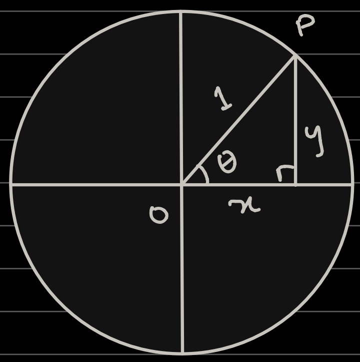
Trigonometry

$\boxed{}$ = identities

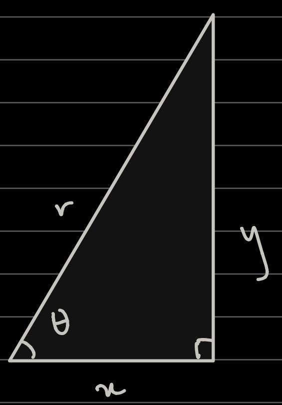


$$x^2 + y^2 = 1^2$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos\theta = x$$
$$\sin\theta = y$$



$$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$$

$$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$$

$$\tan\theta = \frac{y}{x} \rightarrow \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

OR

$$(\tan\theta)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$\cos\theta$

Ex 20. m) $(1+\tan^2\theta)(1-\sin^2\theta) \equiv 1$

(a) $(1+\tan^2\theta)(\cos^2\theta) \equiv 1$

(b) $\dfrac{(1-\cos^2\theta)}{\tan^2\theta} \equiv \cos^2\theta$

(c) $\dfrac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

(d) $\dfrac{1}{\cos^2\theta} - \tan^2\theta \equiv 1$

(e) $\tan^2\theta - \sin^2\theta \equiv \dfrac{\sin^4\theta}{\cos^2\theta}$

f) $\sin^4\theta - \cos^4\theta \equiv 1 - 2\cos^2\theta$

g) $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 \equiv 2$

h) $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$

i) $\dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{1}{\tan\theta}$

j) $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\tan\theta\sin\theta}$

k) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} \equiv \dfrac{2}{\cos^2\theta}$

l) $(\tan\theta + \sin\theta)(\tan\theta - \sin\theta) \equiv \tan^2\theta\sin^2\theta$

n) $\dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} \equiv 1 + \sin\theta\cos\theta$

o) $\dfrac{1+\sin\theta+\cos\theta}{\cos\theta} \equiv \dfrac{1-\sin\theta+\cos\theta}{1-\sin\theta}$

Trig functions practice → from attached screenshot

a. $(1+\tan^2\theta)(\cos^2\theta) \equiv 1$

$= \left(1 + \dfrac{\sin^2\theta}{\cos^2\theta}\right)(\cos^2\theta)$

$= \cos^2\theta + \sin^2\theta$

$= \cos^2\theta + 1 - \cos^2\theta$

$= 1 \longrightarrow$ shown

b. $\dfrac{(1-\cos^2\theta)}{\tan^2\theta} \equiv \cos^2\theta$

$= \left(\dfrac{\sin^2\theta}{\tan^2\theta}\right)$

$= \dfrac{\sin^2\theta}{\dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\sin^2\theta}{\sin^2\theta} \times \cos^2\theta$

$= 1 \times \cos^2\theta$

$= \cos^2\theta \longrightarrow$ shown.

c. $\dfrac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

$= \dfrac{1-\cos^2\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \sin\theta$

$= \tan\theta\sin\theta \longrightarrow$ shown

d. $\dfrac{1}{\cos^2\theta} - \tan^2\theta \equiv 1$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1-\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

$= 1 \longrightarrow$ shown

e. $\tan^2\theta - \sin^2\theta = \dfrac{\sin^4\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$

$= \dfrac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta}$

$= \dfrac{\sin^4\theta}{\cos^2\theta} \longrightarrow$ shown

f. $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

$= (\sin^2\theta)^2 - (\cos^2\theta)^2 \longrightarrow a^2 - b^2 = (a+b)(a-b)$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= (1 - \cos^2\theta + \cos^2\theta)(1 - \cos^2\theta - \cos^2\theta)$

$= (1)(1 - 2\cos^2\theta)$

$= 1 - 2\cos^2\theta \rightarrow$ shown

g. $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 \equiv 2$

$= (\cos\theta + \sin\theta)(\cos\theta + \sin\theta) + (\cos\theta - \sin\theta)(\cos\theta - \sin\theta)$

$= \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2(1)$

$= 2 \longrightarrow$ shown

h. $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$

$= \sin\theta - \sin^2\theta\cos\theta + \cos\theta - \sin\theta\cos^2\theta$

$= \sin\theta - \cos\theta\sin^2\theta + \cos\theta - \sin\theta\cos^2\theta$

$= \sin\theta - \cos\theta(1 - \cos^2\theta) + \cos\theta - \sin\theta(1 - \sin^2\theta)$

$= \cancel{\sin\theta} - \cancel{\cos\theta} + \cos^3\theta + \cancel{\cos\theta} - \cancel{\sin\theta} + \sin^3\theta$

$= \cos^3\theta + \sin^3\theta \longrightarrow$ shown

i. $\dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{1}{\tan\theta}$

$$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{1+\cos\theta - \sin^2\theta}{\sin\theta + \cos\theta\sin\theta}$$

$$= \frac{1-\sin^2\theta + \cos\theta}{\sin\theta + \cos\theta\sin\theta}$$

$$= \frac{\cos^2\theta + \cos\theta}{\sin\theta + \cos\theta\sin\theta}$$

$$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} \div \frac{\cos\theta}{\cos\theta}$$

$$= \frac{1}{\tan\theta} \rightarrow \text{shown}$$

j. $$\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = \frac{4}{\tan\theta\sin\theta}$$

$$= \frac{(1+\cos\theta)^2 - (1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+2\cos\theta+\cos^2\theta) - (1-2\cos\theta+\cos^2\theta)}{1-\cos^2\theta}$$

$$= \frac{\cancel{1} + 2\cos\theta + \cancel{\cos^2\theta} \cancel{-1} + 2\cos\theta - \cancel{\cos^2\theta}}{1-\cos^2\theta}$$

$$= \frac{4\cos\theta}{\sin^2\theta}$$

$$= 4 \times \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}$$

$$= \frac{4}{\tan\theta\sin\theta} \rightarrow \text{shown}$$

k. $$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{2}{\cos^2\theta}$$

$$= \frac{1-\sin\theta + 1+\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta} \longrightarrow \text{shown}$$

l. $(\tan\theta + \sin\theta)(\tan\theta - \sin\theta) \equiv \tan^2\theta \sin^2\theta$

$$= \tan^2\theta - \cancel{\sin\theta\tan\theta} + \cancel{\sin\theta\tan\theta} - \sin^2\theta$$

$$= \tan^2\theta - \sin^2\theta$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{(\sin^2\theta)(\sin^2\theta)}{\cos^2\theta}$$

$$= \tan^2\theta\sin^2\theta \longrightarrow \text{shown}$$

n. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} \equiv 1 + \sin\theta\cos\theta$

$$= \frac{\sin\theta\sin^2\theta - \cos\theta\cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-\cos^2\theta) - \cos\theta(1-\sin^2\theta)}{\sin\theta - \cos\theta}$$

$\sin^2\theta\cos\theta - \cos^2\theta\sin\theta$

$$= \frac{\sin\theta - \cos^2\theta\sin\theta - \cos\theta + \sin^2\cos\theta}{\sin\theta - \cos\theta}$$

$\sin\theta\cos\theta(\sin\theta - \cos\theta)$

$$= \frac{\sin\theta - \cos\theta - \cos^2\theta\sin\theta + \sin^2\theta\cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta) + \sin\theta\cos\theta(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta}$$

$$= \frac{\cancel{(\sin\theta - \cos\theta)}(1 + \sin\theta\cos\theta)}{\cancel{\sin\theta - \cos\theta}}$$

$$= 1 + \sin\theta\cos\theta \longrightarrow \text{shown}$$

0. $\dfrac{1+\sin\theta+\cos\theta}{\cos\theta} \equiv \dfrac{1-\sin\theta+\cos\theta}{1-\sin\theta}$

$= \dfrac{(1+\sin\theta+\cos\theta)(\cos\theta)}{\cos^2\theta}$

$= \dfrac{\cos\theta+\sin\theta\cos\theta+\cos^2\theta}{1-\sin^2\theta}$

$= \dfrac{\cos\theta+\sin\theta\cos\theta+1-\sin^2\theta}{1-\sin^2\theta} \longrightarrow (1+\sin\theta)(1-\sin\theta)$

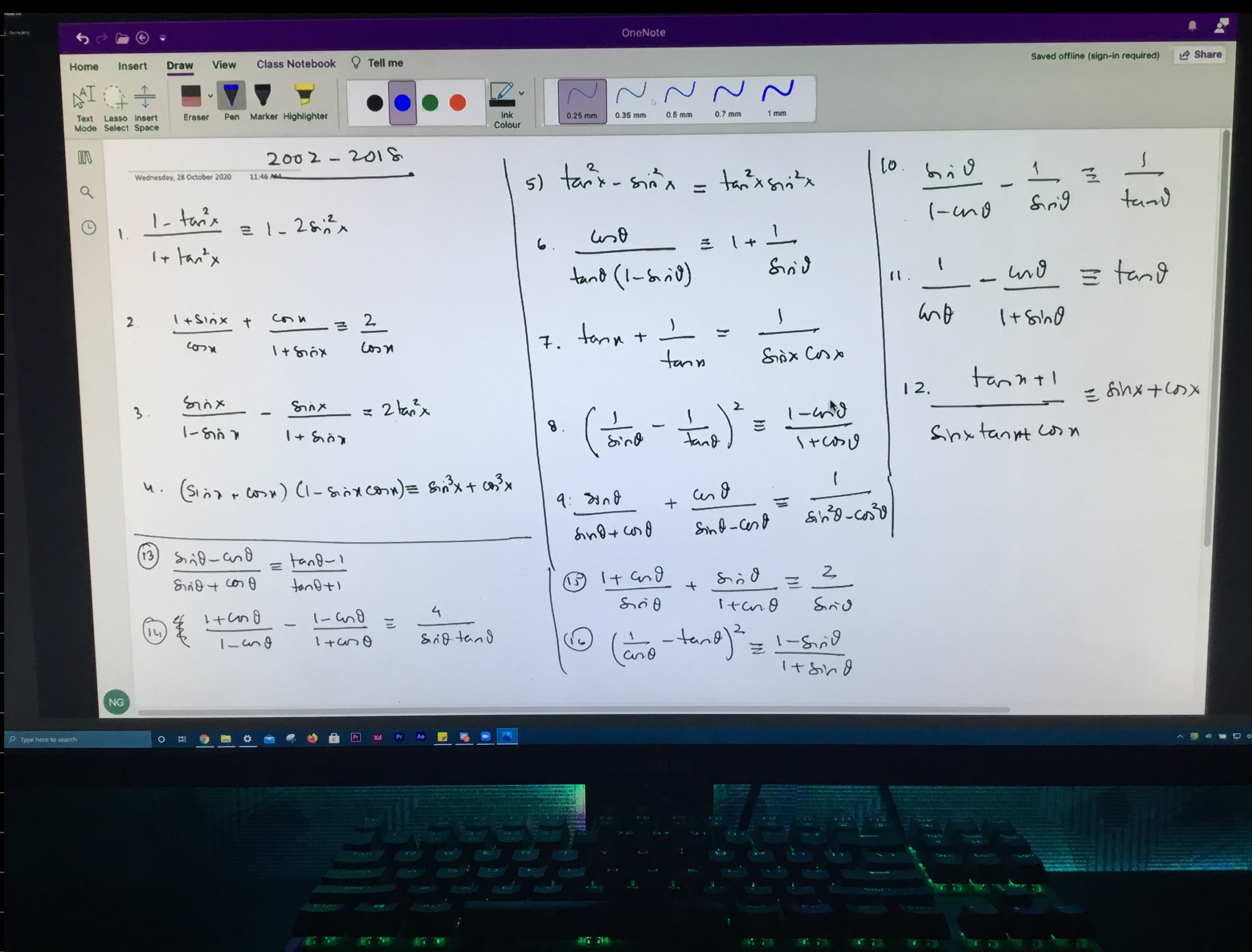
$= \cos\theta+\sin\theta\cos\theta+(1+\sin\theta)(1-\sin\theta)\ /\ 1-\sin^2\theta$

