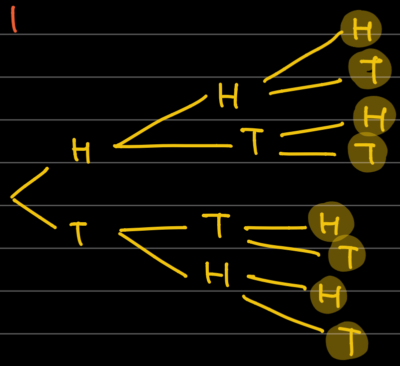
PERMUTATIONS & COMBINATION

"permutations" is synonymous with "arrangements"

Ex. 13.1

Q1.



| 2 | 2 | 2 | = 8

Q3. On the way there → 4 options : $\underbrace{A, B, C}_{\text{Bus}}$, $\underbrace{W}_{\text{walk}}$

On the way back → 3 options : $\underbrace{A, B, C}_{\text{Bus}}$

$4 \times 3 = 12$ possible options

Q.4a) Arrangement for 5 unique elements : $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ possible arrangements

b) For 6 unique elements : $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

Q5. Lunch
Letter
Bank
Papers
→ 4 elements = $4! = 4 \times 3 \times 2 \times 1$

$= 24$ possible combinations

Q6. 6 men + 5 women

6×5

$= 30$ possible pairings

Q7. 5 restaurants + 4 shopping centres

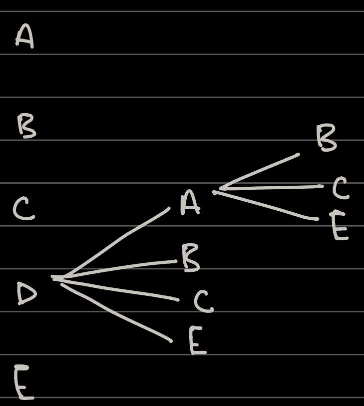
5×4

$= 20$ possible combinations

Q8.

Example

5 letters, arrange 3 at a time



$5 \times 4 \times 3 = 60$ possible arrangements

${}^{n}P_{r}$ → from n things permute/arrange 3 letters at a time

↳ $\dfrac{n!}{(n-r)!}$

${}^{5}P_{3} \rightarrow \dfrac{5!}{2!} = \dfrac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 5 \times 4 \times 3$

$= 60$ possible solutions

CONCEPT 1 (HISTORY)

1. Find the number of different arrangements that can be made from the letters of the word "history".

Total letters : 7
Total spaces : 7

${}^{7}P_{7} = \dfrac{7!}{0!} = 7! = 5040$ possible arrangements

2. Find the number of distinct four letter arrangements that can be made from the letters of the word "history"

Total letters : 7
Total spaces : 4

$^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$ possible solutions

3. Find the number of 7-letter arrangements in which O and R are always together

Total 7-letter arrangements = 7! = 5040

can either have arrangements with OR together or with OR not together

Q. How to find arrangements with OR together?

Method 1: Step1 – Bundle OR together

H I S T OR Y
1 2 3 4 5 6 → 6 elements → 6! possibilities

6! possibilities but OR cannot be arranged either OR or RO

Hence, $\underbrace{6! \times 2}_{1440}$ 2 → no. of ways the "bundle" can be arranged

4. Find the no. of arrangements in which O & R are always separated

Total no. of Arrangements w/out – All arrangements with OR together

5040 – 1440 = 3600 → All arrangements where OR are separated

Method 2: Forget OR, arrange remaining letters

H I S T ~~O~~ ~~R~~ Y
1 2 3 4 5 = $\boxed{5!}$ possibilities

_ H _ I _ S _ T _ Y _
1 2 3 4 5 6 → $\boxed{6}$ places where O can go

Once O is placed, R can go into $\boxed{5}$ places

Hence, $5! \times 6 \times 5$

5. Find the number of arrangements in which the letters HOR are always together

HOR	I	S	T	Y
1	2	3	4	5

$\rightarrow 5! \times 3! = \underline{720}$

can be arranged in 3! ways

6. Find the no. of ways in which HOR are always separated

HOR

_ S _ I _ T _ Y _

1 2 3 4 5

$4! \times 5 \times 4 \times 3$

If question was...

