Engineering Physics (2025)
Course code 25PY101
Unit 1: Metals and Semiconductors

Course Instructor:
Dr. Sreekar Guddeti
Assistant Professor in Physics
Department of Science and Humanities
Vignan's Foundation for Science, Technology and Research

September 25, 2025

Unit 1 Plan

1. Condensed matter
2. Metals
3. Classical free electron theory
4. Expression of electrical conductivity

Unit 1 Plan

1. Condensed matter
2. Metals
3. Classical free electron theory
4. Expression of electrical conductivity

Problem at hand

$$V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r} \qquad V(r) \propto -\frac{1}{r}$$

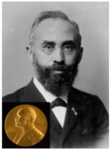
- The problem at hand is to solve the motion of interacting particles– electrons of the order of 10^{23} and ions of the order of 10^{23}.
- This is a tough problem. The idea is to simplify the problem by making **approximations**.

Classical free electron theory

- Ohm's law was discovered in 1827 and needed explanation.
- Proposed by Paul Drude in 1900 using analogy of gas of molecules.
- Mutual repulsion between electrons is neglected i.e. electrons are independent – **independent electron approximation**.
- Assumed "gas" is **free** i.e. not under influence of lattice – **free electron approximation**
- Role of lattice is to redistribute the velocity distribution that obeys **classical** Maxwell-Boltzmann statistics – classical thermodynamics.
- The theory explains Ohm's law
- Hendrik Lorentz extended theory to explain thermal conductivity – hence Drude-Lorentz electron theory.

G. Ohm

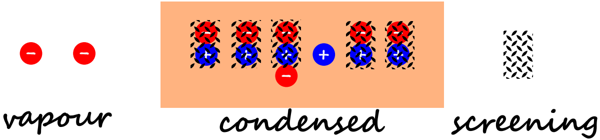
P. Drude



H. Lorentz

Independent electron approximation

$$V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r} \qquad \cancel{V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r}}$$

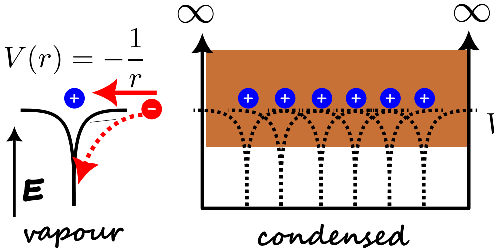


- In the condensed phase, there are of the order of 10^{23} electrons.
- The repulsive Coulomb potential between an electron and other electron is **screened** due to large numbers and results in an **independent** electron.
- The problem of motion of $\mathcal{O}(10^{23})$ electrons is reduced to the problem of motion of **single** electron.

Key Insight

The property of single electron determines the properties of electron gas.

Free electron approximation

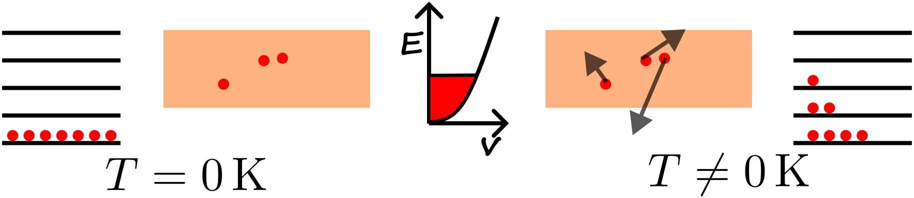


$$V = \begin{cases} 0 & \text{inside} \\ \infty & \text{outside} \end{cases}$$

- The attractive Coulomb potential between ions and an electron is smeared due to large numbers and results in constant potential in the interior of metal with **infinite** barrier at the surface. This potential is called **infinite potential well**.
- This potential traps the electrons analogous to gas atoms in a container.
- The electrons are free to move in the potential well. The kinetic energy K of an electron with mass m and velocity $\mathbf{v}$ is

$$K = \frac{1}{2}mv^2$$

Classical thermodynamics

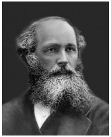


- At temperature $T = 0$, electrons are frozen. At $T \neq 0$, due to the large number of electrons and collisions, the kinetic energy of electrons keeps changing.
- However, the probability to find electron with with lower energy is higher than the probality to find electron with higher energy. This is the essence of Maxwell-Boltzmann statistics.
- The average kinetic energy $\langle T \rangle$ at temperature is

$$\langle K \rangle = \frac{3}{2} k_B T,$$

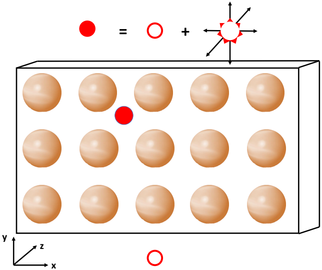
where k_B is the Boltzmann constant.

