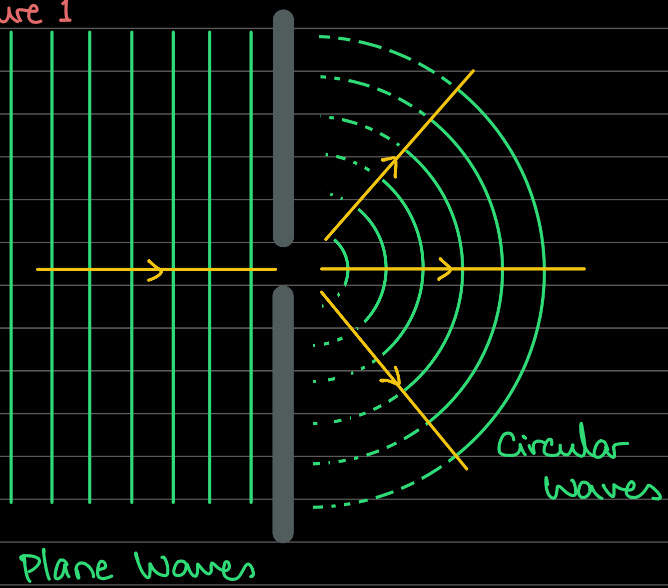
CONCEPT OF DIFFRACTION : WAVES

Definition

The term "diffraction" refers to the spreading of waves when they travel through a narrow gap, small opening, slit, or an aperture.

- Experiments have shown that for significant diffraction to occur, the size of the gap / aperture / opening must be comparable to the wavelength of the waves
- Less diffraction occurs when the size of the gap is significantly larger as compared to the wavelength.

Figure 1



- Diffraction is observed in Fig 1
- During diffraction, wavelength (λ), speed (v) and frequency (f) remains unchanged
- A simplified diagram is shown in Fig 2

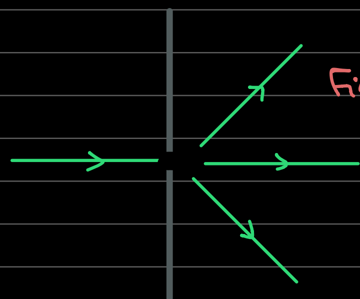


Figure 2

Figure 3

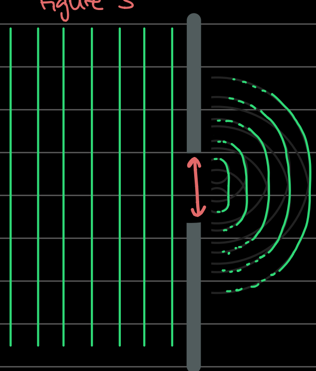
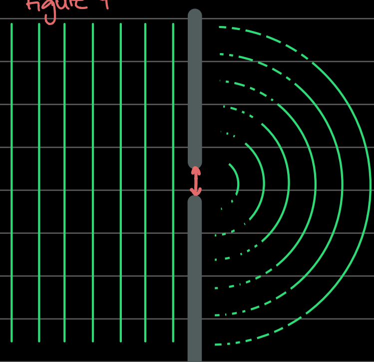
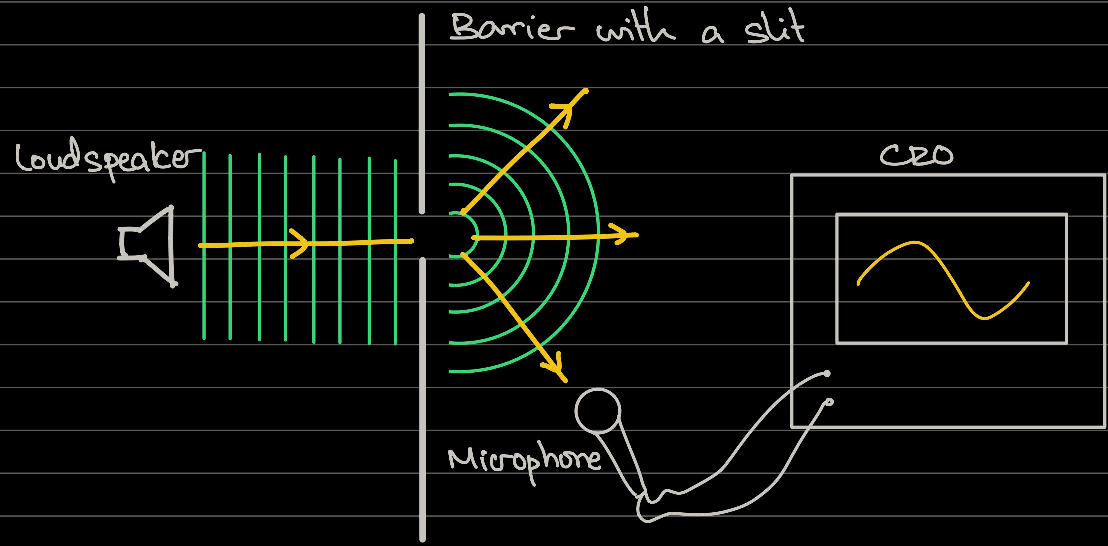


