSN azimuths $\Theta_k$ are assumed as $\Theta_k = [-24^\circ, -21^\circ] \cup [21^\circ, 24^\circ], \forall k$,
- The threshold $\Gamma_k$ of the SSN on $k$-th subcarrier is set as 50dB,
- The initial penalty parameter and the convergence tolerance are set to $\rho_{1,k} = \rho_{2,k} = \rho_{3,k} = 1, \forall k$ and $\hbar_1 = \hbar_2 = \hbar_3 = 10^{-6}$,respectively.

# Comm. Performance

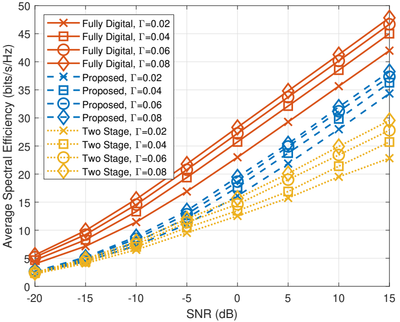



Figure: The achievable spectral efficiency versus number of iteration in an OFDM-DFRC based SU-MIMO system with different ISMR value $\gamma$.

- The attained spectral efficiencies increase along with the $\gamma$ for all algorithms,
- The proposed algorithm always achieve better performance that two stage method with the same ISMR constraint.

# Radar Performance

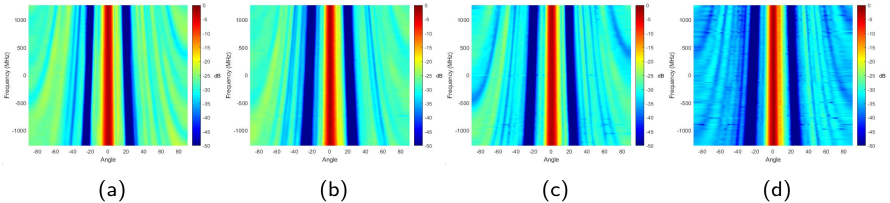



Figure: The space-frequency spectral behaviors of the OFDM-DFRC with different ISMR value (1) $\gamma = 0.08$, (2) $\gamma = 0.06$, (3) $\gamma = 0.04$, (4) $\gamma = 0.02$.

- As the ISMR level $\gamma$ decreases, the side-lobe energy becomes lower and lower,
- The beampattern has desired SSN at interference azimuth for all $\gamma$,
- There exist the coupling effect between the space and frequency.

# Conclusions

- In this paper, we address the problem of the hybrid beamforming design for the OFDM-DFRC system. The corresponding problem is to maximize the spectral efficiency via optimizing the hybrid beamformer, while fulfilling the requirements for the radar side.
- The simulation results shows that:
  - We propose an alternating optimization based method to tackle the non-convex problem.
  - The simulation results show that the proposed method can achieve satisfactory performance and outperform the conventional two-stage method.

# Thank you !!

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