Engineering Physics (2025)
Course code 25PY101-S2
Module 1 Unit 1: Classical Free Electron Theory

Course Instructor:
Dr. Sreekar Guddeti
Assistant Professor in Physics
Department of Science and Humanities
Vignan's Foundation for Science, Technology and Research

February 25, 2026

M1U1 Plan

1. Condensed matter
2. Metals
3. Classical free electron theory
4. Expression of electrical conductivity
5. Introduction to Semiconductors
6. Electrical conductivity of semiconductors
7. Hall effect
8. Concept of Panchabhuta

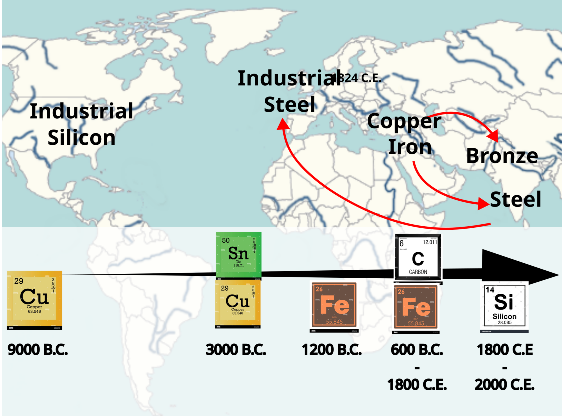
M1U1 Plan

Lecture plan

Learning Objectives

- Condensed matter at macroscopic scale
- Classification of condensed matter based on conductivity
- Nature of metals
- Classical electron theory of metals – Assumptions
- Ohm's law for metals – Conductivity
- Application of metallic conductivity

Discovery of material



Civilization spacetimeline: Copper $\rightarrow$ Bronze $\rightarrow$ Iron $\rightarrow$ Steel $\rightarrow$ Silicon

Key Insight

Material defines the age.

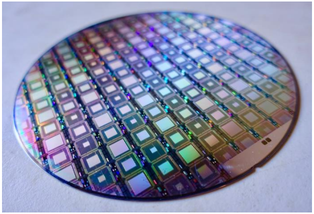
Condensed matter

- Material in the liquid or solid form is called **condensed matter**.
- Condensed matter is further sub-classified based on electrical, optical, magnetic, thermal, mechanical properties at the **macroscopic** scale. In the case of electrical property, we apply electric field and classify the materials based on their conductivity.
- The macroscopic behaviour is related to the **microscopic** behaviour of electrons under applied "forces".

Learning Objectives

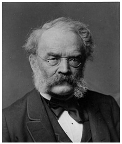
To relate the macroscopic properties with the microscopic behaviour of electrons in condensed matter.

Classification of Condensed Matter by Conductivity


(a) Gold, a metal, (b) Silicon, a semiconductor, (c) Diamond, an insulator

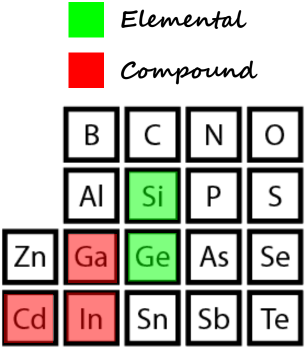
- Conductivity is the measure of how easily electrons move under applied electric field. Its unit is $\Omega^{-1}\,\mathrm{m}^{-1}$ or $\mathrm{S\,m}^{-1}$ (S for Siemens).
- Materials can be classified based on conductivity as:
  1. **Metals**: High ($\sigma \sim 10^7$ S/m).
  2. **Semiconductors**: Intermediate ($\sigma \sim 10^{-4}$ S/m).
  3. **Insulators**: Negligible ($\sigma \sim 10^{-10}$ S/m).



W. Siemens [1816-1892]

Elemental phases

- The electrical state of condensed matter is also called a **phase** – similar to solid phase, liquid phase, etc.

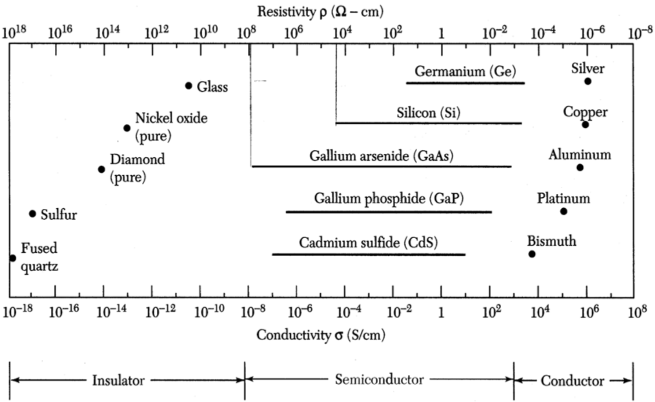


Key Insight

Most elemental phases are metals.

Conductivity of phases

- Conductivity σ is inversely related to resistivity ρ by $\rho = \frac{1}{\sigma}$

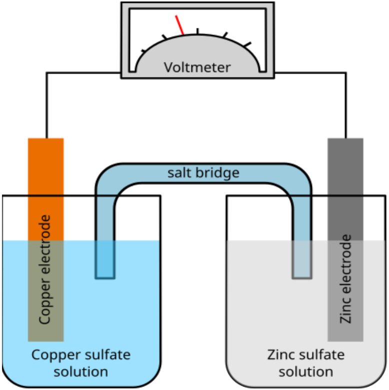


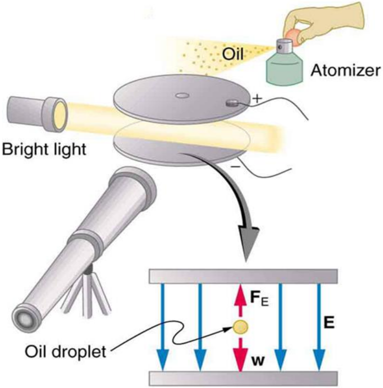
	ρ Ω m
Cu	10^{-7}
Si	10^{4}
SiO_2	10^{10}

Key Insight

Conductivity spans "orders of magnitude" across phases.

Early experiments:Avogadro number and Electron charge

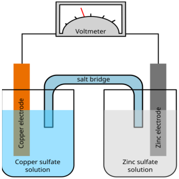




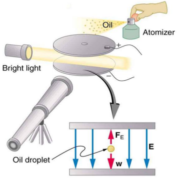
Electrochemical cell

Oil drop experiment

Macroscopic → microscopic

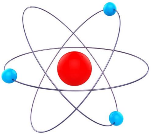


Electrochemical cell



Oil drop experiment

Macroscopic Copper



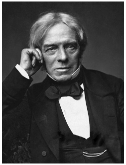
Microscopic Copper

Estimate: Avogadro number N_A

To electroplate 63.5 g of copper, it takes 2 F of charge. [Hint: 1F (F for Faraday)= 96.485 C, charge of electron $e = 1.602 \times 10^{-19}$ C]

Estimate: Radius of atom

The density of copper is $8.96\,\mathrm{g\,cm^{-3}}$.