$= \cos\theta(1+\sin\theta)+(1+\sin\theta)(1-\sin\theta)\ /\ 1-\sin^2\theta$

$= \dfrac{\cancel{(1+\sin\theta)}(\cos\theta+(1-\sin\theta))}{\cancel{(1+\sin\theta)}(1-\sin\theta)}$

$= \dfrac{\cos\theta+(1-\sin\theta)}{1-\sin\theta}$

$= \dfrac{1-\sin\theta+\cos\theta}{1-\sin\theta} \rightarrow$ shown.



1. $\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv 1-2\sin^2\theta$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2}{\cos^2\theta+\sin^2\theta}$$

$$= \frac{\cos^2\theta-(1-\cos^2\theta)}{\cos^2\theta} \times \frac{\cos^2}{\cos^2\theta+\sin^2\theta}$$

$$= \frac{\cos^2\theta-1+\cos^2\theta}{\cos^2\theta} \times \frac{\cos^2}{\cos^2\theta+\sin^2\theta}$$

$$= \frac{2\cos^2\theta-1}{\cos^2\theta} \times \frac{\cos^2}{1-\cancel{\sin^2\theta}+\cancel{\sin^2\theta}}$$

$$= \frac{2\cos^2\theta-1}{\cancel{\cos^2\theta}} \times \cancel{\cos^2}\theta$$

$$= 2\cos^2\theta-1$$
$$= 2(1-\sin^2\theta)-1$$
$$= 2-2\sin^2\theta-1$$
$$= 1-2\sin^2\theta \longrightarrow \text{shown}$$

2. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} \equiv \frac{2}{\cos\theta}$

$$\frac{(1+\sin\theta)^2+\cos^2\theta}{(1+\sin\theta)(\cos\theta)}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{(1+\sin\theta)(\cos\theta)}$$

$$= \frac{1+2\sin\theta+1-\cancel{\cos^2\theta}+\cancel{\cos^2\theta}}{(1+\sin\theta)(\cos\theta)}$$

$$= \frac{2+2\sin\theta}{}$$

$$\overline{(1+\sin\theta)(\cos\theta)}$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}(\cos\theta)}$$

$$= \frac{2}{\cos\theta} \rightarrow \underline{\underline{\text{shown}}}$$

3. $$\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = 2\tan^2\theta$$

$$= \frac{\sin\theta + \sin^2\theta - (\sin\theta - \sin^2\theta)}{1 - \sin^2\theta}$$

$$= \frac{\sin\theta - \sin\theta + 2\sin^2\theta}{1 - \sin^2\theta}$$

$$= \frac{2\sin^2\theta}{\cos^2\theta}$$

$$= 2\tan^2\theta \longrightarrow \underline{\underline{\text{shown}}}$$

4. $$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$$

$$= \sin\theta - \sin^2\theta\cos\theta + \cos\theta - \sin\theta\cos^2\theta$$

$$= \sin\theta - \cos\theta(1-\cos^2\theta) + \cos\theta - \sin\theta(1-\sin^2\theta)$$

$$= \cancel{\sin\theta} - \cancel{\cos\theta} + \cos^3\theta + \cancel{\cos\theta} - \cancel{\sin\theta} + \sin^3\theta$$

$$= \cos^3\theta + \sin^3\theta \rightarrow \underline{\underline{\text{shown}}}$$

5. $$\tan^2\theta - \sin^2\theta \equiv \tan^2\sin^2\theta$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta - \sin^2\theta(1-\sin^2\theta)}{\cos^2\theta}$$

$$= \frac{\cancel{\sin^2\theta} - \cancel{\sin^2\theta} + \sin^4\theta}{\cos^2\theta}$$

$$= \frac{(\sin^2\theta)(\sin^2\theta)}{(\cos^2\theta)}$$

$= \tan^2\theta \sin^2\theta \rightarrow$ shown

6. $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$

$= \dfrac{\cos\theta}{\frac{\sin\theta}{\cos\theta}(1-\sin\theta)}$

$= \dfrac{\cos^2\theta}{\sin\theta(1-\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$

$= \dfrac{(1+\sin\theta)(1-\sin\theta)}{\sin\theta(1-\sin\theta)}$

$= \dfrac{1}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + 1 \rightarrow$ shown

7. $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta} \rightarrow$ shown

8. $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

$= (1-\cos\theta)(1-\cos\theta)$

$$\overline{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)\cancel{(1-\cos\theta)}}{(1+\cos\theta)\cancel{(1-\cos\theta)}}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} \longrightarrow \text{shown}$$

9. $$\frac{\sin\theta}{\sin\theta+\cos\theta} + \frac{\cos\theta}{\sin\theta-\cos\theta} = \frac{1}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{\sin^2\theta - \cancel{\cos\theta\sin\theta} + \cos^2\theta + \cancel{\cos\theta\sin\theta}}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1}{\sin^2\theta - \cos^2\theta} \longrightarrow \text{shown}$$

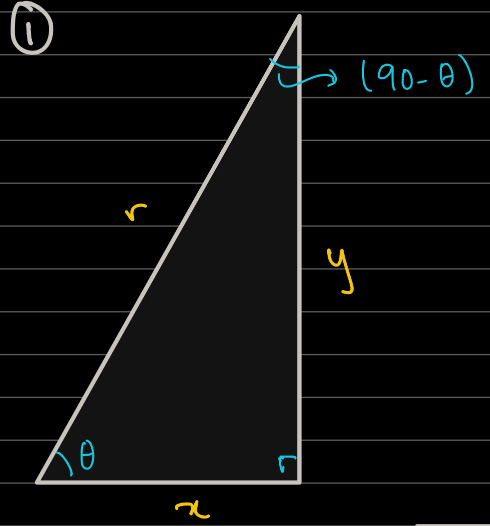
10. $$\frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta} = \frac{1}{\tan\theta}$$

=

TRIGONOMETRY (RELATIONS)

↳ Important things to know + remember

①



θ   90-θ

$\sin\theta = \dfrac{y}{r} \xrightarrow{\text{equal}} \cos(90-\theta) = \dfrac{y}{r}$

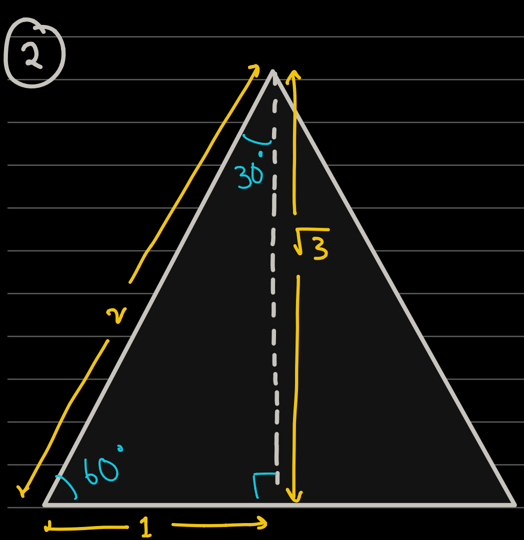
$\cos\theta = \dfrac{x}{r} \xrightarrow{\text{equal}} \sin(90-\theta) = \dfrac{y}{r}$

$\tan\theta = \dfrac{y}{x} \xrightarrow{\text{reciprocal}} \tan(90-\theta) = \dfrac{x}{y}$

$$\boxed{\begin{aligned}\cos\theta &= \sin(90-\theta)\\ \sin\theta &= \cos(90-\theta)\\ \tan\theta &= \frac{1}{\tan(90-\theta)}\end{aligned}}$$

Complementary Angle Relationships

②

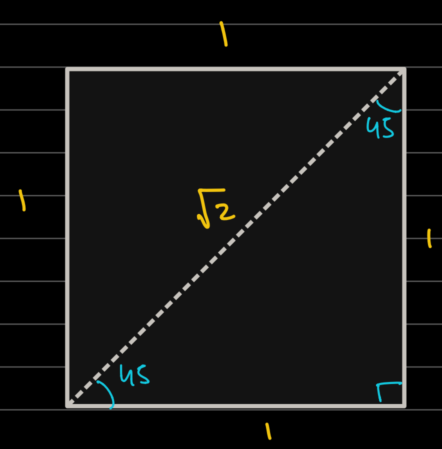


30°   60°

$\sin 30 = \dfrac{1}{2} \xrightarrow{\text{equal}} \cos 60 = \dfrac{1}{2}$

$\cos 30 = \dfrac{\sqrt{3}}{2} \xrightarrow{\text{equal}} \sin 60 = \dfrac{\sqrt{3}}{2}$

$\tan 30 = \dfrac{1}{\sqrt{3}} \xrightarrow{\text{reciprocal}} \tan 60 = \sqrt{3}$



45°

$\sin 45 = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\cos 45 = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\tan 45 = \dfrac{1}{1} = 1$

③ $\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

$* \cos(-\theta) = +\cos\theta$

Explanation for this will come later

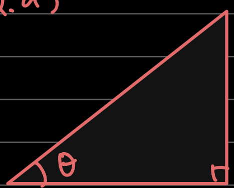
Exercise 10.1 → From Ho Soo Thong

1.a) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

b) $\sin\theta = \frac{\sqrt{5}}{3}$, $\cos\theta = \frac{2}{3}$, $\tan\theta = \frac{\sqrt{5}}{2}$

c) $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

2.a)



	$\cos\theta$	$\sin\theta$	$\tan\theta$
(a)	$\frac{8}{17}$	$\frac{15}{17}$	$\frac{15}{8}$
(b)	$\frac{2}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{2}$
(c)	$\frac{\sqrt{616}}{25}$	$\frac{7}{25}$	$\frac{7}{\sqrt{616}}$
(d)	$\frac{1}{\sqrt{10}}$	$\frac{3}{\sqrt{10}}$	3

