Epidemiology and Big Data

Mixed Models 1: Introduction to Multilevel Models

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Relevant Course Objectives

- At the end of the course, the student will:
    - know when to apply a mixed model in practice
    - be able to perform mixed model analyses using statistical software (R)

LRT = London Reading Test

Objectives for this week

- At the end of this week, the student will:
  - recognize multi-level and longitudinal study designs
  - be able to explain the difference between fixed and random effects, and know when to use random effects
  - know when to apply a linear mixed model
  - be able to perform linear mixed models using R

Overview Lecture 1: Multilevel Modelling

- Introduction to multilevel data
- Example: multilevel data (children within schools)
- The problem, and some possible solutions
- The mixed model solution
- Adding random effects (random intercept, random slope)
- Adding fixed effects (school- and child-level) to the model
- Interpretation of mixed models
- Summary

Examples of multilevel data

- Effect of school environment on exam results
  - Design: hierarchical, where the examination results of a random sample of students within a random sample of schools are compared
- Influence of race and sex on fetal heartbeat during pregnancy
  - Design: repeated measurements on different gestational ages during pregnancy, where the gestational ages were not the same for all women
- Multi-center hypertension trial
  - Design: hierarchical, with 193 patients in 27 centers, DBP measured 5 times per patient over the course of several weeks

- randomly chosen schools and children within the schools
- design is hierarchical because children nested in schools

repeated measures over time
measurements within patients and patients within each hospital - 3 hierarchical

Characteristics of multilevel data

- Hierarchical structure of data
  - children within (classrooms within) schools
  - patients within centers
  - measurements within patients
- Variation at all levels
- "Units" within a level expected to be correlated
- Variables can be measured at different levels
  - Level 2:
    - type of school (mixed vs. single-gender)
    - university vs. community hospital
  - Level 1:
    - reading ability of child at intake
    - gender of patient

potential for variation at all levels
e.g. children vary etc.
units at the lower level are expected to be correlated. so kids within the same schools are more likely to be correlated

Example: London Schools

- Data collected by Goldstein, Rasbash, et al (1993) on 4059 children in 65 schools in Inner London.

  assuming that it's a simple random sample

- Question: is examination achievement related to intake achievement level, pupil gender, school type and exam achievement of school (averaged over all pupils)?

  can we explain how well students will do by the end of their career based on their intake achievement level

- Subquestion: do girls do better at a mixed or all-girls' school?

Example: London Schools

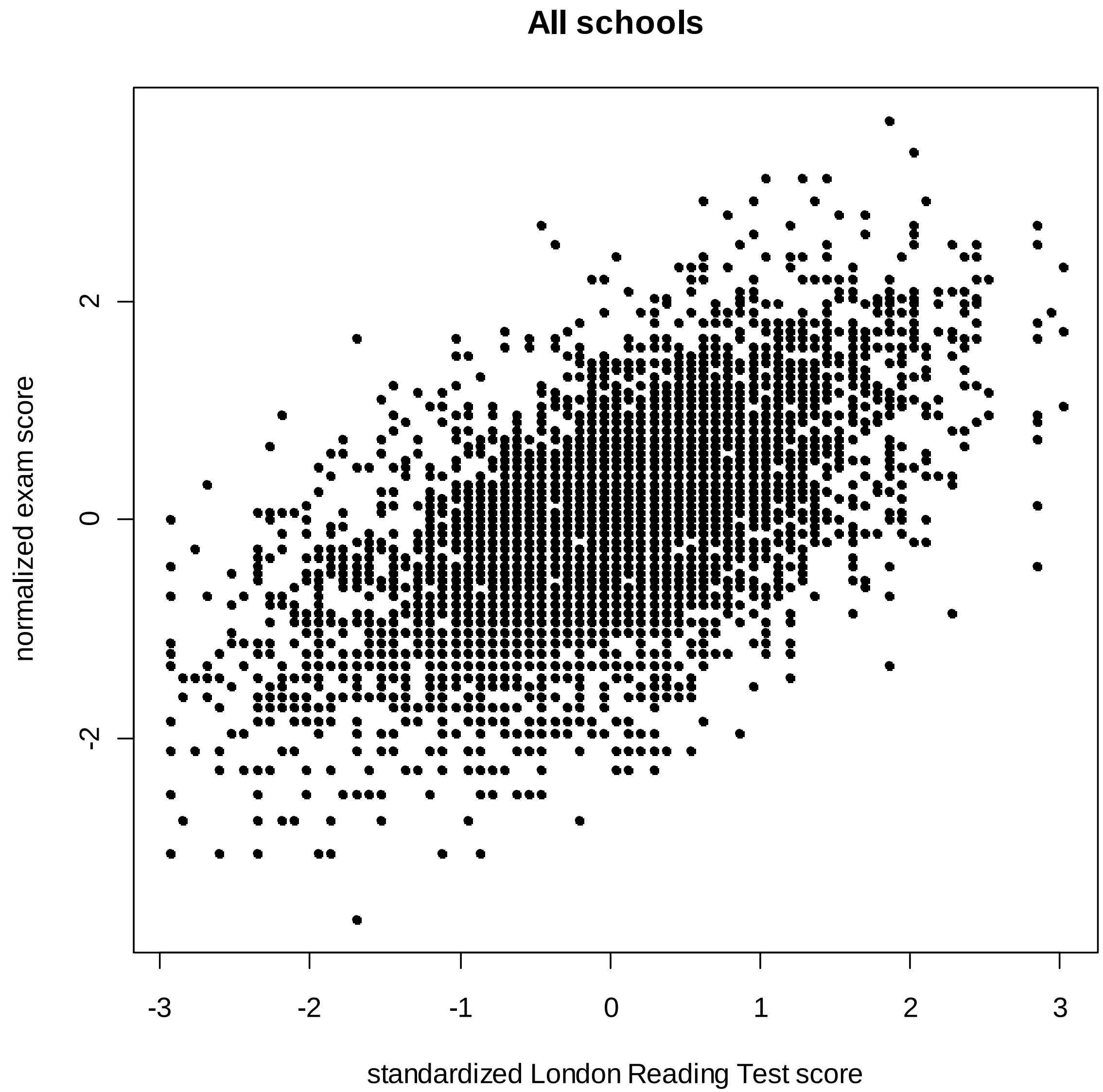
- Variables in dataset:
  - School ID   necessary to have an identifying variable in order to do a mixed model analysis
  - Student ID
  - Normalised exam score (outcome variable)
  - Standardised LR test score   intake to school - can it predict how well kids will be doing at the end?
  - Student gender
  - School gender
  - School average of intake score
  - Student level Verbal Reasoning (VR) score category at intake
  - Category of students' intake score (averaged)   will be used as well (how well school performs on average)

London Schools

school	# boys	# children	school	# boys	# children
1	45	73	13	26	64
2	0	55	14	92	198
3	29	52	15	47	91
4	45	79	16	0	88
5	16	35	17	31	126
6	0	80	18	0	120
7	0	88	19	33	55
8	0	102	20	21	39
9	21	34	21	0	73
10	31	50	22	48	90
11	62	62	.	.	.
12	23	47	.	.	.
			.	.	.

number of children varies significantly

London Schools



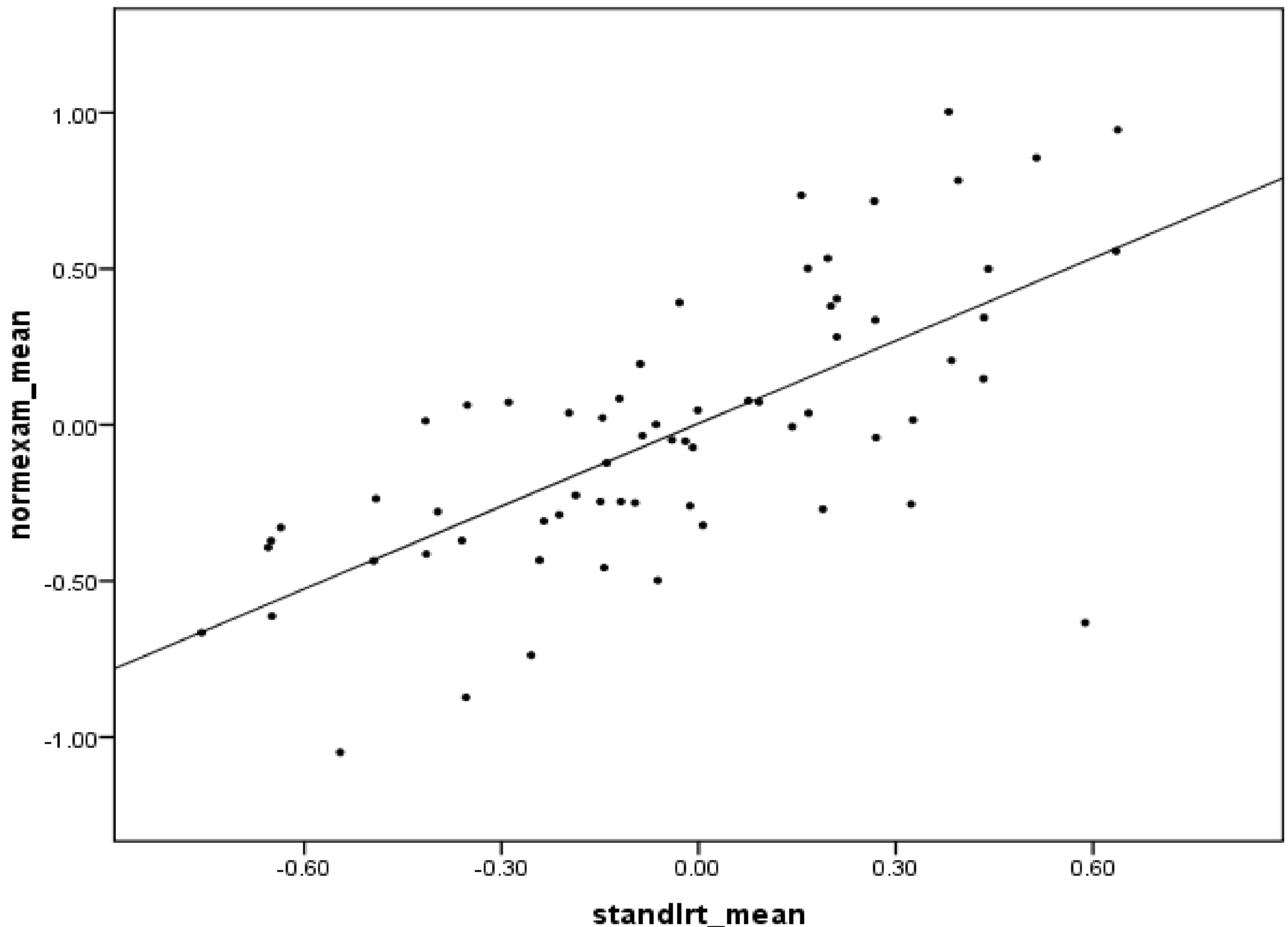
a lot of variation at any given level but on average a positive association
as LRT values go up the exam scores go up as well

