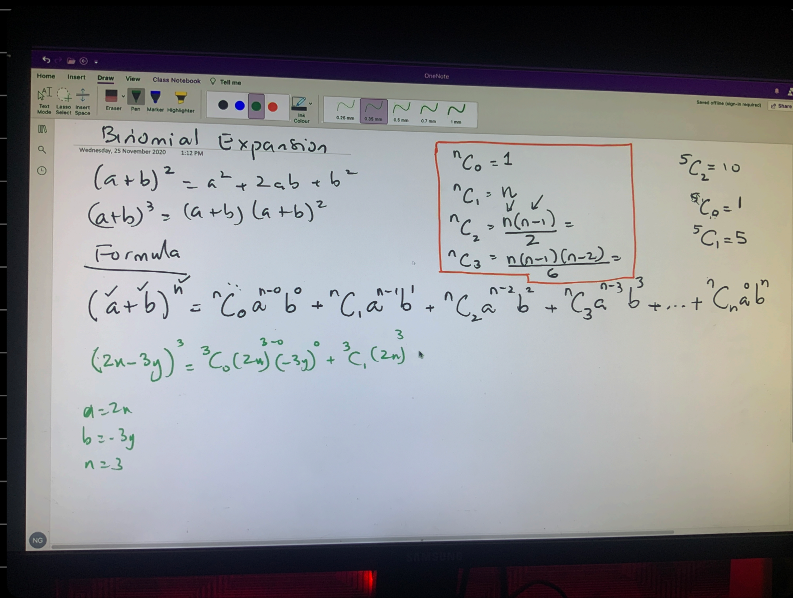
Binomial Expansion

Formula → $(a+b)^n = {}^nC_0 a^{n-0} b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 \ldots {}^nC_n a^0 b^n$

$${}^nC_0 = 1$$
$${}^nC_1 = n$$
$${}^nC_2 = \frac{n(n-1)}{2}$$
$${}^nC_3 = \frac{n(n-1)(n-2)}{6}$$



${}^nC_r a^{n-r} b^r$

Binomial Expansion

Refers to the expansion of binomials (expressions having two terms like $(a+b)$) raised to the n^{th} power.

Example problems:

1. Expand the following

a) $(1+x)^4$

${}^nC_r$ where $n = 4$ and r is incremented by 1 starting from 0

${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$
$\frac{4!}{(4-0)!\,0!}$	$\frac{4!}{(4-1)!\,1!}$	$\frac{4!}{(4-2)!\,2!}$	$\frac{4!}{(4-3)!\,3!}$	$\frac{4!}{(4-4)!\,4!}$
$\frac{4!}{4!\,0!}$	$\frac{4!}{3!\,1!}$	$\frac{4!}{2!\,2!}$	$\frac{4!}{1!\,3!}$	$\frac{4!}{0!\,4!}$
$1x^0$	$4x^1$	$6x^2$	$4x^3$	$1x^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4 \rightarrow$ Ans

b) $(1+x)^5$

$^5C_0$	$^5C_1$	$^5C_2$	$^5C_3$	$^5C_4$	$^5C_5$
$\frac{5!}{5!0!}$	$\frac{5!}{4!1!}$	$\frac{5!}{3!2!}$	$\frac{5!}{2!3!}$	$\frac{5!}{1!0!}$	$\frac{5!}{0!5!}$
1	5	10	10	5	1

$= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5 \rightarrow \underline{\underline{\text{Ans}}}$

c) $(1+3x)^4$

$^4C_0$	$^4C_1$	$^4C_2$	$^4C_3$	$^4C_4$
1	4	6	4	1

$= 1(3x)^0 + 4(3x)^1 + 6(3x)^2 + 4(3x)^3 + 1(3x)^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4 \rightarrow \underline{\underline{\text{Ans}}}$

d) $(1-x)^3$

$^3C_0$	$^3C_1$	$^3C_2$	$^3C_3$
$\frac{3!}{3!0!}$	$\frac{3!}{2!1!}$	$\frac{3!}{1!2!}$	$\frac{3!}{0!3!}$
1	3	3	1

$= 1(-x)^0 + 3(-x)^1 + 3(-x)^2 + 1(-x)^3$

$= 1 - 3x + 3x^2 - x^3 \rightarrow \underline{\underline{\text{Ans}}}$

e) $(1-2x)^4$

$^4C_0$	$^4C_1$	$^4C_2$	$^4C_3$	$^4C_4$
1	4	6	4	1

$= 1 + 4(-2x)^1 + 6(-2x)^2 + 4(-2x)^3 + 1(-2x)^4$

$= 1 - 8x + 24x^2 - 32x^3 + 16x^4 \rightarrow \underline{\underline{\text{Ans}}}$

f) $(1-5x)^3$

$^3C_0$	$^3C_1$	$^3C_2$	$^3C_3$
1	3	3	1

$= 1 - 15x + 75x^2 - x^3 \rightarrow \underline{\underline{\text{Ans}}}$

g) $(1 + \frac{1}{2}x)^4$

$^4C_0$	$^4C_1$	$^4C_2$	$^4C_3$	$^4C_4$
1	4	6	4	1

$= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4 \rightarrow \underline{\underline{\text{Ans}}}$

h) $(1 - \frac{1}{3}x)^3$

$1 \quad 3 \quad 3 \quad 1$

$$= 1 + 3\left(-\frac{1}{3}x\right)^1 + 3\left(-\frac{1}{3}x\right)^2 + 1\left(-\frac{1}{3}x\right)^3$$

$$= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 \rightarrow \text{Ans.}$$

3. Find the coefficient of the term indicated in square brackets in the expansion of each of the expressions below.

a) $(1+x)^7 \quad [x^6]$

$$\frac{n!}{(n-1)!} = n$$

$$^7C_6 = \frac{7!}{1!\,6!} = 7 \rightarrow \text{Ans}$$

$^7C_0$	$^7C_1$	$^7C_2$	$^7C_3$	$^7C_4$	$^7C_5$	$^7C_6$	$^7C_7$
1	2	3	4	5	6	7	8
	x	x^2	x^3	x^4	x^5	x^6.	