Faraday, Millikan

M1U1 Plan

Lecture Plan

Learning Objectives

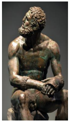
Learn the concept of

- electrical conductivity,
- mobility, and
- relaxation time

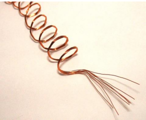
Nature of Metal

(1)

(2)



(3)



(4), (6)

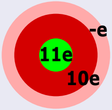
(5)

1. Lustre (Shine)
2. Solid with high 1000 °C melting points
3. Malleable (capable of being shaped)
4. Good electrical conductor
5. Good thermal conductor
6. Ductile (easy to draw wires)

Chemistry

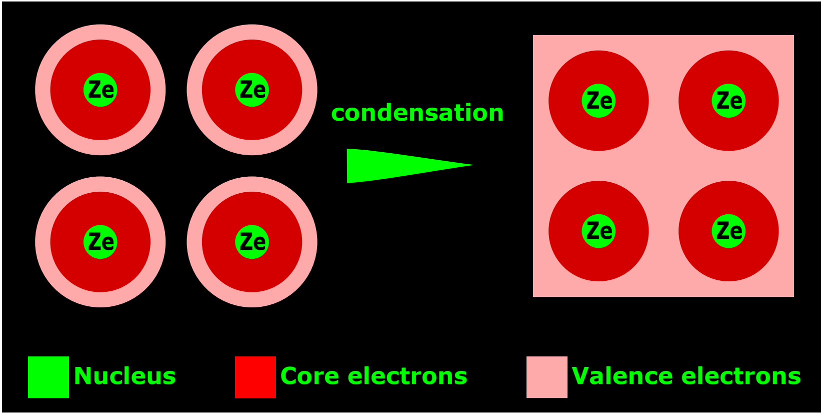
- Metallic bonding
- Screening

Na atom



$$Z_{eff} = +e$$

Metallic bonding ↔ electron gas

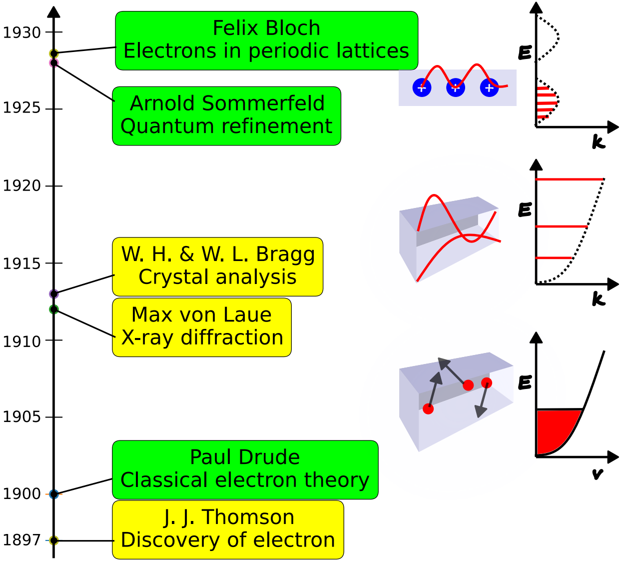


Valence electrons to electron "gas"

Key Insight

The properties of electron "gas" determines the nature of metal.

1. Classical free electron theory
2. Quantum free electron theory [M2 U3]
3. Quantum band theory [M2 U3]



M1U1 Plan

Problem at hand

$$V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r} \qquad V(r) \propto -\frac{1}{r}$$

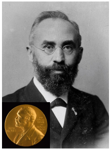
- The problem at hand is to solve the motion of interacting particles–electrons of the order of 10^{23} and ions of the order of 10^{23}.
- This is a tough problem. The idea is to simplify the problem by making **approximations**.

Classical free electron theory

- Ohm's law was discovered in 1827 and needed explanation.
- Proposed by Paul Drude in 1900 using analogy of gas of molecules.
- Mutual repulsion between electrons is neglected i.e. electrons are independent – **independent electron approximation**.
- Assumed "gas" is **free** i.e. not under influence of lattice – **free electron approximation**
- Role of lattice is to redistribute the velocity distribution that obeys **classical** Maxwell-Boltzmann statistics – classical thermodynamics.
- The theory explains Ohm's law
- Hendrik Lorentz extended theory to explain thermal conductivity – hence Drude-Lorentz electron theory.

G. Ohm

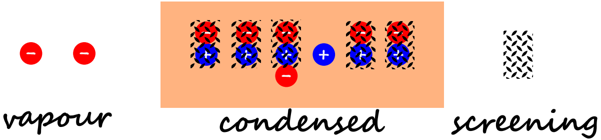
P. Drude



H. Lorentz

Independent electron approximation

$$V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r} \qquad \cancel{V(r) = +\frac{1}{4\pi\epsilon_0}\frac{1}{r}}$$

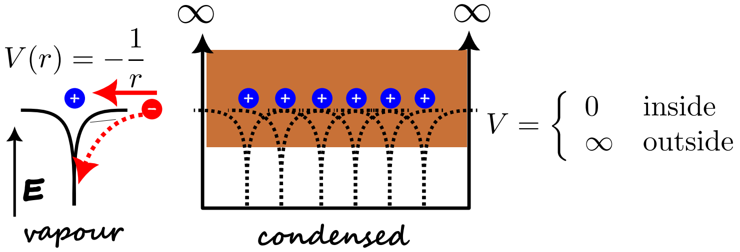


- In the condensed phase, there are of the order of 10^{23} electrons.
- The repulsive Coulomb potential between an electron and other electron is **screened** due to large numbers and results in an **independent** electron.
- The problem of motion of $\mathcal{O}(10^{23})$ electrons is reduced to the problem of motion of **single** electron.

Key Insight

The property of single electron determines the properties of electron gas.

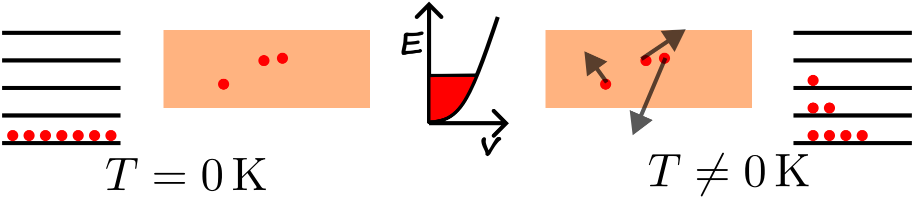
Free electron approximation



- The attractive Coulomb potential between ions and an electron is smeared due to large numbers and results in constant potential in the interior of metal with **infinite** barrier at the surface. This potential is called **infinite potential well**.
- This potential traps the electrons analogous to gas atoms in a container.
- The electrons are free to move in the potential well. The kinetic energy K of an electron with mass m and velocity $\mathbf{v}$ is

$$K = \frac{1}{2}mv^2$$

Classical thermodynamics

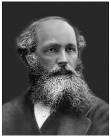


- At temperature $T = 0$, electrons are frozen. At $T \neq 0$, due to the large number of electrons and collisions, the kinetic energy of electrons keeps changing.
- However, the probability to find electron with with lower energy is higher than the probality to find electron with higher energy. This is the essence of Maxwell-Boltzmann statistics.
- The average kinetic energy $\langle T \rangle$ at temperature is

$$\langle K \rangle = \frac{3}{2} k_B T,$$

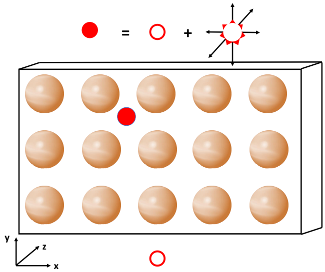
where k_B is the Boltzmann constant.