Boltzmann



Maxwell

Classical free electron theory



- Electron motion is decomposed into **thermal motion** and **drift motion**.
- Thermal motion is due to random collisions with the lattice.
- Drift motion is the net motion of electrons. The electron drifts only between collisions.

Thermal motion

Definition

Mean free path l_{mf} is defined as the average distance the electron travels between collisions.

Definition

Thermal velocity v_{th} is defined as the average velocity of the electron.

- Thermal motion has two properties mean free path l_{mf} and thermal velocity v_{th}.
- Drude assumed that l_{mf} is a material constant.
- At room temperature (300 K), the thermal velocity is of the **order of** $10^5\,\mathrm{m\,s^{-1}}$.

Estimate: Average kinetic energy of electron

Take room temprerature as 27 °C. Express energy in eV. [Hint: $1\,\mathrm{eV} = 1.602 \times 10^{-19}$ J]

Drift motion

- In the absence of external force, the drift motion is zero.
- The external force $\mathbf{F}_{ext}$ on the electron is given by the Lorentz force

$$\mathbf{F}_{ext} = -e\mathbf{E} - e\mathbf{v} \times \mathbf{B},$$

  where $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field.
- The effect of the collisions is treated as a drag or frictional force

$$\mathbf{F}_{fric} = -m\frac{\mathbf{v}}{\tau}$$

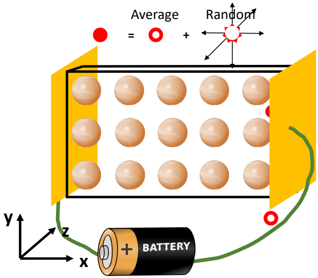
- Since Lorentz extended the theory to incorporate electromagnetic force, the theory is also the Lorentz-Drude theory.

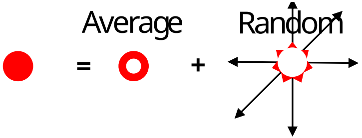
Lorentz-Drude theory – Postulates

Postulates

1. **Particles:** Electrons are **classical** particles with mass m and charge $-e$.
2. **Kinematics:**
   - **Independent electron approximation:** Electrons are independent and mutual repulsion between them is ignored.
3. **Dynamics:**
   - **Free electron approximation:** Electrons are free and move in an **infinite potential well**.
   - **Thermal motion:** Electron undergoes **random** collision with ion cores. Random collisions lead to thermal motion.
   - **Drift motion:** In between collisions, the Lorentz force acts on electron and leads to drift motion.
4. **Classical Thermodynamics:** The average electron energy is given by Maxwell-Boltzmann statistics.

Ohm's law





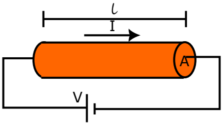
- The motion of electron is decomposed into an **average** part and the **random** part.
- The average part "drifts" with a constant velocity under electric field. The random part has **zero** net motion.

Learning Objectives

Classical electron theory explains Ohm's law qualitatively.

- The **macroscopic** version of the Ohm's law states that potential V applied across a metal is directly proportional to the current I flowing across it. The constant of proportionaly is called the resistance R.
- Its **microscopic** version relates the electric field and current density
- Electric field $\mathbf{E}$ is defined as the potential per unit distance. Its SI unit is V C.
- Current density $\mathbf{j}$ is defined as the current flowing through a wire per unit area, where A is the cross-sectional area of the wire. Its SI unit is $\mathrm{A\,m^{-2}}$.

$$V = IR$$



$$\mathbf{E} = \frac{V}{\ell}\hat{\mathbf{E}}$$

$$\mathbf{j} = \frac{I}{A}\hat{\mathbf{j}}$$

Ohm's law : Microscopic version

Lets start with macroscopic version $V = RI$ and microscopic version of Ohm's law

