EKF Jacobian Definition for ROS-Copter

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Abstract

1 Definitions

The state, x is defined as follows:

$$x = \begin{pmatrix} p_n & p_e & p_d & u & v & w & \phi & \theta & \psi & \alpha_x & \alpha_y & \alpha_z & \beta_x & \beta_y & \beta_z \end{pmatrix} \tag{1}$$

By mechanization, u is defined as accelerometer and gyro inputs, or:

$$u = \begin{pmatrix} a_z & g_x & g_y & g_z \end{pmatrix} \tag{2}$$

where a_z, g_x, g_y, and g_z are defined as the measured acceleration and angular rates.

2 Dynamics

The Dynamics are defined as follows:

$$f(x,u) = \begin{pmatrix} \dot{p_n} \\ \dot{p_e} \\ \dot{p_d} \\ \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\alpha_x} \\ \dot{\alpha_y} \\ \dot{\alpha_z} \\ \dot{\beta_x} \\ \dot{\beta_y} \\ \dot{\beta_z} \end{pmatrix} = \begin{pmatrix} c_\theta c_\psi u + (s_\phi s_\theta c_\psi - c_\phi s_\psi) v + (c_\phi s_\theta c_\psi + s_\phi s_\psi) w \\ c_\theta c_\psi u + (s_\phi s_\theta s_\psi + c_\phi c_\psi) v + (c_\phi s_\theta s_\psi - s_\phi c_\psi) w \\ -s_\theta u + s_\phi c_\theta v + c_\phi c_\theta w \\ rv - qw - g s_\theta \\ pw - ru + g c_\theta s_\phi \\ qu - pv + g c_\theta c_\phi + (a_z + \alpha_z) \\ p + s_\phi t_\theta q + c_\phi t_\theta r \\ c_\phi q - s_\phi r \\ q \frac{s_\phi}{c_\theta} + r \frac{c_\phi}{c_\theta} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Which means the Jacobian, A is defined as

$$
\frac{\partial f}{\partial x} = \begin{pmatrix}
0 & 0 & 0 & c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi & C_{06} & C_{07} & C_{08} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_\phi s_\psi & (s_\phi s_\theta s_\psi + c_\phi c_\psi) & (c_\phi s_\theta s_\psi - s_\phi c_\psi) & C_{16} & C_{17} & C_{18} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -s_\theta & s_\phi c_\theta & c_\phi c_\theta & C_{26} & C_{27} & C_{28} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & r & -q & 0 & -g c_\theta & 0 & 0 & 0 & 0 & 0 & -w & v \\
0 & 0 & 0 & -r & 0 & p & g c_\theta c_\phi & -g s_\theta s_\phi & 0 & 0 & 0 & 0 & w & 0 & -u \\
0 & 0 & 0 & q & -p & 0 & -g c_\theta s_\phi & -g s_\theta c_\phi & 0 & 0 & 0 & 1 & -v & u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{66} & C_{67} & 0 & 0 & 0 & 0 & 1 & s_\phi t_\theta & c_\phi t_\theta \\
0 & 0 & 0 & 0 & 0 & 0 & C_{76} & 0 & 0 & 0 & 0 & 0 & 0 & c_\phi & -s_\phi \\
0 & 0 & 0 & 0 & 0 & 0 & C_{86} & C_{87} & 0 & 0 & 0 & 0 & 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{4}
$$

