TRIG + FUNCTIONS

Questions to attempt:

From worksheets (NG) Slide 100 onwards:

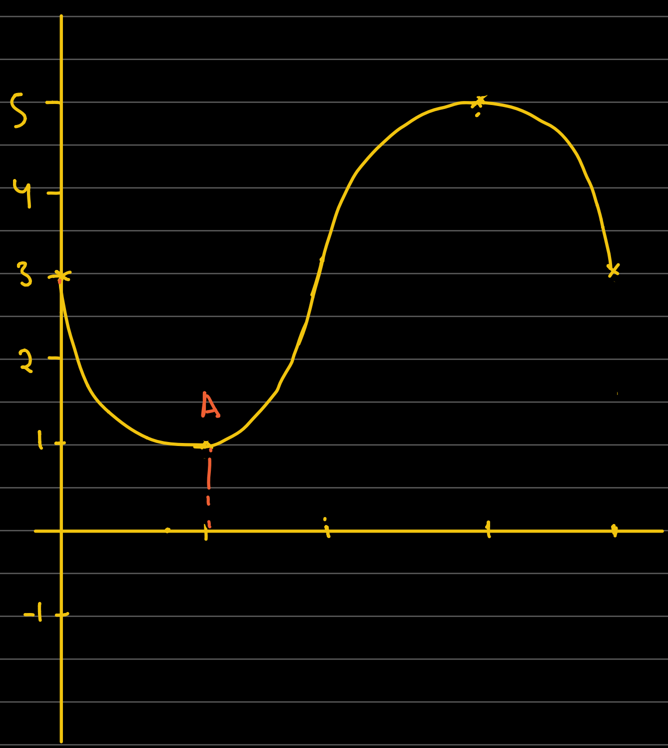
~~8~~, ~~12~~, ~~20~~, ~~21~~, ~~24~~, ~~25~~, 28, 33, 45, 52

8. $f(x) = 3 - 2\sin x \quad , \quad 0 \leq x \leq 360$

i) Range of $f(x) \rightarrow$ Use the "fixing the range method"

$$-1 \leq \sin x \leq 1$$
$$-2 \leq 2\sin x \leq 2$$
$$2 \geq -2\sin x \geq -2$$
$$5 \geq 3 - 2\sin x \geq 1$$
$$1 \leq 3 - 2\sin x \leq 5$$
$$1 \leq f(x) \leq 5 \longrightarrow \text{Ans i) Range of } f(x)$$

ii) $y = f(x)$



iii) $g(x) = 3 - 2\sin x$ for $0 \leq x \leq A$, A constant

Largest value of A for which $g(x)$ has an inverse.

Ans $\longrightarrow$ Largest value = $90°$

iv) Expression for $g^{-1}(x)$

$$y = 3 - 2\sin x$$
$$\frac{2\sin x}{2} = \frac{3-y}{2}$$
$$\sin x = \frac{3-y}{2}$$
$$x = \sin^{-1}\left(\frac{3-y}{2}\right)$$

$$g^{-1}(x) = \sin^{-1}\left(\frac{3-x}{2}\right)$$
Ans iv)
Expression for $g^{-1}(x)$

12. $f(x) = a + b\cos 2x, \quad 0 < x < \pi$, $\quad f(0) = -1$, $f\left(\frac{\pi}{2}\right) = 7$

i) Find a and b

$$f(0) = -1$$
$$-1 = a + b\cos(0)$$
$$-1 = a + b(1)$$
$$-1 = a + b$$
$$-1 = 7 + b + b$$
$$-1 = 7 + 2b$$
$$-8 = 2b$$
$$-4 = b$$

$$f\left(\frac{\pi}{2}\right) = 7$$
$$7 = a + b\cos\left(2\frac{\pi}{2}\right)$$
$$7 = a + b\cos(\pi)$$
$$7 = a - b$$
$$7 + b = a$$
$$7 + (-4) = a$$
$$3 = a$$

$\therefore$ $a = 3$, $b = -4$ $\rightarrow$ Ans (i)

ii) $0 = 3 - 4\cos 2x$

$$\frac{-3}{-4} = \cos 2x$$
$$\cos 2x = \frac{3}{4}$$
$$\cos\theta = \frac{3}{4}$$
$$\alpha = \cos^{-1}\left(\frac{3}{4}\right)$$

$\theta = 0.723 \,/\, 2 \longrightarrow 0.3615 < 3.14$

$\theta = 5.56 \,/\, 2 \longrightarrow 2.78 < 3.14$

$\left[\begin{array}{l} x = 0.362 \\ x = 2.78 \end{array}\right] \rightarrow$ Ans (ii)