Boltzmann



Maxwell

Classical free electron theory



- Electron motion is decomposed into **thermal motion** and **drift motion**.
- Thermal motion is due to random collisions with the lattice.
- Drift motion is the net motion of electrons. The electron drifts only between collisions.

Thermal motion

Definition

Mean free path l_{mf} is defined as the average distance the electron travels between collisions.

Definition

Thermal velocity v_{th} is defined as the average velocity of the electron.

- Thermal motion has two properties mean free path l_{mf} and thermal velocity v_{th}.
- Drude assumed that l_{mf} is a material constant.
- At room temperature (300 K), the thermal velocity is of the **order of** $10^5\ \mathrm{m\,s^{-1}}$.

Estimate: Average kinetic energy of electron

Take room temprerature as 27 °C. Express energy in eV. [Hint: $1\,\mathrm{eV} = 1.602 \times 10^{-19}$ J]

Drift motion

- In the absence of external force, the drift motion is zero.
- The external force $\mathbf{F}_{ext}$ on the electron is given by the Lorentz force

$$\mathbf{F}_{ext} = -e\mathbf{E} - e\mathbf{v} \times \mathbf{B},$$

  where $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field.
- The effect of the collisions is treated as a drag or frictional force

$$\mathbf{F}_{fric} = -m\frac{\mathbf{v}}{\tau}.$$

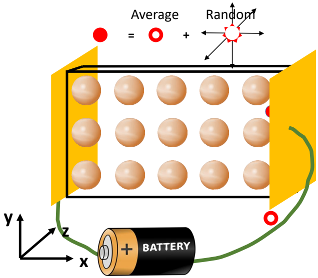
- Since Lorentz extended the theory to incorporate electromagnetic force, the theory is also the Lorentz-Drude theory.

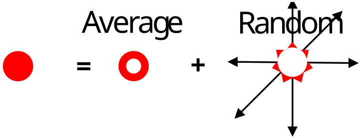
Lorentz-Drude theory – Postulates

Postulates

1. **Particles:** Electrons are **classical** particles with mass m and charge $-e$.
2. **Kinematics:**
   - **Independent electron approximation:** Electrons are independent and mutual repulsion between them is ignored.
3. **Dynamics:**
   - **Free electron approximation:** Electrons are free and move in an **infinite potential well**.
   - **Thermal motion:** Electron undergoes **random** collision with ion cores. Random collisions lead to thermal motion.
   - **Drift motion:** In between collisions, the Lorentz force acts on electron and leads to drift motion.
4. **Classical Thermodynamics:** The average electron energy is given by Maxwell-Boltzmann statistics.

Ohm's law





- The motion of electron is decomposed into an **average** part and the **random** part.
- The average part "drifts" with a constant velocity under electric field. The random part has **zero** net motion.

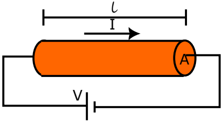
Learning Objectives

Classical electron theory explains Ohm's law qualitatively.

Ohm's law: Macroscopic version

- The **macroscopic** version of the Ohm's law states that potential V applied across a metal is directly proportional to the current I flowing across it. The constant of proportionaly is called the resistance R.
- Its **microscopic** version relates the electric field and current density
- Electric field $\mathbf{E}$ is defined as the potential per unit distance. Its SI unit is $\mathrm{V\,m^{-1}}$.
- Current density $\mathbf{j}$ is defined as the current flowing through a wire per unit area, where A is the cross-sectional area of the wire. Its SI unit is $\mathrm{A\,m^{-2}}$.

$$V = IR$$



$$\mathbf{E} = \frac{V}{\ell}\hat{\mathbf{E}}$$

$$\mathbf{j} = \frac{I}{A}\hat{\mathbf{j}}$$

Ohm's law : Microscopic version

Lets start with the macroscopic version $V = RI$ and derive the microscopic version of Ohm's law

Derivation

Upon substitution of V, I *in terms of* $\mathbf{E}, \mathbf{j}$, *we have*

$$\frac{\ell}{V}\mathbf{E} = \frac{A}{I}\mathbf{j} \quad \Rightarrow \quad \mathbf{j} = \frac{\ell}{A}\frac{I}{V}\mathbf{E} \quad \Rightarrow \quad \mathbf{j} = \frac{\ell}{AR}\mathbf{E} = \sigma\mathbf{E}$$

Thus, the microscopic version of Ohm's law is given by

$$\boxed{\mathbf{j} = \sigma\mathbf{E}}$$

where σ *is given by*

$$\sigma = \frac{\ell}{AR}$$

Conductivity – σ

Definition

The conductivity σ is defined as the current density carried by the wire per unit of electric field.

$$\sigma = \frac{j}{E} \qquad \text{SI unit} \quad [\sigma] = \left[\frac{\ell}{AR}\right] = \Omega^{-1}\,\mathrm{m}^{-1}.$$

Question

A copper wire of length $L = 2\,\mathrm{m}$ and diameter $d{=}1\,\mathrm{mm}$ is connected across a DC source. When a voltage of $V{=}0.5\,\mathrm{V}$ is applied across the ends of the wire, an electric current of $I{=}22\,\mathrm{A}$ is observed. Using these measurements, find the electrical conductivity σ of copper.

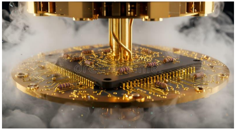
Key Insight

Metals have high conductivity.

Applications of metallic conductivity

Due to high conductivity, metals have many applications ranging from power electronics to microelectronics, from mechanical to aerospace industry.

(a) Power transmission



(b) Chip Interconnects

(c) Heat sinks

(d) Functional alloys

- Power transmission: Copper/Aluminum cables.
- Electronic devices: Contacts, interconnects.
- Thermal applications: Heat sinks, cookware.
- Alloys: Enhanced strength and corrosion resistance.

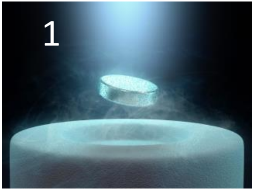
Summary of Lecture

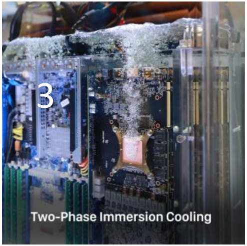
- Materials are classified into conductors, semiconductors, and insulators.
- Metals have high conductivity due to free electrons.
- Drude's classical electron theory introduced microscopic picture of conduction.
- Conductivity is related to resistivity.

Materials: Beyond metals

Condensed	Elec.	Therm.	Opt.	Ductile	Material	Application
Solid	$\Uparrow$	-	-	$\downarrow$	Superconductor	Levitation
Solid	$\Downarrow$	$\Uparrow$	-	-	Diamond	Chips
Liquid	$\Downarrow$	$\Uparrow$	-	-	Dielectric coolants	Heatsinks

Table: New materials with mixed properties.