London Schools

- How to analyze relation between exam score and LRT score?
  1. linear regression, mean exam per school vs mean LRT (“aggregated data”) average scores
  2. linear regression, all schools together (“disaggregated data”) like previous slide
  3. linear regression per school
  4. linear regression, all schools together, regression with main effect and interactions to allow for different intercepts and slopes
  5. Linear Mixed Model

London Schools:

1. linear regression, aggregated mean exam vs mean LRT



bad idea because:

intercept is when LRT is zero which in this case would be the mean value of the test. Everythin under 0 is schools that are under the mean

London Schools:

1. linear regression, aggregated mean exam vs mean LRT

```
> agglondon= aggregate(london, by= list(london$school), FUN=mean)
> head(agglondon)
  Group.1 school student   normexam    standlrt    gender schgend   avslrt schav   vrband mixed
1       1      1    37.0 0.50120348 0.16617305 0.3835616       1 0.166170     2 1.712329     1
2       2      2    28.0 0.78309603 0.39514738 1.0000000       3 0.395150     3 1.636364     0
3       3      3    26.5 0.85543873 0.51415485 0.4423077       1 0.514160     3 1.519231     1
4       4      4    40.0 0.07362567 0.09176214 0.4303797       1 0.091764     2 1.746835     1
5       5      5    18.0 0.40360263 0.21052226 0.5428571       1 0.210520     3 1.657143     1
6       6      6    40.5 0.94456957 0.63765269 1.0000000       3 0.637660     3 1.462500     0
> aggmeanmodel= lm(normexam ~ standlrt, agglondon)
> summary(aggmeanmodel)

Call:
lm(formula = normexam ~ standlrt, data = agglondon)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.15787 -0.13819 -0.00342  0.19873  0.66268 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.004563   0.039737   0.115    0.909    
standlrt    0.883721   0.116016   7.617 1.67e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3191 on 63 degrees of freedom
Multiple R-squared:  0.4794,	Adjusted R-squared:  0.4712 
F-statistic: 58.02 on 1 and 63 DF,  p-value: 1.668e-10
```

estimate for intercept: 0.005 (se 0.040)

estimate for slope: 0.884 (se 0.116)

significant association between mean exam scores and mean reading test

London Schools:

1. linear regression, aggregated mean exam vs mean LRT

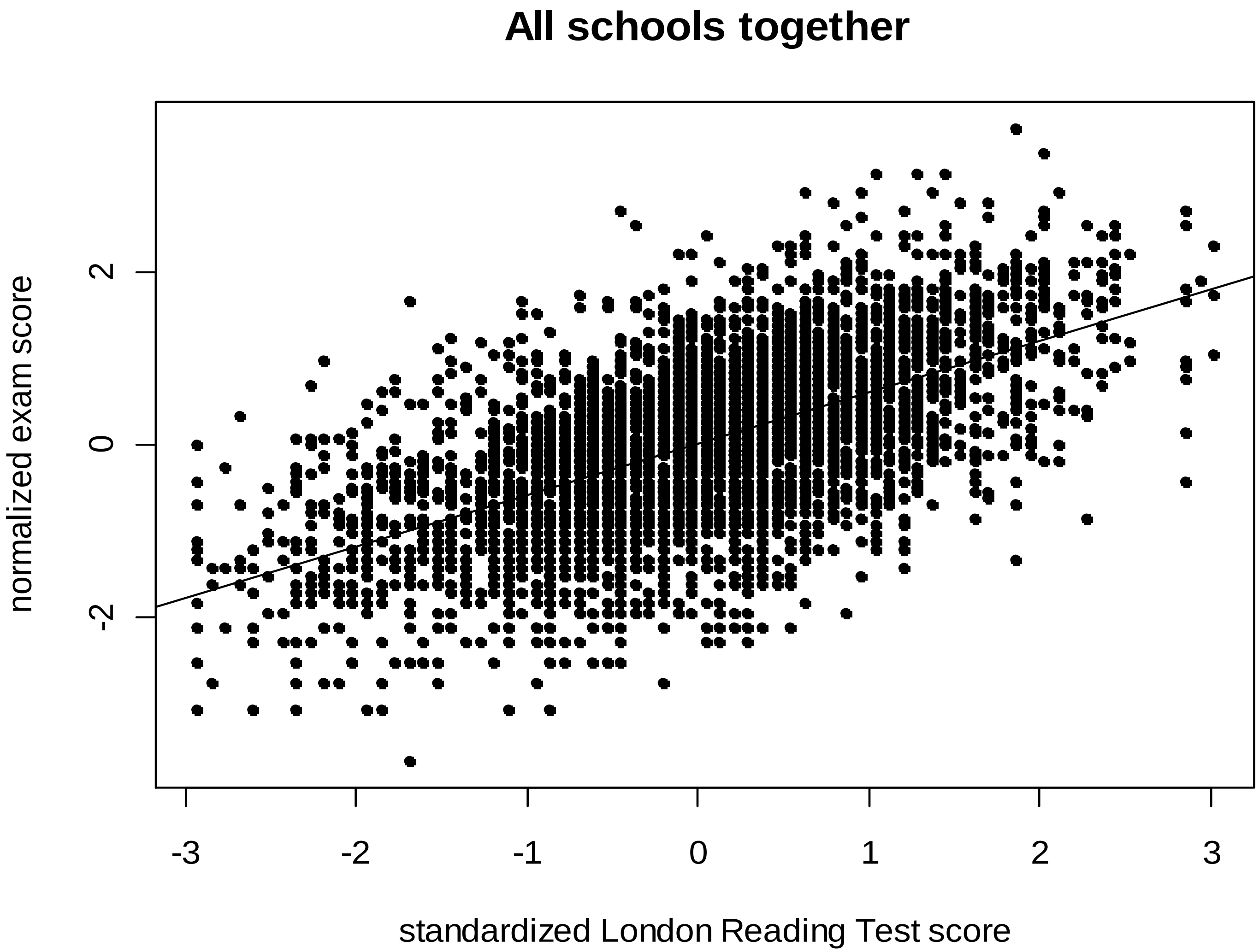
- Disadvantages:
  - every school (regardless of sample size) given equal weight
  - N = 65 sample size reduced significantly - losing power
  - school-level variables possible, but not child-level variables
  - we can only make inference at school level, not child-level
  - possibility of "ecological fallacy"

can't talk about girls vs. boys because they're averaged out so only inference at the school level

when averaging we would tend to see stronger association than it's actually there
= looking at higher level and trying to apply what info we got there to a lower level
= occurs when inferences about the nature of individuals are deduced from inferences about the group to which those individuals belong

London Schools

2. linear regression, all schools together



London Schools

2. linear regression, all schools together

```
> disagmod= lm(normexam ~ standlrt, data=london)
> summary(disagmod)

Call:
lm(formula = normexam ~ standlrt, data = london)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.65617 -0.51847  0.01265  0.54397  2.97399 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.001195   0.012642  -0.095    0.925    
standlrt     0.595055   0.012730  46.744   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8054 on 4057 degrees of freedom
Multiple R-squared:   0.35,	Adjusted R-squared:  0.3499 
F-statistic:  2185 on 1 and 4057 DF,  p-value: < 2.2e-16
```

the normalized exam score is going up just about one half sd for every one unit going up in the standardized reading test

estimate for intercept: - 0.001 (se 0.013)

estimate for slope: 0.595 (se 0.013)

London Schools:

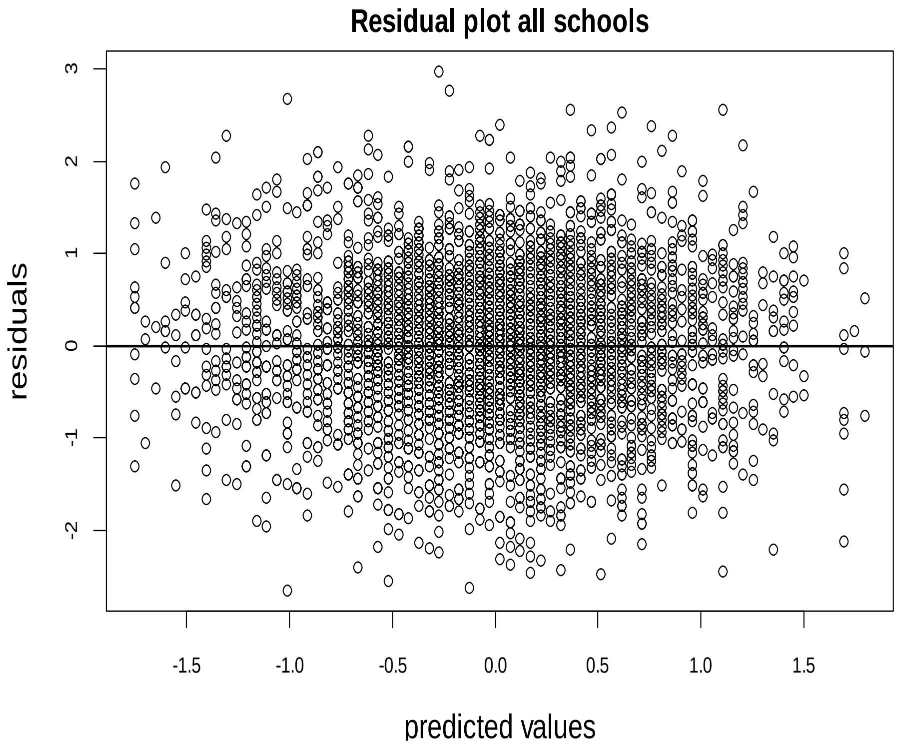
2. linear regression, all schools together

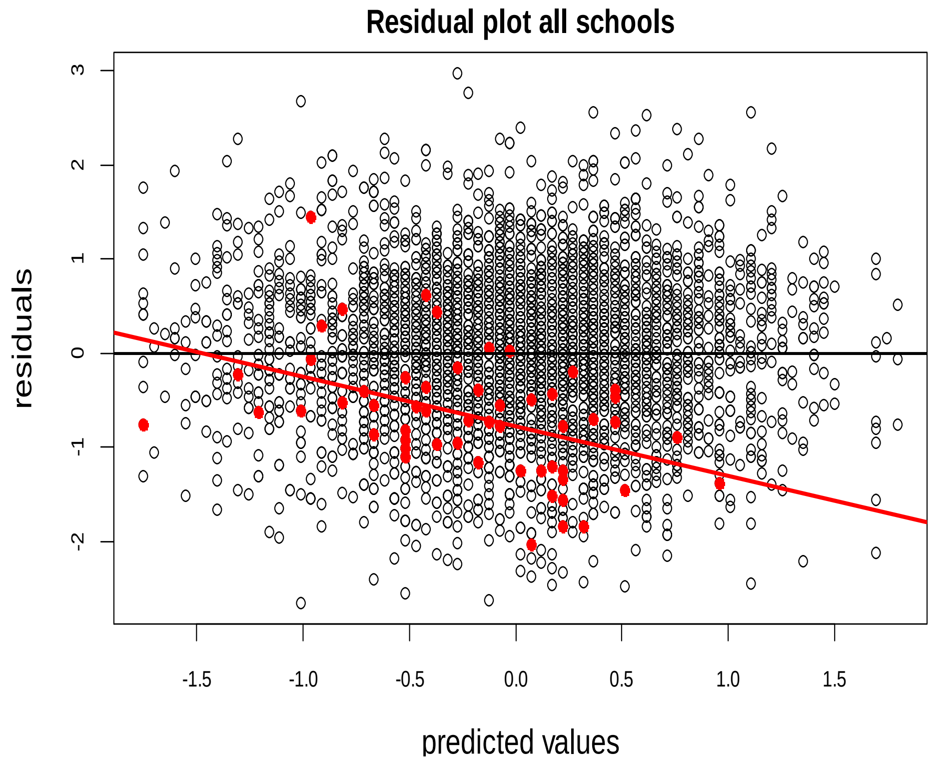
- Disadvantages:
  - inflates sample size, especially for level-2 variables
    - SE's of level-2 variables tend to be underestimated → p-values too small, CI's too narrow (type I error inflated)
    - SE's of level-1 variables may be over- or underestimated
  - ignore correlated residuals (correlation of children within schools)

assuming uncorrelated residuals when doing linear regression - simple random sample and no correlation between the kids/samples - which is not realistic as there is correlation between the kids within the schools

London Schools:

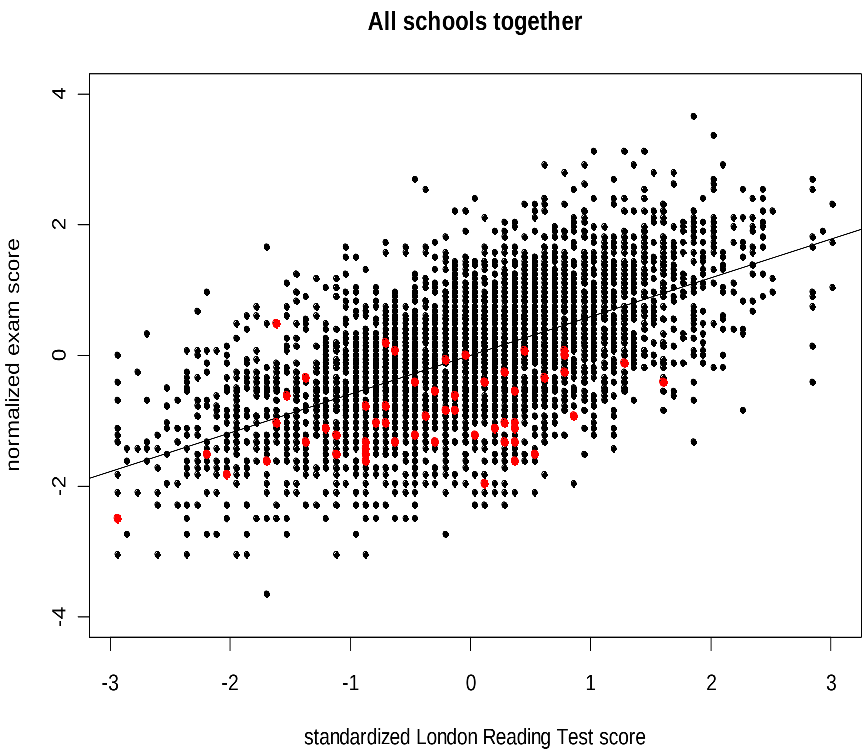
2. linear regression, all schools together

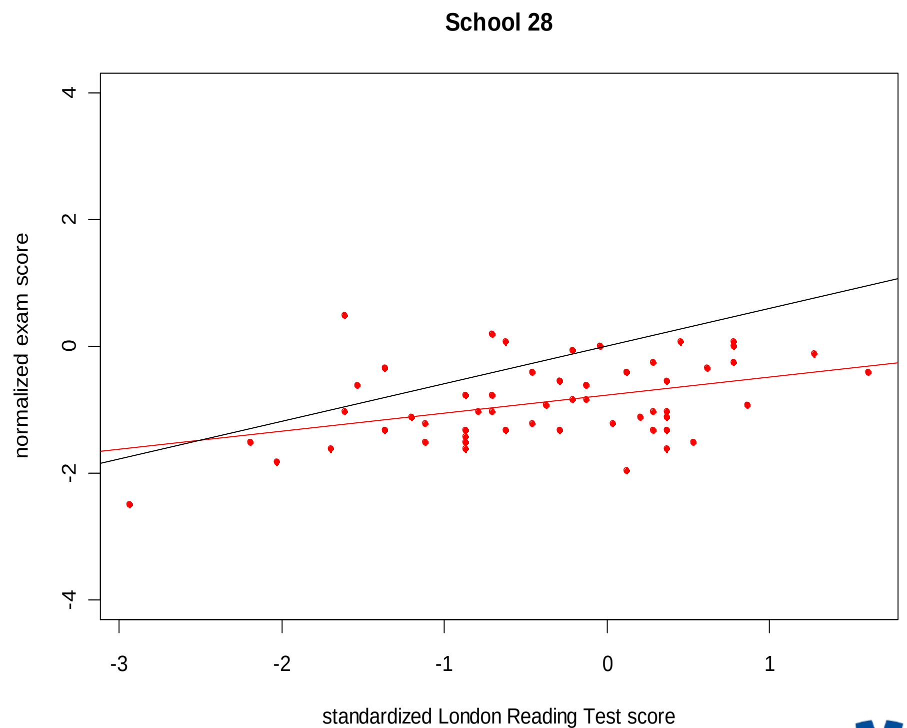




London Schools:

2. linear regression, all schools together

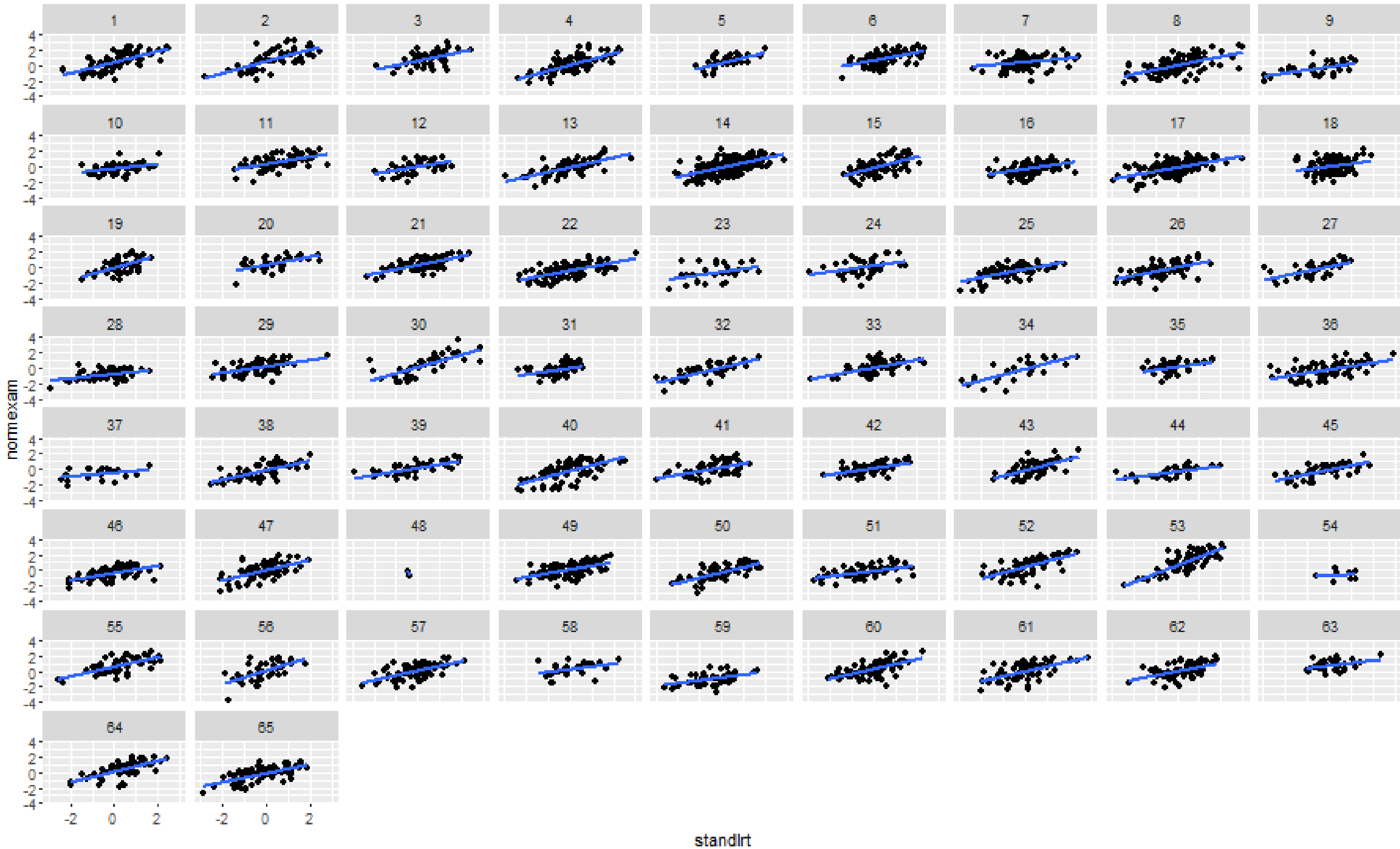




London Schools

correlated residuals would not be a problem because it's for every school individually

3. linear regression per school



London Schools

3. linear regression per school

School	Intercept	slope
1	0.383330189	0.70934058
2	0.482275275	0.76128749
3	0.557750538	0.57898548
4	0.003753722	0.76144638
5	0.260443999	0.68001660
6	0.603206568	0.53534316
....		

- summary intercepts:
  - mean = -0.068; sd = 0.519; sem = 0.064
- summary slopes:
  - mean = 0.425; sd = 0.939; sem = 0.116

London Schools

3. linear regression per school

- Disadvantages:
  - 65 different regressions, how to combine the results?
    - mean slope: every school has equal weight
    - standard error of parameter estimate correct?
  - child-level variables possible, but not school-level variables

we cannot make an analysis as though all students were individual but we have to look at the number of schools to analyze them - (level 2) Minute 38

now we cannot include school levels anymore because there's no variation at the school level as we look at every school individually

London Schools

4. all schools together, main effects and interactions for every school

- Advantage over previous analysis:
  - now we can include both child- and school-level variables
  - residuals probably normally distributed (with constant variance?) around individual lines
- Disadvantages:
  - We wanted 1 intercept and 1 slope for LRT, but:
  - 65 schools, so 1 reference category and dummy variables 64 estimates for intercepts (main effects per school) + 64 estimates for interactions (slopes per school)! but I only want ONE general comparison. Which one would I use as reference?
    - Which school is the reference?
  - We can't generalize beyond these 65 schools
  - This model uses 128 extra df for all those intercepts & slopes

should be independent of one another because within the school I am assuming independence between kids

London Schools: models so far

Model	overall/fixed slope LRT	s.e.
1. aggregated data	0.884	0.116
2. disaggregated data	0.595	0.013
3. regr. per school	0.425	0.116
4. school*LRT interactions	??	??

London Schools

5. Mixed Models

- Advantages:
  - sample size correct, account for correlation of children within schools
    - so: correct SE’s/p-values/CI’s
  - no need for 64 main effects and interactions
    - differences between schools captured one or more ‘variance components’
  - both child-level and school-level variables simultaneously
    - so: inference for both children and schools and cross level interactions
    - interactions between child- and school-level variables possible
  - examine variation at different levels
  - models work well in presence of missing outcomes (longitudinal)

    if missing values only in outcome we can still use the model

Mixed Models

- Mixed models made up of
  - fixed effects
  - random effects

"mixed"

- Sometimes (inaccurately) called “random effects models”
- Also sometimes called “random coefficient” models
- Some variables (or: their coefficients) can be included as both “fixed” (of interest) and “random” (random variation across the level-2 units)

not sure but there is variation

Mixed Models: what is a "fixed effect"?

variable into model to see outcome - in fixed model those are the itneresting variables

- Fixed effect: variable of interest
  - overall intercept (not really of interest)
  - overall slope for LRT (to help make predictions of exam performance)
  - other fixed effects of interest:
    - gender (difference between boys and girls?)
    - type of school (boys', girls', mixed)
    - "achievement level" of school
    - ...

variables added to the model because we want to estimate them
- make inference about them

Mixed Models: what is a "random effect"?

- A random intercept per school allows schools to have different intercepts one school is lower/higher than other schools
- A random effect for LRT per school allows the effect of LRT on exam score to differ per school ("random slope for LRT" = different slope for exam-LRT relation for each school)

unlikely that all schools have the same slope

like difference between boys/girls

- Random effect ("slope") can also be for a categorical variable

allow for the slope to differ for each school (overall line but still allow for variation around this slope)

  - difference between boys and girls on exam score could differ per school

random treatment effect - metanalysis

  - treatment effect on an outcome can be thought to vary per center in a multi-center study
- All variables of interest are added as fixed
- Depending on theory, none/one/some fixed variables may also be modelled as random some variables modeled as random

Mixed Models: what is a “random effect”?

- Why “random effect”?
- Schools are *random* sample of all Inner London schools
  - intercepts (and LRT slopes) from these schools are a random sample from all possible intercepts and slopes
  - intercepts (and LRT slopes?) differ from one another, but
  - interest not in estimating the intercept and slope per school, thus
  - sufficient to estimate the variances of the intercepts and slopes
  - intercepts (and slopes) thought to come from normal distributions with mean 0 and variances $\sigma^2_{\upsilon 0}$ and $\sigma^2_{\upsilon 1}$, and covariance $\sigma_{\upsilon 01}$
  - in this way we only have to estimate 3 extra parameters, not 128

Interlude: some notation

01:05

exam score per child = intersect based on school + slope school x

- level-1 (child) model: $y_{ij} = b_{0i} + b_{1i} \cdot x_{1ij} + \varepsilon_{ij}$
- level-2 (school) model: $b_{0i} = \beta_0 + \upsilon_{0i}$; $b_{1i} = \beta_1 + \upsilon_{1i}$

fixed

- combine the two: $y_{ij} = \beta_0 + \upsilon_{0i} + \beta_1 \cdot x_{1ij} + \upsilon_{1i} \cdot x_{1ij} + \varepsilon_{ij}$
    - rewrite: $y_{ij} = (\beta_0 + \upsilon_{0i}) + (\beta_1 + \upsilon_{1i}) \cdot x_{1ij} + \varepsilon_{ij}$

intercept dependent on the school and a slope + random slope per school x stand. test + residual

