RELATED RATES Questions on slide 68 and 78 of worksheets (NG)

- Basically $\frac{dy}{dx}$ equations that are related to each other

Example : Slide 78 worksheets (NG) → ~~1~~, ~~3~~, ~~4~~, ~~8~~

Q1. $\frac{dl}{dt} = 0.05$ $\quad \frac{dR}{dt} = ?$ $\quad \frac{dR}{dl} = -\frac{10}{l^2}$

The three related rates

$$\frac{dl}{dt} \times \frac{dR}{dl} = \frac{dR}{dt}$$

$$0.05 \times \left(-\frac{10}{l^2}\right) = \frac{dR}{dt}$$

$$\frac{-0.5}{l^2} = \frac{dR}{dt}$$ → the gradient fuction for the graph of R against time

$R = \frac{10}{l}$

$5 = \frac{10}{l}$

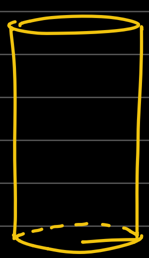
$l = 2$

$\frac{-0.5}{2^2}$ = rate of change of R

$\frac{-0.5}{4} = -0.125$ → Ans

Rate of change of R when R = 5

Q3.



10 cm

base Area = πr^2

$= 100\pi$

Volume = (height)(100π)

$$\frac{360}{100\pi} = \frac{(100\pi)H}{100\pi}$$

$1.15\text{cm} = H$

$$\frac{dH}{dt} = 1.15\text{cm}$$

$$\frac{40}{100\pi} = \frac{(100\pi)H}{100\pi}$$

$0.127 = H_L$

$$\frac{dH_L}{dt} = 0.127\text{cm}$$

Every second, water rises 1.15cm and falls 0.127cm simultaneously

$+1.15 - 0.127 = 1.023$ cm

↳ Ans: The water level is changing at 1.02 cm/s

Different way of doing it

(Gain) $\frac{dV}{dt} = 360$ (Net) $\boxed{\frac{dV}{dt} = 320}$

$\boxed{\frac{dH}{dt} = ??}$

(Loss) $\frac{dV}{dt} = 40$

$V = \pi r^2 H$

$V = 100\pi H$

$\boxed{\frac{dV}{dH} = 100\pi}$

$$\frac{dH}{dt} \times \frac{dV}{dH} = \frac{dV}{dt}$$

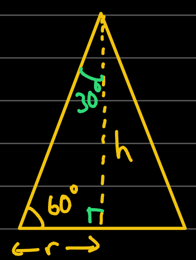
$$\frac{dH}{dt} \times 100\pi = 320$$

$$\frac{dH}{dt} = \frac{320}{100\pi}$$

$$\frac{dH}{dt} = 1.018$$

$= 1.02$ cm per second → Roughly the same answer as previous approach

Q4.



a) $\tan 30° = \frac{1}{\sqrt{3}}$ $\tan 30° = \frac{r}{h}$

$$\frac{1}{\sqrt{3}} = \frac{r}{h}$$

$$\frac{h}{\sqrt{3}} = r$$ → shown

b) Volume of cone $= \frac{\pi r^2 h}{3}$

$$= r^2 \cdot \frac{\pi h}{3}$$

$$= \left(\frac{h}{\sqrt{3}}\right)^2 \cdot \frac{\pi h}{3}$$

$$= \frac{h^2}{3} \cdot \frac{\pi h}{3}$$

$$V = \frac{\pi h^3}{9} \rightarrow \text{shown}$$

c) Rate at which height of pile is increasing when h = 20cm.

$$\frac{dV}{dh} = \frac{3(\pi)}{9} h^2 \qquad \text{(Given)} \boxed{\frac{dV}{dt} = 1000}$$

$$\boxed{\frac{dV}{dh} = \frac{\pi h^2}{3}} \qquad \boxed{\frac{dh}{dt} = ??}$$

$$\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt}$$

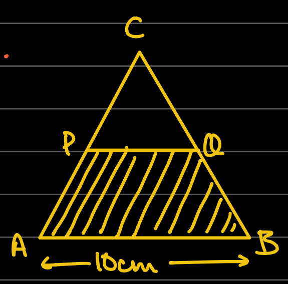
$$\frac{\pi h^2}{3} \times \frac{dh}{dt} = 1000$$

$$\left.\frac{dh}{dt}\right|_{h=20} = \frac{3000}{\pi h^2} = \frac{3000}{\pi 20^2}$$

$$= \boxed{7.44 \text{ cm/s}} \rightarrow \text{Ans}$$

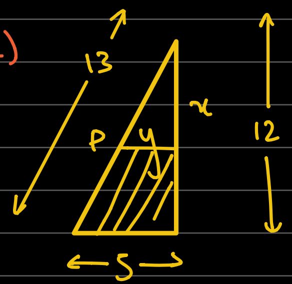
Rate of change of height when height = 20 cm

Q5.