1. Superconductor 2. diamond wafer 3. dielectric coolant

M1U1 Plan

Lecture plan

Learning Objectives

- From qualitative picture of metals to quantitative derivation.
- Derive expression for conductivity.
- Mobility is a material property.

Free electron density – n

Definition

- Electron density n is defined as the number of electrons per unit volume. Its SI unit is m^{-3}.
- For metals, each atom contributes as many electrons as its **valency**. For example, Na has valency 1.

Estimate: Electron density of metals

1. Take the radius r_s of atom is nearly $3a_0$, where $a_0 = 0.529\,\text{Å}$ is the Bohr radius.
2. Deduce n is of the order $10^{22}\,\mathrm{cm}^{-3} = 10^{28}\,\mathrm{m}^{-3}$.
3. For Cu, the experimental observed $n = 8.47 \times 10^{28}\,\mathrm{m}^{-3}$.

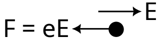
Key Insight

$\frac{r_s}{a_0}$ determines the free electron density.

Drift velocity – v_d

Definition

- **Drift velocity $\mathbf{v_d}$** is defined as the average velocity of an electron in the presence of electric field.
- **relaxation time** τ is defined as the average time between collisions.



Derivation

- *The equation of motion of electron is*
- *During time duration τ, electron reaches drift velocity*

$$\mathbf{a} = -\frac{e}{m}\mathbf{E}$$

$$\boxed{\mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}}$$

Key Insight

Drift velocity is proportional to the applied electric field.

$y = mx$ and $y = m/x$ relations

- We have seen many examples of **linear** relations between physical quantities till now.
- The dependent variable y relates to the independent variable x. The relation is given by the **constant of proportionality**.
- We also say that "y is directly proportional to x".

Law	Equation	Const. of prop.
Ohm's (macroscopic)	$I = \frac{1}{R}V$	Conductance
Ohm's (microscopic)	$\mathbf{j} = \sigma\mathbf{E}$	Conductivity
Capacitor	$E = \frac{1}{d}V$	1/Separation
Drift motion	$\mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}$	Mobility
Photoelectric effect	$E = \hbar\nu$	Planck's constant

- Sometimes, physical quantities are also inversely related i.e. $y \propto \frac{1}{x}$.
- We also say "y is inversely proportional to x".

Law	Equation	Const. of prop.
Light	$\nu = c\frac{1}{\lambda}$	velocity

Mobility – μ

Definition

Mobility μ is defined as the drift velocity per unit electric field

$$\mu = \frac{v_d}{E}.$$

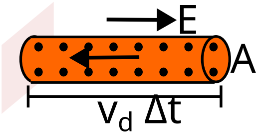
- It is a material property as

$$\mu = \frac{e\tau}{m} \qquad \left[\because \mathbf{v_d} = -\frac{e\tau}{m}\mathbf{E}\right]$$

- Its SI unit is

$$[\mu] = \frac{[v_d]}{[E]} = \frac{[v_d \ell]}{[V]} = \mathrm{m^2\,V^{-1}\,s^{-1}}$$

Conductivity σ



In a time duration Δt, the number of electrons N in a volume of $V = v_d \Delta t A$ will pass through the cross section of area A.

Derivation

1. *Current I:*
2. *Current density j:*
3. *Conductivity σ:*

$$I = -\frac{Ne}{\Delta t} = -\frac{n \cdot V}{\Delta t} = -nev_d A \tag{1}$$

$$j = \frac{I}{A} = -nev_d = \frac{ne^2\tau}{m} E \tag{2}$$

$$\sigma = \frac{ne^2\tau}{m} = ne\left(\frac{e\tau}{m}\right) = ne\mu \tag{3}$$

Learning Objectives

We have derived the expression for conductivity.

Relaxation Time – τ

Assumption

Relaxation time τ is assumed to be a **constant**.

Estimate: Relaxation time in metals

Estimate the electron relaxation time in metals.

1. Take conductivity $\sigma \sim 10^7\,\mathrm{S\,m^{-1}}$, number density $n \sim 10^{28}\,\mathrm{m^{-3}}$
2. Take electron mass $m = 9.1 \times 10^{-31}\,\mathrm{kg}$, charge $e = 1.609 \times 10^{-19}\,\mathrm{C}$.
3. Deduce relaxation time $\tau \sim \boxed{10^{-14}\,\mathrm{s} \text{ or } 10\,\mathrm{fs}}$.

- Larger $\tau \rightarrow$ fewer collisions $\rightarrow$ higher conductivity.

Trends

1. As temperature increases, τ decreases. Why?
2. As impurity concentration increases, τ decreases. Why?

Resistivity – ρ

Definition

Resistivity ρ is defined as the inverse of conductivity

$$\rho = \frac{1}{\sigma}$$

Its SI unit is $\Omega\,\mathrm{m}$.

- Resistivity $\rho = \dfrac{1}{\sigma} = \dfrac{m}{ne^2\tau}$.
- Its SI unit is $\Omega\,\mathrm{m}$.

Trends

1. As number density increases, ρ decreases. Why?
2. As temperature increases, ρ increases. Why?

Summary of physical quantities

Physical quantity	Symbol	Units	Material Property
Resistance	R	Ω	No
Conductivity	σ	$\Omega^{-1}\,\mathrm{m}^{-1}$ or $\mathrm{S\,m}^{-1}$	Yes
Mobility	μ	$\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$	Yes
Relaxation Time	τ	s	Yes
Resistivity	ρ	$\Omega\,\mathrm{m}$	Yes

Theory vs. experiment

Merit: Conductivity order of magnitude

- For metals, classical free electron theory (CEFT) predicts order of magnitude of conductivity correctly.

$$\sigma_{\text{experiment}} \sim \sigma_{\text{theory}} \sim 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$$

Demerit: Conductivity vs valency

- Conductivity is proportional to number density. Number density is proportional to valency. So theory predicts conductivity increases with valency. But experiment shows monovalent metals (Na, K) have higher conductivity than divalent metals (Mg, Ca).

Theory vs Experiment contd...

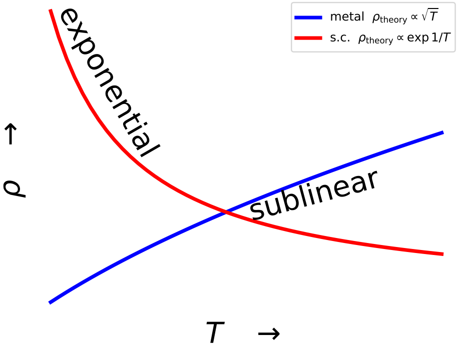
Demerit: Hall coefficient sign

- In metals, there are only electrons as charge carriers.
- It is expected that the **sign of Hall coefficient is negative**.
- However, experiments show metals like Zn show positive Hall coefficient. CEFT cannot explain this **anomaly**.

Under-estimation of mean free path

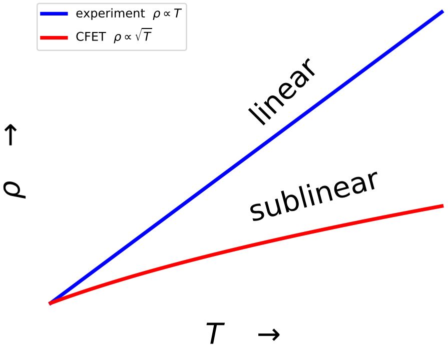
- The mean free path is considered a constant in CEFT and is taken as the interatomic distance that is approximately 5 Å.
- However, experiment shows mean free path to be of the order 50 Å.
- Thus, CEFT underestimates the mean free path by one order of magnitude.