b) $(1+x)^5 \quad [x^2]$

$$^5C_2 = \frac{5!}{3!\,2!} = \frac{5\times4}{2} = 10 \rightarrow \text{Ans}$$

c) $(1+2x)^5 \quad [x^3]$

$$^5C_3 = \frac{5!}{2!\,3!} = \frac{5\times4}{2} = 10$$

$$10(2x)^3 = 10(8x^3) = 80 \rightarrow \text{Ans}$$

d) $(1+5x)^8 \quad [x^2]$

$$^8C_2 = \frac{8!}{6!\,2!} = \frac{8\times7}{2} = \frac{56}{2} = 28$$

$$28(5x)^2 = 28(25x^2) = 700x^2 \rightarrow \text{Ans}$$

e) $(1-3x)^4 \quad [x^3]$

$$^4C_3 = \frac{4!}{1!\,3!} = 4(-3x)^3 = -108x^3 \rightarrow \text{Ans}$$

f) $(1-6x)^7 \quad [x]$

$$^7C_1 = \frac{7!}{6!\,1!} = 7(-6x)^1 = -42x \rightarrow \text{Ans}$$

g) $(1-4x)^4 \quad [x^2]$

$$^4C_2 = \frac{4!}{2!\,2!} = \frac{4\times3}{2} = 6(-4x)^2 = 96x^2 \rightarrow \text{Ans}$$

h) $(1+2x)^5$ $[x^4]$

$^5C_4 = \frac{5!}{1!4!} = 5(2x)^4 = 5(16x^4) = 80x^4 \rightarrow \underline{\underline{Ans}}$

i) $(1-\frac{1}{2}x)^3$ $[x^2]$

$^3C_2 = \frac{3!}{1!2!} = 3(-\frac{1}{2}x)^2 = 3(\frac{1}{4}x^2) = \frac{3}{4}x^2 \rightarrow \underline{\underline{Ans}}$

4. Expand the following $\quad {}^nC_r a^{n-r} b^r$

a) $(2+x)^3$

$= {}^3C_0(2)^3(x)^0 + {}^3C_1(2)^2(x)^1 + {}^3C_2(2)^1(x)^2 + {}^3C_3(2)^0(x)^3$

$= (1 \times 8 \times 1) + (3 \times 4 \times x) + (3 \times 2 \times x^2) + (1 \times 1 \times x^3)$

$= 8 + 12x + 6x^2 + x^3 \rightarrow \underline{\underline{Ans}}$

b) $(3+x)^4$

$= {}^4C_0(3)^4(x)^0 + {}^4C_1(3)^3(x)^1 + {}^4C_2(3)^2(x)^2 + {}^4C_3(3)^1(x)^3 +$
${}^4C_4(3)^0(x)^4$

$= (1 \times 81 \times 1) + (4 \times 27 \times x) + (6 \times 9 \times x^2) + (4 \times 3 \times x^3) + (1 \times 1 \times x^4)$

$= 81 + 108x + 54x^2 + 12x^3 + x^4 \rightarrow \underline{\underline{Ans}}$

c) $(6-5x)^3$

$= {}^3C_0(6)^3(-5x)^0 + {}^3C_1(6)^2(-5x)^1 + {}^3C_2(6)^1(-5x)^2 + {}^3C_3(6)^0(-5x)^3$

$= (1 \times 108 \times 1) + (3 \times 36 \times -5x) + (3 \times 6 \times 25x^2) + (1 \times 1 \times -125x^3)$

$= 108 - 540x + 450x^2 - 125x^3 \rightarrow \underline{\underline{Ans}}$

d) $(2+\frac{1}{2}x)^4$

$= {}^4C_0(2)^4(\frac{1}{2}x)^0 + {}^4C_1(2)^3(\frac{1}{2}x)^1 + {}^4C_2(2)^2(\frac{1}{2}x)^2 + {}^4C_3(2)^1(\frac{1}{2}x)^3$
$+ {}^4C_1(2)^0(\frac{1}{2}x)^4$

$= (1 \times 16 \times 1) + (4 \times 8 \times \frac{1}{2}x) + (6 \times 4 \times \frac{1}{4}x^2) + (4 \times 2 \times \frac{1}{8}x^3)$
$+ (1 \times \frac{1}{16}x^4)$

$= 16 + 16x + 6x^2 + x^3 + \frac{1}{16}x^4 \rightarrow \underline{\underline{Ans}}$

e) $(3x+2y)^3$

$= {}^3C_0(3x)^3(2y)^0 + {}^3C_1(3x)^2(2y)^1 + {}^3C_2(3x)^1(2y)^2 + {}^3C_3(3x)^1(2y)^3$

$= (1 \times 27x^3) + (3 \times 9x^2 \times 2y) + (3 \times 3x \times 4y^2) + (1 \times 8y^3)$

$= 27x^3 + 54x^2y + 36xy^2 + 8y^3 \rightarrow$ Ans

f) $(2x-y)^5$

$= {}^5C_0(2x)^5(-y)^0 + {}^5C_1(2x)^4(-y)^1 + {}^5C_2(2x)^3(-y)^2 + {}^5C_3(2x)^2(-y)^3$
$+ {}^5C_4(2x)^1(-y)^4 + {}^5C_5(2x)^0(-y^5)$

$= (1 \times 32x^5) + (5 \times 16x^4 \times -y) + (10 \times 8x^3 \times y^2) + (10 \times 4x^2 \times -y^3)$
$+ (5 \times 2x \times y^4) + (1 \times -y^5)$

