↳ A theoretical distribution

BINOMIAL DISTRIBUTION : STATS AND PROBABILITY

Started on 9th March, 2021

Example of a situation that can be modelled using binomial distribution

Student A has a 0.4 chance of passing any test.

X = a discrete random variable
X = the number of tests passed

Q(i) What's the probability that the student passes exactly 7 of the next 10 tests?

All possible values x can take $\longrightarrow 0 \leq x \leq 10$

$X \sim B(10, 4)$

X = no. of successful "trials"
n = no. of trials
p = probability of a successful trial
q = probability of a failure

$X \sim B(n, p)$

Formula :

$$P(X = x) = {}^{n}C_{x}\, q^{n-x}\, p^{x}$$

↳ Probability that out of n total trials, x trials are successful given a success rate of p and a failure rate of q

$$P(X = 7) = {}^{10}C_{7}\, q^{3}\, p^{7}$$

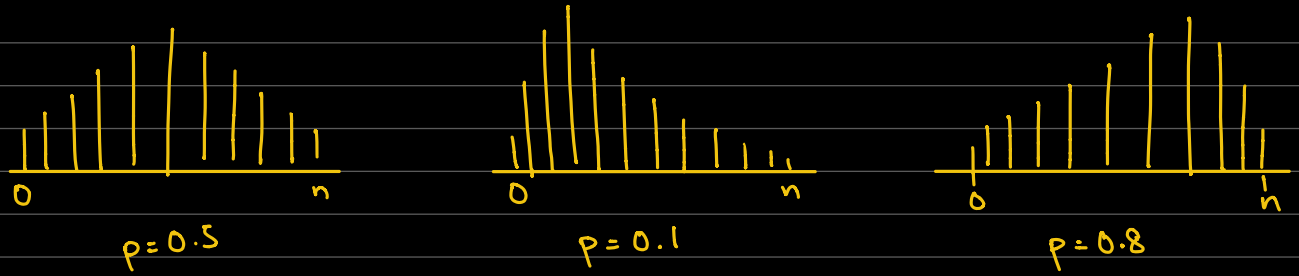
$$= {}^{10}C_{7}\,(0.6)^{3}\,(0.4)^{7}$$

$$= 0.042$$

= 4.2% chance that the student passes 7 out of the next 10 tests

Features of a situation that allow it to be modelled using binomial distribution:

1. The random variable X, must be discrete
   The no. of successful trials must be a discrete number
2. There must only be two outcomes : success or failure
3. The probability of success or failure must remain the same from trial to trial



- The two parameters that determine the binomial distribution are:

1. n = the total number of trials
2. p = the probability of a trial being successful

Two more things that we can determine from a binomial distribution

1. the mean (or avg.)

$$E(X) = np$$

2. the variance

$$\text{var}(X) = npq$$

3. Standard Deviation

$$SD(X) = \sqrt{npq}$$

Q(ii) Calculate the probability that the student passes at least 8 of the next 10 tests.

$$P(X \geq 8) = P(8) + P(9) + P(10)$$
$$= {}^{10}C_8 (0.6)^2 (0.4)^8 + {}^{10}C_9 (0.6)^1 (0.4)^9 + {}^{10}C_{10} (0.4)^{10}$$

$$= 0.01229$$
$$= 1.23\%$$ chance that the student passes at least 8 (8 or 9 or 10) of the next 10 tests

Q7 from Binomial Distribution Worksheet

X = no. of cracked eggs

$p = 0.04 \rightarrow$ chance that the egg is cracked
$q = 0.96 \rightarrow$ chance that the egg is not cracked

$X \sim B(12, 0.04)$

$P(x) = {}^{12}C_2 (0.96)^{10} (0.04)^2$

$= 0.07$

$= 7\%$ chance that exactly two of the 12 eggs chosen are cracked

Practice Qs from Binomial Worksheet

ExA (5-10)
ExB (2-6)