with

$$
\begin{aligned}
C_{06} &= (c_\phi s_\theta c_\psi + s_\phi s_\psi)v + (-s_\phi s_\theta c_\psi + c_\phi s_\psi)w \\
C_{07} &= -s_\theta c_\psi u + s_\phi c_\theta c_\psi v + c_\phi c_\theta s_\psi w \\
C_{08} &= -c_\theta s_\psi u + (-s_\phi s_\theta s_\psi - c_\phi c_\psi)v + (-c_\phi s_\theta s_\psi + s_\phi c_\psi)w \\
C_{16} &= (c_\phi s_\theta s_\psi - s_\phi c_\psi)v + (-s_\phi s_\theta s_\psi - c_\phi c_\psi)w \\
C_{17} &= -s_\theta c_\psi u + (s_\phi c_\theta s_\psi)v + (c_\phi c_\theta s_\psi)w \\
C_{18} &= -c_\theta s_\psi u + (s_\phi s_\theta c_\psi - c_\phi s_\psi)v + (c_\phi s_\theta c_\psi + s_\phi s_\psi)w \\
C_{26} &= c_\phi c_\theta v - s_\phi c_\theta w \\
C_{27} &= -c_\theta u - s_\phi s_\theta v - c_\phi s_\theta w \\
C_{28} &= 0 \\
C_{66} &= c_\phi t_\theta q - s_\phi t_\theta r \\
C_{67} &= \frac{s_\phi q + c_\phi r}{c_\theta^2} \\
C_{76} &= -s_\phi q - c_\phi r \\
C_{86} &= \frac{q c_\phi - r s_\phi}{c_\theta} \\
C_{87} &= \frac{-(q s_\phi + r c_\phi) t_\theta}{c_\theta}
\end{aligned} \tag{5}
$$

3 IMU Measurements

Because IMU measurements are defined as

$$
\begin{aligned}
a_{i_{true}} &= a_{i_{meas}} + \alpha_i + \eta \\
a_{i_{meas}} &= a_{i_{true}} - \alpha_i - \eta
\end{aligned} \tag{6}
$$

the measurement model appears as follows:

$$
\begin{aligned}
\begin{pmatrix} a_{x_{true}} \\ a_{y_{true}} \\ a_{z_{true}} \end{pmatrix} &= \begin{pmatrix} \dot{u} + qw - rv + gs_\theta \\ \dot{v} + ru - pw - gc_\theta s_\phi \\ \dot{w} + pv - qu - gc_\theta c_\phi \end{pmatrix} \\
\begin{pmatrix} a_{x_{meas}} \\ a_{y_{meas}} \\ a_{z_{meas}} \end{pmatrix} &= \begin{pmatrix} \dot{u} + qw - rv + gs_\theta \\ \dot{v} + ru - pw - gc_\theta s_\phi \\ \dot{w} + pv - qu - gc_\theta c_\phi \end{pmatrix} - \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} - \begin{pmatrix} \eta \\ \eta \\ \eta \end{pmatrix}
\end{aligned} \tag{7}
$$

And, after making the assumption that $\dot{u}$, $\dot{v}$, $\dot{w}$, and the coriolis terms are small,

$$
h(x,u) = \begin{pmatrix} a_{x_{meas}} \\ a_{y_{meas}} \\ a_{z_{meas}} \end{pmatrix} \approx \begin{pmatrix} gs_\theta \\ -gc_\theta s_\phi \\ -gc_\theta c_\phi \end{pmatrix} - \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \tag{8}
$$

and therefore

$$
\frac{\partial h}{\partial x} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & gc_\theta & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -gc_\theta c_\phi & gs_\theta s_\phi & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & gc_\theta s_\phi & gs_\theta c_\phi & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \tag{9}
$$

4 Motion Capture Measurements

Motion Capture measurements are used to directly update the position and attitude states.

$$
h(x,u) = \begin{pmatrix} p_n \\ p_e \\ p_d \\ \phi \\ \theta \\ \psi \end{pmatrix} \tag{10}
$$

.

Therefore:

$$
\frac{\partial h}{\partial x} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}
$$

5 GPS Measurements

GPS is used to directly update the position states of the filter

The UAV book tells us to take the derivative of GPS to create a pseudo-velocity measurement. I think this is weird, but I may end up doing that too.

$$h(x,u) = \begin{pmatrix} p_n \\ p_e \\ p_d \end{pmatrix} \tag{12}$$

Therefore

$$\frac{\partial h}{\partial x} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

6 VO Measurements

For now, I'm using VO as a pseudo-global measurement. This is wrong, because it is unobservable. To fix this, though requires a relative estimator, which I don't want to implement right now. Instead, I'm going to pretend that VO gives me a global position and attitude measurement. This will cause problems if operated for a long time, but will probably work for small flights and tests. In this case, it's exactly like the motion capture measurement model, with

$$h(x,u) = \begin{pmatrix} p_n \\ p_e \\ p_d \\ \phi \\ \theta \\ \psi \end{pmatrix} \tag{14}$$

.

and

$$\frac{\partial h}{\partial x} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$