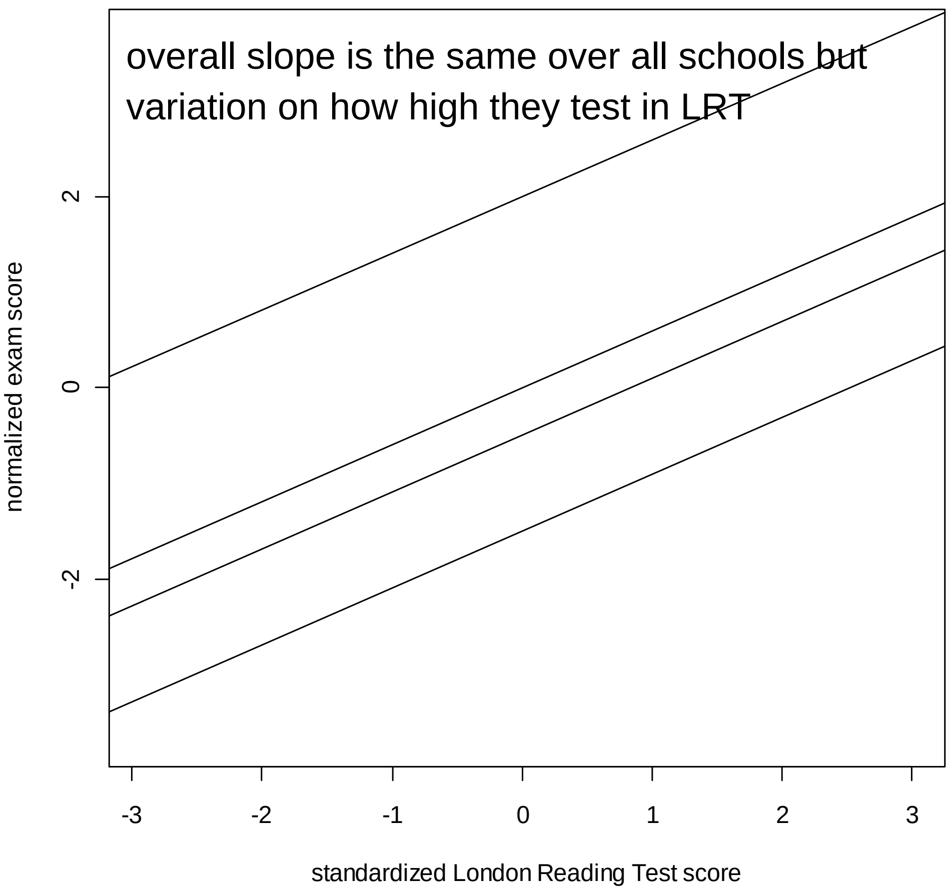
- y_{ij} : outcome (exam score) for jth child in ith school
- x_{1ij}: 1st explanatory var (LRT score) at level 1 (jth child in ith school)
- β_0, β_1, … : regression coeffients for overall effects of explanatory vars ("fixed effects")
- υ_{0i} : individual effect of ith school on intercept ("random effect")
- υ_{1i} : individual effect of ith school on slope (for LRT) ("random effect")
- ε_{ij} : level-1 residual (jth child in ith school)

Mixed Models: what is a “random effect”?

Random intercept only:

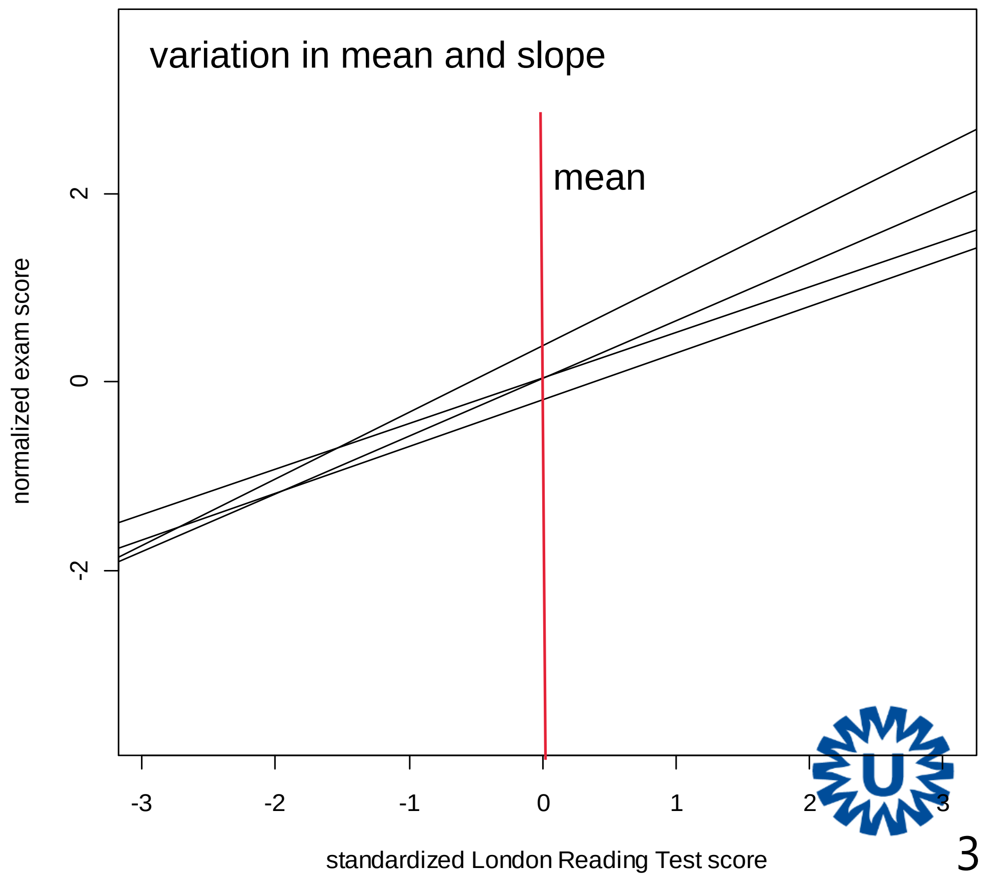
same slope

Between-school variation (simple)

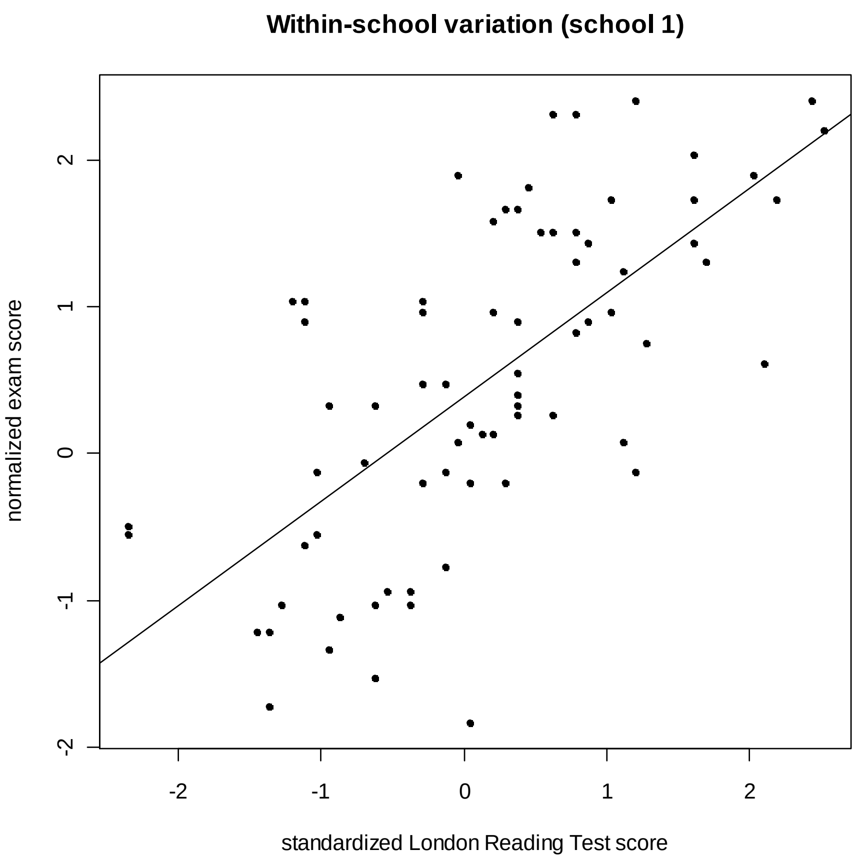


Random intercept + random slope:

Between-school variation (complex)