Figure 4



- In fig 3 and fig 4, we can clearly observe that the amount of diffraction / amount of spreading depends on the size of the gap / slit in comparison with the wavelength.
- Given that the size of the gap is comparable to the wavelength → significant diffraction occurs (Fig 4)
- Given that the size of the gap is much larger than the wavelength → less diffraction occurs (Fig 3)

EXPERIMENT TO SHOW DIFFRACTION OF SOUND WAVES

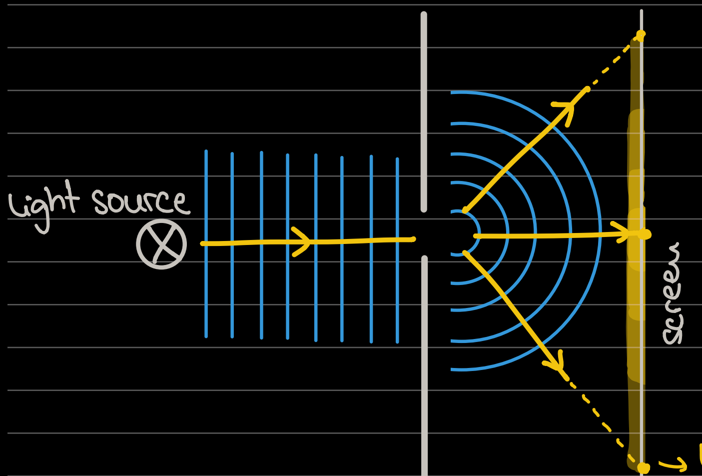


Apparatus Required
- Loudspeaker
- Receiver (Microphone + CRO)
- Barrier with an opening that's about 0.5m wide (roughly the wavelength of sound waves)

Precaution
- Sound proof room

Observation
- A waveform is displayed on the screen of the CRO indicating that sound undergoes diffraction

EXPERIMENT TO SHOW DIFFRACTION OF LIGHT WAVES



Apparatus :
- Lamp/ Light source
- Screen
- Barrier with a very small slit (due to much shorter wavelength of sound)
  - ↳ gap should be of a few cm's or mm's