$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5 \rightarrow$ Ans

g) $(2x+5y)^3$

$= {}^3C_0(2x)^3(5y)^0 + {}^3C_1(2x)^2(5y)^1 + {}^3C_2(2x)^1(5y)^2 + {}^3C_3(2x)^0(5y)^3$
$= (1 \times 8x^3) + (3 \times 4x^2 \times 5y) + (3 \times 2x \times 25y^2) + (1 \times 125y^3)$

$= 8x^3 + 60x^2y + 150xy^2 + 125y^3 \rightarrow$ Ans

h) $(3x-4y)^4$

$= {}^4C_0(3x)^4(-4y)^0 + {}^4C_1(3x)^3(-4y)^1 + {}^4C_2(3x)^2(-4y)^2 + {}^4C_3(3x)^1(-4y)^3$
$+ {}^4C_4(3x)^0(-4y)^4$

$= (1 \times 81x^4) + (4 \times 27x^3 \times -4y) + (6 \times 9x^2 \times 16y^2) + (4 \times 3x \times -64y^3)$
$+ (1 \times 256y^4)$

$= 81x^4 - 432x^3y + 864x^2y^2 - 768xy^3 + 256y^4 \rightarrow$ Ans

Q5. Find the co-efficient of the term indicated in square brackets in the expansion of each of the expressions below.

a) $(2+3x)^5$ $[x^3)$

${}^5C_3(2)^2(3x)^3 = 10 \times 4 \times 27x^3 = 1080x^3 \rightarrow 1080 \rightarrow$ Ans

b) $(5+2x)^8$ $[x^6]$

${}^8C_6(5)^2(2x)^6 = 28 \times 25 \times 64x^6 = 44800x^6 \rightarrow 44800 \rightarrow$ Ans

c) $(3+2x)^7$ $[x^5]$

${}^7C_5(3)^2(2x)^5 = 21 \times 9 \times 32x^5 = 6048x^5 \rightarrow 6048 \rightarrow$ Ans

d) $(7-4x)^5$ $[x^4]$

${}^5C_4(7)^1(-4x)^4 = 5 \times 7 \times 256x^4 = 8690x^4 \rightarrow 8690 \rightarrow$ Ans

e) $(2-7x)^4 \quad [x]$

$^4C_1(2)^3(-7x)^1 = 4 \times 8 \times -7x = -224x \rightarrow -224 \rightarrow$ Ans

f) $(5+2x)^4 \quad [x^3]$

$^4C_3(5)^1(2x)^3 = 4 \times 5 \times 8x^3 = 160x^3 \rightarrow 160 \rightarrow$ Ans

g) $\left(\frac{1}{3} + \frac{3}{2}x\right)^6 \quad [x^3]$

$^6C_3\left(\frac{1}{3}\right)^3\left(\frac{3}{2}x\right)^3 = \frac{\cancel{6}\times 5 \times 4}{\cancel{6}} = 20 \times \frac{1}{27} \times \frac{27}{8}x^3 = 2.5x^3 \rightarrow 2.5 \rightarrow$ Ans

h) $\left(\frac{7}{5} + \frac{2}{5}x\right)^3 \quad [x]$

$^3C_1\left(\frac{7}{5}\right)^2\left(\frac{2}{5}x\right)^1 = 3 \times \frac{49}{25} \times \frac{2}{5}x = 2.352x \rightarrow 2.352 \rightarrow$ Ans

6. Expand each of the following in ascending powers of x, up to and including x^3

a) $(1-3x)^5$

$^5C_0(1)^5(-3x)^0 + {}^5C_1(1)^4(-3x)^1 + {}^5C_2(1)^3(-3x)^2 + {}^5C_3(1)^2(-3x)^3$

$= (1 \times 1 \times 1) + (5 \times 1 \times -3x) + (10 \times 1 \times 9x^2) + (10 \times 1 \times -27x^3)$

$= 1 - 15x + 90x^2 - 270x^3 \rightarrow$ Ans

b) $(1+2x)^{10}$

$^{10}C_0(1)^{10}(2x)^0 + {}^{10}C_1(1)^9(2x)^1 + {}^{10}C_2(1)^8(2x)^2 + {}^{10}C_3(1)^7(2x^3)$

$= (1 \times 1 \times 1) + (10 \times 1 \times 2x) + (45 \times 1 \times 4x^2) + (120 \times 1 \times 8x^3)$

$= 1 + 20x + 180x^2 + 960x^3 \rightarrow$ Ans

c) $(1-5x)^7$

$^7C_0(1)^7(-5x)^0 + {}^7C_1(1)^6(-5x)^1 + {}^7C_2(1)^5(-5x)^2 + {}^7C_3(1)^4(-5x)^3$

$= (1 \times 1 \times 1) + (7 \times 1 \times -5x) + (21 \times 1 \times 25x^2) + (35 \times 1 \times -125x^3)$

$= 1 - 35x + 525x^2 - 4375x^3 \rightarrow$ Ans

d) $(2-3x)^5$

$^5C_0(2)^5(-3x)^0 + {}^5C_1(2)^4(-3x)^1 + {}^5C_2(2)^3(-3x)^2 + {}^5C_3(2)^2(-3x)^3$

$= (1 \times 32 \times 1) + (5 \times 16 \times -3x) + (10 \times 8 \times 9x^2) + (10 \times 4 \times -27x^3)$

$= 32 - 240x + 720x^2 - 1080x^3$

e) $(4-x)^3$

$^3C_0(4)^3(-x)^0 + {}^3C_1(4)^2(-x)^1 + {}^3C_2(4)^1(-x)^2 + {}^3C_3(4)^0(-x)^3$

$= (1 \times 64) + (3 \times 16 \times -x) + (3 \times 4 \times x^2) + (1 \times -x^3)$

$= 64 - 48x + 12x^2 - x^3 \rightarrow$ Ans

f) $(2+3x)^4$

$^4C_0(2)^4(3x)^0 + {}^4C_1(2)^3(3x)^1 + {}^4C_2(2)^2(3x)^2 + {}^4C_3(2)^1(3x)^3$