Derivation

Upon substitution of V, I *in terms of* $\mathbf{E}, \mathbf{j}$*, we have*

$$\frac{\ell}{V}\mathbf{E} = \frac{A}{I}\mathbf{j}, \quad \Rightarrow \quad \mathbf{j} = \frac{\ell}{A}\frac{I}{V}\mathbf{E}, \quad \Rightarrow \quad \mathbf{j} = \frac{\ell}{AR}\mathbf{E} = \sigma\mathbf{E}$$

Thus, the microscopic version of Ohm's law is given by

$$\boxed{\mathbf{j} = \sigma\mathbf{E}}$$

where σ *is given by*

$$\sigma = \frac{\ell}{AR}$$

Conductivity – σ

Definition

The conductivity σ is defined as the current density carried by the wire per unit of electric field.

$$\sigma = \frac{j}{E} \qquad \text{SI unit} \quad [\sigma] = \left[\frac{\ell}{AR}\right] = \Omega^{-1}\,\mathrm{m}^{-1}.$$

Question

A copper wire of length $L = 2\,\mathrm{m}$ and diameter d=1 mm is connected across a DC source. When a voltage of V=0.5 V is applied across the ends of the wire, an electric current of I=22 A is observed. Using these measurements, find the electrical conductivity σ of copper.

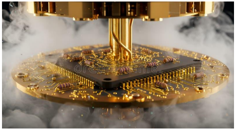
Key Insight

Metals have high conductivity.

Applications of metallic conductivity

Due to high conductivity, metals have many applications ranging from power electronics to microelectronics, from mechanical to aerospace industry.

(a) Power transmission



(b) Chip Interconnects

(c) Heat sinks

(d) Functional alloys

- Power transmission: Copper/Aluminum cables.
- Electronic devices: Contacts, interconnects.
- Thermal applications: Heat sinks, cookware.
- Alloys: Enhanced strength and corrosion resistance.

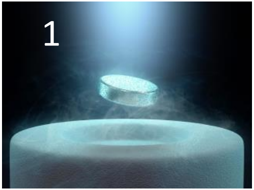
Summary of Lecture

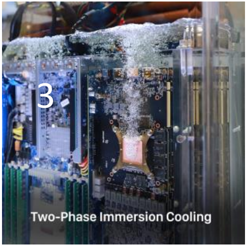
- Materials are classified into conductors, semiconductors, and insulators.
- Metals have high conductivity due to free electrons
- Drude's classical electron theory introduced microscopic picture of conduction.
- Conductivity is related to resistivity.

Materials: Beyond metals

Condensed	Elec.	Therm.	Opt.	Ductile	Material	Application
Solid	$\Uparrow$	-	-	$\downarrow$	Superconductor	Levitation
Solid	$\Downarrow$	$\Uparrow$	-	-	Diamond	Chips
Liquid	$\Downarrow$	$\Uparrow$	-	-	Dielectric coolants	Heatsinks

Table: New materials with mixed properties.





1. Superconductor 2. diamond wafer 3. dielectric coolant

Unit 1 Plan

Lecture plan

Learning Objectives

- From qualitative picture of metals to quantitative derivation.
- Derive expression for conductivity
- Mobility is a material property.

Free electron density – n

Definition

- Electron density n is defined as the number of electrons per unit volume. Its SI unit is m^{-3}.
- For metals, each atom contributes as many electrons as its **valency**. For example, Na has valency 1,

Estimate: Electron density of metals

1. Take the radius r_s of atom is nearly $3a_0$, where $a_0 = 0.529\,\text{Å}$ is the Bohr radius.
2. Deduce n is of the order $10^{22}\,\mathrm{cm}^{-3} = 10^{28}\,\mathrm{m}^{-3}$.
3. For Cu, the experimental observed $n = 8.47 \times 10^{28}\,\mathrm{m}^{-3}$.

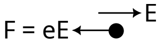
Key Insight

$\frac{r_s}{a_0}$ determines the free electron density.

Drift velocity – v_d

Definition

- **Drift velocity $\mathbf{v_d}$** is defined as the average velocity of an electron in the presence of electric field.
- **relaxation time** τ is defined as the average time between collisions.