3. $\sin\theta \cos\theta (90-\theta)$

$= \sin\theta \sin\theta$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

4. $\tan A = 2$

$2\tan A + \tan(90-a)$

$= 2(2) + \frac{1}{2}$

$= 4 + \frac{1}{2}$

$= 4\frac{1}{2}$

5. (a) $\frac{\sin 45^\circ}{\cos 30^\circ + \sin 60^\circ}$

$\frac{1}{\sqrt{2}} \times \frac{1}{2\frac{\sqrt{3}}{2}}$

(b) $\tan 45^\circ + \tan 30^\circ \tan 60^\circ$

$= 1 + \frac{1}{\sqrt{3}} \times \sqrt{3}$

$= 1 + 1$

$= 2$

$= \dfrac{1}{\sqrt{2} \times \sqrt{3}}$

$= \dfrac{1}{\sqrt{6}}$

6. a) $\dfrac{\sin 65}{\cos 25}$ → equal to each other

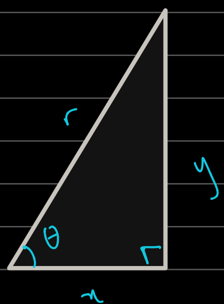
$= 1$

(b) $\tan 75 \tan 15$ ← reciprocals of each other

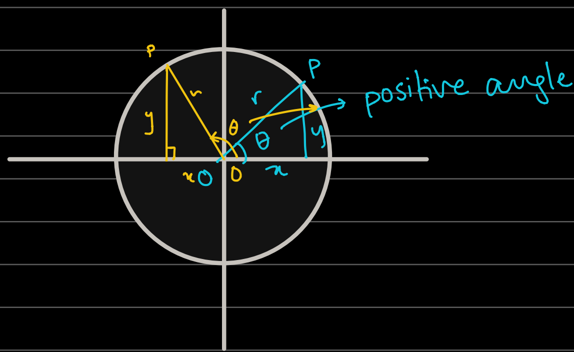
$= 1$

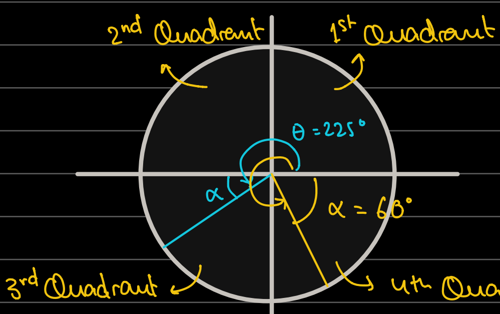
7. → Ask Sir Nasir to explain [particularly : What's a basic α angle?]

8. a) $\theta = 130°$ c) $210°$
b) $\theta = 300°$ d) $-135°$

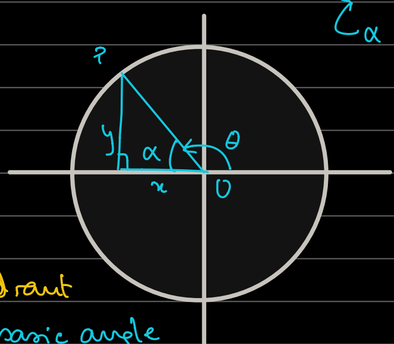


$\sin 120° = ?$
$\sin 225° = ?$
$\sin 300° = ?$





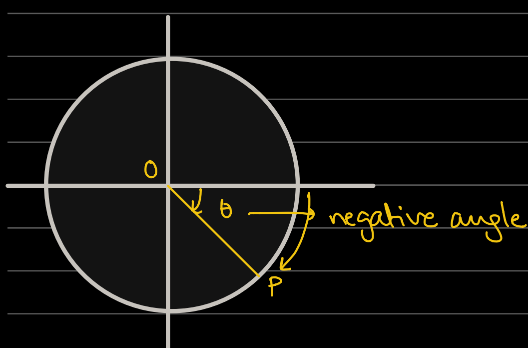
α = 45° → basic angle



Reference angle
α = basic angle
↳ always acute,
always positive

α is always between the segment OP and the x axis

for now ... $\sin \alpha = \sin \theta$



7. (a) $\alpha = 70^\circ$
3rd Quadrant

(b) $\alpha = 30^\circ$
1st Quadrant

(c) $\alpha = 60^\circ$
4th Quadrant

(d) $\alpha = 80^\circ$
3rd Quadrant

8 → Done.

9. (a) $\theta = 20^\circ$
$\theta = 160^\circ$
$\theta = 200^\circ$
$\theta = 340^\circ$

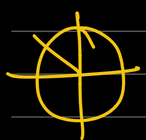
(b) $\theta = 70^\circ$
$\theta = 110^\circ$
$\theta = 250^\circ$
$\theta = 290^\circ$

(c) $\theta = 35^\circ$
$\theta = 145^\circ$
$\theta = 215^\circ$
$\theta = 325^\circ$

10. $\theta = 10^\circ$
$\theta = 170^\circ$
$\theta = -10^\circ$
$\theta = -170^\circ$

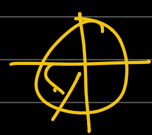
11. $-360^\circ < A < 720$, α of $A = 80^\circ$

(a) 2nd Quadrant



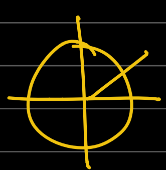
$A = 100^\circ$
$A = -260^\circ$
$A = 460^\circ$

(b) 3rd Quadrant



$A = 260^\circ$
$A = 620^\circ$
$A = -100^\circ$

(c) 1st Quadrant



$A = 80^\circ$
$A = 440^\circ$
$A = -280^\circ$

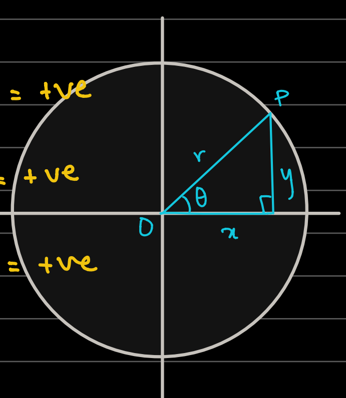
Quadrants and the polarity of trigonometric ratios

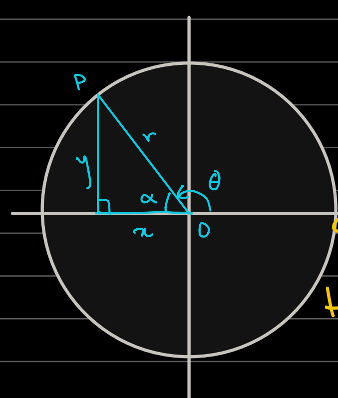
Quadrant 1

$\sin\theta = \frac{y}{r} = \frac{+ve}{+ve} = +ve$

$\cos\theta = \frac{x}{r} = \frac{+ve}{+ve} = +ve$

$\tan\theta = \frac{y}{x} = \frac{+ve}{+ve} = +ve$





Quadrant 2

$\sin\theta = \frac{y}{r} = \frac{+ve}{+ve} = +ve$

$\cos\theta = \frac{x}{r} = \frac{-ve}{+ve} = -ve$

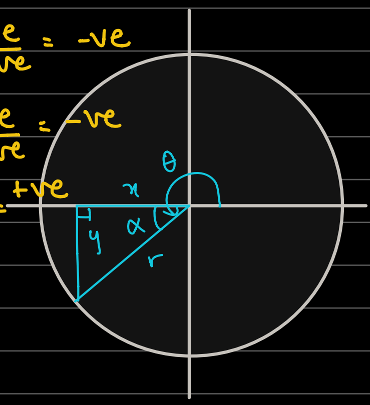
$\tan\theta = \frac{y}{x} = \frac{+ve}{-ve} = -ve$

Quadrant 3

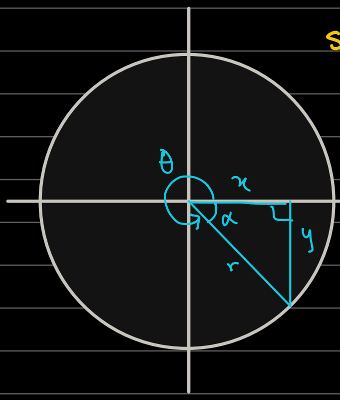
$\sin\theta = \frac{y}{r} = \frac{-ve}{+ve} = -ve$

$\cos\theta = \frac{x}{r} = \frac{-ve}{+ve} = -ve$

$\tan\theta = \frac{y}{x} = \frac{-ve}{-ve} = +ve$



Quadrant 4



$\sin\theta = \frac{y}{r} = \frac{-ve}{+ve} = -ve$

$\cos\theta = \frac{x}{r} = \frac{+ve}{+ve} = +ve$

$\tan\theta = \frac{y}{x} = \frac{-ve}{+ve} = -ve$

Exercise 10.2 from Ho Soo Thong

1 a) negative b) negative c) positive d) positive e) negative f) positive
g) negative h) positive

2. a) I, III b) IV c) I, II d) II, III

3. a) $\tan 300°$
$= \tan(30°)$
$= \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

b) $\cos 330°$
$= \cos 60°$
$= \frac{1}{2}$

c) $\sin 150°$
$= \sin 30°$
$= \frac{1}{2}$

d) $\tan 315°$
$= \tan 45°$
$= 1$

e) $\sin 225°$
$= \sin 45°$
$= \frac{\sqrt{2}}{2}$

f) $\cos 210°$
$= \cos 30°$
$= \frac{\sqrt{3}}{2}$

g) $\tan(-120°)$
$= -\tan(120)$
$= -\tan(60)$
$= -\sqrt{3}$

h) $\sin 405°$
$= \sin 45°$
$= \frac{\sqrt{2}}{2}$

4. $\theta < 90$, $\cos\theta = \frac{4}{5}$, find $\sin\theta$ and $\tan\theta$

$\cos\theta = \frac{4 \rightarrow adj}{5 \rightarrow hyp.}$

$\sqrt{5^2 - 4^2} = \sqrt{25-16}$
$= \sqrt{9}$
$= 3$

$\sin\theta = \frac{3}{5}$

$\tan\theta = \frac{3}{4}$

5. $A > 90$ (obtuse), $\tan A = -\frac{1}{2}$, find $\cos A$ and $\sin A$

lies in the 2nd quadrant

$\tan A = \frac{1 \rightarrow opp}{-2 \rightarrow adj.}$

$\sqrt{1^2 + 2^2} = \sqrt{5} \rightarrow hyp.$

$\sin A = \frac{1}{\sqrt{5}}$ $\cos A = \frac{-2}{\sqrt{5}}$

6. $180° < A < 270°$, $\sin A = -\frac{5}{13}$, find $\tan A$ and $\cos(-A)$

↳ Q III

$\sin A = -\frac{5 \to \text{opp}}{13 \to \text{hyp.}}$

$\sqrt{13^2 - 5^2} = \sqrt{169 - 25}$
$= \sqrt{144}$
$= 12 \to \text{adj}$

$\cos(-A) = \cos A$
$= -\frac{12 \to \text{adj}}{13 \to \text{hyp.}}$

$\tan A = \frac{5}{12}$

7. $90 < A < 180$, $\sin A = \frac{2}{\sqrt{5}}$, find $\cos A$ and $\tan A$

↳ Q II

$\sin A = \frac{2 \to \text{opp.}}{\sqrt{5} \to \text{hyp.}}$

$\sqrt{(\sqrt{5})^2 - 2^2} = \sqrt{5-4}$
$= \sqrt{1}$
$= 1 \to \text{adj} \to (-)$

$\cos A = \frac{-1}{\sqrt{5}}$ $\tan A = -2$

8. $\underbrace{\cos A = \frac{1}{2}, \sin A > 0}_{\text{↳ Q I}}$, find $\sin(-A)$ and $\tan A$

$\cos A = \frac{1 \to \text{adj}}{2 \to \text{hyp.}}$

$\sqrt{2^2 - 1^2} = \sqrt{4-1}$
$= \sqrt{3} \to \text{opp.}$

$\sin(-A) = -\sin A$
$= -\frac{\sqrt{3}}{2}$

$\tan A = \sqrt{3}$

9. $\underbrace{\tan A = -\frac{5}{12}, \cos A > 0}_{\to \text{Q IV}}$, find $\cos A$ and $\cos(90 - A)$

$\tan = -\frac{5 \to \text{opp.}}{12 \to \text{adj.}}$

$\sqrt{5^2 + 12^2} = \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13 \to \text{hyp.}$

$\cos A = \frac{12}{13}$

$\cos(90 - A) = \sin A = \frac{-5}{13}$

10. $\cos A = \sqrt{\frac{2}{3}}$, $180 < A < 360$, find

a) $\sin A$

$\cos A = \frac{\sqrt{2} \rightarrow \text{adj}}{\sqrt{3} \rightarrow \text{hyp.}}$

$\sqrt{(\sqrt{3})^2 - (\sqrt{2})^2} = \sqrt{3-2}$

$= \sqrt{1}$

$= 1 \rightarrow$ opp.