Demerit: Classification of materials



- CEFT **does not** explain the classification of materials into conductors, semi-conductors and insulators.
- For example, the resistivity of metals varies linearly $\rho_{\text{conductor}} \propto T$ but resistivity of semiconductors varies exponentially $\rho_{\text{semi-conductor}} \propto \exp\left(\frac{1}{T}\right)$.

CFET Demerit: Resistivity vs temperature



- Wrong prediction of temperature dependence of conductivity. Theory predicts $\sigma(T) \propto 1/\sqrt{T}$, but experimental trend is $\sigma(T) \propto T^{-1}$.

Demerit: Specific heat capacity

- Metals heat "quickly". When we heat a metal, the heat energy is transferred to electrons. The quickness is measured in terms of heat capacity.
- Specific heat capacity is defined as the rate of change of energy per unit temperature per unit mole.
- Energy of electron at temperature T is $E_{\mathrm{el}} = \frac{3}{2}k_B T$.
- Energy of a mole of electrons is

$$E = E_{\mathrm{el}} N_A = \frac{3}{2} k_B N_A T = \frac{3}{2} RT. \qquad \left[R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}\right]$$

- Therefore, specific heat capacity is

$$C_{\mathrm{theory}} = \frac{\mathrm{d}E}{\mathrm{d}T} = \frac{3}{2} R \sim 12\,\mathrm{J\,mol^{-1}\,K^{-1}}.$$

- But experimental values of specific heat capacity $C_{\mathrm{experiment}}$ are **smaller by two orders** of magnitude.

Key Insight

Mercury thermometer is a good instrument to measure body temperature.

Theory vs experiment contd...

Reason

- Does not account for quantum effects (Fermi–Dirac statistics).
- Improved later by Sommerfeld's free electron theory (quantum).

Summary of lecture

- Derived $\sigma = \dfrac{ne^2\tau}{m}$.
- Connected microscopic electron properties to macroscopic Ohm's law.
- Classical free electron theory has qualitative and quantitative agreement with experiment values of conductivity.
- However, the theory has drawbacks. Some of them are
    - Incorrect prediction for conductivity vs valency.
    - Cannot explain anomalous sign of Hall coefficient in some metals.
    - Underestimation of mean free path.
    - Cannot explain classification of materials into conductors. semi-conductors and insulators.
    - Wrong prediction of conductivity vs temperature.
    - Overestimation of heat capacity.

Lecture 3 plan

Learning Objectives

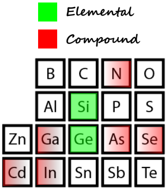
- Classification of semiconductors
- Nature of bonding in semiconductor
- Doping of semiconductor

Semiconductors: Classification

Definition

Semiconductor is a class of condensed matter having electrical conductivity ($10^{-4}\,\mathrm{S\,m^{-1}}$ to $1\,\mathrm{S\,m^{-1}}$) greater than that of insulator ($<10^{-10}\,\mathrm{S\,m^{-1}}$) but lower than that of conductor ($10^{7}\,\mathrm{S\,m^{-1}}$).

- In the periodic table, there are 11 elements that are semiconductors. These are called **elemental semiconductors**. Si and Ge from Group IV are widely used elemental semiconductors.
- Certain combinations of elements of Groups III and V or Groups II and VI also are semiconductors. These are called **compound semiconductors**. GaAs, InP are some of the well known examples.
- The unique feature of semiconductor is that two charge carriers, namely **electrons** and **holes**, transport current.



Nature of bonding: Silicon crystal

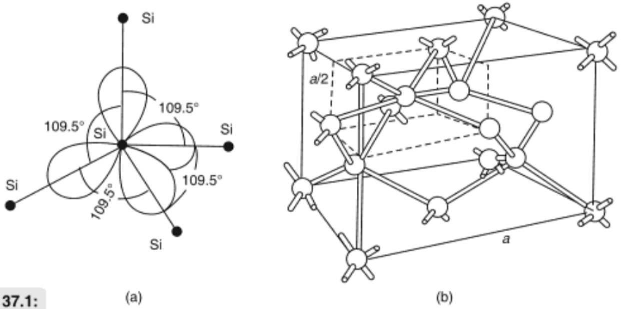


Fig. 37.1:

(a) Tetrahedral arrangement (b) Diamond crystal structure. Taken from Avadhanulu, Chapter 37.

- ^{18}Si atom has electronic configuration $1\,s^2\,2\,s^2\,2\,p^6\,3\,s^2\,3\,p^2$.
- In Si crystal, the four valence electrons undergo **hybridization** to form four sp^3 molecular orbitals. These orbitals form covalent bonds with neighbouring atoms arranged in tetrahedral arrangement. The resulting structure is the **diamond cubic crystal structure**.

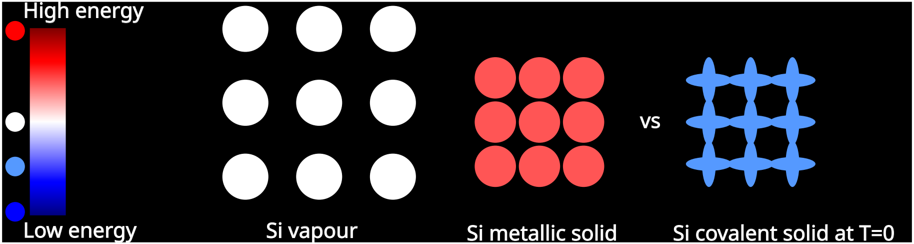
Intrinsic Semiconductor

- Chemically pure semiconductor is called intrinsic semiconductor.
- Practically, a semiconductor is considered pure if there is less than one impurity atom per billion host atoms i.e. < 1 ppb (part per billion).
- It has five properties
  1. Perfect insulator at absolute zero temperature
  2. Generation of charge carriers by thermalization
  3. Existence of energy band gap (M2U3)
  4. Conductivity is **highly** influenced by temperature
  5. Recombination of charge carriers
- For Si, the valence electron density is of the order of $10^{29}\,\mathrm{m}^{-3}$. However, at room temperature the conduction electron density is only of the order of $10^{18}\,\mathrm{m}^{-3}$.

Estimate: Valence electron density of Si

The lattice constant of Si crystal is $5.43\,\text{Å}$.

Nature of intrinsic semiconductor at $T = 0$ (property 1)

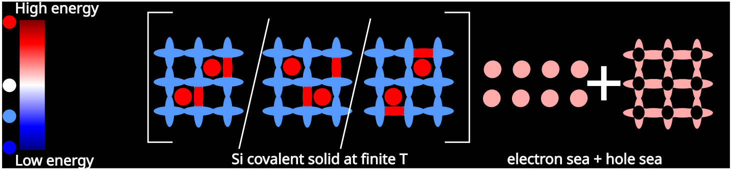


- Valence electrons are strongly attracted to the nucleus. This prevents the formation of free electron gas.
- Instead the valence electrons are shared between neighbouring atoms resulting in **covalent bond**.
- As all the valence electrons are involved in bonding, there are no "free" electrons at absolute zero temperature.

