Homework #1: Line Search — ME 575

due 1/21/2015 before class — 50 possible points

1.1 [3 pts] Answer the "Getting to Know You" questions posted on Digital Dialog within Learning Suite. You don't need to turn them in. When you answer on Learning Suite I will see your completion.

1.2 [2 pts] Complete the FERPA "Quiz" on Learning Suite. (Similarly, I will see your completion online.)

1.3 [5 pts] Read Chapter 1 (Introduction) and Chapter 2 (Single Variable Minimization) posted on Learning Suite. Also read through the Homework Submission Instructions.

1.4 [10 pts] Browse through different journal articles on engineering design optimization and select one that catches your interest. Write a short critique (maximum of one page). Briefly summarize the paper's findings, evaluate its importance, provide constructive criticism, and see if you can infer new insights. Pay particular attention to:

- Problem formulation (choice of objective, constraints and design variables)
- Optimization method used
- Practicality of the results
- Conclusions

Include a copy of the article in your submission as a separate file.

1.5 [15 pts] Let $f(x) = -x + \gamma x^2$, where $\gamma > 0$ is a parameter. For this question you will investigate three values for the parameter: $\gamma = 0.5$, 10, and 10^4.

(a) Starting from $x_0 = 0$, use a backtracking line-search — with initial step length $\alpha_0 = 1$ and reduction parameter $\rho = 0.5$ — to find a step that satisfies the sufficient decrease condition with $\mu_1 = 10^{-4}$. How many function evaluations are required for the three different values of γ?

(b) How does the performance of the backtracking line-search compare with Newton's method for the different values of γ?

Remark: this question illustrates the impact of poor scaling, a problem that often affects practical optimization problems.

1.6 [15 pts] Apply the code you developed in the previous problem to an application of your choice. This must be an application that can be solved with a 1-dimensional unconstrained optimization. If you cannot think of anything, I have provided an example for a simple aircraft wing drag minimization below. For your problem complete the steps below.

(a) Write out the objective as a function of the design variable.

(b) Compare a backtracking line search that satisfies the strong Wolfe conditions with any exact line search method. You may use any exact line search such as golden section method, secant method, Newton's method, Brent's method, etc. (for extra credit include more than one exact line search method in your comparison).

(c) Discuss the relative performance of these two methods. Try different starting points/intervals and different reduction parameters. Compare convergence rates, number of iterations and any other metrics you find suitable.

Wing Drag Minimization. The following describes a very simple model for drag minimization of a wing. Consider a rectangular wing with span b and chord c. Its planform area is thus $S = bc$ and its aspect ratio is $A = b^2/S$. The drag of this wing can be approximated as [1]

$$C_D = kC_f \frac{S_{\text{wet}}}{S} + \frac{C_L^2}{\pi A e} \tag{1}$$

The first term corresponds to the parasite drag. For a fully turbulent boundary layer, the skin friction coefficient (C_f) can be approximated as [2]

$$C_f = \frac{0.074}{Re^{0.2}} \tag{2}$$

In this equation the Reynolds number is based on the wing chord ($Re = \rho V c/\mu$). The form factor (k) accounts for the effects of pressure drag. The wetted area (S_{wet}) is the area over which the skin friction drag acts, and we divide the expression by the reference area S because of the way the drag coefficient is defined.

The second term in Eq. (1) is the induced drag, where e is the Oswald efficiency factor. The lift coefficient C_L and the wing planform area S are to be kept constant. The values for all the constants are listed in Table 1. For this simple model we want to minimize the drag coefficient C_D with respect to the aspect ratio A.

Remark: the optimal aspect ratio will be much larger than is practical because we are using an extremely simple model that does not include structures or other important disciplines.

Table 1: Flow conditions and other fixed variables

Quantity	Value	Units	Description
ρ	1.23	kg/m^3	density of air
μ	17.8×10^{-6}	kg/(m sec)	viscosity of air
V	35	m/s	airspeed
S	11.8	m^2	planform area
S_{wet}	$2.05S$	m^2	wing wetted area
k	1.2		form factor
C_L	0.3		lift coefficient
e	0.96		Oswald efficiency factor

References

[1] Shevell, R. S., *Fundamentals of Flight*, Pearson College Division, 1989.

[2] Anderson, J. D., *Introduction to Flight*, McGraw-Hill Medical Publishing, 2011.