Topics in Macroeconomics

Unit 3 - Introduction

Richard Foltyn

University of Glasgow

Outline

Outline of unit 3

- Introduction (this mini-lecture)
 - Recursive methods in quantitative macroeconomics
 - Infinite-horizon vs. life-cycle solution methods
- Value function iteration (VFI)
- 3 Endogenous grid-point method (EGM)

Recursive formulation of household problem

Recall that we can write a household problem in two ways:

Sequential formulation

$$V(a_0, y_0) = \max_{\{c_t\}_{t=0}^{\infty}, \{a_{t+1}\}_{t=0}^{\infty}} \mathbb{E} \left[\left. \sum_{t=0}^{\infty} \beta^t u(c_t) \right| y_0 \right]$$
s.t. $c_t + a_{t+1} = (1+r)a_t + y_t \quad \forall \ t$
 $c_t \ge 0, \ a_{t+1} \ge 0 \quad \forall \ t$

Recursive formulation

$$V(a, y) = \max_{c, a'} \left\{ u(c) + \beta \mathbf{E} \left[V(a', y') \middle| y \right] \right\}$$

s.t. $c + a' = (1 + r)a + y$
 $c \ge 0, \ a' \ge 0$

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Recursive methods

- The sequential formulation is quite useless for solving heterogeneous-agent models numerically
 - ⇒ We exclusively deal with recursive formulation
- We want to find functions that characterise the solution:
 - 1 The value function V(a, y)
 - The policy functions
 - c = C(a, y) Optimal consumption
 - a' = A(a, y) Optional savings

These functions are defined on discretised grids $a \in \mathcal{G}_a$ and $y \in \mathcal{G}_y$.

Iteration and backwards induction

Two main types of household problems:

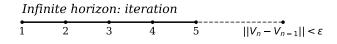
Infinite-horizon problems

- Need to start with a guess for the solution; often this is just $V_0(a, y) = 0$
- Iterate on some object until consecutive iterations V_n , V_{n+1} are sufficiently close
- We can iterate either on value functions (VFI) or policy functions (PFI, EGM: endogenous grid-point method)

Finite-horizon problems

- Life-cycle and OLG models
- Solve for last period T
- Use backward induction to solve previous periods T 1, T 2, ...

Infinite horizon vs. life-cycle



Life-cycle: backward induction
$$T = T - 1 \quad T - 2 \quad T - 3 \quad T - 4$$

Figure 1: Solving infinite-horizon vs. life-cycle models

Overview of unit 3

Outline of remaining mini-lectures

- We exclusively solve household problems
 - Ignore distribution of households
 - Ignore general equilibrium
- Next mini-lectures:
 - 1 Lecture 1: Value function iteration (VFI)
 - Grid search
 - Interpolation
 - 2 Lecture 2: Endogenous grid-point method (EGM)
- Slides and pre-recorded lectures: general concepts, algorithms, results
- Live sessions: implement examples discussed in slides
- Hands-on approach to complement units 1–2

Overview of unit 3

Source code

- Github repository: https://github.com/richardfoltyn/mres-macro-topics
- Python and Matlab source code for examples discussed in lectures / live sessions
- We use Matlab in live sessions (since you continue with Matlab in unit 4)