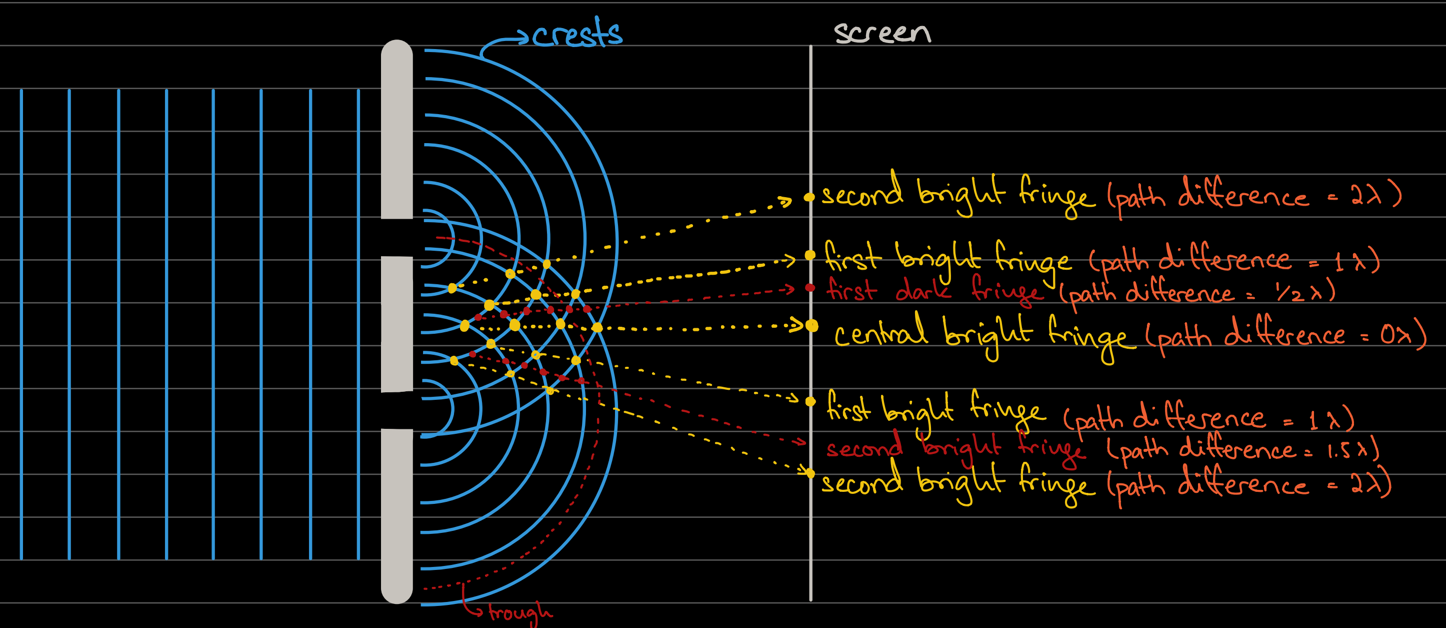
Observation : A large area on the screen is "Lit up", indicating that the light waves are spreading much beyond the width of the slit

YOUNG'S DOUBLE SLIT INTERFERENCE PATTERN : WAVES

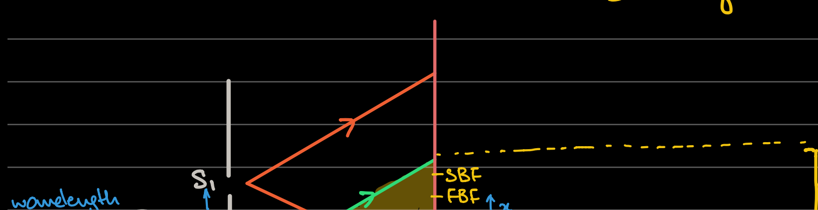
Purpose :
To observe the interference of light

Procedure :
- Light is allowed to pass through two slits S_1 and S_2.
- Diffraction occurs as shown below
- This causes light from one slit to interfere with the light from the other slit

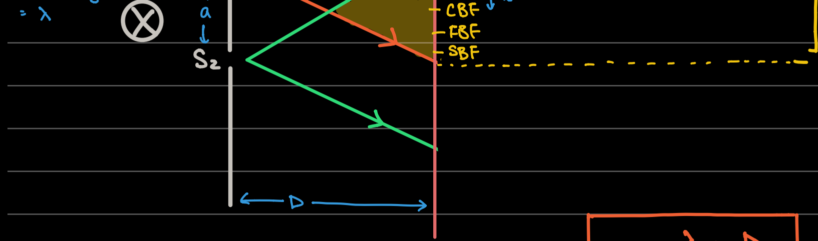
Observation :
- Bright and dark spots, also called fringes, are seen on the screen
- Bright fringes will be observed due to constructive interference
- Dark fringes will be due to destructive interference



SIMPLIFIED VERSION
of the diagram



An interference pattern



is observed on the screen

$$\boxed{x = \frac{\lambda \cdot D}{a}}$$

Typical values for λ, D, and a so that an interference pattern can be observed on the screen?

