Propagation of uncertainties to flow records

This document describes how errors from the rating curve (parametric errors and structural/remnant errors) and those from the stage series (non-systematic and systematic stage measurement errors) are propagated to the flow series by BaRatin.

The estimation of a rating curve by BaRatin is actually based on 500 rating curves, each corresponding to a set of feasible parameters (parameters are those of the equation of the rating curve, $\boldsymbol{\theta}$, and that $\gamma_1$et γ_2 used to define the standard deviation of the normal distribution used to sample the structural (remnant) error. The stage series is a time series of recorded water levels $\tilde{h}(t)$. Standard deviations σ_A^h and σ_B^h (which correspond to non-systematic and systematic errors affecting the stage series) can be used to generate 500 stage series. The method is described below for a rating curve i (i.e., a set of parameters) estimated by BaRatin:

(1) At each time step t, an error $\varepsilon_i^h(t)$ is sampled according to the Gaussian distribution $N\left(0, \sigma_A^h\right)$. The error is added to the measured stage ($\tilde{h}(t)$).

(2) For each period in which the bias affecting the stage series is assumed to be constant, an error δ_i^h is sampled according to the Gaussian distribution $N\left(0,\ \sigma_B^h\right)$. The error is added to the measured stage already affected by non-systematic errors. We obtain the stage series i as: $h_i(t) = \tilde{h}(t) + \varepsilon_i^h(t) + \delta_i^h$

(3) At each time step t, a discharge $\tilde{Q}_i(t)$ is computed from the stage series $h_i(t)$, the rating curve equation f and the set of parameters $\boldsymbol{\theta_i}$.

(4) At each time step t, a remnant (structural) rating curve error $\varepsilon_i^f(t)$ is finally added to the computed discharge $\tilde{Q}_i(t)$. The error is sampled according to the Gaussian distribution $N\left(0,\ \gamma_{1,i} +\ \gamma_{2,i}\tilde{Q}_i(t)\right)$.

The equation combining these various steps is given below

$$Q_i(t) = \underbrace{f\left(\overbrace{\tilde{h}(t) + \varepsilon_i^h(t) + \delta_i^h}^{h_i(t)}\ |\ \boldsymbol{\theta_i}\right)}_{\tilde{Q}_i(t)} + \varepsilon_i^f(t)$$

Where

- $\tilde{h}(t)$ = recorded stage
- $\varepsilon_i^h(t)$ = non-systematic stage measurement error
- δ_i^h = systematic stage measurement error
- $\varepsilon_i^f(t) \sim N\left(0,\ \gamma_{1,i} +\ \gamma_{2,i}\tilde{Q}_i(t)\right)$ = remnant (structural) rating curve error.

Figure 1 shows and sums up the various computational steps.

To obtain the *MaxPost* (most probable) flow series, all the errors are ignored:

$Q_{MP}(t) = f\left(\tilde{h}(t)\ |\ \boldsymbol{\theta_{MP}}\right)$ where $\boldsymbol{\theta_{MP}}$ corresponds the set of parameters of *MaxPost* rating curve.

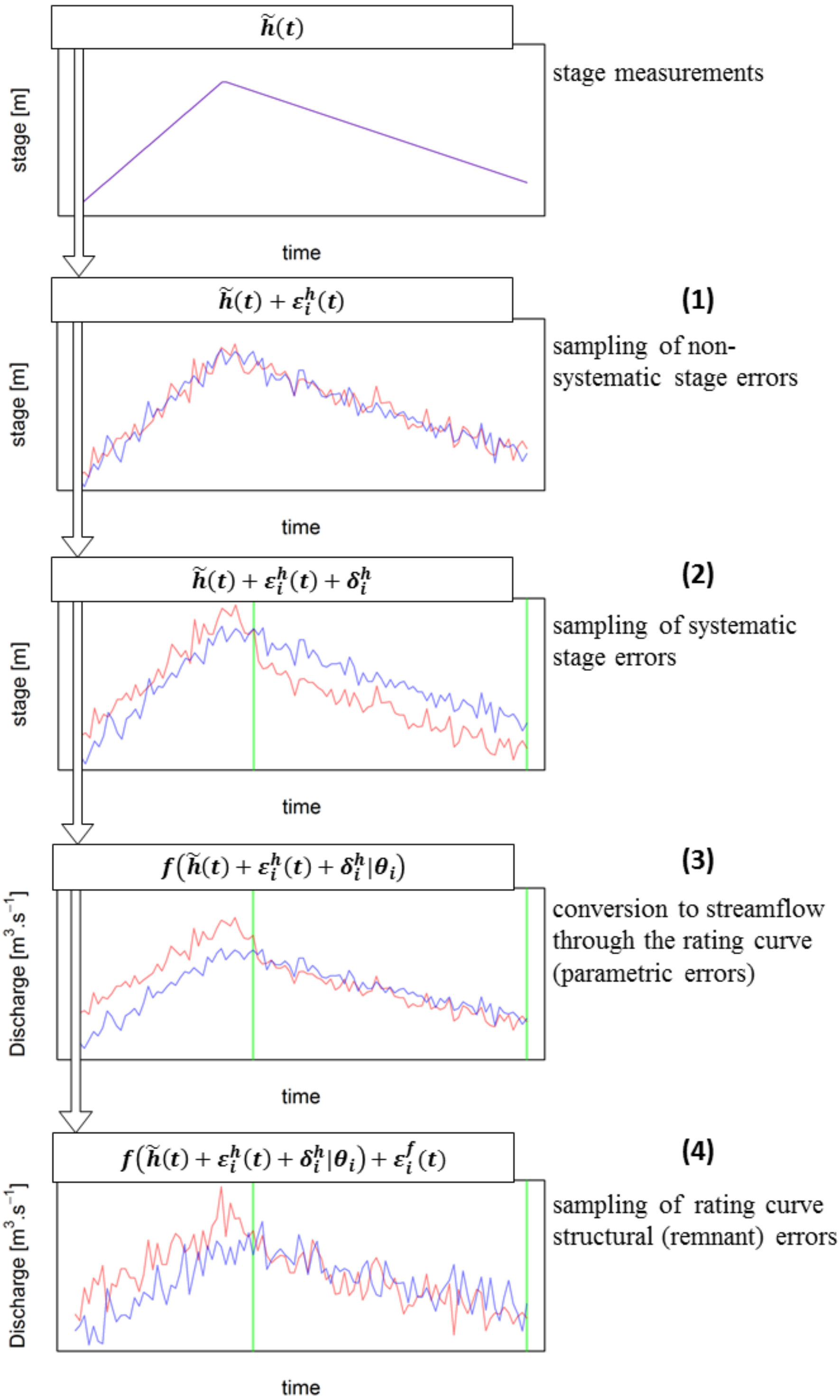

Figure 1: Principle of the sampling method considering two sets of parameters (θ_{i_1} in red and θ_{i_2} in blue): from the measured stage series up to the two flow series (each corresponding to a set of parameters, i.e. a possible rating curve and a possible stage series)