Proofs: Using Resolution

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Resolution

Resolution is a proof technique used to automate and streamline proofs. There are two key parts of this technique. First, convert all premises to Conjunctive Normal Form (CNF). Second, only use "Resolution" from all of our possible rules of inference. Resolution has several forms, which we will show and discuss.

Standard Form

In Table 1, Section 1.6.3*, the book shows the standard form for resolution:

$$\begin{array}{l} \quad p \lor q \\ \neg p \lor r \\ \hline \therefore q \lor r \end{array}$$

The key aspect of this rule of inference is that if we have two separate disjunction premises that use the same variable, but one premise has that variable negated, we can combine the two disjunction premises and remove that variable.

Resolution with Boolean Literal

Here is another form of resolution:

$$\begin{array}{ll} \quad p \lor q & \\ \neg p \lor F & \\ \hline \quad q \lor F & \\ \therefore q & \text{by Identity law} \end{array}$$

Disjunctive Syllogism

From the same table in the book* you will find a rule of inference called Disjunctive Syllogism:

$$\begin{array}{l} \quad p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Note that this is similar to the previous form of resolution, but we dropped the disjunction with false. When you use Disjunctive Syllogism in your proofs you may label it as "Resolution" for this course.

Resolution without Disjunction

Yet another form of resolution is as follows:

$$\begin{array}{l} \quad p \\ \neg p \\ \hline \therefore F \end{array}$$

Note that this follows from the standard form where q and r are substituted with false:

$$\begin{array}{l} \quad p \vee F \\ \neg p \vee F \\ \hline \quad F \vee F \\ \therefore F \end{array}$$

This form of resolution will be used when we use the technique of Proof by Contradiction using Resolution. In your proofs, just label this step as "Resolution."

Resolution with Multiple Disjunctions

Finally, the number of disjunctions in a premise is irrelevant:

$$\begin{array}{l} \quad p \vee q \vee s \\ \neg p \vee r \vee t \\ \hline \therefore q \vee r \vee s \vee t \end{array}$$

Conclusion

Whenever you see any of these forms, use "Resolution" in your proofs.

**Discrete Mathematics and Its Applications*, by Kenneth H. Rosen.