Reputation and Competition

MRes IO Writing Assignment II
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1 Paper Summary

This paper is concerned with one specific type of equilibrium in the setup of repeated interaction between consumers and firms of hidden types and hidden actions. Instead of analysing the general picture, this paper pinpoints on the specially-defined competitive equilibrium, which features zero-profit conditions and satisfies standard Perfect Bayesian Equilibrium (PBE henceforth) conditions. This report devotes a considerable amount to clarifying the equilibrium structure, and aims to discuss mainly the equilibrium path.

The question modelled is important and interesting. The traditional literature of perfect competition do not consider atomic firm's strategic behaviour. It requires homogenous firms, common knowledge of consumers and firms types (and potential actions), and thus prices are Walrasian in the sense that no individual has market power. As a result, zero-profit condition is carefully protected. None of these conditions holds when information is incomplete. In specific, hidden actions leads to moral hazard, encouraging firms to shirk.

Studies introducing reputational concern into firm's decision problem has been successful in solving moral hazard issues. In fear of the bad or in favour of the good, a taste for reputation enables equilibriums where firm exerts high effort (Klein and Leffer 1981, Shapiro 1983). Indeed, high effort could rationally be sustained in an infinite horizon setup by preventing the hidden types from perfectly revealed (Holmström 1999). But it need to be associated with market power granted to the party holding reputation, e.g., firms setting prices or managers getting the surplus.

This model contributes to the literature as it combines model of reputation and competition. On the one hand, it shows the existence of equilibrium where **reputation** ensures high effort, without firms having market power and thus binding zero-profit. On the other hand, **competition** ensures the sustainable high effort as types are not revealed. Types are not perfectly revealed because firms are almost surely producing bad products sometime, and once they became unlucky they have to compete as if they are a new entrant.

The report will develop as followed. Section 2 will discuss the model setup, where I focus on the **rich model** only. Section 3 clarifies Bayesian updates. Section 4 defines the desired equilibrium. Section 5 lays out why the desired equilibrium is a PBE. I hope to discuss with readers some limitations of this paper in Section 6, and we will conclude shortly.

2 Model Setup with Comments

Index a consumer by $k \in [0,1]$, a firm by $j \in [0,1]$. Consumer decides whether to:

(1) stay loyal (S) or to quit and switch to another firm (Q), and

(2) trade (T) or chose the outside option (O).

A firm's decisions consist of two part: practice

(i) the price to offer $p^j \in \mathbb{R}$ or to exit, and

(ii) the effort $\{H, L\}$. A bad-type firm can only practice low effort.

If a consumer stayed loyal, her information set at the start of this period consists of this firm's price p^j. If she decided to quit and switch, however, she gains better information: the price distributions over all firms $F : j \in [0,1] \to \mathbb{R}$, and the existence of each firm's customer base $Y : j \in [0,1] \to \{Y, N\}$. At the end of this period, she observes the outcome $\in \{G, B\}$ of her chosen firm. Her history is private—**no communication among consumers** allowed.

A firm's information set at the start of this period is the measure of loyal consumers. At the middle of this period, it knows the price distribution F. At the end of this period, it observes its outcome. Therefore, **prices are the only signals for communication** between firms, while the G/B outcome is known only between a firm and its consumers.

It is worth special notice that **the measure of the firms,** λ are in the following way, **flexible**

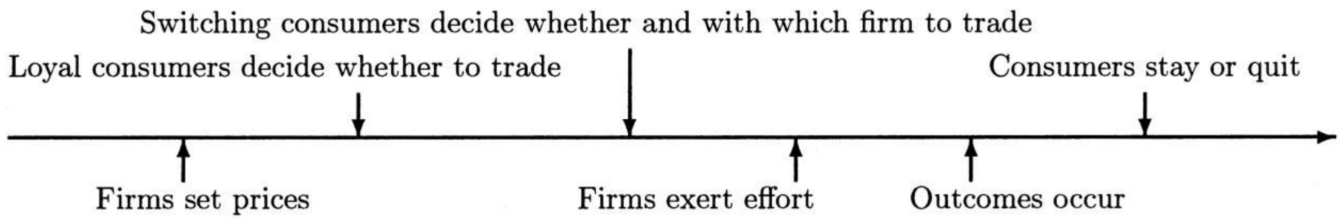

Figure 1: Timeline of Moves in a Period

as desired. On the equilibrium path, firms which just made a bad outcome will lose all its consumer base. Also, it has continuation value 0. It therefore is indifferent between exiting the market or not. W.l.o.g, we could **assume all of them exit, and some of them together with other really-new entrants, enter the market again**. They all have the same continuation value 0, and a consumer do not distinguish a specific previous firm j from other new entrants—prices are the only signals. This translates the original assumption that n_0 measure of new firms enter each period, and justifies why we do not care about firms' mixing strategies.