Key Insight

Intrinsic semiconductor is a perfect insulator at absolute zero!

Nature of intrinsic semiconductor at $T \neq 0$ (property 2)

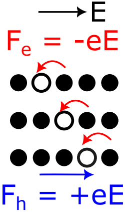


- However, at finite temperature, due to thermalization, few bonds are broken, releasing the electrons to the **free electron sea**.
- In the process of bond breaking, the crystal lattice deficient with one electron can be considered as a new type of charge carrier. This is defined as the **hole**.
- The holes are released into the **free hole sea**.
- Every free electron is associated with a free hole. Hence they are called **electron-hole pair**.

Definition

An intrinsic semiconductor is a semiconductor crystal in which the electrical conduction arises due to **thermally excited** electrons- hole pairs.

Nature of hole: Drift motion (property 2)



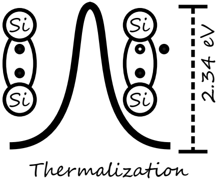
(a) Analogy for a hole: Bubbles (b) Drift motion of holes

- A hole is a **quantum vacancy** that is described as an absence of electron.
- Analogy is the bubbles appearing in carbonated water. The bubbles are absence of water.
- Under application of electric field, the bonded electrons move in the opposite direction. The hole appears to move along the electric field.

Key Insight

The motion of hole is along the electric field.

Energy band gap: Chemistry view (property 3)

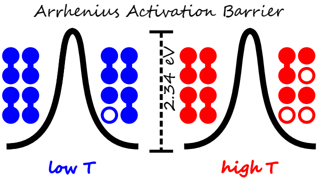


- The electron of the covalent bond needs to overcome bonding strength to become free.
- If all the bonds are broken, then the crystal vapourizes. The energy needed to vapourize a crystal is called **cohesive energy**.
- The actual energy to free electron is $1.12\,\mathrm{eV}$ and is called the **energy band gap** (M2U3).

Estimate: Bonding strength

The cohesive energy of Si crystal is $450\,\mathrm{kJ\,mol^{-1}}$. Estimate the Si−Si bond strength.

Charge carriers vs temperature (property 4)



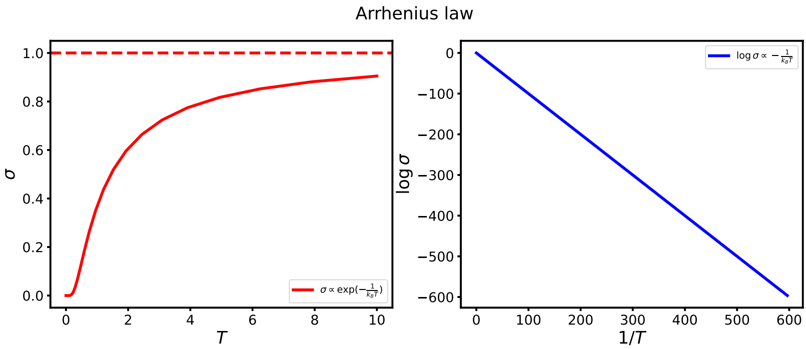
- The scale of thermal energy is given by $k_B T$. At room temperature, the thermal energy of electron is of the order of 26 meV. [Estimate]
- The bond strength can be considered an **activation barrier** for the generation of electron-hole pair. This is analogous to rate of chemical reaction

$$\text{covalent bond} + \text{thermal energy} \rightarrow \text{electron} + \text{hole}$$

- Electron-hole pair density increases exponentially with temperature. This follows the **Arrhenius law**.

$$n \propto \exp\left(-\frac{1}{k_B T}\right) \quad p \propto \exp\left(-\frac{1}{k_B T}\right)$$

Conductivity vs temperature (property 4)



- Since conductivity is proportional to number density, conductivity also follows Arrhenius law.

$$\sigma \propto n \quad \sigma \propto p$$

Key Insight

Conductivity exponentially depends on temperature.

Recombination of carriers (property 5)

- We have seen thermal energy to generate electron-hole pairs. This is an example of **equilibrium** generation
- Light can also be used to generate electron-hole pairs.

  $$\text{covalent bond} + \text{light} \rightarrow \text{electron} + \text{hole}$$

  However, this is a **non-equilibrium** generation process.
- The reverse process where the electron-hole pairs combine to become part of the covalent lattice is called **recombination**.

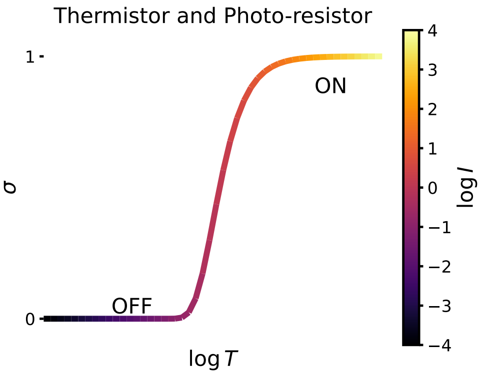
  $$\text{electron} + \text{hole} \rightarrow \text{covalent bond} + \text{light/lattice vibration}$$

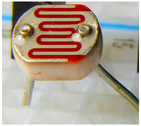
  The additional energy is released either as light (**photon**) or lattice vibration (**phonon**).
- The generation-recombination of charge carriers is a **dynamic** two way process.

Key Insight

For analysis, a semiconductor is a collection of electron, holes, photons and phonons and the interaction between them.

Applications of intrinsic semiconductor: Switch





- Thermistor (Temperature dependent resistor). Generation of carriers by temperature.
- Photo-resistor: (Light Dependent Resistor (LDR)). Generation of carriers by light.

App	Material
Thermistor	Fe_3O_4
Photoresistor	CdSe

Extrinsic semiconductor

- The electrical properties of intrinsic semiconductor can be altered by adding impurities.
- The technique of adding controlled impurity is called **doping** and the impurity added is called **dopant**.
- Doped semiconductor is called **extrinsic** semiconductor.
- Dopant atom substitutes the position of parent atom. The dopants are of two types – donor and acceptor.
- For Si, donors are P, As, etc and acceptors are B, Al, etc.
- For P as a donor dopant, four valence electrons contribute to covalent bonding. The fifth electron is loosely bound to the phosphorus atom
- For B as an acceptor dopant, three valence electrons contribute to covalent bonding.

Estimate: Dopant concentration

P doping is 1 ppm. $n_{valence} \sim 2 \times 10^{23}\,\mathrm{cm}^{-3}$

Ionization energy of dopants

Definition

The energy required to ionize a dopant atom is called ionization energy.

- Lattice vibrations (phonons) supply the ionization energy (I.E.)
- For P as donor dopant,

$$\mathrm{P} + \mathrm{I \cdot E \cdot} \longrightarrow \mathrm{P}^{+} + \mathrm{e}$$

- Similarly, for B as acceptor dopant,

$$\mathrm{B} + \mathrm{I \cdot E \cdot} \longrightarrow \mathrm{B}^{-} + \mathrm{h}$$

- If thermal energy (E_{th}) is greater than I.E. E_{ion}, then ions are completely ionized. This is called **complete** ionization.
- For $E_{th} \ll E_{ion}$, there is no ionization. This is called **freeze out**.
- **Partial** ionization occurs when $E_{th} \lesssim E_{ion}$
- In Si, $E_{ion} \sim 25\,\mathrm{meV}$ and $E_{th} \sim 25\,\mathrm{meV}$ at room temperature. Thus, the ions are **completely ionized**.