Derivation

- *The equation of motion of electron is*
- *During time duration τ, electron reaches drift velocity*

$$\mathbf{a} = -\frac{e}{m}\mathbf{E},$$

$$\boxed{\mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}}$$

Key Insight

Drift velocity is proportional to the applied electric field.

$y = mx$ and $y = m/x$ relations

- We have seen many examples of **linear** relations between physical quantities till now.
- The dependent variable y relates to the independent variable x. The relation is given by the **constant of proportionality**.
- We also say that "y is directly proportional to x".

Law	Equation	Const. of prop.
Ohm's (macroscopic)	$I = \frac{1}{R}V$	Conductance
Ohm's (microscopic)	$\mathbf{j} = \sigma\mathbf{E}$	Conductivity
Capacitor	$E = \frac{1}{d}V$	1/Separation
Drift motion	$\mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}$	Mobility
Photoelectric effect	$E = \hbar\nu$	Planck's constant

- Sometimes, physical quantities are also inversely related i.e. $y \propto \frac{1}{x}$.
- We also say "y is inversely proportional to x".

Law	Equation	Const. of prop.
Light	$\nu = c\frac{1}{\lambda}$	velocity

Mobility – μ

Definition

Mobility μ is defined as the drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

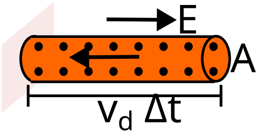
- It is a material property as

$$\mu = \frac{e\tau}{m} \qquad \left[\because \mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}\right]$$

- Its SI unit is

$$[\mu] = \frac{[v_d]}{[E]} = \frac{[v_d \ell]}{[V]} = \mathrm{m^2\,V^{-1}\,s^{-1}}$$

Conductivity σ



In a time duration Δt, the number of electrons N in a volume of $V = v_d \Delta t A$ will pass through the cross section of area A.

Derivation

1. *Current I:*
2. *Current density j:*
3. *Conductivity σ:*

$$I = -\frac{Ne}{\Delta t} = -\frac{n \cdot V}{\Delta t} = -nev_d A \tag{1}$$

$$j = \frac{I}{A} = -nev_d = \frac{ne^2\tau}{m} E \tag{2}$$

$$\sigma = \frac{ne^2\tau}{m} = ne\left(\frac{e\tau}{m}\right) = ne\mu \tag{3}$$

Learning Objectives

We have derived the expression for conductivity.

Relaxation Time – τ

Assumption

Relaxation time τ is assumed to be a **constant**.

Estimate: Relaxation time in metals

Estimate the electron relaxation time in metals.

1. Take conductivity $\sigma \sim 10^7\,\mathrm{S\,m^{-1}}$, number density $n \sim 10^{28}\,\mathrm{m^{-3}}$
2. Take electron mass $m = 9.1 \times 10^{-31}\,\mathrm{kg}$, charge $e = 1.609 \times 10^{-19}\,\mathrm{C}$.
3. Deduce relaxation time $\tau \sim \boxed{10^{-14}\,\mathrm{s} \text{ or } 10\,\mathrm{fs}}$

- Larger $\tau \rightarrow$ fewer collisions $\rightarrow$ higher conductivity.

Trends

1. As temperature increases, τ decreases Why?
2. As impurity concentration increases, τ decreases Why?

Resistivity – ρ

Definition

Resistivity ρ is defined as the inverse of conductivity

$$\rho = \frac{1}{\sigma}$$

Its SI unit is $\Omega\,\mathrm{m}$.

- Resistivity $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$.
- Its SI unit is $\Omega\,\mathrm{m}$.

Trends

1. As number density increases, ρ decreases — Why?
2. As temperature increases, ρ increases — Why?

Summary of physical quantities

Physical quantity	Symbol	Units	Material Property
Resistance	R	Ω	No
Conductivity	σ	$\Omega^{-1}\,\mathrm{m}^{-1}$ or $\mathrm{S\,m}^{-1}$	Yes
Mobility	μ	$\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$	Yes
Relaxation Time	τ	s	Yes
Resistivity	ρ	$\Omega\,\mathrm{m}$	Yes

Theory vs. experiment

Merit: Conductivity order of magnitude

- For metals, classical free electron theory (CEFT) predicts order of magnitude of conductivity correctly.

$$\sigma_{\text{experiment}} \sim \sigma_{\text{theory}} \sim 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$$

Demerit: Conductivity vs valency

- Conductivity is proportional to number density. Number density is proportional to valency. So theory predicts conductivity increases with valency. But experiment shows monovalent metals (Na, K) have higher conductivity than divalent metals (Mg, Ca).