With regard to reputation, we introduce some notations here. Let $i \in Z$ denote the *age* of a firm. A simple explanation of age is the periods that a firm consecutively produces a good outcome. Then λ_i denotes the measure of firms that have age i, and n_i denotes the consumer base of all firms of age i. Price p_i then will be the same price charged by all firms on the equilibrium path.

3 Bayesian Updates

The crucial updating is a consumer's belief of firms type. Let φ_i denote the probability that a firm of age i is a good firm. Notice, i **is not observable**. Instead, consumers rely on the equilibrium strategy where different age firms set different prices to tell i.

It is specifcally delicate that in this model, φ_i is the **actual proportion of firms** at age i who are good types. This is a clever simplification because of a continuum of firm, and allows φ_i to be a part of both consumers' and firms' belief system (we will discuss later). Since high effort produces a good outcome with probability α, and a low effort with β, it is common knowledge that φ_i evolves at the equilibrium path according to

$$\varphi_{i+1} = \frac{\alpha\varphi_i}{\alpha\varphi_i + \beta(1-\varphi_i)},\ \varphi_0 \text{ exogenous} \tag{1}$$

4 Equilibrium Structure: as Desired

Definition 1 *Competitive Equilibrium*

A stationary competitive equilibrium with reputation is a

- *non-revealing: consumers (mixed) strategies are identical, as is the restriction to prices of firms' pure strategies*
- *high-effort: good firms are always indifferent between H and L*
- *zero-profit: payoff of firms entering the market is zero*
- *Markovian: histories are summarised as beliefs*
- *PBE: beliefs are correct and non-deviation conditions satisfy*

4.1 Equilibrium Strategies

Theorem 1 *Consumer's Equilibrium Strategy*

Consumers' strategy consists of two part: (see item (1) and (2) in section 2)

(1) $\rho : \varphi \in [0,1] \to [0,1]$: the probability consumer k quit-and-switches

$$\begin{cases} \rho(\varphi_{i+1}) = 0 \; for \; i \leqslant i^* - 1 \\ \rho(\varphi_{i+1}) = 1 - \frac{n_{i+1}}{n_i} \; for \; i \geqslant i^* \\ \rho() = 1 \; otherhwise \end{cases} \tag{2}$$

(2) $\xi_Q : \mathcal{F} \times (p^j \in Supp\ F) \times \{Y, N\} \to [0,1]$: probability a quit-switch consumer k picks firm j

$$\begin{cases} \xi_Q(F, p_{i+1}, Y) = \frac{\lambda_{i+1}(n_{i+1} - n_i)}{\sum_{l=-1}^{i^*-1} \lambda_{l+1}(n_{l+1} - n_l)} \quad 0 \leqslant i \leqslant i^* - 1 \\ \xi_Q(F, p_0, N) = \frac{\lambda_0(n_0 - n_{-1})}{\sum_{l=-1}^{i^*-1} \lambda_{l+1}(n_{l+1} - n_l)} \quad n_{-1} = 0 \\ \xi_Q() = 0 \; otherwise \end{cases} \tag{3}$$

(simplified. If rigorous, we also consider O. Recall: consumer observes a price distribution $F \in \mathcal{F}$)

$\xi_S : (\varphi_i \in [0,1]) \times (p^j \in \mathbb{R}) \to \{T, O\}$: *a loyal consumer trades or not, knowing i*

$$\begin{cases} \xi_S(\varphi_i, p^j) = T \;\; if \; \alpha\varphi_i + \beta(1-\varphi_i) - p^j \geqslant \gamma \\ \xi_S(\varphi_i, p^j) = O \;\; otherwise \end{cases} \tag{4}$$

The consumers strategy $\rho()$ is interpreted as followed. Normally, if a firm makes good outcome, φ_i is updated correctly, so consumer k sticks to this firm. On the equilibrium path, she pays higher price p_{i+1} next period as reputation premium.