5.56

20 $f(x) = 5 - 3\sin 2x$ for $0 \le x \le \pi$

i) Find range of f

$-1 \le \sin x \le 1$

Affects period, doesn't affect range / amplitude

$-1 \le \sin 2x \le 1$

$-3 \le 3\sin 2x \le 3$

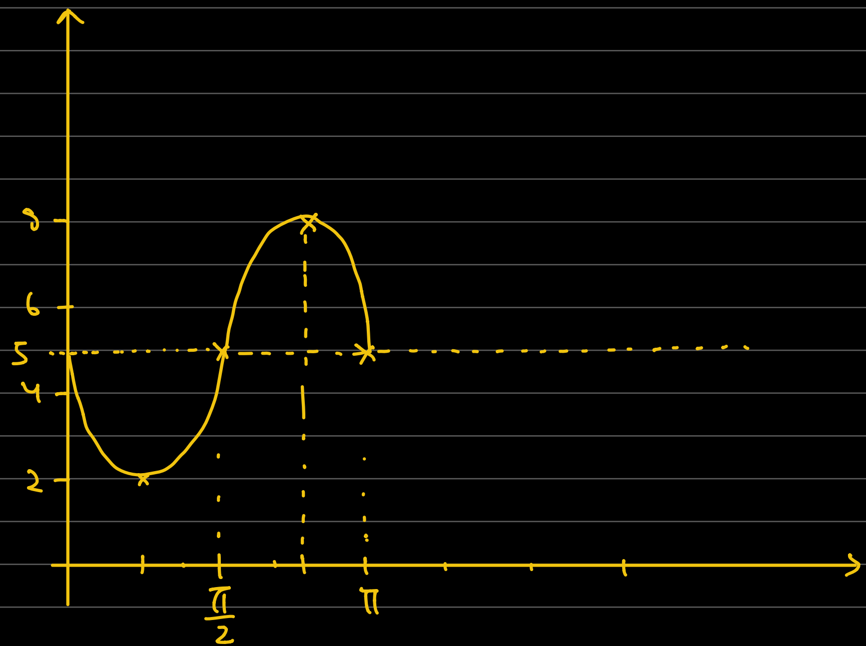
$3 \ge -3\sin 2x \ge -3$

$8 \ge 5 - 3\sin 2x \ge 2$

$2 \le 5 - 3\sin 2x \le 8$

$2 \le f(x) \le 8 \longrightarrow$ Ans (i)

ii)



iii) $f(x)$, for the defined domain, does not have an inverse because it is not a 1 to 1 function and does not pass the horizontal line test.

21. $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \le x \le \pi$

i) $a + b\cos^2 x$

$= 2(1-\cos^2 x) - 3\cos^2 x$

$= 2 - 2\cos^2 x - 3\cos^2 x$

$= 2 - 5\cos^2 x \longrightarrow$ Ans(i) $\longleftarrow \left[\begin{array}{l} a = 2 \\ b = -5 \end{array} \right.$

(ii) $-1 \le \cos x \le 1$

$0 \le \cos^2 x \le 1$

$0 \geq -5\cos^2 x \geq -5$

$2 \geq 2 - 5\cos^2 x \geq -3$

$-3 \le 2 - 5\cos^2 x \le 2$

$\therefore$, greatest value $= -3$
least value $= 2$
$\hookrightarrow$ Ans (ii)

(iii) $f(x) + 1 = 0$

$3 - 5\cos^2 x = 0$

$$\frac{3}{5} = \frac{5\cos^2 x}{5}$$

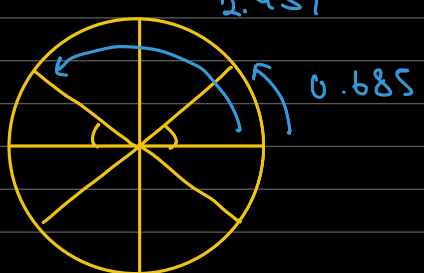
$$\sqrt{\frac{3}{5}} = \cos x$$

$$\cos \alpha = \pm\sqrt{\frac{3}{5}}$$

$$\alpha = \cos^{-1}\left(\sqrt{\frac{3}{5}}\right)$$

$$\alpha = 0.685$$

2.457



$\left[\begin{array}{l} x = 2.46 \\ x = 0.685 \end{array} \right] \longrightarrow$ Ans (iii)