Theory vs Experiment contd...

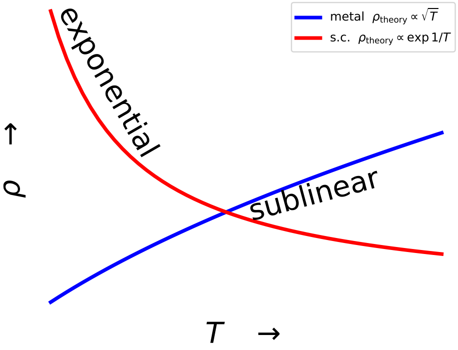
Demerit: Hall coefficient sign

- In metals, there are only electrons as charge carriers.
- It is expected that the **sign of Hall coefficient is negative**.
- However, experiments show metals like Zn show positive Hall coefficient. CEFT cannot explain this **anomaly**.

Under-estimation of mean free path

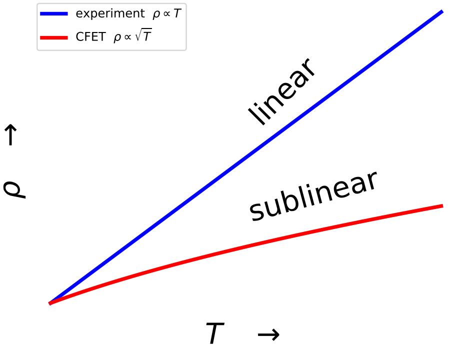
- The mean free path is considered a constant in CEFT and is taken as the interatomic distance that is approximately 5 Å.
- However, experiment shows mean free path to be of the order 50 Å.
- Thus, CEFT underestimates the mean free path by one order of magnitude.

Demerit: Classification of materials



- CEFT **does not** explain the classification of materials into conductors, semi-conductors and insulators.
- For example, the resistivity of metals varies linearly $\rho_{\text{conductor}} \propto T$ but resistivity of semiconductors varies exponentially $\rho_{\text{semi-conductor}} \propto \exp\left(\frac{1}{T}\right)$

CFET Demerit: Resistivity vs temperature



- Wrong prediction of temperature dependence of conductivity. Theory predicts $\sigma(T) \propto 1/\sqrt{T}$ but experimental trend is $\sigma(T) \propto T^{-1}$

Demerit: Specific heat capacity

- Metals heat "quickly". When we heat a metal, the heat energy is transferred to electrons. The quickness is measured in terms of heat capacity.
- Specific heat capacity is defined as the rate of change of energy per unit temperature per unit mole.
- Energy of electron at temperature T is $E_{\mathrm{el}} = \frac{3}{2} k_B T$
- Energy of a mole of electrons is

$$E = E_{\mathrm{el}} N_A = \frac{3}{2} k_B N_A T = \frac{3}{2} RT \qquad \left[R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}\right]$$

- Therefore, specific heat capacity is

$$C_{\mathrm{theory}} = \frac{\mathrm{d}E}{\mathrm{d}T} = \frac{3}{2} R \sim 12\,\mathrm{J\,mol^{-1}\,K^{-1}}$$

- But experimental values of specific heat capacity $C_{\mathrm{experiment}}$ are **smaller by two orders** of magnitude.

Theory vs experiment contd...

Reason

- Does not account for quantum effects (Fermi–Dirac statistics).
- Improved later by Sommerfeld's free electron theory (quantum).

Summary of lecture

- Derived $\sigma = \dfrac{ne^2\tau}{m}$.
- Connected microscopic electron properties to macroscopic Ohm's law.
- Classical free electron theory has qualitative and quantitative agreement with experiment values of conductivity.
- However, the theory has drawbacks. Some of them are
  - Incorrect prediction for conductivity vs valency
  - Cannot explain anomalous sign of Hall coefficient in some metals.
  - Underestimation of mean free path
  - Cannot explain classification of materials into conductors, semi-conductors and insulators,
  - Wrong prediction of conductivity vs temperature,
  - Overestimation of heat capacity