$= (1 \times 16 \times 1) + (4 \times 8 \times 3x) + (6 \times 4 \times 9x^2) + (4 \times 2 \times 27x^3)$

$= 16 + 96x + 216x^2 + 216x^3 \rightarrow$ Ans

7. Expand each of the following in ascending powers of x, up to and including the term in x^2

a) $(2+x)(1+x)^5$

$^5C_0(1)^5(x)^0 + {}^5C_1(1)^4(x)^1 + {}^5C_2(1)^3(x)^2$

$= (1 \times 1 \times 1) + (5 \times 1 \times x) + (10 \times 1 \times x^2)$

$= (1 + 5x + 10x^2)(2 + x)$

$= 2 + 10x + 20x^2 + x + 5x^2 + 10x^3$

$= 2 + 11x + 25x^2 + 10x^3$

Drop this term bc. x^3

$= 2 + 11x + 25x^2 \rightarrow$ Ans

b) $(5-x)(1+x)^4$

$^4C_0(1)^4(x)^0 + {}^4C_1(1)^3(x)^1 + {}^4C_2(1)^2(x)^2$

$= (1 \times 1 \times 1) + (4 \times 1 \times x) + (6 \times 1 \times x^2)$

$= (1 + 4x + 6x^2)(5 - x)$

$= 5 + 20x + 30x^2 - x - 4x^2 - 6x^3$

$= 5 + 19x + 26x^2 - 6x^3$

$= 5 + 19x + 26x^2 \rightarrow$ Ans

c) $(5+4x)(1-2x)^7$

${}^7C_0(1)^7(-2x)^0 + {}^7C_1(1)^6(-2x)^1 + {}^7C_2(1)^5(-2x)^2$

$= (1 \times 1 \times 1) + (7 \times 1 \times -2x) + (21 \times 1 \times 4x^2)$

$= (1 - 14x + 84x^2)(5 + 4x)$
$= 5 - 70x + 420x^2 + 4x - 56x^2 + 336x^3$
$= 5 - 66x + 364x^2 + 336x^3$

$= 5 - 66x + 364x^2 \longrightarrow$ Ans

d) $(6+5x)(3+4x)^4$

${}^4C_0(3)^4(4x)^0 + {}^4C_1(3)^3(4x)^1 + {}^4C_2(3)^2(4x)^2$

$= (1 \times 81) + (4 \times 27 \times 4x) + (6 \times 9 \times 16x^2)$

$= (81 + 432x + 864x^2)(6 + 5x)$

$= 486 + 2592x + 5184x^2 + 405x + 2160x^2 + 4320x^3$
$= 486 + 2997x + 7344x^2 \longrightarrow$ Ans

9. Expand the following

a) $(1+x^3)^4$

$= {}^4C_0(1)^4(x^3)^0 + {}^4C_1(1)^3(x^3)^1 + {}^4C_2(1)^2(x^3)^2 + {}^4C_3(1)^1(x^3)^3 + {}^4C_4(1)^0(x^3)^4$

$= (1 \times 1 \times 1) + (4 \times 1 \times x^3) + (6 \times 1 \times x^6) + (4 \times 1 \times x^9) + (1 \times 1 \times x^{12})$

$= 1 + 4x^3 + 6x^6 + 4x^9 + x^{12} \longrightarrow$ Ans

b) $(1+3x^2)^3$

${}^3C_0(1)^3(3x^2)^0 + {}^3C_1(1)^2(3x^2)^1 + {}^3C_2(1)^1(3x^2)^2 + {}^3C_3(1)^0(3x^2)^3$

$= (1 \times 1 \times 1) + (3 \times 1 \times 3x^2) + (3 \times 1 \times 9x^4) + (1 \times 1 \times 27x^6)$

$= 1 + 9x^2 + 27x^4 + 27x^6 \longrightarrow$ Ans

c) $(3-2x^3)^3$

$= {}^3C_0(3)^3(-2x^3)^0 + {}^3C_1(3)^2(-2x^3)^1 + {}^3C_2(3)^1(-2x^3)^2 + {}^3C_3(3)^0(-2x^3)^3$

$= (1 \times 27 \times 1) + (3 \times 9 \times -2x^3) + (3 \times 3 \times 4x^6) + (1 \times 1 \times -8x^9)$

$= 27 - 54x^3 + 36x^6 - 8x^9 \rightarrow \underline{\underline{\text{Ans}}}$

d) $(1 + x + x^2)^2$

${}^2C_0(1)^2(x+x^2)^0 + {}^2C_1(1)^1(x+x^2)^1 + {}^2C_2(1)^0(x+x^2)^2$

$= (1 \times 1 \times 1) + (2(x+x^2)) + (1(x+x^2)^2)$

$= 1 + 2x + 2x^2 + x^2 + 2x^3 + x^4$

$= 1 + 2x + 3x^2 + 2x^3 + x^4 \rightarrow \underline{\underline{\text{Ans}}}$

13. a) Expand each of the following in ascending powers of x up to and including the term in x^2.

i) $(3 + 2x)^5$

${}^5C_0(3)^5(2x)^0 + {}^5C_1(3)^4(2x)^1 + {}^5C_2(3)^3(2x)^2$

$= (1 \times 243 \times 1) + (5 \times 81 \times 2x) + (10 \times 27 \times 4x^2)$

$= 243 + 810x + 1080x^2 \rightarrow \underline{\underline{\text{Ans}}}$

ii) $(1 - 3x)^5$

${}^5C_0(1)^5(-3x)^0 + {}^5C_1(1)^4(-3x)^1 + {}^5C_2(1)^3(-3x)^2$

$= (1 \times 1 \times 1) + (5 \times 1 \times -3x) + (10 \times 1 \times 9x^2)$

$= 1 - 15x + 90x^2 \rightarrow \underline{\underline{\text{Ans}}}$

b) By first factorising the quadratic $3 - 7x - 6x^2$, deduce the first three terms in the binomial expansion of $(3 - 7x - 6x^2)^5$ → First three terms of this would be lowest powers of x since terms inside are arranged in an ascending order

$-6x^2 - 7x + 3$

$-6x^2 - 9x + 2x + 3$

$-3x(2x+3) + 1(2x+3)$

$= (1 - 3x)(2x + 3)$

$[(1-3x)(2x+3)]^5$

$= (1-3x)^5 \cdot (2x+3)^5$

To get the lowest terms of x for $(3 - 7x - 6x^2)^5$, we have to find the three lowest terms of x for it's factors, and to do that, we need the terms of x inside to be in the ascending order.