$\sin A = \frac{1}{\sqrt{3}}$

$= -\frac{1}{\sqrt{3}}$ bc. QIII or QIV

b) $\sin(90-A)$

$= \cos A = \sqrt{\frac{2}{3}}$

c) $\tan(90-A)$

$= \frac{1}{\tan A}$

$= \frac{1}{-\frac{1}{\sqrt{2}}}$

$= -\sqrt{2}$

11. $\sin 20 = k$, express the following in terms of k

a) $\sin 200$

$= \sin(\alpha)$

$= \sin(20) = k$

b) $\cos 20$

$\sin 20 = \frac{k \rightarrow \text{opp.}}{1 \rightarrow \text{hyp}}$

$\cos 20 = \frac{\sqrt{1-k^2} \rightarrow \text{adj}}{1}$

c) $\tan(-20)^\circ$

$= -\tan(20)$

$= \frac{-k \rightarrow \text{opp}}{\sqrt{1-k^2} \rightarrow \text{adj}}$

d) $\sin 70$

$\sin(90-\theta) = \cos\theta$

$\sin 70 = \sin(90-20) = \cos 20$

$= \frac{\sqrt{1-k^2}}{1}$

12. $0 < x < 360$, find x such that

a) $\cos x = -0.71$

Step 1: find basic angle

$\cos\alpha = 0.71$

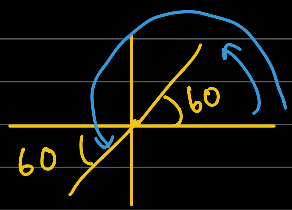
$\alpha = \cos^{-1}(0.71)$ calculate α without regards for sign

$\alpha = 44.8^\circ$

Step 2: place α in appropriate quads.

b) $\tan x = 1.732$

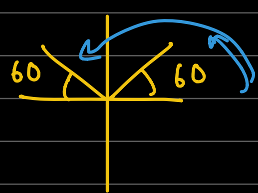
$\alpha = \tan^{-1}(1.732)$
$\alpha = 60.0$



$\theta = 60°$, and $240°$

c) $\sin x = 0.866$

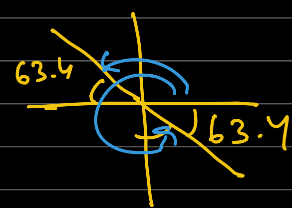
$\alpha = \sin^{-1}(0.866)$
$\alpha = 60.0$



$\theta = 60°, 120°$

d) $\tan x = -2$

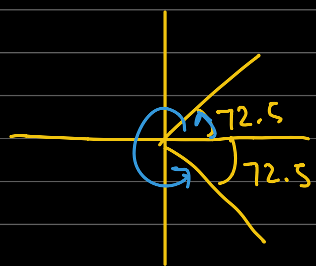
$\alpha = \tan^{-1}(2)$
$\alpha = 63.4°$



$\theta = 296.6°, 116.6°$

e) $10\cos x - 3 = 0$
$\cos x = \dfrac{3}{10}$
$\alpha = \cos^{-1}\left(\dfrac{3}{10}\right)$

$\alpha = 72.5$



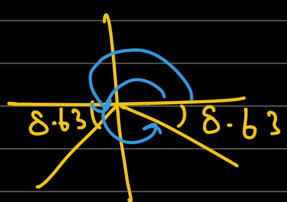
$\theta = 287.5°, 72.5°$

f) $4(\tan x - 1) = 3(5 - 2\tan x)$
$4\tan x - 4 = 15 - 6\tan x$

$$10 \tan x = 15 + 4$$
$$\tan x = \frac{19}{10}$$
$$\tan x = 1.9$$
$$x = \tan^{-1}(1.9)$$
$$\alpha = 62.2^\circ$$

$\theta = 242.2^\circ, 62.2^\circ$

g) $2\sin(-x) = 0.3$
$$-2\sin(x) = 0.3$$
$$\sin x = -0.15$$
$$\alpha = \sin^{-1}(0.15)$$
$$\alpha = 8.63$$

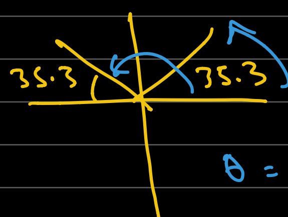


$\theta : 351.4^\circ, 188.6^\circ$

h) $2\cos^2 x = 1$
$$\cos^2 x = \frac{1}{2}$$
$$\cos x = \frac{1}{\sqrt{2}}$$
$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\alpha = 45$$

$\theta = 315^\circ, 45^\circ$

i) $3\sin x + 2 = \tan 75^\circ$
$$\sin x = \frac{\tan 75 - 2}{3}$$
$$\alpha = \sin^{-1}\left(\frac{\tan 75 - 2}{3}\right)$$
$$\alpha = 35.3^\circ$$



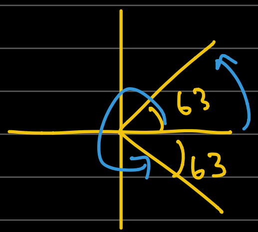
$\theta = 35.3^\circ, 144.7^\circ$

j) $\dfrac{8\cos x + 1}{2 - \cos x} = 3$
$$8\cos x + 1 = 6 - 3\cos x$$
$$11\cos x = 5$$

$\cos x = \frac{5}{11}$

$\alpha = \cos^{-1}\left(\frac{5}{11}\right)$

$\alpha = 63.0$



$\theta = 297°, 63°$

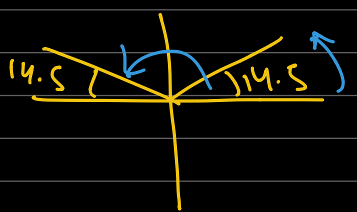
Exercise A. Q4 all parts

4. $0 \leq \theta \leq 360$

a) $\sin\theta = 0.3$

$\alpha = \sin^{-1}(0.3)$

$\alpha = 14.5°$

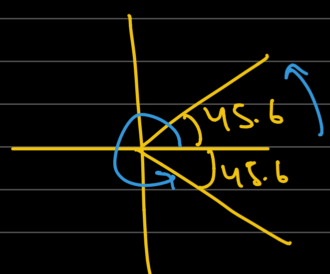


$\theta = 14.5°, 165.5°$

b) $\cos\theta = 0.7$

$\alpha = \cos^{-1}(0.7)$

$\alpha = 45.6$

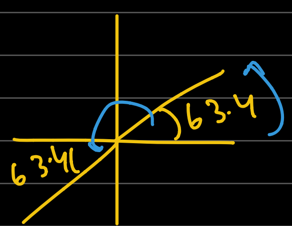


$\theta = 45.6°, 314.4°$

c) $\tan\theta = 2$

$\alpha = \tan^{-1}(2)$

$\alpha = 63.4$

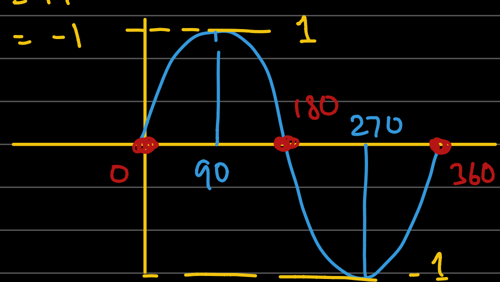


$\theta = 63.4°, 243.4°$

$\sin\theta = 0$

$\sin\theta = +1$

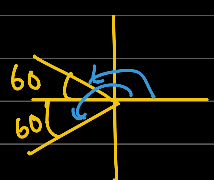
$\sin\theta = -1$



d) $\cos\theta = -0.5$

$\alpha = \cos^{-1}(0.5)$

$\alpha = 60°$

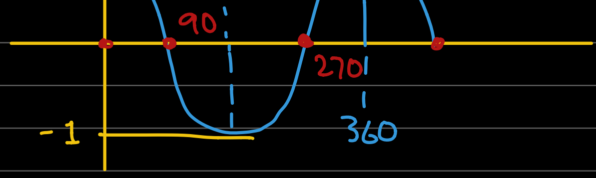


$\theta = 120°, 240°$

$\cos\theta = 0$

$\cos\theta = +1$

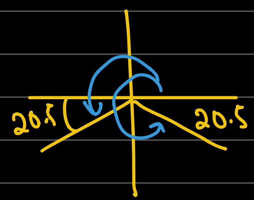
$\cos\theta = -1$



e) $\sin\theta = -0.35$

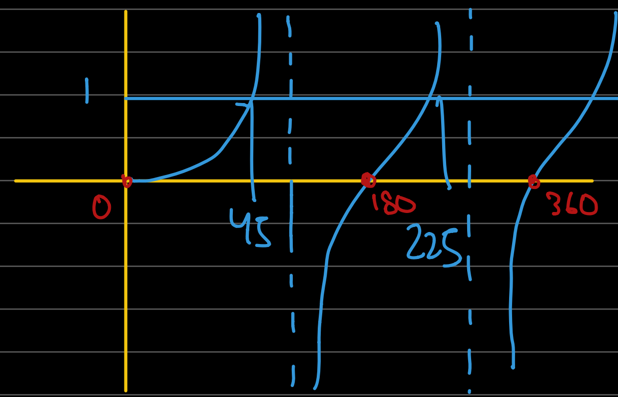
$\alpha = \sin^{-1}(0.35)$

$\alpha = 20.5°$



$\theta = 200.5°,$
$339.5°$

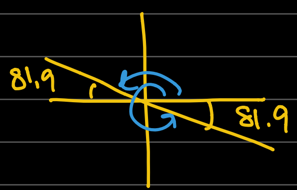
$\tan\theta = 0$



f) $\tan\theta = -7$

$\alpha = \tan^{-1}(7)$

$\alpha = 81.9°$

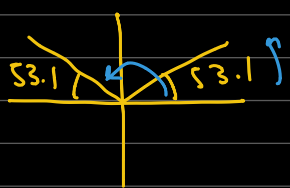


$\theta = 98.1°,$
$278.1°,$

g) $\sin\theta = 0.8$

$\alpha = \sin^{-1}(0.8)$

$\alpha = 53.1°$



$\theta = 53.1°,$
$= 126.9°$

h) $\sin\theta = -1$

$\theta = 270$ (from graph)