Find the number of arrangements in which the letters HOR are not all together

Total − HOR together possibilities = 5040 − 720
= 1440 possibilities

HOR

I	S	T	Y
1	2	3	4

$4! \times 3!$

7! → Attempt later

vii) Find the number of arrangements which begin with a consonant and end with the vowel

Consonant (5 options)

Vowel (2 options)

5	5	4	3	2	1	2

5 letters remaining

$= 5!$

$= 5 \times 5! \times 2 \rightarrow$ Ans

viii) Find the number of arrangements that begin with a consonant and end with a consonant

5! for remaining five characters

5	5	4	3	2	1	4

Consonant

$= 5 \times 5! \times 4 \rightarrow$ Ans.

ix) Find the number of arrangements that begin with letters H, I, in that order

H	I	5	4	3	2	1

5! for remaining five characters

$= 5! \rightarrow$ Ans

x) Find the number of arrangements in which an even position can only be occupied by a consonant

1	2	3	4	5	6	7
0	5	0	4	0	3	0

Consonant (5 options

4! for remaining 4 spaces/characters

$= \frac{5!}{2!} \times 4! \rightarrow$ Ans

Classwork — Ex. 13.2 — Q18 - 24

Q18

A, B, C, D, E

a) EC group

EC, A, B, D $\rightarrow$ 4! arrangements

2! arrangements

$= 4! \times 2!$

$= 48$ arrangements → Ans (a)

b) BD → $4! \times 2!$

Total arrangements = $5!$

$5! - 4! \times 2!$

$= 72$ arrangements → Ans (b)

19. Total possibilities = $7! = 5040$

Girls

a)

4	3	3	2	2	1	1
1	2	3	4	5	6	7

Boys (4 options)

$= 4! \times 3!$

$= 144$ → Ans (a)

b)

3	5	4	3	2	1	4

$5!$ for 5 remaining children

$= 3 \times 5! \times 4$

$= 1440$ → Ans (b)

20. I N C L U D E

a)

$\underbrace{\text{NCLD}}_{4!}$, I, U, E → $4!$

$= 4! \times 4!$

$= 576$ → Ans (a)

b)

4	3	3	2	2	1	1

Vowel

$= 4! \times 3!$

$= 144$ → Ans (b)

c)

4						3

$\underbrace{\quad\quad\quad\quad\quad}_{5!}$

$= 4 \times 5! \times 3$

$= 1440$ → Ans (c)

21.

$$\overbrace{\boxed{2}\boxed{5}\boxed{4}\boxed{3}}^{}$$

$5 \times 4 \times 3$ over the last three boxes; "2 or 4" under the first box.

$= 2 \times \dfrac{5!}{2!}$

$= 120 \rightarrow$ Ans. 21.

22.
a)

$$\underbrace{\boxed{0}\boxed{0}\boxed{0}}_{3!}\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }}_{4!}$$

$= 4! \times 3!$

$= 144 \rightarrow$ Ans (a)

b)

$$\boxed{3}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{2}$$

$3 \times \quad 6! - 5! \times 2$

$= 3 \times (6! - 5! \times 2)$

$= 1440 \rightarrow$ Ans (b)

23. a) $6! = 720 \rightarrow$ Ans (a)

b) $5! \times 2! = 240 \rightarrow$ Ans (b)

c)

$$\boxed{3}\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }}_{4!}\boxed{3}$$

$= 3 \times 4! \times 3$

$= 216 \rightarrow$ Ans (c)

24. a) $7! \times 3! = 30240 \rightarrow$ Ans (a)

b) $9! - (8! \times 2!)$

$= 282240 \rightarrow$ Ans (b)

c) $9! - (7! \times 3!) - 2(8! \times 2!)$ → Review Sir Nasir's way of doing it

$= 196566 \rightarrow$ Ans (c)