London Schools



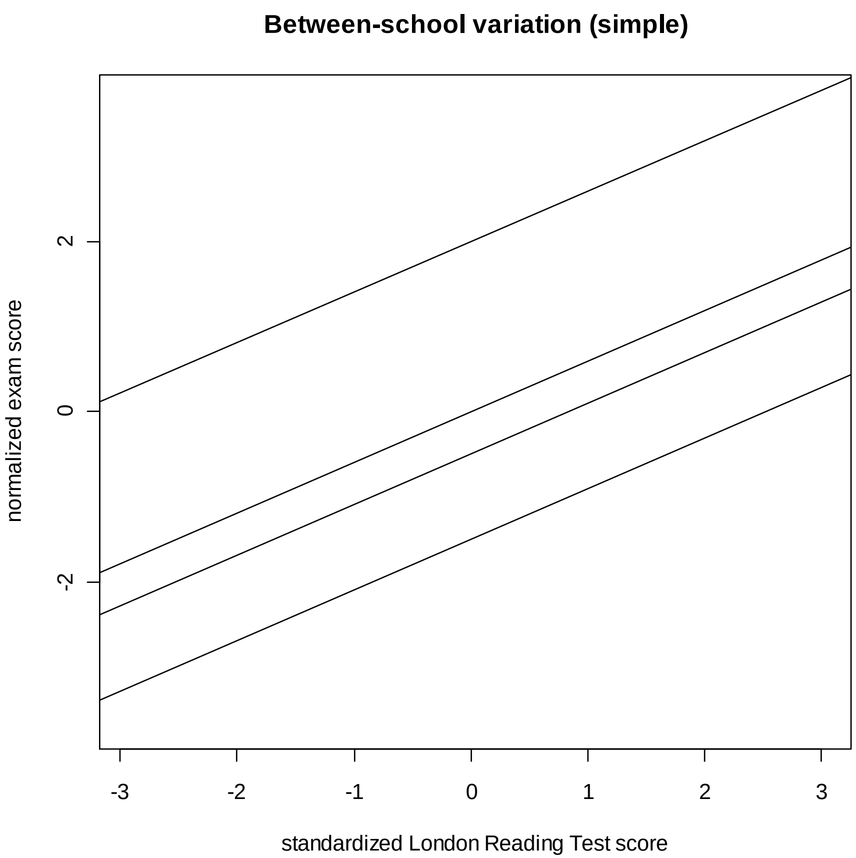
$$y_{1j} = \beta_0 + \beta_1 X_{11j} + \varepsilon_{1j}$$

within school variation

lines within the schools still have variation - not all kids are exactly on the regression line

estimate of the variation

London Schools



estimate of between school variation
allowing the intercepts to vary or adding this variation v

$$y_{1j} = \beta_0 + \upsilon_{01} + \beta_1 X_{11j} + \varepsilon_{1j}$$

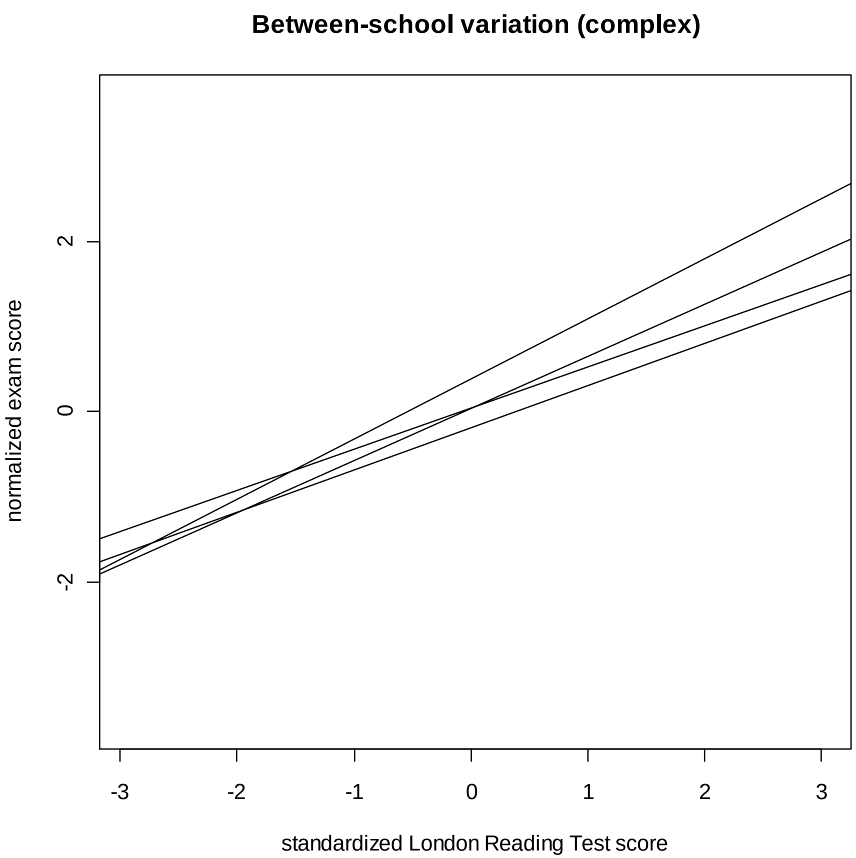
$$y_{2j} = \beta_0 + \upsilon_{02} + \beta_1 X_{12j} + \varepsilon_{2j}$$

$$y_{ij} = \beta_0 + \upsilon_{0i} + \beta_1 X_{1ij} + \varepsilon_{ij}$$

between school variation

keeping slopes the same and allowing the intercept to vary

London Schools



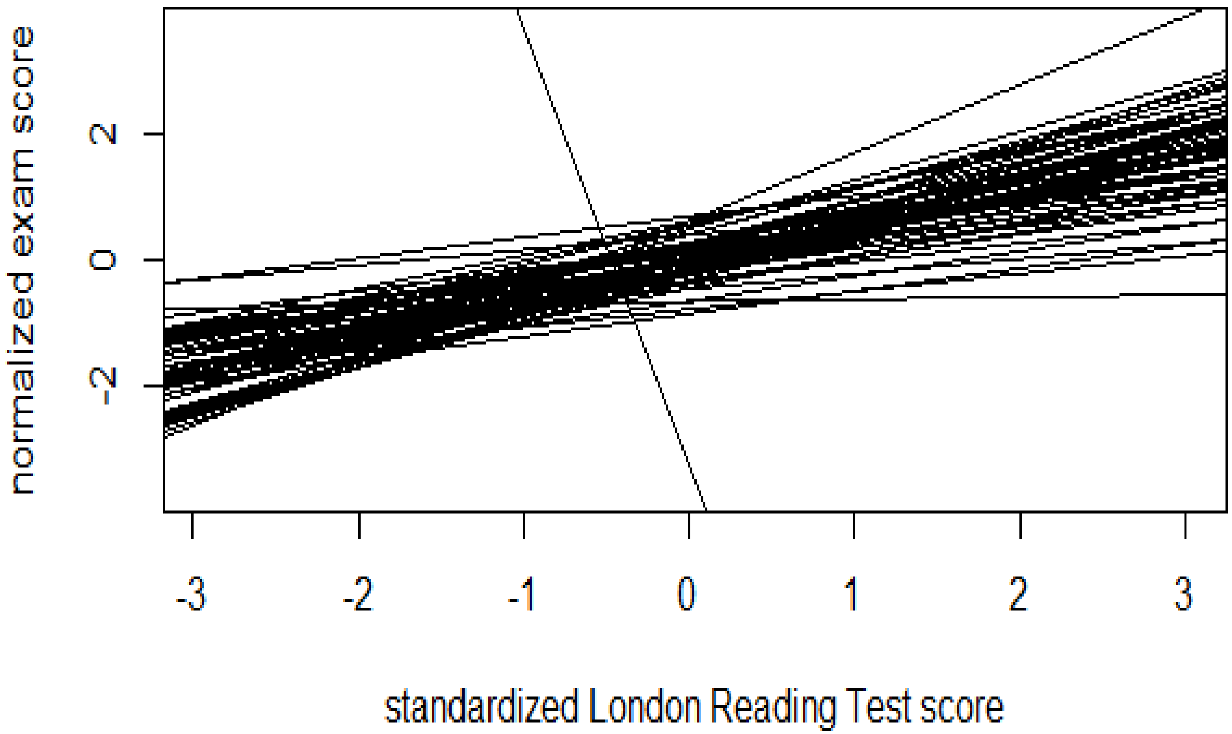
$$y_{1j} = \beta_0 + \upsilon_{01} + \beta_1 X_{11j} + \upsilon_{11} X_{11j} + \varepsilon_{1j}$$

$$y_{2j} = \beta_0 + \upsilon_{02} + \beta_1 X_{12j} + \upsilon_{12} X_{12j} + \varepsilon_{2j}$$

$$y_{ij} = \beta_0 + \upsilon_{0i} + \beta_1 X_{1ij} + \upsilon_{1i} X_{1ij} + \varepsilon_{ij}$$

London Schools

Graph per school (“spaghetti plot”):



each schools estimated intercept and slope plotted
variation in intercept and the schools differ in slope (steeper etc.)
-> random slope would be an appropriate fit

Mixed Models: the model

- $y_{ij} = (\beta_0 + \upsilon_{0i}) + (\beta_1 + \upsilon_{1i}) \cdot x_{1ij} + \cdots + \varepsilon_{ij}$ residual per child/school
- Where:
  - y_{ij}: outcome (exam score) for j^{th} child in i^{th} school
  - x_{1ij}: first explanatory variable (LRT score) at level 1 (j^{th} child in i^{th} school)
  - β_0, β_1, ... : regression coefficients for explanatory variables ("fixed effects")
  - υ_{0i}: random effect for the intercept in i^{th} school
  - υ_{1i} : random effect for the slope (for LRT) in i^{th} school
  - ε_{ij}: level-1 residual (j^{th} child in i^{th} school)
- Model assumptions:
  - $\varepsilon_{ij} \sim N(0, \sigma_e{}^2)$; $\upsilon_{0i} \sim N(0, \sigma_{\upsilon 0}{}^2)$; $\upsilon_{1i} \sim N(0, \sigma_{\upsilon 1}{}^2)$
  - ε_{ij} independent
  - $cov(\upsilon_{0i}, \upsilon_{1i}) = \sigma_{\upsilon 01}$ allowing covariation
  - $cov(\varepsilon_{ij}, \upsilon_{0i}) = cov(\varepsilon_{ij}, \upsilon_{1i}) = 0$

sigma squared over residuals
is expected to be the same over all schools

Mixed models in R

Two packages used most frequently

- Package nlme
  - lme() for Gaussian models
  - gls() function for models with correlated errors
  - approximate (Wald) CI's via intervals() function in same package