$(1-3x)^5$
first three terms (from part a) $= (1 - 15x + 90x^2)$
↳ arranged in ascending powers of x)
$(3+2x) \rightarrow (243 + 810x + 1080x^2$

$(1 - 15x + 90x^2)(243 + 810x + 1080x^2)$

↳ The first three terms of this multiplication are the answer(s).

$= 243 + 810x + 1080x^2 - 3645x - 12150x^2 + 21870x^2$
$= 243 + 810x - 3645x + 1080x^2 - 12150x^2 + 21870x^2$

$= 243 - 2835x + 10770x^2 \rightarrow$ Ans

List of questions to be done:

From worksheets (NG) slide 54

→ After all those done above:
~~17~~, ~~12~~, ~~16~~

From 7- series ApGp :
1, 3, 7, 10, 18, 20, 30, 26, 14, 28, 32, 35, 38, 44, 50, 46, 56,
58, 61, 62, 72, 76, 70

FROM worksheets (NG)

12.a) Expand each of the following up to and including the term in x^2

i) $(1+3x)^4$
$= {}^4C_0(1)^4(3x)^0 + {}^4C_1(1)^3(3x)^1 + {}^4C_2(1)^2(3x)^2$
$= (1 \times 1 \times 1) + (4 \times 1 \times 3x) + (6 \times 1 \times 9x^2)$

$= 1 + 12x + 54x^2 \rightarrow$ Ans

ii) $(1-4x)^4$
$= {}^4C_0(1)^4(-4x)^0 + {}^4C_1(1)^3(-4x)^1 + {}^4C_2(1)^2(-4x)^2$
$= (1 \times 1 \times 1) + (4 \times 1 \times -4x) + (6 \times 1 \times 16x^2)$

$= 1 - 16x + 96x^2 \rightarrow$ Ans

b) By first factorising the quadratic $1 - x - 12x^2$, deduce the first three terms in the binomial expansion of $(1 - x - 12x^2)^4$

$-12x^2 - x + 1$
$= -12x^2 - 4x + 3x + 1$
$= -4x(3x+1) + 1(3x+1)$
$= (1-4x)(1+3x)$

$(1-4x)^4$ first 3 terms $\rightarrow (1 + 12x + 54x^2)$
$(1+3x)^4$ first 3 terms $\rightarrow (1 - 16x + 96x^2)$

$(1 + 12x + 54x^2)(1 - 16x + 96x^2) \rightarrow$ Only multiply for values where the power of $x \leq 2$.

$= 1 - 16x + 96x^2 + 12x - 192x^2 + 54x^2$
$= 1 - 16x + 12x + 96x^2 - 19x^2 + 54x^2$

$= 1 - 4x + 131x^2 \rightarrow$ Ans

16 a) First 3 terms in the binomial expansion of $(1-3x)^8$

${}^8C_0 (1)^8 (-3x)^0 + {}^8C_1 (1)^7 (-3x)^1 + {}^8C_2 (1)^6 (-3x)^2$

$= (1 \times 1 \times 1) + (8 \times 1 \times -3x) + (28 \times 1 \times 9x^2)$

$= 1 - 24x + 252x^2 \rightarrow$ Ans

b) Hence evaluate $(0.997)^8$ correct to 5 decimal places

$0.997 = 1 - 3x$
$3x = 1 - 0.997$
$3x = 0.003$
$x = 0.001$

$(0.997)^3 = 1 - 24(0.001) + 252(0.001)^2$

$= 1 - 0.024 + 0.000252$

$= 0.976252$
$= 0.97625 \rightarrow$ Ans

17. a) First three terms in the binomial expansion of $(2+5x)^9$

${}^9C_0 (2)^9 (5x)^0 + {}^9C_1 (2)^8 (5x)^1 + {}^9C_2 (2)^7 (5x)^2$

$= (1 \times 512 \times 1) + (9 \times 256 \times 5x) + (36 \times 128 \times 25x^2)$

$= 512 + 11520x + 115200x^2 \rightarrow$ Ans

b) Hence evaluate $(2.005)^9$ correct to 2 decimal places

$2 + 5x = 2.005$
$5x = 0.005$
$x = 0.001$

$(2.005)^9 = 512 + 11520(0.001) + 115200(0.001)$
$= 512 + 11.52 + 115.2$
$= 638.72 \rightarrow$ Ans

From the 7-SeriesApGp WS :

~~1~~, ~~3~~, ~~7~~, ~~10~~, ~~18~~, ~~20~~, ~~30~~, ~~26~~, ~~14~~, ~~28~~, ~~32~~, ~~35~~, ~~38~~, ~~44~~, ~~50~~, ~~46~~, ~~56~~, ~~58~~, ~~61~~, ~~62~~, ~~72~~, 76, ~~70~~

1. $\left[x + \frac{3}{x}\right]^4$

$\left[x + 3x^{-1}\right]^4$

$\left[3x^{-1} + x\right]^4$ [independent of x]