M1U1 Plan

Conductivity

In a semiconductor, there are two types of charge carriers – electrons and holes. Each carrier is associated with corresponding mobility. Thus, the mobility of electrons is μ_e and that of holes is μ_h.
From the classical free electron theory, the conductivity in terms of mobility and charge density is given by

$$\sigma = ne\mu$$

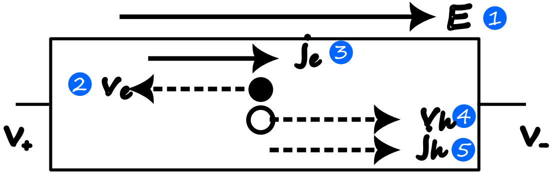
In semiconductors, there are **two channels** for conduction. Thus, the conductivity adds up and is given by

$$\sigma = ne\mu_e + pe\mu_h$$

Key Insight

Both electrons and holes contribute to conductivity

Drift motion of electrons and holes



- Under electric field, electron and hole drift motion is **anti-parallel**.

$$\mathbf{F}_e = -q_e\mathbf{E}, \qquad \mathbf{F}_h = +q_h\mathbf{E}.$$

- The sign of their charges are opposite.
- Electron and hole drift current direction is **parallel**.
- The total drift current density is given by $\mathbf{j}_{\mathrm{drift}} = \mathbf{j}_e + \mathbf{j}_h$.
- Drift current is proportional to electric field. $\mathbf{j}_{\mathrm{drift}} = \sigma_e\mathbf{E} + \sigma_h\mathbf{E}$
- Conductivity is written in terms of carrier density and mobility

$$\sigma_e = ne\mu_e, \qquad \sigma_h = pe\mu_h,$$

- The drift conductivity is given by

$$\sigma_{\mathrm{drift}} = ne\mu_e + pe\mu_h$$

Intrinsic semiconductor

For the intrinsic case, the number density of electrons equals the number density of holes

$$n = p = n_i$$

Thus, the conductivity is given by

$$\sigma = n_i e \left(\mu_e + \mu_h\right)$$

Carrier density	cm^{-3}
$n_{\mathrm{i-Si}}$	10^{10}
n_{Cu}	10^{22}

Mobility	$\mathrm{cm}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$
μ_e [Si]	1400
μ_h [Si]	500
μ [Cu]	40

Estimate: Conductivity of intrinsic Si

Compare it with conductivity of copper.

Key Insight

Though the carrier mobility in metals is less than in semiconductors, **very much higher** carrier density leads to higher metallic conductivity.

Extrinsic semiconductor

- n-type: Electrons are **majority** charge carriers

$$n \gg p, \quad \text{so that} \quad ne\mu_e \gg pe\mu_h$$

$$\therefore \sigma \simeq ne\mu_e$$

- p-type: Holes are **majority** charge carriers

$$p \gg n, \quad \text{so that} \quad pe\mu_h \gg ne\mu_e$$

$$\therefore \sigma \simeq pe\mu_h$$

Material	n (in $\mathrm{cm^{-3}}$)	σ (in $\mathrm{S\,m^{-1}}$)
i-Si	10^{10}	10^{-4}
1 ppm n-Si	10^{16}	2

Estimate: Conductivity of extrinsic n-type Si

P doping is $5 \times 10^{16}\,\mathrm{cm^{-3}}$. Compare it with conductivity of intrinsic-Si.

Key Insight

Doping of the order of 1 ppm increases conductivity of silicon by six orders of magnitude.

M1U1 Plan

Hall effect

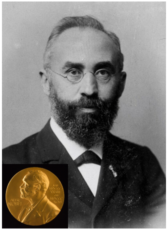
- In the presence of magnetic field acting perpendicular to the flow of electrons, the electrons experience a Lorentz force that deviates their path in the transverse direction. Accumulation of electrons along the edges leads to generation of voltage. This is called **Hall effect**.
- Hall effect is prominent in two dimensional geometry and can be considered the advent of research on thin films.
- It was discovered before the discovery of electron!
- In the context of semiconductors, Hall effect has contribution from holes, in addition to electrons. It also helps determine
  - the type of semiconductor (n or p),
  - the majority carrier concentration, and
  - the majority carrier mobility.

E. Hall

Hall effect: Lorentz force

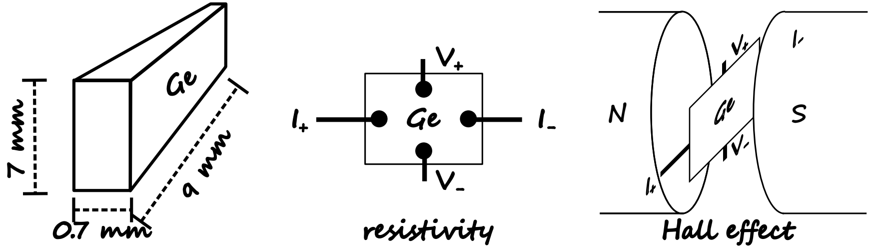
- In the presence of electric field **E** and magnetic field **B**, the electron experiences a force given by

$$\begin{aligned}\mathbf{F}_{\mathrm{Lorentz}} &= \mathbf{F}_{\mathrm{elec}} + \mathbf{F}_{\mathrm{mag}} \\ &= -e\mathbf{E} - e\mathbf{v} \times \mathbf{B}\end{aligned}$$



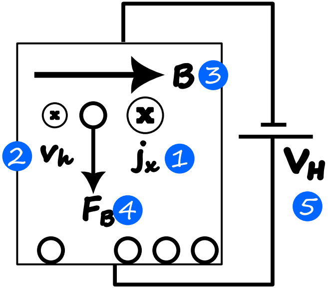
H. Lorentz

Hall effect: experiment



- The aim is to find the charge carrier type of Germanium crystal of dimensions 9 mm x 7 mm x 0.7 mm.
- The resistivity is measured in the **four-probe geometry**.
- Current is applied along x direction.
- Magnetic field of 0.1 T is applied along z direction. [$1\,\mathrm{T}=10^4\,\mathrm{G}$]
- The Hall voltage is measured along y direction.

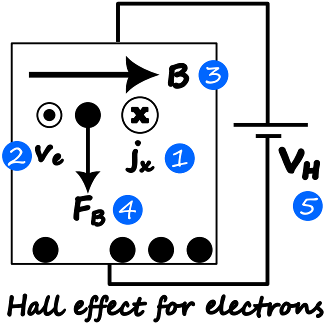
Hall effect in p-type semiconductor



Hall effect for holes

1. Current along x.
2. Electron drift along x.
3. Magnetic field along z.
4. Magnetic force along $-y$.
5. The buildup of holes **induces** an electric field in the $+y$ direction.
6. The corresponding voltage across the width is positive.