Q6. a) $2\sin^2\theta - \sin\theta = 0$

$(\sin\theta)(2\sin\theta - 1) = 0$

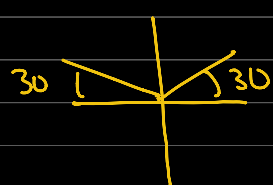
$\sin\theta = 0$ or $2\sin\theta - 1 = 0$

$\theta = 180, 360$ or $\sin\theta = \frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\therefore, \theta = 180°, 360°, 30°,$
$150°$

$\alpha = 30$



$\theta = 30, 150$

b) $3\cos^2\theta = \cos\theta$

$3\cos^2\theta - \cos\theta = 0$

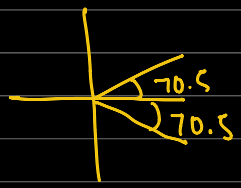
$(\cos\theta)(3\cos\theta - 1) = 0$

$\cos\theta = 0$ or $3\cos\theta - 1 = 0$

$\theta = 90, 270$ $\quad$ $\cos\theta = \frac{1}{3}$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$\therefore, \theta = 70.5°, 90°, 270°, 289.5°$ $\quad$ $\alpha = 70.5°$



$\theta = 70.5°, 289.5°$

c) $5\sin\theta\cos\theta - \sin\theta = 0$

$(\sin\theta)(5\cos\theta - 1) = 0$

$\sin\theta = 0$ or $5\cos\theta - 1 = 0$

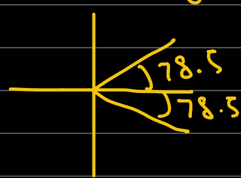
$\theta = 180, 360$ $\quad$ $\cos\theta = \frac{1}{5}$

$\alpha = \cos^{-1}\left(\frac{1}{5}\right)$

$\therefore, \theta = 180°, 360°, 78.5°, 281.5°$

$\alpha = 78.5°$

$\theta = 78.5, 281.5$



d) $\tan^2\theta + 4\tan\theta = 0$

$(\tan\theta)(\tan\theta + 4) = 0$

$\tan\theta = 0$ or $\tan\theta + 4 = 0$

$\theta = 180, 360$ $\quad$ $\tan\theta = -4$

$\alpha = \tan^{-1}(4)$

$\alpha = 76°$

$\therefore, \theta = 180°, 360°, 104°, 284°$

$\theta = 104, 284$

e) $6\sin^2\theta + \sin\theta = 0$

$(\sin\theta)(6\sin\theta + 1) = 0$

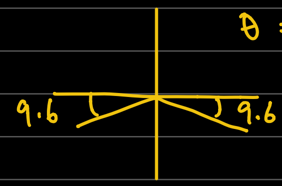
$\sin\theta = 0$ or $6\sin\theta + 1 = 0$

$\theta = 180, 360$ $\quad$ $\sin\theta = \frac{-1}{6}$

$\alpha = \sin^{-1}\left(\frac{1}{6}\right)$

$\therefore, \theta = 180, 360, 189.6, 350.4°$ $\alpha = 9.6°$

$\theta = 189.6, 350.4$



i) $(\tan\theta + 1)^2 = 9$

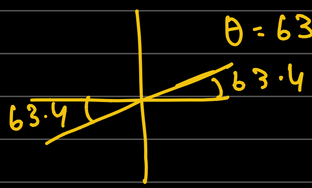
$\tan\theta + 1 = 3$ or $\tan\theta + 1 = -3$

$\tan\theta = 2$ $\tan\theta = -4$

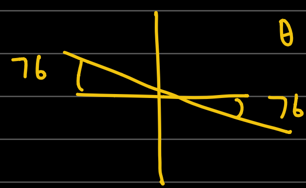
$\alpha = \tan^{-1}(2)$ $\alpha = \tan^{-1}(4)$

$\alpha = 63.4°$ $\alpha = 76°$

$\theta = 63.4, 243.4$



$\theta = 104, 284$



$\therefore, \theta = 63.4°, 243.4°, 104°, 284°$

Q8 from Exercise A

8.a) $\sin(\theta + 20°) = 0.4$ find x

fix range. $0 \le \theta \le 360$

$20 \le \theta + 20 \le 380°$

substitute $\sin(\theta + 20) = 0.4$

$\sin x = 0.4$

$20 \le x \le 380$

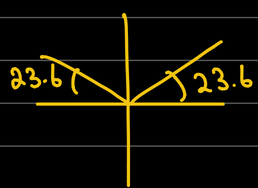
$\theta + 20 = 23.6$

$\theta = 3.6°$

$\theta + 20 = 156.4$

$\theta = 136.4°$

$\therefore, \theta = 3.6°, 136.4°$



$\alpha = \sin^{-1}(0.4)$

$\alpha = 23.6°$

$x = 23.6°, 156.4°$

b) $\cos(\theta - 50°) = -0.3$ find x

fix range

$-50 \le \theta - 50° \le 310°$

$\theta - 50° = 107.5$

$\theta = 157.5°$

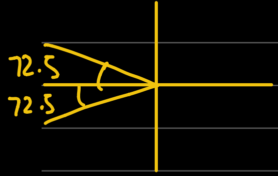
substitute $-50 \le x \le 310$

$\cos(x) = -0.3$

$\alpha = \cos^{-1}(0.3)$

$\theta - 50° = 252.5$

$\theta = 302.5°$



$\alpha = 72.5°$

$x = 107.5, 252.5$

$\therefore, \theta = 157.5°, 302.5°$

c) $1 + 5\sin(\theta - 100°) = 0$

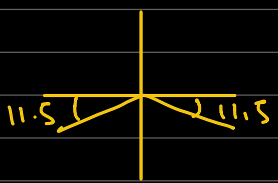
$\sin(\theta - 100°) = \frac{-1}{5}$

fix range $-100° \le \theta - 100 \le 260°$

substitute $-100° \le x \le 260°$

$\sin x = -\frac{1}{5}$

$\alpha = \sin^{-1}\left(\frac{1}{5}\right)$



$\alpha = 11.5$

$x = 191.5°, 348.5$

find x

$\theta - 100° = 191.5$

$\theta = 291.5°$

$\theta - 100° = 348.5$

$\theta = 448.5°$ x

$\therefore, \theta = 291.5°$

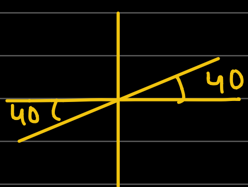
d) $\tan(\theta - 162°) = 0.6$

$-162° \le \theta \le 198°$

$\tan(x) = 0.6$

$\alpha = \tan^{-1}(0.6)$

$\alpha = 40.0$



$x = 40, 220$

find x

$\theta - 162 = 40$

$\theta = 202°$

$\theta - 162 = 220$

$\theta = 382°$ x

$\therefore, \theta = 202°$

Ex B, Q7

Q7. $0 \le \theta \le 360$

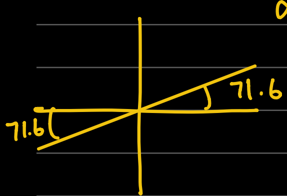
a) $\sin\theta = 3\cos\theta$

$\frac{\sin\theta}{\cos\theta} = 3$

$\tan\theta = 3$

$\alpha = \tan^{-1}(3)$

$\alpha = 71.6°$



$\underline{\theta} = 71.6°, 251.6°$

b) $5\cos\theta = 3\sin\theta$

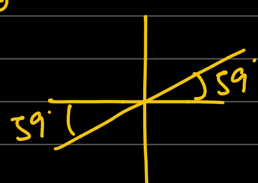
$5 = \frac{3\sin\theta}{\cos\theta}$

$5 = 3\tan\theta$

$\frac{5}{3} = \tan\theta$

$\tan^{-1}\left(\frac{5}{3}\right) = \alpha$

$59° = \alpha$



$\theta = 59°, 239°$

c) $\sin\theta + \cos\theta = 0$

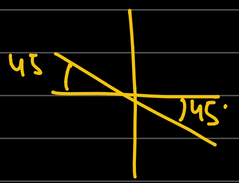
$\sin\theta = -\cos\theta$

$\sin\theta = (-1)(\cos\theta)$

$\tan\theta = -1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45°$



$\theta = 135°, 315°$

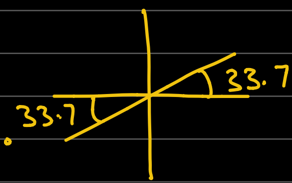
d) $2\cos\theta - 3\sin\theta = 0$

$2\cos\theta = 3\sin\theta$

$\frac{2}{3} = \tan\theta$

$\tan^{-1}\left(\frac{2}{3}\right) = \alpha$

$33.7° = \alpha$



$\theta = 33.7°, 213.7°$

g) $\sin^2\theta - 5\sin\theta\cos\theta = 0$

$(\sin\theta)(\sin\theta - 5\cos\theta) = 0$

$\sin\theta = 0$ or $\sin\theta - 5\cos\theta = 0$

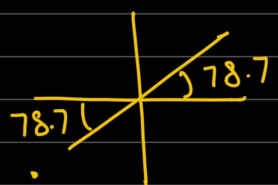
$\theta = 0°, 180°, 360°$

$\sin\theta = 5\cos\theta$

$\tan\theta = 5$

$\alpha = \tan^{-1}(5)$

$\alpha = 78.7°$



$\theta = 78.7°, 258.7°$

$\therefore, \theta = 0°, 180°, 360°,$
$78.7°, 258.7°$

h) $3\cos^2\theta = 7\sin\theta\cos\theta$

$3\cos^2\theta - 7\sin\theta\cos\theta = 0$

$(\cos\theta)(3\cos\theta - 7\sin\theta) = 0$

$\cos\theta = 0$ or $3\cos\theta - 7\sin\theta = 0$

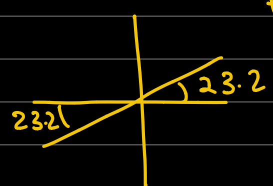
$\theta = 90°, 270°$

$3\cos\theta = 7\sin\theta$

$\frac{3}{7} = \tan\theta$

$\tan^{-1}\left(\frac{3}{7}\right) = \alpha$

$23.2 = \alpha$



$\theta = 23.2°, 103.2°$

$\therefore, \theta = 90°, 270°, 23.2°, 103.2°$

i) $4\sin^2\theta = \cos^2\theta$

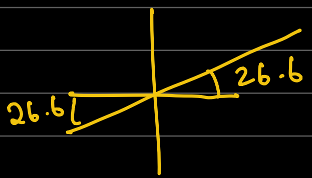
$4\frac{\sin^2\theta}{\cos^2\theta} = 1$