There is an age i^* where the price is so higher that being loyal, even if safer, is suboptimal. In fact, for consumers in n_{i^*+1} and above, they are indifferent between being loyal or quitting, therefore they only stay with some probability.

An 'easy' explanation for the probability $1 - \frac{n_{i+1}}{n_i}$ is because the equilibrium is **designed** by the author to be such that $\{n_i\}_0^\infty$ follows the equilibrium path. By design, we want only n_{i+1} consumers to stay, therefore $n_i - n_{i+1}$ will leave. To dig deeper, $\{n_i\}_0^\infty$ is, in turn, desired by definition of competitive equilibrium, in specific, the binding of firms incentive constraints (equation 13), and the indifference principle of consumer's strategy ξ_Q. Let's put the calculation to the next section.

Intuitively, $\xi_Q()$ tells a quit-and-switch consumer where to go. She is indifferent between all firms because of market's competitive nature. If a firm attracts more agents than expected, it will instantly realize that and raise price. As a usual but strict restriction to sustain a high-effort level equilibrium, consumers should ignore any firms setting an off-equilibrium price. This is rationalized by believing any off-equilibrium pricing firms are bad, a part of the belief system never to be tested.

Similarly, the probabilities in equation 3 is also 'by design'. Notice no consumers exit on equilibrium path. Those quitting due to either a bad outcome or too high a price should be attached to a lower i firm. This amount to a total measure of $\sum_{l=-1}^{i^*-1} \lambda_{l+1}(n_{l+1} - n_l)$. The reallocation is therefore simply their change in n, required by the equilibrium design.

Theorem 2 *Firm's Equilibrium Strategy*

(i) Firm with age i sets a price $p_i = (\alpha - \beta)(\varphi_i - \varphi_0) - \frac{\beta}{\alpha - \beta} c$

(ii) Good firms always practice high effort

4.2 Equilibrium Beliefs

Theorem 3 *Consumers' Equilibrium Belief*

A consumer k's belief is a φ_j^k, her belief on any firm $j \in [0,1]$ being a good type. With some simplification, her belief should be updated each period given her information gained in this period:

(S) Recall if the consumer stayed loyal, then her information gained in this period is first p^j:

$$\begin{cases} \varphi_j^k(\varphi_j^k, p^j) = \varphi_i \ \ if \ (\varphi_j^k, p^j) = (\varphi_i, p_i) \ \textit{for some } i \\ \varphi_j^k() = 0 \qquad\qquad\qquad \textit{otherwise} \end{cases} \tag{5}$$

and outcome (G/B) if she decides to trade. $(\varphi_j^k)'$ is updated according to Bayes's Law.

(Q) Recall a quit-and-switch consumer get a rich information about the distribution and the existence of consumer base. Her update:

$$\begin{cases} \varphi_j^k(F, p^j, Y) = \varphi_i \ \ if \ p^j = p_i \ \textit{for some } i, \ \ \forall F \in \mathcal{F} \\ \varphi_j^k(F, p^j, N) = \varphi_0 \ \ if \ p^j = p_0, \ \ \forall F \in \mathcal{F} \\ \varphi_j^k() = 0 \qquad \textit{otherwise} \end{cases} \tag{6}$$

If she trades, she will observe outcome (G/B), and updates $(\varphi_j^k)'$ according to Bayes's Law.

Consumer's belief is straightforward. They only update beliefs based on one-period information ($(\varphi_j^k)'$ to indicate one period forward in equation 5). If they observe a price off-equilibrium they resort to 0.

Theorem 4 *Firms' Equilibrium Belief*

Firms' belief are subtler: a firm j forms a belief on the distribution of $\{\varphi_j^k\}_{k\in[0,1]}$.

Nevertheless, firm's belief on the equilibrium path is the same as the reality, i.e., the true distribution of φ_j^k. Since the desired equilibrium requires a symmetric belief system of consumers, then firms' belief is essentially equivalent to 'they know consumers believe φ_j^k'.