- Package lme4
  - lmer() for Gaussian models
  - glmer() for generalized linear mixed models (day 4)
  - "profile likelihood" CI's via confint()

- See information on Blackboard

London Schools: mixed model

random intercept only

random intercept per school

```
> sch.lme.1 <- lme(fixed=normexam~standlrt, random=~1 | school,
data=london, method="ML")
```

maximum likelihood

```
> summary(sch.lme.1)
Linear mixed-effects model fit by maximum likelihood
 Data: london
       AIC       BIC    logLik
  9365.213 9390.447 -4678.606

Random effects:
 Formula: ~1 | school
        (Intercept)  Residual
StdDev:   0.3035269 0.7521481
```

sd for residuals

sd for random intercepts and

"`fixed=`" is optional; you could also just use:

```
lme(normexam~standlrt,
random=~1|school,
data=london, method="ML")
```

Watch out! R gives the standard deviation of the random effects, not the variance. Var(rand int) = $0.3035^2 = 0.092$; res var=$0.7521^2 = 0.565$

London Schools: mixed model

random intercept only

```
Fixed effects: normexam ~ standlrt
                Value  Std.Error   DF  t-value p-value
(Intercept) 0.0023871 0.04003241 3993  0.05963  0.9525
standlrt    0.5633697 0.01246844 3993 45.18366  0.0000
 Correlation:
        (Intr)
standlrt 0.008

Standardized Within-Group Residuals:
       Min         Q1        Med         Q3        Max
-3.7161719 -0.6304245  0.0286690  0.6844298  3.2680306

Number of Observations: 4059
Number of Groups: 65
```

estimated fixed effect of the standardized london reading test on exam scores taken in account how many kids were taken into account

London Schools: mixed model

simplest model: only random intercept

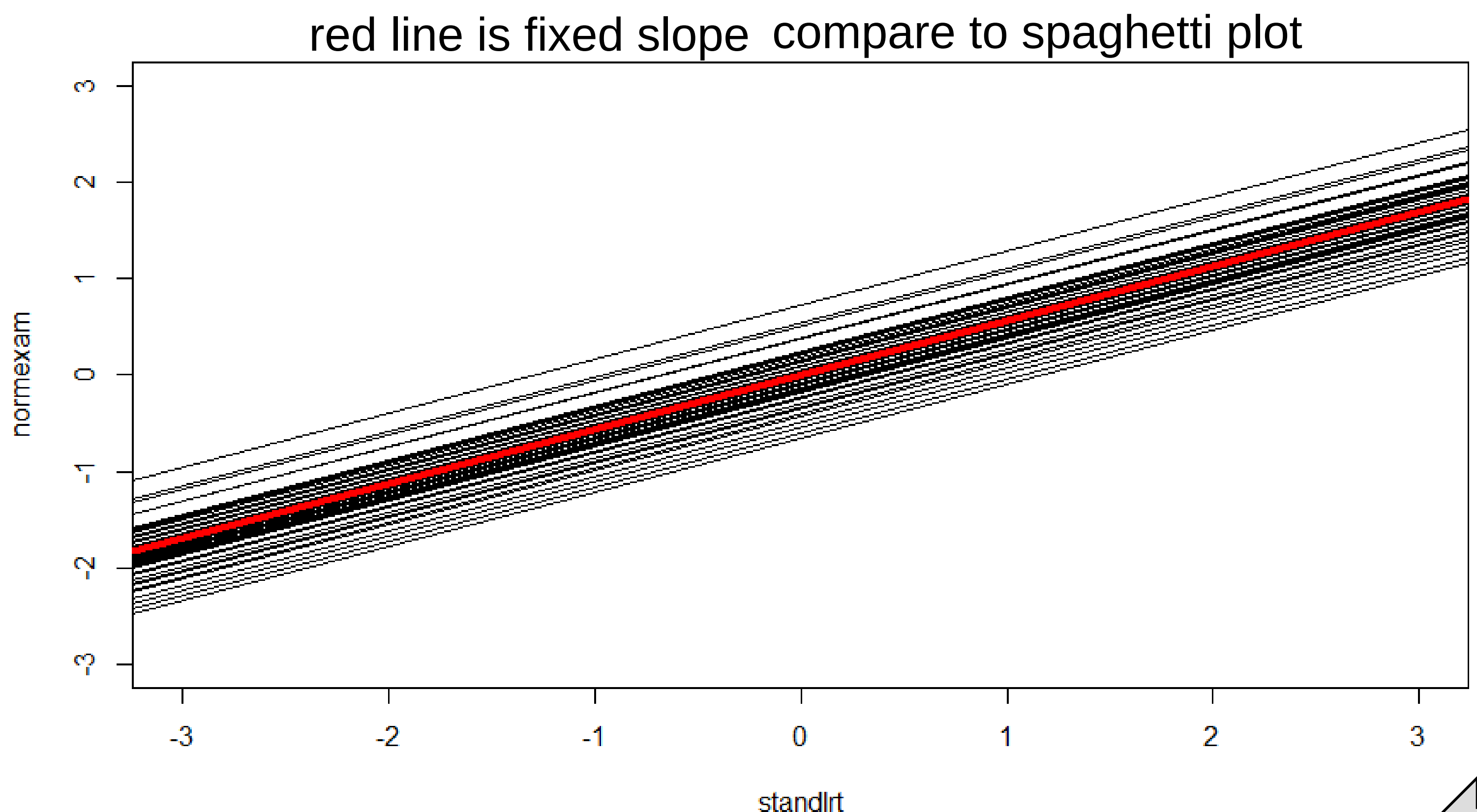
- Estimate for fixed intercept is 0.0024
  - (est.) mean exam score for a child with standardized LRT = 0 (mean)
- Estimate for fixed slope is 0.563
  - for every unit (1 sd) increase in LRT score, the exam score increases on average by 0.563 sd (= units of exam score, because normalized)
- Estimate for random intercept (between-school) variance is 0.092
- Estimate for within-school (residual) variance is 0.566
  - In this model, more unexplained variance within than between schools

    more variance within schools than between schools

London Schools: mixed model

simplest model: only random intercept

Fitted model



probably not appropriate model as it differs a lot in intercept etc. from the spaghetti plot

London Schools: mixed model

random intercept + random slope

1+standlrt more specific

```
> sch.lme.2 <- lme(fixed=normexam~standlrt, random=~standlrt | school,
data=london, method="ML")
> summary(sch.lme.2)

Linear mixed-effects model fit by maximum likelihood
 Data: london
      AIC      BIC    logLik
  9328.84 9366.693 -4658.42

Random effects:
 Formula: ~standlrt | school
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 0.3007313 (Intr)
standlrt    0.1205753 0.497
Residual    0.7440777
```

sd

estimated sd of intercept

of slope

fair amount in slopes as well

schools that are higher tned to have steeper slope

London Schools: mixed model

random intercept + random slope

estimate of overall average didn't change much but standard error increased

```
Fixed effects: normexam ~ standlrt
                 Value  Std.Error   DF   t-value p-value
(Intercept) -0.0115074 0.03979173 3993 -0.289192  0.7724
standlrt     0.5567279 0.01994287 3993 27.916142  0.0000
 Correlation:
        (Intr)
standlrt 0.365
```

uncertainty around intercept and slope = less certainty on overall fixed slope

```
Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-3.83123233 -0.63247485  0.03404163  0.68320636  3.45617450

Number of Observations: 4059
Number of Groups: 65
```