Hall effect in n-type semiconductor



1. Current along x.
2. Electron drift along $-x$.
3. Magnetic field along z.
4. Magnetic force along $-y$.
5. The buildup of electrons **induces** an electric field in the $-y$ direction.
6. The corresponding voltage across the width is negative.

Hall field and Hall voltage

- In the steady state, the magnetic field force is balanced by the induced electric field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(E_y - v_x B_z) = 0$$

- The induced electric field, called the **Hall field** is given by

$$\boxed{E_y = v_x B_z.}$$

- The Hall field produces a voltage across the semiconductor, which is called the **Hall voltage** given by

$$V_H = E_H w$$

where w is the width of the semiconductor.

- The Hall voltage is positive for p-type and negative for n-type semiconductor.

Hall coefficient

Definition

Hall coefficient R_H is defined as the Hall field per unit current density per unit magnetic field.

- R_H is given by

$$R_H := \frac{E_H}{j_x B_z} = \frac{v}{j_x}$$

- E_H is positive for holes and negative for electrons.
- However, the current density j is related to the drift velocity v by

$$j = nev \qquad [j = pev \quad \text{for holes}]$$

- Therefore,

$$\boxed{R_H(\text{electron}) = -\frac{1}{ne}, \qquad R_H(\text{hole}) = +\frac{1}{pe}}$$

Charge carrier properties

- Semiconductor type: The sign of Hall voltage determines the type of majority carrier in a semiconductor
- Majority carrier density: The Hall coefficient gives the majority carrier density as

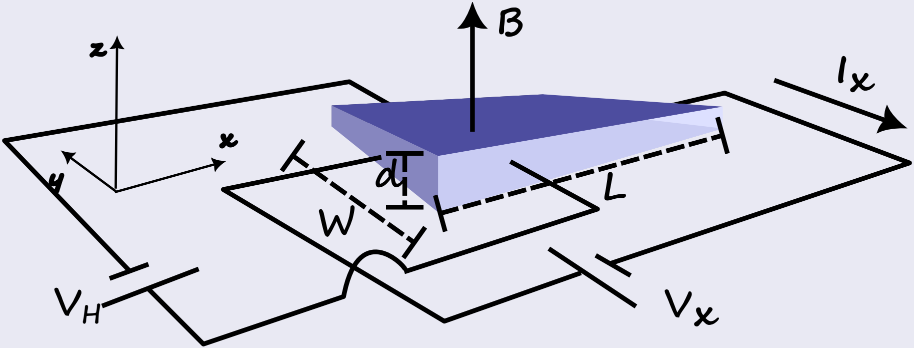
$$\boxed{n = \frac{1}{R_H e}}$$

- Majority carrier mobility: If the conductivity of the semiconductor σ is measured in the absence of magnetic field, then the mobility of majority carriers is given by

$$\boxed{\mu = \frac{\sigma}{ne}}$$

Test your Understanding

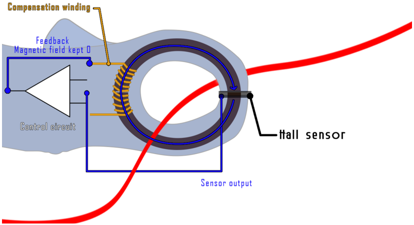
Problem

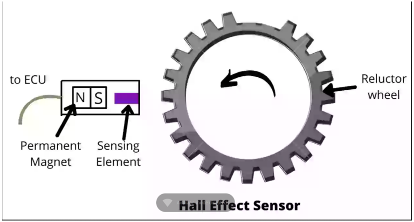


Determine the majority carrier type, concentration and mobility.
$L = 10^{-1}\,\text{cm},\ W = 10^{-2}\,\text{cm},\ d = 10^{-3}\,\text{cm}.\ I_x = 1\,\text{mA},\ V_x = 12.5\,\text{V},$
$B = 500\,\text{G},\ V_H = -6.25\,\text{mV}.$

Applications of Hall effect

- Magnetic sensor: Hall voltage is directly proportional to Magnetic field
    - Gaussmeter
    - Compass sensor in smartphones
    - Current measurement without direct contact



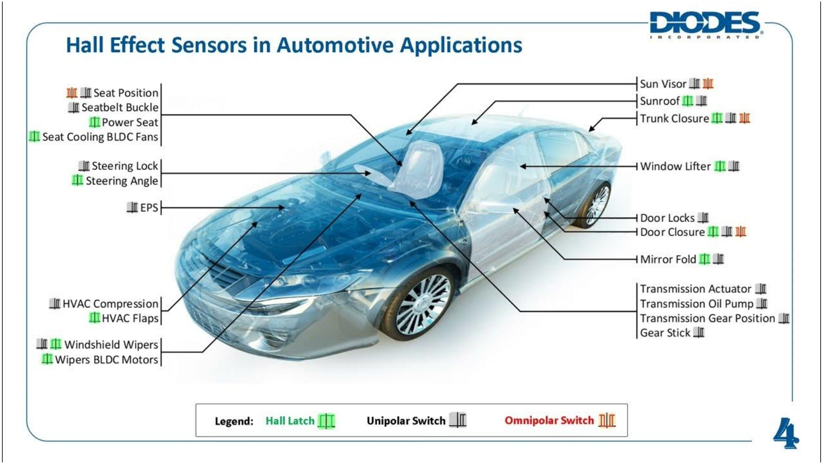


(a) Non contact Ammeter, (b) Rotation sensor

Applications of Hall effect

- Magnetic latch (💋)
- Magnetic switch (👋)

Proximity sensor
Rotation sensor



Reference: Youtube link

Summary of Hall effect

- The Hall effect is a consequence of charged carrier moving in the presence of **perpendicular** electric and magnetic fields.
- The charged carrier is deflected, **inducing** a Hall voltage.
- The polarity of the Hall voltage depends on type of the semiconductor.
- The majority carrier concentration and mobility can be determined from the Hall voltage.

M1U1 Plan

Panchabhuta – Five elements

- Etymology is Pancha (Sanskrit for five) and bhuta (Sanskrit for elements)
- Fundamental idea in Indian philosophy, Ayurveda, Yoga, and traditional sciences. It explains the composition of the universe, including the human body and mind, in terms of five basic elements.
- The Panchabhuta are considered the building blocks of the cosmos. Everything in the universe (macrocosm) and in the human body (microcosm) is made of these five elements in varying proportions.

Panchabhuta

Bhuta	Translation	Associated Sense	Body part
Ākāśa	Space/Ether	Hearing (śabda)	Cavities (Mouth) Channels (Nostrils) Spaces (Thorax, Abdomen)
Vāyu	Air	Touch (sparśa)	
Agni/Tejas	Fire	Sight (rūpa)	digestion, vision body temperature intelligence
Āpa / Jala	Water	Taste (rasa)	blood, saliva, digestive juices plasma, reproductive fluids
Pṛthvī	Earth	Smell (gandha)	bones, teeth, flesh, muscles, skin, nails, hair

Table: 1. Ākāśa represents emptiness, vastness, and the medium that allows sound to travel. 2. Vāyu symbolizes movement, motion, and dynamism. 3. Agni represents transformation, heat, energy, and metabolism. 4. Jala symbolizes fluidity, cohesion, and life-sustaining properties. 5. Pṛthvī represents solidity, stability, and structure.

End of Unit 1