Evaluating computational methods for modeling off-normal operation of gas centrifuge cascades

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Abstract

This work compares and evaluates different computational approaches for modeling off-normal operation of a gas centrifuge enrichment cascade.

The goal of this work focuses on developing the necessary understanding of potential misuse of enrichment cascades to design more effective and efficient international safeguards approaches. While it is straightforward to design an enrichment cascade under ideal conditions as a function of the theoretical feed, product and tails assays, it is very difficult to find reliable information about the behavior of a given cascade when the feed assay does not match the design value. Several methods have been developed to assess the behavior of an enrichment cascade in such circumstances, those methods evaluate the cut, the feed to product, feed to tail and the product to tail enrichment ratio, respectively, alpha, beta and gamma, as a function of the cascade feed assay. As those four parameters depends on each other, determining two of them allows to compute the other. The first approach consists of fixing the cut and alpha recomputing the corresponding assays at each stages of the cascade. The second one maintains the ideal condition of the cascade (alpha and beta fixed across the whole cascade), modifying the cut value at each stage accordingly. Both approaches have been implemented into the Cyclus fuel cycle simulator[1, 2]. The third fixes the cut and gamma, using both alpha and beta at each stage as free parameters. The third method has been investigated in [3].

Following a description of each method and an evaluation of differences between each approach, this work compares the results produced by these methods within scenarios involving misuse of enrichment cascades simulated using the dynamic nuclear fuel cycle simulator, Cyclus.

1 theory

1.1 Centrifuge properties

The present work uses the analytical solution by Räetz [4] of the differential equation for the gas centrifuge as described in [5]. Centrifuge parameters, such as average gas temperature, T, peripheral speed, v, height, h, diameter, d, pressure ratio, x, feed flow rate, F, counter-current flow ratio, L/P, and efficiency, e have been chosen (Table 1) to match the cascade design describe in [5] and [3]. These parameters for a P1-type centrifuge are used to estimate the JCPOA-compliant IR-1 centrifuge.

Tab. 1: Summary of the centrifuge parameters.

T[K]	v[m/s]	h[m]	d[m]	x	F[mg/s]	L/F	e
320	320	1.8	0.105	$1e3$	13	2	1.0

1.2 Cascade Design

The cascade is built to be ideal, defined by $\alpha = \beta = const$ for all stage of the cascade, where α and β respectively represent the feed to product and the feed to tail enrichment factors. α can be expressed as a function of the feed rate F, the separative performance $\delta U(\theta)$, the cut θ, and β as a function of $alpha$ and θ:

$$\alpha = \sqrt{\frac{2\delta U}{F} \frac{1-\theta}{\theta} + 1} \tag{1a}$$

$$\beta = \left(1 - \frac{N - \theta \dfrac{\alpha R}{1+\alpha R}}{1-\theta}\right) \left(\frac{R}{\dfrac{N - \theta \dfrac{\alpha R}{1+\alpha R}}{1-\theta}}\right) \tag{1b}$$

From equation (1a) and (1b) it is possible to determine the cut, or the ratio of product flow to feed flow required to build an ideal cascade.

Since α_i and β_i remain constant, only the value of the cut, θ_i, changes in each stage i of a cascade.

It can be shown that θ_i can be computed from α, β values and the feed assay, N_i:

$$\theta_i = \frac{N_i - \dfrac{1}{1+\beta/R_i}}{\dfrac{\alpha R_i}{1+\alpha R_i} - \dfrac{1}{1+\beta/R_i}} \tag{2}$$

This algorithm assumes that the corresponding separative power δU (not re-computed) can be achieved with the chosen centrifuge design. Once θ_i is determined, it is possible to compute the product and the tail assay.

The design of the cascade is performed through 2 steps. First one determines the configuration and number of stages, adding stages until the product assay of the final stage is greater than or equal to than the desired assay, and the tails assay is similarly less than or equal to the desired tails assay. This determines the number of enriching and stripping stages as well as their enrichment properties (N_i, N_i', N_i'', θ_ii).

The second step determines how to populate the cascade with the user-defined maximum number of centrifuges. One then solves the linear flow equation to determine the theoretical flow in the cascade.

$$
\begin{bmatrix}
-1 & 1-\theta_{S+1} & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
\theta_S & -1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
& & & & & \ldots & & & & & & \\
0 & 0 & 0 & \ldots & \theta_{-2} & -1 & 1-\theta_0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & \theta_{-1} & -1 & 1-\theta_1 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & \theta_0 & -1 & 1-\theta_2 & 0 & \ldots & 0 \\
& & & & & \ldots & & & & & & \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & -1 & 1-\theta_E \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & \theta_{E-1} & -1
\end{bmatrix}
\times
\begin{bmatrix}
F_S \\
F_{S+1} \\
\ldots \\
F_{-1} \\
F_0 \\
F_1 \\
\ldots \\
F_{E-1} \\
F_E
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
\ldots \\
0 \\
F \\
0 \\
\ldots \\
0 \\
0
\end{bmatrix}
\quad (3)
$$

Once the relative flow of each stage has been determined, the cascade can be populated with actual machines up the stages until either the maximum number available of machines or the maximum feed flow is reached.

References

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