Topics in Macroeconomics

Unit 3 – Introduction

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Outline

Outline of unit 3

1. Introduction (this mini-lecture)
   - Recursive methods in quantitative macroeconomics
   - Infinite-horizon vs. life-cycle solution methods
2. Value function iteration (VFI)
3. Endogenous grid-point method (EGM)

Recursive formulation of household problem

Recall that we can write a household problem in two ways:

1. Sequential formulation

$$V(a_0, y_0) = \max_{\{c_t\}_{t=0}^{\infty}, \{a_{t+1}\}_{t=0}^{\infty}} \mathbf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t) \,\middle|\, y_0 \right]$$
$$\text{s.t.} \quad c_t + a_{t+1} = (1+r)a_t + y_t \quad \forall\, t$$
$$c_t \geq 0, \; a_{t+1} \geq 0 \quad \forall\, t$$

2. Recursive formulation

$$V(a, y) = \max_{c,\, a'} \left\{ u(c) + \beta \mathbf{E}\left[V(a', y') \,\middle|\, y \right] \right\}$$
$$\text{s.t.} \quad c + a' = (1+r)a + y$$
$$c \geq 0, \; a' \geq 0$$

Recursive methods

- The sequential formulation is quite useless for solving heterogeneous-agent models numerically

  $\Rightarrow$ We exclusively deal with recursive formulation
- We want to find functions that characterise the solution:
  1. The value function $V(a, y)$
  2. The policy functions

     $c = C(a, y)$ Optimal consumption

     $a' = A(a, y)$ Optional savings

  These functions are defined on discretised grids $a \in \mathcal{G}_a$ and $y \in \mathcal{G}_y$.

Iteration and backwards induction

Two main types of household problems:

Infinite-horizon problems

- Need to start with a guess for the solution; often this is just $V_0(a, y) = 0$
- Iterate on some object until consecutive iterations V_n, V_{n+1} are sufficiently close
- We can iterate either on value functions (VFI) or policy functions (PFI, EGM: endogenous grid-point method)

Finite-horizon problems

- Life-cycle and OLG models
- Solve for last period T
- Use backward induction to solve previous periods $T-1, T-2, \ldots$

Infinite horizon vs. life-cycle

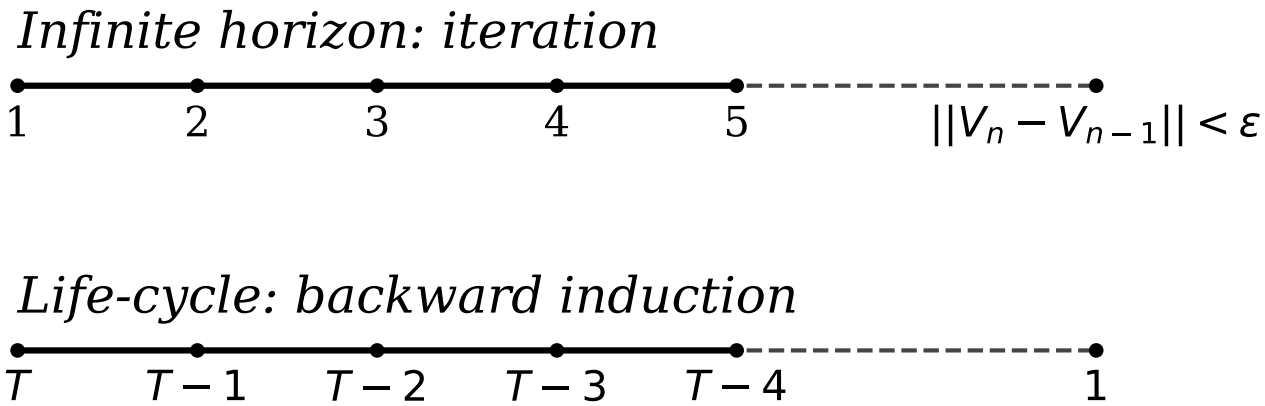

Figure 1: Solving infinite-horizon vs. life-cycle models

Overview of unit 3

Outline of remaining mini-lectures

- We exclusively solve household problems
  - Ignore distribution of households
  - Ignore general equilibrium
- Next mini-lectures:
  1. Lecture 1: Value function iteration (VFI)
     - Grid search
     - Interpolation
  2. Lecture 2: Endogenous grid-point method (EGM)
- Slides and pre-recorded lectures: general concepts, algorithms, results
- Live sessions: implement examples discussed in slides
- Hands-on approach to complement units 1–2

Overview of unit 3

Source code

- Github repository: https://github.com/richardfoltyn/mres-macro-topics
- Python and Matlab source code for examples discussed in lectures / live sessions
- We use Matlab in live sessions (since you continue with Matlab in unit 4)