London Schools: mixed model

random intercept + random slope

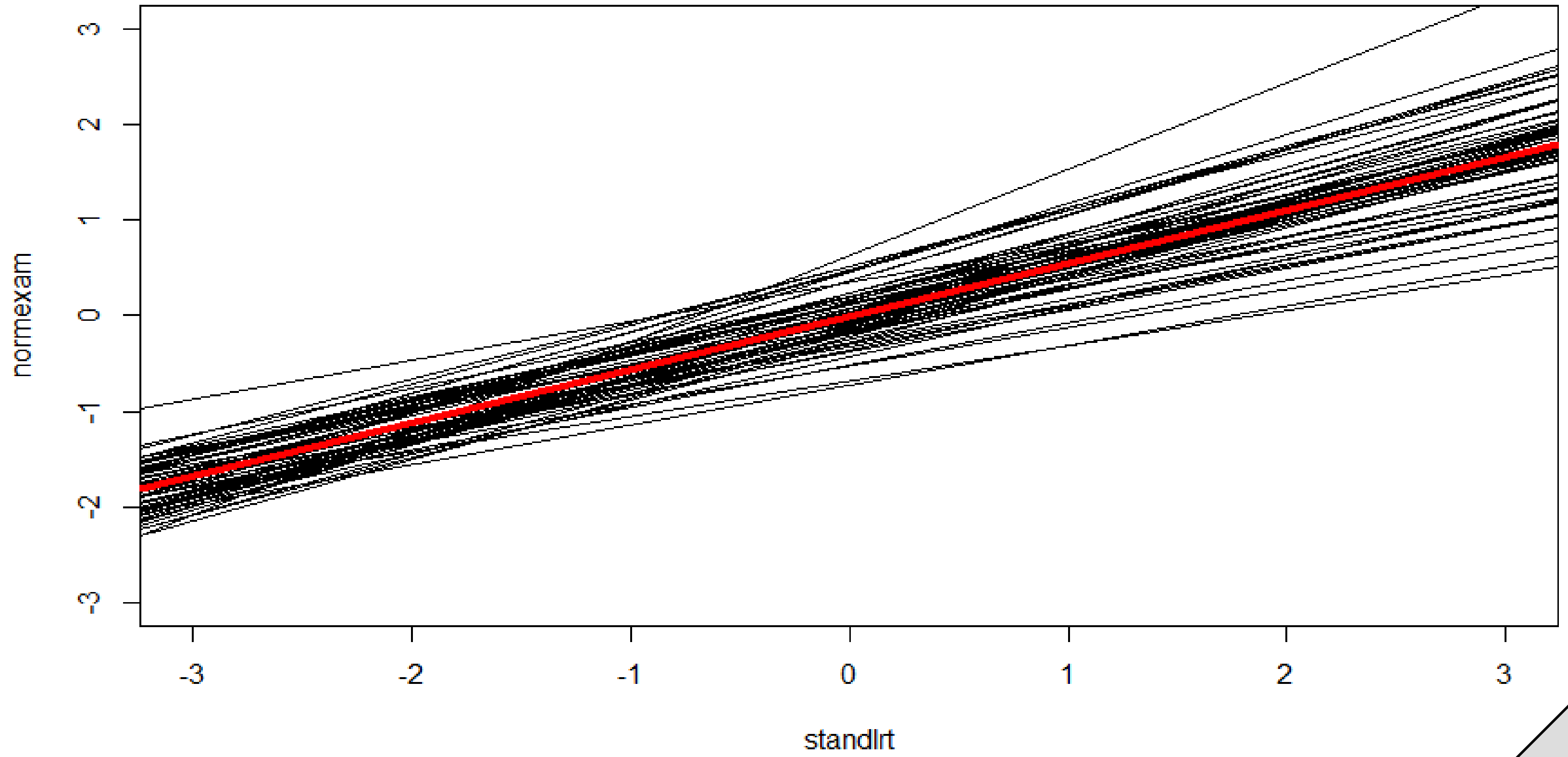
- Interpreting the model:
    - Fixed intercept = -0.01: average exam score when stdLRT = 0 (so for a child with an average LRT score)
    - Fixed effect LRT = 0.56: for two children who differ by 1 SD in LRT score, the exam score will be (on average) 0.56 SD higher for the child with the higher LRT score
    - SD of random intercepts (0.30) and slopes (0.12) is much smaller than residual sd (0.74) - more variance within than between schools
    - Correlation intercept-slope (0.497) usually not interesting, but:
        - schools with higher mean exam score when stdLRT=0 (mean LRT) tend to have higher slope

London Schools: mixed model

random intercept + random slope

Fitted model

the school with very few students and weird slope was assigned a new 0 so that the school probably still has a lower slope but was pulled towards the average



London Schools: comparing right & wrong models

Model	overall/fixed slope LRT	s.e.
1. aggregated data	0.884	0.116
2. disaggregated data	0.595	0.013
3. regr. per school	0.425	0.116
4. school*LRT interactions	??	??
5a. mixed model (random intercept) 5b. mixed model (random int + random slope LRT)	0.563 0.557	0.012 0.020

difference in the model mostly seen in standard errors

not as much in the fixed effects

Mixed model with random intercept and random slope is prob. the more appropriate model

London Schools data

so far one child level variable

Aside: coding of categorical variables

reference group

- Gender: 0=boy, 1=girl
- Schavg (school average of intake score): 1=low, 2=mid, 3=high
- Schgend: 1= mixed school, 2=boys' school, 3=girls' school

school gender

London Schools:

adding a (fixed) child-level covariate

adding child level gender

```
> sch.lme.3 <- lme(fixed=normexam~standlrt + factor(gender), random=~standlrt |
school, data=london, method="ML")
> summary(sch.lme.3)
Linear mixed-effects model fit by maximum likelihood
 Data: london
       AIC      BIC    logLik
  9301.358 9345.518 -4643.679

Random effects:
 Formula: ~standlrt | school
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 0.2936242 (Intr)
standlrt    0.1212575 0.533
Residual    0.7416710

Fixed effects: normexam ~ standlrt + factor(gender)
                     Value  Std.Error   DF   t-value p-value
(Intercept)     -0.1117670 0.04305229 3992 -2.596075  0.0095
standlrt         0.5529634 0.01998634 3992 27.667060  0.0000
factor(gender)1  0.1757988 0.03225659 3992  5.450011  0.0000
```

estimated covariance and variance sd and cor.

residual variation

in addition to overall intercept

difference between boys and girls

girls would be 0.17 better on the test on average

(boys reference group)

London Schools:

adding a child-level covariate

- On average, girls score 0.176 SD higher on exam than boys (holding stdLRT constant)

London Schools

adding (fixed) school-level covariates

```
> sch.lme.4 <- lme(normexam~standlrt + factor(gender) + factor(schgend) + factor(schav),
      random=~standlrt | school, data=london, method="ML")
> summary(sch.lme.4)

Random effects:
 Formula: ~standlrt | school
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 0.2660309 (Intr)
standlrt    0.1212542 0.499
Residual    0.7417279

Fixed effects: normexam ~ standlrt + factor(gender) + factor(schgend) +
factor(schav)
                      Value  Std.Error   DF   t-value p-value
(Intercept)      -0.2647657 0.08159434 3992 -3.244902  0.0012
standlrt          0.5515520 0.02006950 3992 27.482097  0.0000
factor(gender)1   0.1671313 0.03385088 3992  4.937282  0.0000
factor(schgend)2  0.1869684 0.09777600   60  1.912211  0.0606
factor(schgend)3  0.1570156 0.07780641   60  2.018029  0.0481
factor(schav)2    0.0668879 0.08534936   60  0.783696  0.4363
factor(schav)3    0.1742650 0.09876108   60  1.764511  0.0827
```

comparing all boys/ all girls schools to mixed schools

London Schools: results from previous slides

Adding child- and school-level covariates

normally only fixed effects are presented

Effect	estimate	se	p
Fixed Effects			
Intercept	-0.265	0.082	0.0012
norm. LRT	0.552	0.020	< 0.0005
girls (vs. boys)	0.167	0.034	< 0.0005
school avg: low	(ref)	0.100	
school avg: mid	0.067	0.085	0.436
school avg: high	0.174	0.099	0.083
school gender: mixed	(ref)		
school gender: boys	0.187	0.098	0.061
school gender: girls	0.157	0.078	0.048
(Co)variance Parameters:			
school intercept	0.266^2		
school slope	0.121^2		
corr int-slope	0.499		
residual variance	0.742^2		

London Schools: conclusions (so far)

- The reading score is a significant predictor of exam score
  - for every 1 SD higher on reading score, average increase of 0.552 SD on exam score
- Boys do significantly worse than girls on exam
  - boys score, on average, 0.167 SD lower on exam than girls
- School "level" (average exam score) does not appear to be predictive of exam score
- School gender may be predictive
  - average exam score at girls' schools is 0.157 SD higher than at mixed schools
  - average exam score at boys' schools is 0.174 SD higher than at mixed schools
- Note: these conclusions are based on the "Wald" p-values and are not necessarily to be trusted!

lme function for Wald values
slide 50

London Schools: conclusions (so far)

- Because the LRT score has been centered, the estimate for the intercept (-0.265) is the estimated average (normalized) exam score for:
  - a boy (ref) with
  - avg LRT score from
  - a school with low average score (ref) and
  - mixed school (ref)
- The residual variance is 0.550, much larger than the variances for the random intercept (0.071) and random slope (0.015), indicating more variation within schools than between.

  variance of random intercept is slightly decreased compared to the simpler model
- Adding child- and school-level covariates explains some of the variance between schools (variance intercepts 0.09 → 0.07)

London Schools: still to do

- We've made model assumptions, need to check them!
  - distribution of residuals
  - distribution of random effects (?)
- How to choose among models? which variables are associated
- How to answer subquestion (does gender of school have influence on effect of gender of pupil?) are girls in mixed schools doing worse than girls in all girls schools?

Multilevel modelling, summary

- Account for correlation of measurements at different levels
    - children within schools, measurements within patients
- Allow us to include variables measured at different levels
    - child’s gender, school’s achievement or SES level
- We can model variation at different levels
    - more variation within than between schools
- Longitudinal data is a specific example of multi-level data
    - lecture 2: mixed models for longitudinal data
- How to build models, check assumptions?
    - lecture 3: technical issues in multilevel/longitudinal modelling
- Outcomes don’t have to be continuous
    - lecture 4: models for Poisson, binomial ~~and survival data~~