4.3 Equilibrium Path

Theorem 5 *Equilibrium Path of Prices and Consumer Base*

On the equilibrium path where both consumers and firms believe in their system and behaves like above strategies, firms of the same age set the same prices

$$p_i = (\alpha - \beta)(\varphi_i - \varphi_0) - \frac{\beta}{\alpha - \beta}c \tag{7}$$

The consumer base evolves according to

$$n_i = \frac{cn_{i-1}}{\delta(\alpha - \beta)^2(\varphi_i - \varphi_0)}, \ i \in N^* \tag{8}$$

The measure of firms at each age i evolves

$$\lambda_i = (\alpha\varphi_{i-1} + \beta(1 - \varphi_{i-1}))\lambda_{i-1} \ i \in N^* \tag{9}$$

and n_0, the consumer base we want to ensure all new-entrants to have, is adjusted s.t.

$$\sum_0^\infty n_i\lambda_i = 1 \tag{10}$$

Such an equilibrium exists as long as

(1) $\frac{c\alpha}{\delta(\alpha-\beta)^2} \leqslant (1 - \varphi_0)$
(2) $\gamma \leqslant \alpha\varphi_0 + \beta(1 - \varphi_0) + \frac{\beta}{\alpha+\beta}c$

The equilibrium path has straightforward intuitions. From the price eqution (7), we see that when i is higher, prices are higher, which corresponds to higher reputation premium. Also, $p_0 < 0$, which ensures zero-profit condition.

The evolution of consumer base captures the trade-off of consumers between picking a safer firm and affording the higher price. From equation 5, we see n_i/n_{i-1} decreases and it should fall below 1 when $i \to \infty$. This trade-off ensures that consumers are, as desired by the equilibrium, indifferent between older and new firms.

In order to ensure the successful construction of such an equilibrium, condition (1) imposes on the firm-side that there should be enough bad firms for good firms to want to distinguish themselves apart. Suppose, to the contrary, all firms are good ($\varphi_0 = 1$), then there is no incentive for firms to continue building reputations by practising high effort.

Condition (2), on the other hand, poses constraint on the consumers. As desired by the equilibrium, outside option O has to be unattractive. In fact, if $\gamma = \beta$, condition (2) always holds.

We will explain the conditions in the following section. It is, however, worth explaining why n_0 is adjustable. Recall that we have assumed, w.l.o.g., all firms that got an unlucky bad result will exit the market and re-enter with some measure of new firms. This total mass is normalized to $\lambda_0 = 1$. This is **not to say we inject mass** 1 **firms** into the economy. We inject new firms such that $n_0\lambda_0 + n_1\lambda_1 + ... = 1$, the exogenous time-invariant measure 1 of consumers. Therefore, as long as $n_0 > 0$, we could always implement it by adjusting the number of actual new firms.

5 Showing the Equilibrium Satisfies PBE

Essentially, we do the other way round: we show what the equilibrium path should look like given all the desired properties, and design a strategy that satisfies it.

Proof.

First we show the above equilibrium satisfies the consumer's rationality. A consumer k will choose to trade always as long as condition (2) in theorem 5 holds. We need to check consumer's choice of firm j. By equilibrium strategy, she has to be indifferent among firms,

$$\alpha\varphi_i + \beta(1-\varphi_i) - p_i = U(\text{choosing any age i firm}) = \alpha\varphi_0 + \beta(1-\varphi_0) - p_0 \tag{11}$$

which requires prices by the firm of age i to follow equation 7 in theorem 5, for $i \geqslant 1$. ■

Proof.

Next we show the firm's rationality. This consists of several parts: for one thing we want firms to exert high-effort rationally. A good firms utility is expressed in the following Bellman equation

$$V_t = p_t + \max\{-c + \alpha\delta\frac{n_{t+1}}{n_t}V_{t+1}, \beta\delta\frac{n_{t+1}}{n_t}V_{t+1}\} \tag{12}$$

By high-effort condition, we want an equilibrium which ensure

$$-c + \alpha\delta\frac{n_{t+1}}{n_t}V_{t+1} \geqslant \beta\delta\frac{n_{t+1}}{n_t}V_{t+1} \tag{13}$$

which, by re-arranging and lagging one period,

$$V_{t+1} = \frac{n_t}{n_{t+1}}\frac{c}{\delta(\alpha-\beta)} \tag{14}$$

$$p_t = \frac{c}{\delta(\alpha-\beta)}(\frac{n_t}{n_{t+1}} - \beta\delta) \tag{15}$$

If we combine equation 15 and equation 7, we get the evolution of consumer base (equation 5) in theorem 5. Hence firm's rational effort choice is ensured.