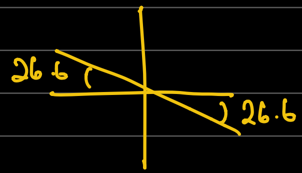
$\tan^2\theta = \frac{1}{4}$

$\tan\theta = +\frac{1}{2}$ or $\tan\theta = -\frac{1}{2}$

$\alpha = \tan^{-1}(1/2)$ $\qquad$ $\alpha = \tan^{-1}(1/2)$

$\alpha = 26.6°$ $\qquad$ $\alpha = 26.6°$





$\theta = 26.6, 206.6°$ $\qquad$ $\theta = 333.4°, 153.4°$

$\therefore, \theta = 26.6°, 206.6°, 153.4°, 333.4°$

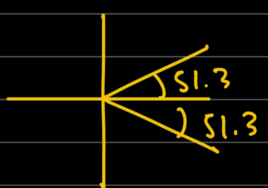
k) $25\cos\theta = 16\sin\theta\tan\theta$

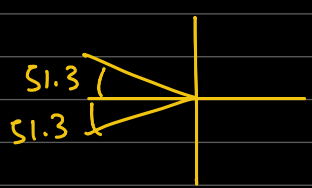
$$25\cos\theta - 16\sin\theta\tan\theta = 0$$
$$25\cos\theta - 16\sin\theta\frac{\sin\theta}{\cos\theta} = 0$$
$$25\cos\theta - \frac{16\sin^2\theta}{\cos\theta} = 0$$
$$25\cos^2\theta - 16\sin^2\theta = 0$$
$$25\cos^2\theta - 16(1-\cos^2\theta) = 0$$
$$25\cos^2\theta - 16 + 16\cos^2\theta = 0$$
$$41\cos^2\theta = 16$$
$$\cos^2\theta = \frac{16}{41}$$

$\cos\theta = \frac{4}{6.403}$ $\qquad$ $\cos\theta = \frac{-4}{6.403}$

$\alpha = \cos^{-1}\left(\frac{4}{6.403}\right)$ $\qquad$ $\alpha = \cos^{-1}\left(\frac{4}{6.403}\right)$

$\alpha = 51.3°$ $\qquad$ $\alpha = 51.3°$





$\theta = 51.3°, 308.7°, 128.7°, 231.3°$

Q8 from Ex. B

a) $2\cos^2\theta + 3\sin\theta - 3 = 0$

$$2(1-\sin^2\theta) + 3\sin\theta - 3 = 0$$
$$2 - 2\sin^2\theta + 3\sin\theta - 3 = 0$$
$$-2\sin^2\theta + 3\sin\theta - 1 = 0$$
$$2\sin^2\theta - 3\sin\theta + 1 = 0 \qquad y = \sin\theta$$
$$2y^2 - 2y - 1y + 1 = 0$$
$$2y(y-1) - 1(y-1) = 0$$

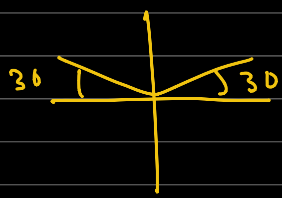
$(2y-1)(y-1) = 0$

$2\sin\theta = 1$ or $\sin\theta = 1$

$\sin\theta = \frac{1}{2}$ $\qquad \theta = 90^\circ$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = 30$



$\therefore, \theta = \pm 90^\circ, 30^\circ, 150^\circ$

b) $3\sin^2\theta - 5\cos\theta - 1 = 0$

$3(1-\cos^2\theta) - 5\cos\theta - 1 = 0$
$3 - 3\cos^2\theta - 5\cos\theta - 1 = 0$
$-3\cos^2\theta - 5\cos\theta + 2 = 0$
$3\cos^2\theta + 5\cos\theta - 2 = 0$
$3\cos^2\theta + 6\cos\theta - \cos\theta - 2 = 0$
$3\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$
$(3\cos\theta - 1)(\cos\theta + 2) = 0$

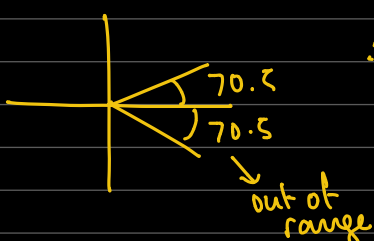
$\cos\theta = \frac{1}{3}$ or $\cos\theta = -2$

$\alpha = \cos^{-1}(2)$

Ans. not possible

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$\alpha = 70.5$



$\therefore, \pm 70.5$ ✓

c) $8\sin^2\theta = 11 - 10\cos\theta$

$8(1-\cos^2\theta) = 11 - 10\cos\theta$
$8 - 8\cos^2\theta = 11 - 10\cos\theta$
$8 - 11 = 8\cos^2\theta - 10\cos\theta$
$0 = 8\cos^2\theta - 10\cos\theta + 3$
$= 8\cos^2\theta - 4\cos\theta - 6\cos\theta + 3$
$= 4\cos\theta(2\cos\theta - 1) - 3(2\cos\theta - 1)$
$0 = (4\cos\theta - 3)(2\cos\theta - 1)$

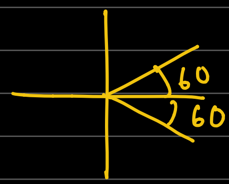
$4\cos\theta - 3 = 0$ or $2\cos\theta - 1 = 0$

$\cos\theta = \frac{3}{4}$ $\cos\theta = \frac{1}{2}$

$\alpha = \cos^{-1}\left(\frac{3}{4}\right)$ $\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\alpha = 41.4°$ $\alpha = 60°$

$\pm 41.4°$



$\pm 60°$

$\therefore,\ \theta = \pm 41.4°,\ \pm 60°$

d) $\sin^2\theta - 2 = 2\cos^2\theta - 4\sin\theta$

$\sin^2\theta - 2 = 2(1-\sin^2\theta) - 4\sin\theta$

$\sin^2\theta - 2 = 2 - 2\sin^2\theta - 4\sin\theta$

$3\sin^2\theta + 4\sin\theta - 4 = 0$

$3\sin^2\theta + 6\sin\theta - 2\sin\theta - 4 = 0$

$3\sin\theta(\sin\theta + 2) - 2(\sin\theta + 2) = 0$

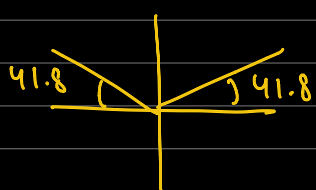
$(3\sin\theta - 2)(\sin\theta + 2) = 0$

$3\sin\theta - 2 = 0$ $\sin\theta + 2 = 0$

$\sin\theta = \frac{2}{3}$ ↳ No answer

$\alpha = \sin^{-1}\left(\frac{2}{3}\right)$

$\alpha = 41.8$



$\theta = 41.8°,\ 138.2°$ ✓

e) $4(2 + \cos^2\theta) = \sin\theta(11 + \sin\theta)$

$8 + 4\cos^2\theta = 11\sin\theta + \sin^2\theta$

$8 + 4(1-\sin^2\theta) = 11\sin\theta + \sin^2\theta$

$8 + 4 - 4\sin^2\theta = 11\sin\theta + \sin^2\theta$

$0 = 5\sin^2\theta + 11\sin\theta - 12$

$= 5\sin^2\theta + 15\sin\theta - 4\sin\theta - 12$

$= 5\sin\theta(\sin\theta + 3) - 4(\sin\theta + 3)$

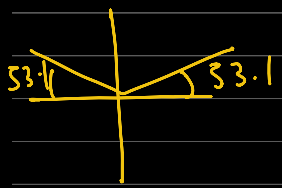
$0 = (5\sin\theta - 4)(\sin\theta + 3)$

$\sin\theta = \frac{4}{5}$ or $\sin\theta = -3$

↳ No answer

$\alpha = \sin^{-1}\left(\frac{4}{5}\right)$

$= 53.1°$



$\theta = 53.1°, 126.9°$

f) $2\cos^3\theta - 5\cos^2\theta - 3\cos\theta = 0$

$(\cos\theta)(2\cos^2\theta - 5\cos\theta - 3) = 0$

$\cos\theta = 0$ or $2\cos^2\theta - 5\cos\theta - 3 = 0$

$\theta = \pm 90°$

$2\cos^2\theta - 6\cos\theta + \cos\theta - 3 = 0$

$2\cos\theta(\cos\theta - 3) + 1(\cos\theta - 3) = 0$

$(2\cos\theta + 1)(\cos\theta - 3) = 0$

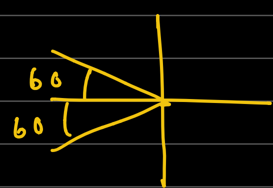
$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 3$

↳ No answer

$\therefore, \theta = \pm 90°, \pm 120°$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\alpha = 60$



$\theta = \pm 120°$

g) $2\cos^3\theta = 3\sin\theta\cos\theta$

$2\cos^3\theta - 3\sin\theta\cos\theta = 0$

$(\cos\theta)(2\cos^2\theta - 3\sin\theta) = 0$

$\cos\theta = 0$ or $2\cos^2\theta - 3\sin\theta = 0$

$\theta = \pm 90°$

$2 - 2\sin^2\theta - 3\sin\theta = 0$

$2\sin^2 + 3\sin\theta - 2 = 0$

$2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$

$2\sin\theta(\sin\theta + 2) - 1(\sin\theta + 2) = 0$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$\therefore, \theta = \pm 90°, 30°, 150°$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -2$

↳ No Ans

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = 30$

30, 150

h) $4\sin\theta\cos\theta(1+\sin\theta) = 11\cos^3\theta - 7\cos\theta$

$4\sin\theta\cos\theta + 4\sin^2\cos\theta - 11\cos^3\theta + 7\cos\theta = 0$

$(\cos\theta)(4\sin\theta + 4\sin^2\theta - 11\cos^2\theta + 7) = 0$

$\cos\theta = 0$
$\theta = \pm 90^\circ$

$4\sin\theta + 4\sin^2\theta - 11(1-\sin^2\theta) + 7 = 0$
$4\sin^2 + 4\sin\theta - 11 + 11\sin^2\theta + 7 = 0$
$15\sin^2\theta + 4\sin\theta - 4 = 0$
$15\sin^2\theta + 10\sin\theta - 6\sin\theta - 4 = 0$
$5\sin\theta(3\sin\theta + 2) - 2(3\sin\theta + 2) = 0$

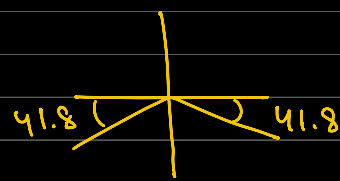
$(5\sin\theta - 2)(3\sin\theta + 2) = 0$

$\sin\theta = \frac{2}{5}$ or $\sin\theta = -\frac{2}{3}$

$\alpha = \sin^{-1}\left(\frac{2}{5}\right)$ $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$

$\alpha = 23.6^\circ$ $\alpha = 41.8$

23.6, 156.4



−41.8, −138.2

$\therefore, \theta = \pm 90^\circ, 23.6^\circ, 156.4^\circ, -41.8^\circ, -138.2^\circ$

Trig past papers from worksheets [NG]

1i) $3\tan\theta = 2\cos\theta \rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\frac{3\sin\theta}{\cos\theta} = 2\cos\theta$
$3\sin\theta = 2\cos^2\theta$
$3\sin\theta = 2(1-\sin^2\theta)$
$3\sin\theta = 2 - 2\sin^2\theta$
$2\sin^2\theta + 3\sin\theta - 2 = 0 \rightarrow$ shown

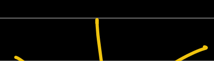
(ii) $2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$
$2\sin\theta(\sin\theta + 2) - 1(\sin\theta + 2) = 0$
$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$\sin\theta = -2$
NS

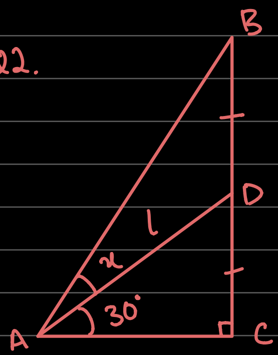
$\sin\theta = \frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$
$\alpha = 30$

$\therefore, \theta = 30^\circ, 150^\circ$



Q2.