↳ r must be such that $n - r = r$ or $2r = n$

$\frac{4}{2} = 2$, hence, $r = 2$

${}^4C_2 \left(\frac{3}{x}\right)^2 (x)^2$

$= 6 \times \frac{9}{x^2} \times x^2$

$= 6 \times 9$
$= 54 \rightarrow$ Ans

3. coefficient of $\frac{1}{x}$ in $\left(2x - \frac{1}{x}\right)^5$

$(2x - 1x^{-1})^5$

$(-1x^{-1} + 2x)^5$, value of r must be such that $n - r = r + 1$

$5 - r = r + 1$
$4 = 2r$
$2 = r$

$^5C_2\left(\frac{-1}{x}\right)^3(2x)^2$

$= 10 \times \frac{-1}{x^3} \times 4x^2$

$= 40 \times \frac{-1}{x^3} \times x^2$

$= 40 \times \frac{-1}{x}$

$= 40 \times -1\left(\frac{1}{x}\right)$

$= -40\left(\frac{1}{x}\right)$ $\therefore$, Ans $= -40$

7. Find coefficients of x^3 in the following expansions:

i) $(1+2x)^6$

$^6C_3(1)^3 + (2x)^3$

$= 40 \times 1 \times 8x^3$

$= 320x^3$

$= 320 \rightarrow$ Ans

ii) $(1-3x)(1+2x)^6$

$^6C_1(2x)^0 + {}^6C_1(2x)^1 + {}^6C_2(2x)^2 + {}^6C_3(2x)^3$

$= 1 + 12x + 60x^2 + 320x^3$

$(1-3x)(1+12x+60x^2+320x^3)$

↳ only concerned with multiplication resulting in x^3 values

$= 320x^3 - 180x^3$

$= 140x^3$

$= 140 \rightarrow$ Ans

10. i) $(2-x)^6$

$^6C_0(2)^6(-x)^0 + {}^6C_1(2)^5(-x)^1 + {}^6C_2(2)^4(-x)^2$

$= (1 \times 64 \times 1) + (6 \times 32 \times -x) + (15 \times 16 \times x^2)$

$= 64 - 192x + 240x^2 \rightarrow$ Ans

ii) $(1+kx)(2-x)^6$, value of k for which there is no term in x^2

$(1+kx)(64-192x+240x^2)$

$(kx)(-192x) = 240x^2$

$-192kx^2 = 240x^2$

$-192k = 240$

$k = \frac{240}{-192}$

$k = -1.5 \rightarrow$ Ans

18. i) $(2+u)^5$

$^5C_0(2)^5(u)^0 + {}^5C_1(2)^4(u)^1 + {}^5C_2(2)^3(u)^2$

$= (1 \times 32) + (5 \times 16 \times u) + (10 \times 8 \times u^2)$

$= 32 + 80u + 80u^2 \rightarrow$ Ans

ii) $u = x + x^2$, find coefficient of x^2 in $(2 + x + x^2)^5$

$(2+x+x^2)^5 = 32 + 80(x + x^2) + 80(x + x^2)^2$

$\hookrightarrow (x^2 + 2x^3 + x^4)$

$= 32 + 80(x + x^2) + 80(x^2 + 2x^3 + x^4)$

only concerned with these 2 terms

$= 80x^2 + 80x^2$

$= 160x^2$

$= 160 \rightarrow$ Ans

20. i) First three terms, $(2 + x^2)^5$

$^5C_0(2)^5(x^2)^0 + {}^5C_1(2)^4(x^2)^1 + {}^5C_2(2)^3(x^2)^2$

$= (1 \times 32 \times 1) + (5 \times 16 \times x^2) + (10 \times 8 \times x^4)$

$= 32 + 80x^2 + 80x^4 \rightarrow$ Ans

ii) $(1+x^2)^2$

$= (1+x^2)(1+x^2)$

$= (1 + 2x^2 + x^4)$

2

$(1 + 2x^2 + x^4)(32 + 80x^2 + 80x^4)$

3

1

$= 80x^4 + 160x^4 + 32x^4$

$= 272x^4$

$= 272 \rightarrow$ Ans

26. i) First three terms, $(2-x)^6$

$^6C_0(2)^6(-x)^0 + {^6C_1}(2)^5(-x)^1 + {^6C_2}(2)^4(-x)^2$

$=(1\times 64)+(6\times 32\times -x)+(15\times 16\times x^2)$

$= 64 - 192x + 240x^2 \rightarrow \underline{\underline{\text{Ans}}}$

ii) $(1+2x+ax^2)(2-x)^6$, coefficient of x^2 is 48, find a.

3

$(1+2x+ax^2)(64-192x+240x^2)$

1

2

$48 = 64a + 240 - 384$
$48 = 64a - 144$
$48+144 = 64a$
$\frac{192}{64} = \frac{64a}{64}$
$3 = a$

$\therefore a = 3 \rightarrow \underline{\underline{\text{Ans}}}$

30. i) First three terms, descending powers of x, $\left(2x-\frac{3}{x}\right)^5$

$^5C_0(2x)^5\left(-\frac{3}{x}\right)^0 + {^5C_1}(2x)^4\left(-\frac{3}{x}\right)^1 + {^5C_2}(2x)^3\left(-\frac{3}{x}\right)^2$

$=(1\times 32x^5)+\left(5\times 16x^4\times \frac{-3}{x}\right)+\left(10\times 8x^3\times\frac{9}{x^2}\right)$

$= 32x^5 - 240x^3 + 720x \rightarrow \underline{\underline{\text{Ans}}}$

ii) $\left(1+\frac{2}{x^2}\right)\left(2x-\frac{3}{x}\right)^5$, coefficient of x