For another, we want firms to charge a price that is rational. This is easier because we have designed the consumer strategy ξ_Q that is highly non-continuous, ensuring any deviation from equilibrium-pricing would be suboptimal. Therefore, firms' rationality problem simplifies to a participation constraint

$$V_0 \geqslant 0 \tag{16}$$

■

Proof.

To satisfy the zero-profit condition, we then plug in the Bellman equation

$$0 = V_0 = p_0 - c + \alpha\delta\frac{n_1}{n_0}V_1 \tag{17}$$

which simplifies to $p_0 = -\frac{\beta c}{\alpha-\beta}$, so equation 7 holds for $i = 0$ too. ■

Proof.

Thirdly, we still have condition (1) to prove in theorem 5. Thie comes from the logic we have shown previously. Since

$$\sum_0^\infty n_i\lambda_i = 1 \tag{18}$$

has to hold for some $n_0 > 0$, we need

$$\frac{n_i}{n_{i-1}} = \frac{c}{\delta(\alpha-\beta)^2(\varphi_i - \varphi_0)} \tag{19}$$

to at least be smaller than 1 in the limits. In the limits, $\varphi_i = 1$, thus condition (1) ■

Proof.

Finally, we are left to check that the beliefs are correct on the equilibrium path. These are hardwired into our construction of beliefs already. For example, a consumer will only observe p_i's for some i, and updates the belief φ_j^k on the girds of φ_i, and therefore the belief is the actual proportion at each age. Similarly, the firms will "know that consumers have $\varphi_j^k()$", and since the off-equilibrium beliefs are never observed, firms belief is correct

It is worth notice that the distribution, $F \in \mathcal{F}$, does not have a material impact (except for confirming its support Supp F). In the equilibrium belief system consumers will form φ_j^k based on prices p^j only, disregarding how many firms are setting this price. ■

6 Limitations and Discussions

The contribution of this paper is self-convincing. The fact that reputation could embrace competition and competition sustains the long-run practice of high effort is igniting and opens up further researches on this field (Morrison and Wilhelm 2004, Dilme 2019). This is not to say there are no limitations.

First, firms are still price-setters. It would be interesting to see the robustness of the result if prices are offered by the consumers instead. Under definition of perfect competition, prices should ideally be independent from the equilibrium strategy.

Second, it suffers from the critiques of PBE, where the unobserved belief system is not tested. For example, an off-equilibrium firm offering a lower price even with good reputation, could potentially be optimal if consumers are attracted to it.

Third, the design is very restrictive. There is no guarantee that the game will converge to such equilibrium; if there is anything, controlling the injection of n_0 each period is implying that this equilibrium is probably hard to be achieved. Another restriction on this equilibrium is that consumers are not allowed to communicate. As we have seen in Hömstom (2000), a commonly shared feedback history would quickly reveal a firm's type, and destroying further incentive to sustain high effort. Both critique concerns broadly with the ‘desirability’ os the equilibrium design, however they reflects that more sophisticated equilibria exist.

One way to extend this model and relax the restrictiveness is to allow for better memories of the consumer. For example, a consumer could identify a previous firm j and forgive its ‘mistake’. As mentioned in section 2, a consumer is assumed to **‘forget’ about their previous firm** j in her detailed history structure. This is however arguable. If a consumer could remember a specific firm that once she cooperated with, she would distinguish this firm from new entrants, unlink in this model where all firms are uniformly unfamiliar to her.

To conclude, we have analysed the rich model considering repeated interaction between hidden type, hidden information agents. The main results are impressive, that reputation could be sustained with the help of competition. Some maths details are discussed, but we do not intend to go over-the-table. Although structure of history is essential for rigour, a belief system described using φ captures better the intuition and conveys the design of this delicate, non-revealing, high-effort, competitive Markovian PBE.

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