(i) $\cos 30 = \frac{AC}{l}$

$\frac{\sqrt{3}}{2} = \frac{AC}{l}$

$\frac{l\sqrt{3}}{2} = AC$

$\sin 30 = \frac{DC}{l}$

$l \sin 30 = DC$

$l \cdot \frac{1}{2} = DC$

$BC = 2DC$

$BC = l$

$AC^2 + CB^2 = AB^2$

$\left(\frac{l\sqrt{3}}{2}\right)^2 + l^2 = AB^2$

$\frac{3l^2}{4} + l^2 = AB^2$

$\frac{7l^2}{4} = AB^2$

$\frac{l\sqrt{7}}{2} = AB \rightarrow$ shown

i) $x = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 30$

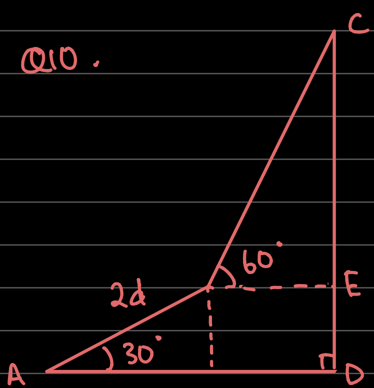
$\tan(x+30) = \frac{\cancel{l}}{l\frac{\sqrt{3}}{2}}$

$\tan(x+30) = \frac{2}{\sqrt{3}}$

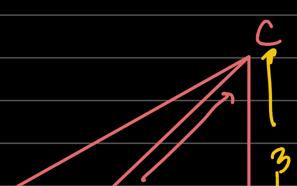
$x + 30 = \tan^{-1}\left(\frac{2}{3}\right)$

$x = \tan^{-1}\left(\frac{2}{3}\right) - 30 \rightarrow$ shown.

Q10.

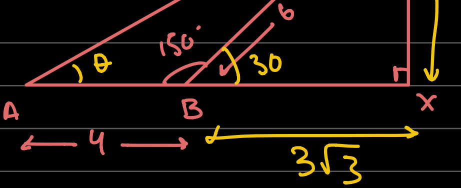


Q12.



(i) $\cos 30 = \frac{BX}{6}$

$\frac{\sqrt{3}}{2} = \frac{BX}{6}$



$$\frac{6\sqrt{3}}{2} = BX$$

$$3\sqrt{3} = BX$$

$$\sin 30 = \frac{CX}{6}$$

$$\frac{1}{2} = \frac{CX}{6}$$

$$3 = CX$$

$$\tan\theta = \frac{CX}{AX}$$

$$\tan\theta = \left(\frac{3}{4+3\sqrt{3}}\right)$$

$$\theta = \tan^{-1}\left(\frac{3}{4+3\sqrt{3}}\right)$$

↑

CAB

↳ <u>shown</u>

(ii) $(4+3\sqrt{3})^2 + 3^2 = AC^2$

$$16 + 24\sqrt{3} + 27 + 9 = AC^2$$

$$\sqrt{(52 + 24\sqrt{3})} = AC \rightarrow \text{shown}$$

Q13. $x = \sin^{-1}\left(\frac{2}{5}\right)$

(i) $\cos^2 x$

$$\sqrt{25-4} = \sqrt{21}$$

$$\sin x = \frac{2}{5}$$

$$\cos x = \frac{\sqrt{21}}{5}$$

$$\cos^2 x = \frac{21}{25} \rightarrow \text{Ans.}$$

(ii) $\tan^2 x$

$$\tan x = \frac{2}{\sqrt{21}}$$

$$\tan^2 x = \frac{4}{21}$$

Q14. $\dfrac{1-\tan^2 x}{1+\tan^2 x} = 1-2\sin^2 x$

$1-\left(\dfrac{\sin^2 x}{\cos^2 x}\right) \times \dfrac{1}{1+\left(\dfrac{\sin^2 x}{\cos^2 x}\right)}$

$= \dfrac{\cos^2 - \sin^2 x}{\cos^2 x} \times \dfrac{\cos^2 x}{\cos^2 x + \sin^2 x}$

$= \dfrac{\cancel{(\cos^2 x)}(\cos^2 x - \sin^2 x)}{\cancel{(\cos^2 x)}(\cos^2 x + \sin^2 x)}$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$

$= \dfrac{1 - \sin^2 x - \sin^2 x}{1 - \cancel{\sin^2 x} + \cancel{\sin^2 x}}$

$= 1-2\sin^2 x \rightarrow$ shown

Q15. $3\sin x \tan x = 8$

$3\sin x\left(\dfrac{\sin x}{\cos x}\right) = 8$

$\dfrac{3\sin^2 x}{\cos x} = 8$

$3\sin^2 x = 8\cos x$

$3 - 3\cos^2 x = 8\cos x$

$0 = 3\cos^2 x + 8\cos x - 3 \rightarrow$ shown

(ii) $0 \leq x \leq 360$

$3\cos^2 x + 9\cos x - \cos x - 3 = 0$

$3\cos x(\cos x + 3) - 1(\cos x + 3) = 0$

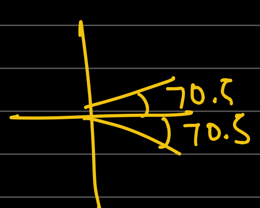
$3\cos x - 1 = 0$ or $\cos x + 3 = 0$

$\cos x = -3 \rightarrow$ NS.

$\cos x = \dfrac{1}{3}$

$\alpha = \cos^{-1}\left(\dfrac{1}{3}\right)$

$= 70.5$



$\theta = 70.5^\circ, 289.5^\circ$

Q17. $2\tan^2\theta \cos\theta = 3$

$$2\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\cos\theta = 3$$

$$\frac{2\sin^2\theta}{\cos\theta} = 3$$

$$2\sin^2\theta = 3\cos\theta$$
$$2 - 2\cos^2\theta = 3\cos\theta$$

$$0 = 2\cos^2\theta + 3\cos\theta - 2 \rightarrow \underline{\text{shown}}$$

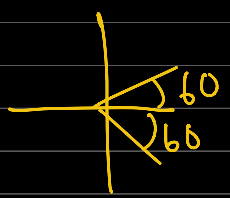
(ii) $2\cos^2\theta + 4\cos\theta - \cos\theta - 2$
$= 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2)$

$= (2\cos\theta - 1)(\cos\theta + 2)$

$\cos\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = -2 \rightarrow$ NS.

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\alpha = 60°$



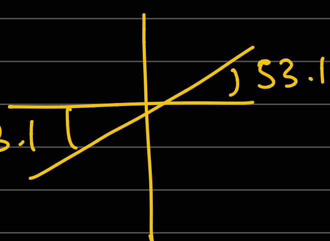
$\theta = 60°, 300°$

Q22 $3\tan(2x + 15) = 4$, $\quad 0 \leq x \leq 180$

$$\tan(2x + 15) = \frac{4}{3}$$

$$15 \leq \theta \leq 375$$

$\theta = 2x + 15$



$\tan\theta = \frac{4}{3}$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$= 53.1$

$\theta = 53.1, 233.1$

53.1 = 53.1
233.1 = 233.1
360 + 53.1 = x
360 + 233.1 = x

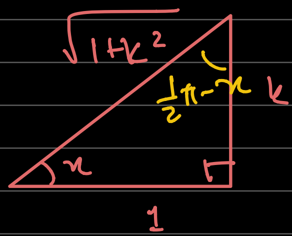
$53.1 = 2x + 15$
$38.1 = 2x$
$19.05 = x$

$233.1 = 2x + 15$
$218.1 = 2x$
$109.5 = x$

$\therefore,\ 19.05,\ 109.5$

$= \therefore,\ 19.1^\circ,\ 109.5^\circ \rightarrow$ Ans

Q25.



(i) $\tan(\pi - x) = -k$

(ii) $\tan(\frac{1}{2}\pi - x) = \frac{1}{k}$

(iii) $\sin x = \frac{k}{\sqrt{1+k^2}}$

Q30. $15\sin^2 x = 13 + \cos x \qquad 0 < x \leq 180$

$15 - 15\cos^2 x = 13 + \cos x$

$0 = 15\cos^2 + \cos x - 2$

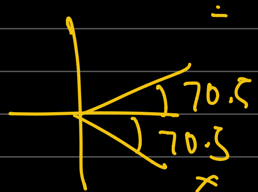
$0 = 15\cos^2 x + 6\cos x - 5\cos x - 2$

$= 3\cos x(5\cos x + 2) - 1(5\cos x + 2)$

$\cos x = \frac{1}{3}$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$= 70.5$



$\theta = 70.5$

$\cos x = -\frac{2}{5}$

$\alpha = \cos^{-1}\left(\frac{2}{5}\right)$

$\alpha = 66.4$

$\theta = 113.6$

$\therefore,\ \theta = 70.5^\circ,\ 113.6^\circ$

Q26. $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$

(i) $6\sin x - 3\cos x = 2\sin x - 6\cos x$

$$4\sin x = -3\cos x$$

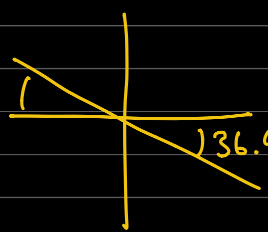
$$\frac{\sin x}{\cos x} = -\frac{3}{4}$$

$$\tan x = -\frac{3}{4} \rightarrow \text{shown}$$

(ii) $\tan x = -\frac{3}{4}$

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\alpha = 36.9^\circ$$



$\theta = 323.1^\circ, 143.1^\circ \rightarrow \underline{\underline{\text{Ans}}}$

Solving trig problems in Radians
(WS on G ClassRoom)