✗

$= 151200$

COMBINATIONS

Permutations → Arrangements where order matters

Combinations → The presence of the elements matter, and not the order in which they are present.

ie. ABC, ACB, BAC, BCA, CAB, CBA → 1 combination

Example:

A B C D E → five letters

Q. Find number of three letter combinations

1. ABC
2. ABD
3. ABE
4. ACD
5. ACE
6. ADE
7. BCD
8. BCE
9. BDE
10. CDE

10 combinations × 3! permutations for each combination

= number of total permutations (ie. $^5P_3$)

$10 \times 6 = 60$

Formula for combinations: $^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

ie. with the example above

^nC_r: total elements ← n; C → C for "Combinations"; r → how many are being chosen

$$^5C_3 = \frac{^5P_3}{3!} = \frac{\frac{5!}{(5-3)!}}{3!} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\cdot 2!} = \frac{5\times 4\times \cancel{3!}}{\cancel{3!}\times 2!}$$

$$= \frac{5\times 4}{2!}$$

$= 10$ → no. of combinations

CONCEPT 2 President / Secretary

Q. In a chess club, there are 8 members, 1 president, and 1 secretary

a) Find the no. of 4 member teams that can be sent, including both the president and secretary

P S __ __ (2 spots left) → order doesn't matter

$$^{8}C_2 = \frac{8!}{6!\,2!} = \frac{8\times 7}{2} = \frac{56}{2} = 28 \text{ possibilities}$$

ii) Find the no. of 4 member teams that can be formed that include neither the president nor the secretary

$\cancel{P}$, $\cancel{S}$ __ __ __ __ (1 2 3 4) →

$$^{8}C_4 = \frac{8!}{4!\,4!} = \frac{8\times 7\times 6\times 5}{4\times 3\times 2} = 70 \text{ possibilities}$$

iii) Find the no. of teams of 4 which include the president but not the secretary

$\cancel{S}$ P __ __ __ (1 2 3)

$$^{8}C_3 = \frac{8!}{3!\,5!} = \frac{8\times 7\times 6}{6} = 56 \text{ possibilities}$$

iv) Find the no. of teams that include the secretary but not the president

$\cancel{P}$ S __ __ __ (1 2 3)

$$^{8}C_3 = 56 \text{ possibilities}$$

v) Total without restrictions

$$^{10}C_4 = 210 \text{ possibilities} \longrightarrow \text{Sum of all above cases}$$

CONCEPT 3

6 men, 5 women   4 member team has to be formed

i) Total possibilities

$$^{11}C_4 = \frac{11!}{4!\,7!} = 330 \text{ possibilities}$$

CONCEPT 4

i) 11 people → make 2 groups : one group of 5, one group of 6

$^{11}C_5 \times {}^6C_6$ or $^{11}C_6 \times {}^5C_5$

$= 462$ possibilities $= 462$ possibilities

ii) 11 people → 3 groups : group of 2, group of 3, group of 6

$^{11}C_2 \times {}^9C_3 \times {}^6C_6 = 4620$ possibilities

Ex. 13.3 from Ho Soo Thong, slide 317, Q11 – Q18

Q11. a) $^5C_3 \times {}^2C_2 = 10$ possibilities

b) $^9C_5 \times {}^4C_4 = 126$ possibilities

Hence, $^9C_3 \times {}^6C_2 \times {}^4C_4 = 1260$ possibilities

Q12. $^{10}C_5 \times {}^5C_3 \times {}^2C_2 = 2520$ possibilities

Q13. a) $^8C_2 = 28$ possibilities

b) $^8C_3 = 56$ possibilities

Q14. i) total without restrictions

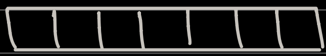
$^{15}C_6 = 5005$ total possibilities without restrictions

8 men and 7 women

ii) 3 by men and 3 by women

$^8C_3 \times {}^7C_3 = 1960$ possibilities iii)