AC = BC = 13cm

a)



Ratio of $\frac{5}{12}$ remains constant because similar triangles

$$PQ = 2y$$

$$\frac{y}{x} = \frac{5}{12}$$

$$y = \frac{5x}{12}$$

$$PQ = 2\left(\frac{5x}{12}\right)$$

$PQ = \frac{5x}{6} \longrightarrow$ shown

b) Shaded Area

$= A_{ABC} - A_{PQC}$

$= \frac{10 \times 12}{2} - \frac{5x^2}{12}$

$= \frac{120}{2} - \frac{5x^2}{12}$

$= \frac{720}{12} - \frac{5x^2}{12}$

$A = \frac{5}{12}(144 - 5x^2) \longrightarrow$ shown

$A = 60 - \frac{5}{12}x^2$

$\frac{dA}{dx} = -\frac{5}{6}x$

c) $\boxed{\frac{dA}{dx} = -\frac{5x}{6}}$ $\boxed{\frac{dA}{dt} = ??}$

$\boxed{\frac{dx}{dt} = 0.5}$

$\frac{dA}{dx} \times \frac{dx}{dt} = \frac{dA}{dt}$

$-\frac{5x}{6} \times 0.5 = \frac{dA}{dt}$

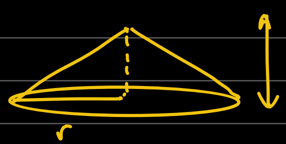
$-\frac{2.5x}{6} = \frac{dA}{dt}$

$\left.\frac{dA}{dt}\right|_{x=6} = -\frac{2.5(6)}{6} = -2.5\,cm^2/s$

Ans
Rate of decrease of shaded area

From slide 68 worksheets (NG)

Q1. $\boxed{\frac{dV}{dt} = 4}$ $\boxed{\frac{dr}{dt} = ??}$



$\frac{3r}{4}$

r

$V = \frac{1}{3} \cdot \pi r^2 \cdot \frac{3r}{4}$

$= \frac{\pi r^2 \cdot 3r}{12}$

$= \frac{3\pi}{12} \times r^3$

$V = \frac{\pi}{4} r^3$

$\boxed{\frac{dV}{dr} = \frac{3\pi r^2}{4}}$

$\frac{dr}{dt} \times \frac{dV}{dr} = \frac{dV}{dt}$

$\frac{dr}{dt} \times \frac{3\pi r^2}{4} = 4$

$\left.\frac{dr}{dt}\right|_{r=4} = \frac{16}{3\pi r^2}$

$= \frac{16}{3\pi(4)^2}$

$= \frac{\cancel{16}}{3\pi \times \cancel{16}}$

$\frac{dr}{dt} = \frac{1}{3\pi} \rightarrow$ Ans

Rate of change

of radius

Q2. $pv = 240$

$p = 240v^{-1}$

$\boxed{\frac{dp}{dv} = \frac{-240}{v^2}}$

when $p = 60$, $\boxed{\frac{dp}{dt} = 5}$

$\boxed{\frac{dv}{dt} = ??}$ when $p = 60$?

$pv = 240$

$\frac{dv}{dt} \times \frac{dp}{dv} = \frac{dp}{dt}$

$60v = 240$

$v = 4$

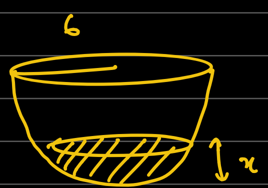
$\frac{dv}{dt} \times \frac{-240}{v^2} = 5$

$$\left.\frac{dv}{dt}\right|_{v=4} = \frac{5v^2}{-240} = \frac{5(16)}{-240}$$

$$= -\frac{80}{240}$$

$$= -\frac{1}{3}$$ Ans

Q3.



$\boxed{\frac{dV}{dt} = 3}$

$V = \frac{1}{3}\pi x^2(18 - x)$

$\boxed{\frac{dx}{dt} = ??}$

$V = \frac{\pi}{3}(18x^2 - x^3)$

$V = \frac{18\pi x^2}{3} - \frac{\pi}{3}x^3$

$\frac{dV}{dx} \times \frac{dx}{dt} = \frac{dV}{dt}$

$\boxed{\frac{dV}{dx} = 12\pi x - \pi x^2}$

$(12\pi x - \pi x^2)\frac{dx}{dt} = 3$

$$\left.\frac{dx}{dt}\right|_{x=2} = \frac{3}{12\pi x - \pi x^2}$$

$$= \frac{3}{24\pi - 4\pi}$$

$$= \frac{3}{20\pi}$$

$$= 0.0477$$

$= 0.048 \text{ cm/s}$

$\downarrow$

Ans

Q4. $\boxed{\frac{dA}{dt} = 5}$ $\boxed{\frac{dr}{dt} = ??}$

$A = \pi r^2$

$\boxed{\frac{dA}{dr} = 2\pi r}$

a) $20 \times 5 = 100 \text{ cm}^2$

$100 = \pi r^2$

$\frac{100}{\pi} = r^2$

$\frac{10}{\sqrt{\pi}} = r$

b) $\frac{dA}{dr} \times \frac{dr}{dt} = \frac{dA}{dt}$

$2\pi r \times \frac{dr}{dt} = 5$

$\frac{dr}{dt}\Big|_{r = \frac{10}{\sqrt{\pi}}} = \frac{5}{2\pi r} = \frac{5\sqrt{\pi}}{2\pi 10}$

$= \frac{5}{20} \cdot \frac{\cancel{\sqrt{\pi}}}{(\cancel{\sqrt{\pi}})(\sqrt{\pi})}$

$= \frac{5}{20\sqrt{\pi}} \rightarrow$ Ans