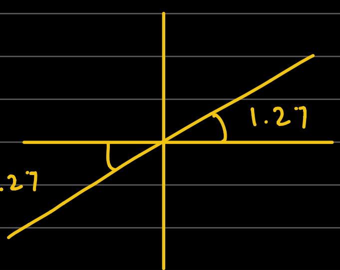
1. $y < 4$, largest value of y (in radians)

$$5\tan(2y+1) = 16$$

$$\tan(2y+1) = \frac{16}{5}$$

$$\alpha = \tan^{-1}\left(\frac{16}{5}\right)$$

$$\alpha = 1.27$$



$\theta = 1.27, \pi + 1.27$
$1.27, 4.41$

$-2\pi + 1.27 = -5.01 = 2y + 1 = -3.01$
$-2\pi + 4.41 = -1.87 = 2y + 1 = -1.44$

$0 + 1.27 = 1.24 = 2y + 1 = 0.12$
$0 + 4.41 = 4.41 = 2y + 1 = 1.71$ x

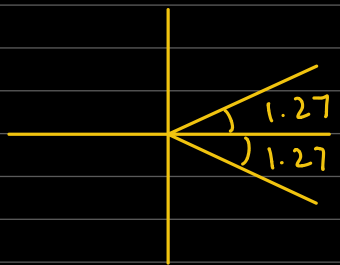
$2\pi + 1.27 = 7.55 = 2y + 1 = \boxed{3.28}$ → The largest value of y that satisfies the equation
$2\pi + 4.41 = 10.69 = 2y + 1 = 4.85$ x

2. $x > 10$, smallest value of x

$10\cos\left(\frac{x+1}{2}\right) = 3$

$\cos\left(\frac{x+1}{2}\right) = 0.3$

$\alpha = \cos^{-1}(0.3)$
$\alpha = 1.27$



$\theta = 1.27,\ 5.01$

$-2\pi + 1.27 = -5.01$
$-2\pi + 5.01 = -1.27$

$0 + 1.27 = 1.27$
$0 + 5.01 = 5.01 \rightarrow \frac{x+1}{2}$ 9.02

$2\pi + 1.27 = \boxed{}$ → oughta be the Ans.
$2\pi + 5.01 = 11.29 \rightarrow 21.6$ x
GUESS

$2\pi + 1.27 = 14.1$ ✓

$\therefore,\ x = 14.1$ → Ans

3. $0 < y < 4$

$\frac{1}{\tan 2y} = 0.2$

$\tan 2y = 5$

$\alpha = \tan^{-1}(5)$
$\alpha = 1.37$

$\theta = 1.37,\ 4.51$

$-2\pi + 1.37$ x
$-2\pi + 4.51$ x

$0 + 1.37 = 1.37 = 2y \rightarrow 0.69$
$0 + 4.51 = 4.51 = 2y \rightarrow 2.26$
$2\pi + 1.37 = 7.65 = 2y \rightarrow 3.83$
$0 < y < 4$
$2\pi + 4.51 = 10.79 = 2y \rightarrow 5.40$ x

$\therefore,\ y = 0.69,\ 2.26,\ 3.83$

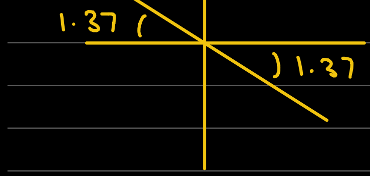
5. $y > 3$, smallest value of y

$\tan(3y - 2) = -5$
$\alpha = \tan^{-1}(5)$
$\alpha = 1.37$

-2π x
-2π x

0 x
$0 + 4.91 = 4.91 = 3y - 2$ x

$2\pi + 1.77 = 8.05 = 3y - 2 = 3.35$ ✓



$\theta = 1.77, 4.91$

$\therefore, y = 3.35$

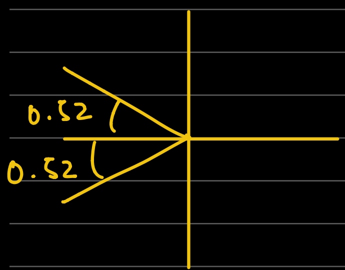
6b) $4 \le y \le 6$

$2\cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0$

$\cos\left(\frac{2y}{3}\right) = \frac{-\sqrt{3}}{2}$

$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\alpha = 0.52$



$\theta = 2.62, 3.66$

-2π x
-2π x
0 x
$0 + 3.66 = 3.66 = \frac{2y}{3} \rightarrow 5.49$ ✓
$2\pi + 2.62 = 8.9 = \frac{2y}{3} =$ x
2π x x

$\therefore, y = 5.49$
$= 5.5$ rad.

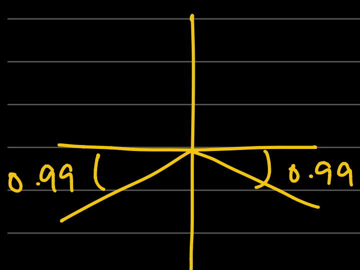
5b). $y > 0$, two smallest possible values

$6\sin(2y+1) = -5$

$\sin(2y+1) = -\frac{5}{6}$

$\alpha = \sin^{-1}\left(\frac{5}{6}\right)$

$\alpha = 0.99$



$\theta = 4.13, 5.29$

-2π x
-2π x
$0 + 4.13 = 4.13 = 2y+1 = 1.57$
$0 + 5.29 = 5.29 = 2y+1 = 2.15$

$\therefore, y = 1.57, 2.15$ rad

9. $0 \le x \le 2$

$1 + 5\cos 3x = 0$

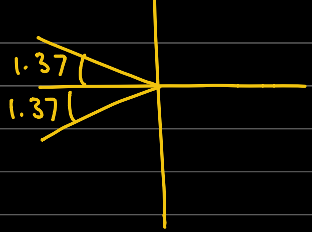
$\cos 3x = \frac{-1}{5}$

$\alpha = \cos^{-1}\left(\frac{1}{5}\right)$

$\alpha = 1.37$

-2π x
-2π x
0 ✓
0 ✓
2π x
2π x

$1.77 = 3y \rightarrow 0.59$
$4.51 = 3y \rightarrow 1.5$



$\theta = 1.77, 4.51$

$\therefore, x = 0.59, 1.5$ rad.

Q.37.

$3\sin^2 x - 8\cos x - 7 = 0$

$\cos x = \frac{-2}{3}$

$3 - 3\cos^2 x - 8\cos x - 7 = 0.$

$3\cos^2 x + 8\cos x + 4 = 0$

$3\cos^2 x + 6\cos x + 2\cos x + 4 = 0$

$3\cos x(\cos x + 6) + 2(\cos x + 2) = 0$

$\frac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$

$\frac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta = 5$

$\cos\theta + 4 + 5\sin^2\theta + \cancel{5\sin\theta} = \cancel{5\sin\theta} + 5$

$5\sin^2\theta + \cos\theta - 1 = 0$

$5 - 5\cos^2\theta + \cos\theta - 1 = 0$

$-5\cos^2\theta + \cos\theta + 4 = 0$

$5\cos^2\theta - \cos\theta - 4 = 0 \rightarrow$ shown

$5\cos^2\theta - 5\cos\theta + 4\cos\theta - 4 = 0$

$5\cos\theta(\cos\theta - 1) + 4(\cos\theta - 1) = 0$

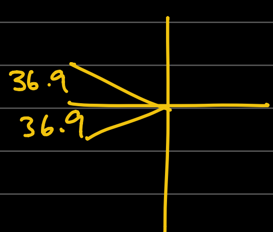
$5\cos\theta + 4 = 0$ or $\cos\theta = 1$

$\cos\theta = \frac{-4}{5}$ $\qquad \cos\theta = 1$

$0°$ and $360°$

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$= 36.9°$



$\theta = 143.1°, 216.9°, 0°, 360°$

7a. i) $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1} = \dfrac{\dfrac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}}$

$= \dfrac{\cancel{(\cos^2\theta)}(\sin^2\theta - \cos^2\theta)}{\cancel{(\cos^2\theta)}(\sin^2\theta + \cos^2\theta)}$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta}$

$= \dfrac{\sin^2\theta - (1 - \sin^2\theta)}{\sin^2\theta + (1 - \sin^2\theta)}$

$= \dfrac{\sin^2\theta - 1 + \sin^2\theta}{\cancel{\sin^2\theta} + 1 - \cancel{\sin^2\theta}}$

$= 2\sin^2\theta - 1 \rightarrow$ shown.

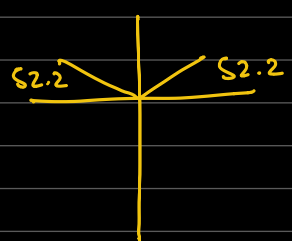
(ii). $\dfrac{1}{4} = 2\sin^2\theta - 1$

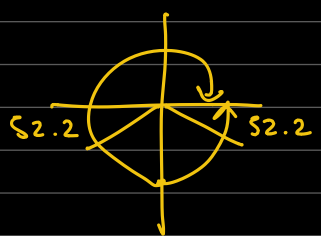
$\dfrac{5}{8} = \sin^2\theta$

$\sqrt{\dfrac{5}{8}} = \sin\theta$

$\sin^{-1}\left(\sqrt{\dfrac{5}{8}}\right) = \alpha$

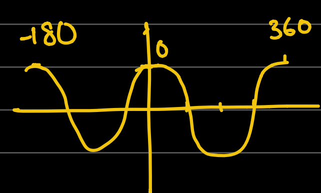
$52.2 = \alpha.$





$\theta = -52.2^\circ$

$3\cos^2\theta - 2\cos\theta - 1 = 0$



$3\cos^2\theta - 3\cos\theta + \cos\theta - 1 = 0$

$3\cos\theta(\cos\theta - 1) + 1(\cos\theta - 1) = 0$

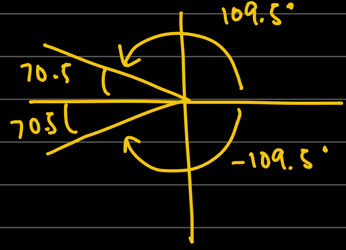
$(3\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\dfrac{1}{3}$ $\qquad$ $\cos\theta = 1$

$\theta = 0^\circ$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

$\theta = 70.5^\circ$



$109.5^\circ, \ -109.5^\circ$

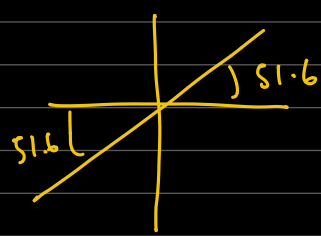
$\therefore, \ \theta = 0^\circ, \ 109.5^\circ, -109.5^\circ$

$$\sin^3\theta + \cos^3\theta = 3\cos^3\theta$$
$$\sin^3\theta = 2\cos^3\theta$$

$$\tan^3\theta = 2$$
$$\tan\theta = \sqrt[3]{2}$$

$$\theta = \tan^{-1}(\sqrt[3]{2}).$$

$$\theta = 51.6$$



$\theta = 51.6^\circ, \ 231.6^\circ$

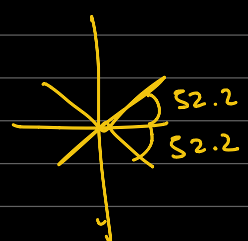
$$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$$

$$\sin\theta - \sin^2\theta\cos\theta + \cos\theta - \sin\theta\cos^2\theta$$
$$\sin\theta - (1 - \cos^2\theta)\cos\theta + \cos\theta - \sin\theta(1 - \sin^2\theta)$$

$= \qquad -\cancel{\cos\theta}$

$= \cancel{\sin\theta} + \cos^3\theta + \cancel{\cos\theta} - \cancel{\sin\theta} + \sin^3\theta$

$= \cos^3\theta + \sin^3\theta$



52.2,