CONCEPT 5 (Simple Seating)



3 people : 1 man
1 woman
1 child

Q. i) Find the no. of ways of seating these 3 people in a row of 7 chairs

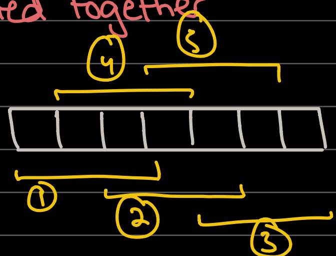
Method 1 : $7 \times 6 \times 5 = 210$

Method 2 : $^{7}C_{3} \times 3! = 210$

$^{7}C_{3}$ ↓ no. of seating patterns

$3!$ ↓ ways of arranging the 3 people in each seating pattern

ii) Find the no. of ways of seating these 3 such that they're always seated together



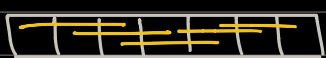
5 possible groups of 3

Can the arrange themselves in 3! ways

$= 5 \times 3!$

$= 30$ possibilities

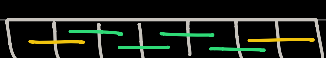
iii) Find the no. of seating arrangements in which all 3 are together and child is always seated in the middle.



$5 \times 2! = 10$ ways

iv) Find the no. of seating arrangements in which the woman and the child are always together but the man must not be next to them

M , WC



$2(2! \times 4) + 4(2! \times 3) = 16 + 24$

$= 40$

CONCEPT 6 (Symmetric Seating)

3 Girls
4 Boys

i) Find no. of symmetric seating arrangements

3	4	3	2	2	1	1

= 144

+

4	3	3	2	1	2	1

= 144

+

4	3	3	2	2	1	1

= 144

432

5 girls 6 boys

i) Find no. of symmetric seating arrangements

$5! \times 6!$

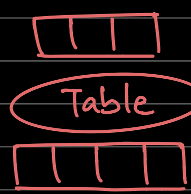
CONCEPT 7 (Complicated Seating)

Q. 9 friends go to a restaurant. To get there, they divide themselves into two groups to fit into two cars

$\boxed{3, x}$ $\boxed{4, y}$

Friend x drives one car with 3 other friends, and friend y drives the other car, with 4 other friends as passengers

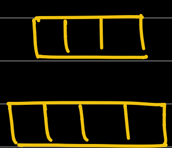
When they get to the restaurant, the only available table has the following configuration



i) Find the number of arrangements that can be made by seating 7 out of the 9 friends

$$^{9}P_7 = \frac{9!}{(9-7)!} = \frac{9!}{2!} = 181440$$

ii) Find the no. of seating arrangements such that X and Y are on the same side of the table



iii) Find the no. of seating arrangements in which not only are X and Y on the same side, but they're seated together

INDEPENDENT

i) How many 5 letter arrangements can you make from INDEPENDENT?

I
NNN
DD
EEE
P
T

All 5 diff $\longrightarrow$ ${}^6C_5 \times 5!$

3 same / 2 diff $\longrightarrow$ ${}^2C_1 \times {}^5C_2 \times \frac{5!}{3!}$

3 same / 2 same $\longrightarrow$ ${}^3C_2 \times {}^2C_1 \times {}^{5!}/{}_{3! \times 2!}$

2 same / 3 diff $\longrightarrow$ ${}^3C_1 \times {}^5C_3 \times {}^{5!}/{}_{2!}$

2 same / 2 same / 1 diff $\longrightarrow$ ${}^3C_2 \times {}^4C_1 \times \frac{5!}{2! \times 2!}$