$\left(1+\frac{2}{x^2}\right)(32x^5-240x^3+720x)$

2

1

$= 720x - 480x$
$= 240x$
$= 240 \rightarrow \underline{\underline{\text{Ans}}}$

14. $(2+ax)^n$

$^nC_0(2)^n(ax)^0 + {^nC_1}(2)^{n-1}(ax)^1 + {^nC_2}(2)^{n-2}(ax)^2$

1 1

$32 \qquad -40x \qquad bx^2$

$2^n = 32$

$2^5 = 32$

$n = 5$

${}^5C_1(2)^4(ax)^1$

$= 5 \times 16 \times ax$

$= 80 \times ax$

$-40 = 80 \times a$

$-0.5 = a$

$= {}^5C_2(2)^3(-0.5x)^2$

$= 10 \times 8 \times 0.25x^2$

$= 80 \times 0.25x^2$

$= 20x^2$

$b = 20$

$\therefore,\ n = 5,$

$a = -0.5, \rightarrow$ Ans

$b = 20$

28. First 4 terms, $(k+x)^8$

$${}^8C_0(k)^8(x)^0 + {}^8C_1(k)^7(x)^1 + {}^8C_2(k)^6(x)^2 + {}^8C_3(k)^5(x)^3$$

$$= (1 \times k^8) + (8 \times k^7 \times x) + (28 \times k^6 \times x^2) + (112 \times k^5 \times x^3)$$

$$= k^8 + 8k^7x + 28k^6x^2 + 112k^5x^3 \rightarrow \text{Ans}$$

ii) coefficients of x^2 and x^3 are equal, find k

$28k^6 = 112k^5$

$\dfrac{k^6}{k^5} = \dfrac{112}{28}$

$k = 4$

$\therefore,\ k = 4 \rightarrow$ Ans

32. i) first 3 terms, $(1+ax)^5$

$${}^5C_0(1)^5(ax)^0 + {}^5C_1(1)^4(ax)^1 + {}^5C_2(1)^3(ax)^2$$

$$= (1 \times 1) + (5 \times 1 \times ax) + (10 \times 1 \times a^2x^2)$$

$$= 1 + 5ax + 10a^2x^2 \rightarrow \text{Ans}$$

ii) $(1-2x)(1+ax)^5$, no term in x, find a

$(1-2x)(1 + 5ax + 10a^2x^2)$

1

2

$0 = 5ax - 2x$

$2x = 5ax$

$2 = 5a$

$0.4 = a$

$\therefore,\ a = 0.4 \rightarrow$ Ans

iii) coefficient of x^2

$= (-2x)(5ax) \quad + \quad 10a^2x^2$
$= (2x)(-2x) \quad + \quad 1.6x^2$
$= -4x^2 + 1.6x^2$
$= -2.6x^2$

$= -2.6 \rightarrow$ Ans

35. First 3 terms, desc, $(x - \frac{2}{x})^6$

${}^6C_0(x)^6(-\frac{2}{x})^0 + {}^6C_1(x)^5(-\frac{2}{x})^1 + {}^6C_2(x)^4(-\frac{2}{x})^2$

$= (1 \times x^6) + (6 \times x^5 \times -\frac{2}{x}) + (15 \times x^4 \times \frac{4}{x^2})$

$= x^6 - 12x^4 + 60x^2 \rightarrow$ Ans

iii) $(1+x^2)(x - \frac{2}{x})^6$

$(1+x^2)(x^6 - 12x^4 + 60x^2)$

1

2

$= 60x^4 - 12x^4$
$= 48x^4$
$= 48 \rightarrow$ Ans

38i) First 3 terms, $(1 - 2x^2)^8$

${}^8C_0(1)^8(-2x^2)^0 + {}^8C_1(1)^7(-2x^2)^1 + {}^8C_2(1)^6(-2x^2)^2$

$= (1 \times 1 \times 1) + (8 \times 1 \times -2x^2) + (28 \times 1 \times 4x^4)$

