№1

$$I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha}\,dx$$

1) $$I_0(\alpha) = \int_0^1 \frac{1}{x+\alpha}\,dx =$$

$$= \ln|x+\alpha|\Big|_0^1 = \ln|1+\alpha| - \ln|\alpha| =$$

$$= \ln\frac{|1+\alpha|}{|\alpha|}$$

$$I_1(\alpha) = \int_0^1 \frac{x}{x+\alpha}\,dx = \cancel{\int_0^1 1\,dx} + \int_0^1 \frac{x}{\alpha}\,dx =$$

$$= \cancel{1 + \frac{x^2}{2\alpha}\Big|_0^1 = 1 + \frac{x^2}{2\alpha}}$$

$$= \cancel{\int_0^1 \frac{1}{1+\frac{\alpha}{x}}\,dx = \int}$$

$$\cancel{\frac{\partial}{\partial\alpha}\frac{x}{x+\alpha} = \frac{-1\cdot x}{(x+\alpha)^2} = -\frac{x}{(x+\alpha)^2}}$$

$$x - \alpha\log(\alpha+x)\Big|_0^1 =$$

$$= 1 - \alpha\log(\alpha+1) + \alpha\log(\alpha) =$$

$$\cancel{1 - \alpha\log} \quad 1 - \alpha\ln\left|\frac{\alpha+1}{\alpha}\right|$$

$$\int_0^1 \frac{x^2}{x+\alpha}\,dx = \alpha^2 \ln(\alpha+x)\Big|_0^1 +$$

$$+ \frac{1}{2}x(x-2\alpha)\Big|_0^1 =$$

$$= \alpha^2 \ln\frac{|\alpha+1|}{|\alpha|} + \frac{1}{2}(1-2\alpha) =$$

$$= \alpha^2 \ln\frac{|\alpha+1|}{|\alpha|} + \frac{1}{2} - \alpha$$

Рекурр. соот. между

i) I_n и I_{n-1}

$-\ln(1+\alpha) + \ln(\alpha)$

$$\int_0^1 \frac{x^3}{x+\alpha}\,dx = \alpha^3(-\ln|\alpha+x|)\Big|_0^1 +$$

$$+ \alpha^2 x\Big|_0^1 - \frac{\alpha x^2}{2}\Big|_0^1 + \frac{x^3}{3}\Big|_0^1 =$$

$$\cancel{= -\alpha^3 \ln\frac{|1+\alpha|}{|\alpha|} + \alpha^2 \cdot 2 - \frac{1}{\alpha} + \frac{2}{6}} =$$

-2

$$\cancel{= -\alpha^3 \ln\frac{|1+\alpha|}{|\alpha|} + \alpha^2} - \frac{1}{6}$$

$$= -\alpha^3 \ln\frac{|1+\alpha|}{|\alpha|} + \alpha^2 - \frac{\alpha}{2} + \frac{1}{3}$$

(ii) Рекуррентное соотношение

$$I_n(\alpha) = \frac{1}{n} - \alpha I_{n-1}(\alpha)$$

Обратная рекурсия $\Downarrow$

$$I_{n-1} = \frac{\frac{1}{n} - I_n}{\alpha}$$

* $\alpha < 1$ прямая рекурсия точнее

$\alpha > 1$ обратная рекурсия точнее

N 4

$$A = \begin{pmatrix} 1 & 10 \\ \delta & 1 \end{pmatrix}$$

$$\lambda = \det A = (1-\lambda)^2 - 10\delta = 0$$

$$(1-\lambda)^2 = 10\delta$$

$$\lambda^2 - 2\lambda + 1 - 10\delta = 0$$

$$D = 40\delta$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{40\delta}}{2}$$

$$\lambda_{1,2} = 1 \pm \sqrt{10\delta}$$

$$\lambda_{max} = 1 + \sqrt{10\delta}$$

$$k(\delta) = \frac{d\varepsilon(\delta)}{d\delta}$$

$$k(10) = 0{,}5$$

$$k(0{,}1) = 5$$

№3

$f(1) = -3$

$f(0) = 1$

$f(2020) = ?$

$f(0) = 1$ $f(1) = -3$ $f(2) = 3+6\cdot 1 = 9$

$f(3) = -27$ $f(4) = 27+6\cdot 9 = 81$

$f(5) = -243$

$\Downarrow$

$f(n) = (-1)^n \cdot 3^n$

$f(2020) = 3^{2020}$