ASSUMED

A
SS
U
M
E
D

All diff : ${}^6C_5 \times 5!$

2 same 3 diff : $1 \times {}^5C_3 \times \frac{5!}{2!}$

TOKYO

3 letter arrangements

T
OO
K
Y

All diff $\longrightarrow$ ${}^4C_3 \times 3!$

Two same $\longrightarrow$ $1 \times {}^3C_1 \times \frac{3!}{2!}$

MMM
A
X
I
U

MAXMUM

no. of arrangements such that all the consonants are together

MMMX, A, I, U

$\frac{4! \times 4!}{3!}$ because there are 3 Ms

SELECTION no. of arrangements

i) such that two Es together

$8!$

ii) such that two Es are not together

ADVANCE

No. of four letter arrangements

a) that include As

$^5C_2 \times \frac{4!}{2!}$

b) that dont include two As

$^5C_4 \times 4!$

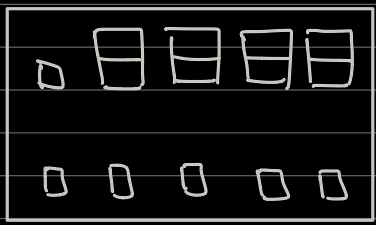
EXAMINATION find 4 letter selections and the corresponding arrangements

E
X
AA
M
II
NN
T
I

all 4 diff $\longrightarrow$ $^8C_4 \times 4!$

2 same / 2 diff $\longrightarrow$ $^3C_1 \times {^7C_2} \times \frac{4!}{2!}$

2 same / 2 same $^3C_2 \times \frac{4!}{2! \times 2!}$



Seat 12 passengers

a) $\underbrace{{}^{14}C_{12}}_{\text{number of seating patterns}} \times 12!$

b)

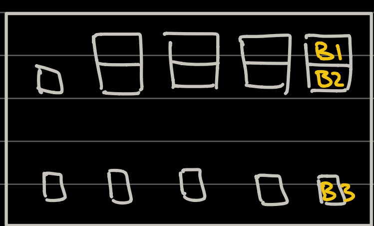
Married Couples → Mr / Mrs. Lin
→ Mr / Mrs. Brown

+ 5 students and 3 business people

Conditions

If two individuals are spouses, they must be on the same side
All students must have window seats
Business people must sit in the front row

Back



Front

$\underbrace{3!}_{\text{Business Ppl}} \times$

c) Mrs. Brown in front row   3 → Mrs. Browns options
Mrs. Lin needs to have a student in front of her

$\underbrace{10}_{\text{Mrs. Lins options}} \times \underbrace{5}_{\text{5 possible students}} \times \underbrace{\boxed{{}^{11}C_9 \times 9!}}_{\text{remaining after conditions}}$

$3 \times 10 \times 5 \times {}^{11}C_9 \times 9!$

MINIBUS

i) $^{17}C_{11} \times 11! \rightarrow$ possible seating arrangements

ii) 5 particular ppl.

$$\underbrace{1}_{\text{reserve 5 spots}} \times \overbrace{5!}^{\text{Arrange 5 particular ppl}} \times \underbrace{^{12}C_6}_{\text{seating patterns}} \times \underbrace{6!}_{\text{Arrange remaining}}$$

iii) $\underbrace{^{3}C_2}_{\text{From 3 couples choose 2}} \times \underbrace{7}_{\text{Choose one person from remaining 7}}$

Q 24.

(i) $^{4}C_3 \times 3!$

A

B

3 seats occupied $\times$ $^{5}C_1 \times {}^{4}C_4 \times 4!$

$\times 4!$

1, 3, 6, 8

3. Pg 42. a)

end w/ 6 $\quad$ $\boxed{3}$ 1 8 $\boxed{6}$ $= 2$ (1 2 1 1)

8 1 $\quad +$

end w/ 8 $\quad = 4$ (2 2 1 1)

$=$ 6 possibilities

b) end w/6 $\boxed{}\boxed{}\boxed{}\boxed{1}$

$2 \times 4 \times 4 \times 1 = 32$

$+$

end w/8 $\boxed{}\boxed{}\boxed{}\boxed{8} = 32$

$2 \times 4 \times 4 \times 1$

$=$

64 possibilities

4\. a) ${}^{10}C_6$ 13.

b) ${}^{10}C_6 \times 6!$

c) ${}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2$

↳ 3 groups of 2