24 $f(x) = 4 - 3\sin x$, $0 \le x \le 2\pi$

i) $f(x) = 2$

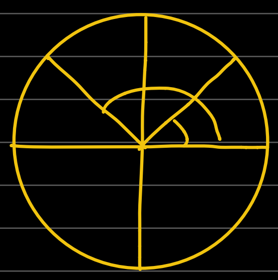
$2 = 4 - 3\sin x$

$3\sin x = 2$

$\sin x = \frac{2}{3}$

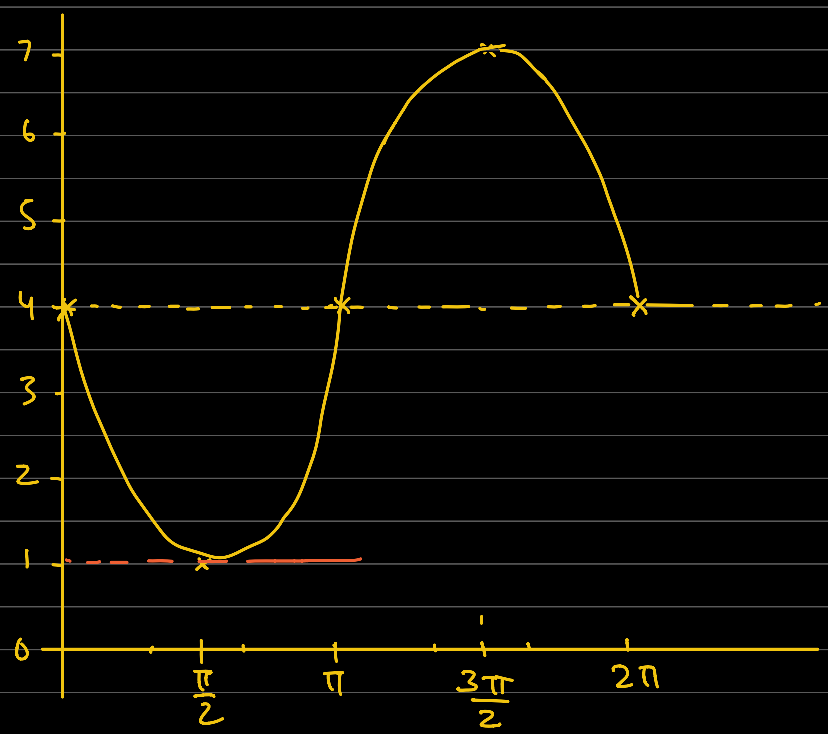
$\alpha = \sin^{-1}\left(\frac{2}{3}\right)$

$\alpha = 0.730$



$\left[\begin{array}{l} x = 0.730 \\ x = 2.41 \end{array}\right] \rightarrow$ Ans (i)

ii) $1 \leq f(x) \leq 7$



iii) $k < 1$, $k > 7$ $\rightarrow$ Ans (ii)

iv) $g(x) = 4 - 3\sin x$ for $\frac{1}{2}\pi \leq x \leq A$

largest value for which $g(x)$ has an inverse $\rightarrow A = \frac{3\pi}{2} \rightarrow$ Ans (iv)

v) $y = 4 - 3\sin x$

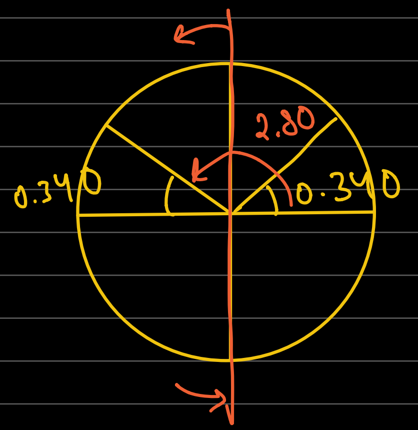
$3\sin x = 4 - y$

$x = \sin^{-1}\left(\frac{4-y}{3}\right)$

$g^{-1}(x) = \sin^{-1}\left(\frac{4-x}{3}\right)$

$g^{-1}(3) = \sin^{-1}\left(\frac{4-3}{3}\right)$

$\alpha = \sin^{-1}\left(\frac{1}{3}\right)$



$\theta = 2.80 \rightarrow$ Ans (v)

25. $f(x) = a + b\cos x$ for $0 \leq x \leq 2\pi$ $f(0) = 10$, $f\left(\frac{2\pi}{3}\right) = 1$

i) values of a and b

$f(0) = 10$

$10 = a + b\cos(0)$

$10 = a + b$

$10 = 1 + 0.5b + b$

$\frac{9}{1.5} = \frac{1.5b}{1.5}$

$6 = b$

$f\left(\frac{2}{3}\pi\right) = 1$

$1 = a + b\cos\left(\frac{2}{3}\pi\right)$

$1 = a + b(-0.5)$

$1 = a - 0.5b$

$1 + 0.5b = a$

$1 + 0.5(6) = a$

$1 + 3 = a$

$4 = a$

$\therefore$, $a = 4$ → Ans i)
$b = 6$

ii) Range of f $\longrightarrow 4 + 6\cos x$

$-1 \leq \cos x \leq 1$

$-6 \leq 6\cos x \leq 6$

$-2 \leq 4 + 6\cos x \leq 10$

$-2 \leq f(x) \leq 10 \longrightarrow$ Ans (ii)

iii) $f(\frac{5}{6}\pi) = 4 + 6\cos(\frac{5}{6}\pi)$

$= 4 + 6(-\frac{\sqrt{3}}{2})$

$= 4 - 3\sqrt{3}$

↳ Ans (iii)

$\cos(x) = -\cos(180-x)$

$\cos(\frac{5}{6}\pi) = -\cos(\frac{1}{6}\pi)$

$\cos(150^\circ) = -\cos(30^\circ)$

$\cos(\frac{5}{6}\pi) = -\frac{\sqrt{3}}{2}$

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