$= 1 - 16x^2 + 112x^4 \rightarrow$ Ans

ii) $(2 - x^2)(1 - 2x^2)^8$

$(2 - x^2)(1 - 16x^2 + 112x^4)$

1

2

$= 16x^4 + 224x^4$

$= 240x^4$

$= 240 \rightarrow$ Ans

44. i) ${}^6C_2(1)^4\left(-\frac{3}{2}x\right)^2$

$= 15 \times 1 \times \frac{9}{4}x^2$

$= 33.75x^2 \rightarrow$ Ans

${}^6C_3(1)^3\left(-\frac{3}{2}x\right)^3$

$= 40 \times 1 \times \frac{-27}{8}x^3$

$= -135x^3 \rightarrow$ Ans

ii) $(k + 2x)(\ldots\ldots\ 33.75x^2 - 135x^3 \ldots\ldots)$

1

2

$0 = 67.5x^3 - 135kx^3$

$135kx^3 = 67.5x^3$

$k = \frac{67.5}{135}$

$k = 0.5$

$\therefore k = 0.5 \rightarrow$ Ans

50. i) $(2-y)^5$

${}^5C_0(2)^5(-y)^0 + {}^5C_1(2)^4(-y)^1 + {}^5C_2(2)^3(-y)^2$

$= (1 \times 32) + (5 \times 16 \times -y) + (10 \times 8 \times y^2)$

$= 32 - 80y + 80y^2 \rightarrow$ Ans

ii) $(2-(2x-x^2))^5$

$y = 2x - x^2$

$32 - \underline{80(2x - x^2)} + \underline{80(2x - x^2)^2}$

↳ potential sources of x^2 ↲

$= 80x^2$

$80(4x^2 - 4x^3 + x^4)$

$= 320x^2$

$80 + 320$

$= 400 \rightarrow$ Ans

46. $(a+x)^5 + (1-2x)^6$, $90x^3$

$(a+x)^5$: x^3 from here means

$^5C_3 (a)^2 (x)^3$

$= 10 \times a^2 \times x^3$

$= 10a^2x^3$

$(1-2x)^6$: $\hookrightarrow$ x^3 from here means

$^6C_3 (1)^3 (-2x)^3$

$= 20 \times 1 \times -8x^3$

$= -160x^3$

$90\cancel{x^3} = 10a^2\cancel{x^3} - 160\cancel{x^3}$

$90 = 10a^2 - 160$

$\frac{250}{10} = a^2$

$25 = a^2$

$5 = a$

$\therefore a = 5 \rightarrow$ Ans

56. $(a+x)^5 + (2-x)^6$, $90x^3$

$^5C_3 (a)^2 (x)^3$

$= 10 \times a^2 \times x^3$

$= 10a^2x^3$

$^6C_3 (2)^3 (-x)^3$

$20 \times 8 \times -x^3$

$-160x^3$

$90 = 10a^2x^3 - 160x^3$

$5 = a \rightarrow$ Ans

58. $(1-4x+4x^2)(1+ax)^6$

$\hookrightarrow$ $^6C_0 (1)^6 (ax)^0 + {}^6C_1 (1)^5 (ax)^1 + {}^6C_2 (1)^4 (ax)^2$

$= 1 + 6ax + 15a^2x^2$

2b 3b 3a 1 2a 3c

$(1-4x+4x^2)(1+6ax+15a^2x^2)$

$1 = 1$

$-x = 6ax - 4x \longrightarrow -x + 4x = 6ax$

$3x = 6ax$

$3 = 6a$

$0.5 = a$

$bx^2 = 15a^2x^2 + 4x^2 - 24ax^2$

$= 15 \times 0.25x^2 + 4x^2 - 12x^2$

$bx^2 = 3.75x^2 + 4x^2 - 12x^2$

$bx^2 = -4.25x^2$

$b = -4.25$

$a = 0.5$ $\Big\} \rightarrow$ Ans

61. i) First 3 terms, $(2x-x^2)^6$

$^6C_0(2x)^6(-x^2)^0 + {}^6C_1(2x)^5(-x^2)^1 + {}^6C_2(2x)^4(-x^2)^2$

$= (1 \times 64x^6) + (6 \times 32x^5 \times -x^2) + (15 \times 16x^4 \times x^4)$

$= 64x^6 - 192x^7 + 240x^8 \rightarrow$ Ans

ii) $(2+x)(64x^6 - 192x^7 + 240x^8)$

2

1

$= 480x^8 - 192x^8$

$= 288x^8$

$= 288 \rightarrow$ Ans

$^nC_r(x)^{n-r}\left(\frac{-a}{x}\right)^r$

62. $(x^2 - ax^{-1})^7$, $-280x^5$

↳ r must be such that

$n-r-r = 5$

$n-2r = 5$

$7-2r = 5$

$7-5 = 2r$

$2 = 2r$

$1 = r$

$-280x^5 = {}^7C_1(x)^6\left(\frac{-a}{x}\right)^1$

$= 7 \times x^6 \times \frac{-a}{x}$

$-280x^5 = -7ax^5$

$-280 = -7a$

$40 = a$

$\therefore$, $a = 40 \rightarrow$ Ans

7. i) First 3 terms, $(2+3x)^6$

$^6C_0(2)^6(3x)^0 + {}^6C_1(2)^5(3x)^1 + {}^6C_2(2)^4(3x)^2$

$= (1 \times 64) + (6 \times 32 \times 3x) + (15 \times 16 \times 9x^2)$

$= 64 + 576x + 2160x^2 \rightarrow$ Ans

ii) $(1+ax)(64 + 576x + 2160x^2)$

$0 = 576ax^2 + 2160x^2$

$-2160\cancel{x^2} = 576a\cancel{x^2}$

$\frac{-2160}{576} = \frac{576a}{576}$

$\therefore$, $a = -3.75 \rightarrow$ Ans

$-3.75 = a$

70. i) First three terms, $(2+ax)^5$

$${}^5C_0(2)^5(ax)^0 + {}^5C_1(2)^4(ax)^1 + {}^5C_2(2)^3(ax)^2$$

$$= (1 \times 32) + (5 \times 16 \times ax) + (10 \times 8 \times a^2x^2)$$

$$= 32 + 80ax + 80a^2x^2 \longrightarrow \text{Ans}$$

ii) $(1+2x)(32+80ax+80a^2x^2)$, $240x^2$

$$240 = 160a + 80a^2$$
$$0 = 80a^2 + 160a - 240$$
$$0 = 8a^2 + 16a - 24$$
$$0 = a^2 + 2a - 3$$
$$= a^2 + 3a - a - 3$$
$$= a(a+3) - 1(a+3)$$
$$= (a-1)(a+3)$$

$\therefore$, $a = 1$ or $a = -3 \longrightarrow$ Ans

$a = 1$ or $a = -3$

76. i) $(x+3x^2)^4$ ${}^4C_r(x)^{4-r}(3x^2)^r$

r must be such that $4 - r + 2r = 8$

$$4 + r = 8$$
$$r = 4$$

$${}^4C_4(x)^0(3x^2)^4$$
$$= 1 \times 1 \times 81x^8$$
$$= 81x^8$$
$$= 81 \longrightarrow \text{Ans}$$

ii) $(x+3x^2)^5$ ${}^5C_r(x)^{5-r}(3x^2)^r$

$$5 - r + 2r = 8$$
$$5 + r = 8$$
$$r = 3$$

$${}^5C_3(x)^2(3x^2)^3$$
$$= 10 \times x^2 \times 27x^6$$
$$= 270x^8$$
$$= 270 \longrightarrow \text{Ans}$$

iii) ${}^nC_r(1)^{n-r}(x+3x^2)^r$

↳ raised to the fourth power $= 81x^8 \times {}^5C_4 = 5$

↳ raised to the fifth power $= 270x^8 \times {}^5C_5 = 1$

$$(81 \times 5) + 270$$
$$= 405 + 270$$
$$= 675 \longrightarrow \text{Ans}$$