Homework 1

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1 Learning exercises

1.1 Exercise 1.1

1.1.1 Medical diagnosis

the input space X: medical history and symptoms
the output space Y: medical diagnosis of patients
the target function f: (X $\rightarrow$ Y): based the medical history and symptoms, finding the formula for identification of the diagnosis for the patients

1.1.2 Handwritten digit recognition

the input space X: pictures of hand digit
the output space Y: zip code recognition
the target function f:(X $\rightarrow$ Y): an algorithm that sorts mail using hand digit pictures

1.1.3 Spam Determination

the input space X: any emails
the output space Y: filters spam emails
the target function f:(X $\rightarrow$ Y): an algorithm that identifies email spam or not

1.1.4 Electric load problem

the input space X: price, temperature and day of the week
the output space Y: choosing the electric load
the target function f:(X $\rightarrow$ Y): predicting the variation of electric board using price, temperature, and day of the week

1.1.5 Data prediction solver

the input space X: past data
the output space Y: empirical solution
the target function f:(X $\rightarrow$ Y): an algorithm to make an empirical solution based on the past data

1.2 Exercise 1.5

1.2.1 Determining the age at which a particular medical test should be performed

learning approach

1.2.2 Classifying numbers into primes and non-primes

design approach

1.2.3 Detecting potential fraud in credit card charges

learning approach

1.2.4 Determining the time it would take a falling object to hit the ground

design approach

1.2.5 Determining the optimal cycle for traffic lights in a busy intersection

learning approach

2 Perceptron Learning Algorithm

2.1 Show that $y(t)w(t)x(t) < 0$

Output can hold only -1 and +1. If our weight have a negative sign or $x(t)$ is misclassified by $w(t)$, so it could be $-y(t) = -w(t)x(t) < 0$. This is telling that $-y(t)y(t) < 0$, so that $y(t)w(t)x(t) < 0$.

2.2 Show that $y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$

If we decompose $w(t+1) = w(t) + y(t)x(t)$. So that,

$$y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$$

$$y(t)(w(t) + y(t)x(t))^T x(t) > y(t)w^T(t)x(t)$$

$$y(t)w(t)^T x(t) + y^2(t)x^2(t) > y(t)w^T(t)x(t)$$

$$y^2(t)x^2(t)x(t) > 0$$

It shows that $y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$.

2.3 As far as classifying x(t) is concerned, argue that the move from $w(t)$ to $w(t+1)$ is a move 'in the right direction

In the case, when $w^T(t)$ is small, $w(t+1) = w(t) + w(0)x(t)$ must increase to the right. Otherwise, $w(t+1) = w(t) - w(0)x(t)$ must decrease to the right.

3 Independence

For discrete random values:
$E[X * Y] = \sum^{i}\sum^{j} x_i * y_j * f_x y * (x_i, y_j)$
$E[X * Y] = \sum^{i}\sum^{j} x_i * y_j * f_x * (x_i) * f_y * (y_j)$
$E[X * Y] = (\sum^{i} x_i * f_x * (x_i)) * (\sum^{i} y_j * f_y * (y_j))$
$E[X * Y] = E[X]E[Y]$

For continuous random variables:
$E[X * Y] = \int_{\infty}^{-\infty}\int_{\infty}^{\infty} x * y * f_x y * dx * dy$
$E[X * Y] = \int_{\infty}^{-\infty}\int_{\infty}^{\infty} x * y * f_x(x) * f_y(y) * dx * dy$
$E[X * Y] = (\int_{\infty}^{-\infty} x * f_x(x) * dx) * (\int_{\infty}^{\infty} y * f_y(y) * dy)$
E[X*Y] = E[X]E[Y]

4 *Spam filtering equation- (optional bonus problem)

$P(S|W) = \frac{P(S)P(W|S)}{P(W)}$

$P(W) = P(W|H)P(H) + P(W|S)P(S)$

$P(S|W) = \frac{P(S)P(W|S)}{P(W|H)P(H)+P(W|S)P(S)}$

5 I.I.D. assumption in spam filters

1. To disable the spam filter so it lets all spam into the inbox
2. To miss a particular ham email filtered away as spam
3. To get a particular spam into the victim's inbox
4. To get any spam from the victim's inbox