Learn the following typical values:

λ = 400nm (red) to 700nm (violet) (4×10^{-7}m to 7×10^{-7}m)

D = 1m to 3m

a = 0.5mm to 1.5mm

Example Question:

Q. Calculate the distance between two successive bright fringes using the following values:

$\lambda = 5 \times 10^{-7}$m, D = 1.2m, a = 0.75mm

$$x = \frac{D \cdot \lambda}{a}$$

$$x = \frac{(5 \times 10^{-7})(1.2)}{(0.75 \times 10^{-3})}$$

x = 0.0008 m or 8×10^{-4}m or 0.8mm

Q. Calculate the distance between two successive dark fringes
It is equal to x.
The distance between two bright fringes is equal to the distance between two successive dark fringes

Q. Calculate the distance between a bright fringe and the next dark fringe.

$\frac{1}{2}x$, because dark fringes occur exactly in the middle of two successive bright fringes, and vice versa

Observations in Young's Double Slit Experiment if the following factors are changed independently

1. Distance between the double slit and the screen (D) is increased
   - Brightness of the fringes decreases

$$I \propto \frac{1}{d^2}$$

   - Fringe separation x increases

$$\uparrow x = \frac{\lambda D \uparrow}{a}$$

2. The light source is now replaced with a sound source
   - Interference pattern disappears, that is, fringes will be replaced with loud sound and soft sound (or zero sound if conducted in a sound proof room)
   - Since wavelength of sound is greater than the wavelength of visible sound, $\therefore \uparrow x = \frac{\uparrow \lambda D}{a}$, hence x will increase

3. Size of each slit is increased while keeping 'λ', 'D', and 'a' constant

Insert diagram
from
Sir's Notes

   - Based on the formula $x = \frac{\lambda D}{a}$, since 'λ', 'D', and 'a' are all unchanged, then fringe separation x also remains unchanged.
   - As size of each slit has increased, more light can pass through the slits, therefore, the brightness of the fringes increases
   - Less diffraction since the size of the slit has increased, therefore, interference pattern will be observed over a limited area on the screen

4. The size of one slit is increased whereas the size of the other slit remains unchanged

Insert diagram
from
Sir's Notes

- Bright fringes will now be brighter because more light would be coming through one of the slits
- At the positions of dark fringes, some light will be observed. Therefore, the term "dark fringes" will now be replaced with the term "less bright fringes".
- Appearance will now consist of "more bright" and "less bright" fringes instead of bright and dark fringes
- $x = \frac{\lambda D}{a}$, fringe separation x remains unchanged because 'λ', 'D' and 'a' are all constant

Note: The distance between the slits, 'a', is measured from the centers of the slit

5. One of the slits is completely closed while the other slit remains open

- No interference pattern observed
- No fringes detected
- Light will emerge from one slit and a large area will be well lit up on the screen

6. The slit separation 'a' is reduced while all other factors remain unchanged

DIAGRAM SPACE

- $\uparrow x = \frac{\lambda D}{a \downarrow}$, fringe separation '$x$' will increase
- The brightness of the Central Bright Fringe (CBF) will remain almost constant
- The brightness of successive fringes (ie. FBF, SBF ...) will decrease

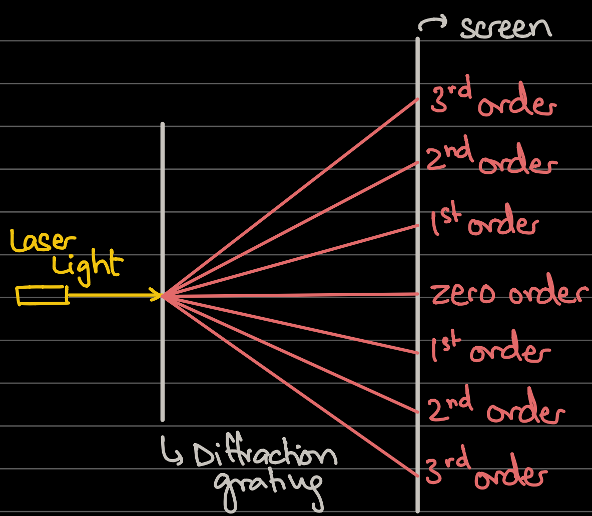
7. A violet source ($\lambda = 400nm$) is replaced with a red source ($\lambda = 700nm$)

- $\uparrow x = \frac{\uparrow \lambda D}{a}$ (λ red $>$ λ violet)
- Fringe separation x increases, and therefore, brightness of successive fringes would marginally decrease

DIFFRACTION GRATING

- A diffraction grating is an optical instrument constructed either using glass or plastic
- This instrument has many microscopic slits, so that when light is allowed to fall on this instrument, diffraction occurs, causing the light to spread
- If a screen is positioned in the background, then the spreading of light can be displayed on the screen
- The angle through which the light spreads is denoted by θ (and is generally measured from the central line)
  ↳ The angle θ can be figured out using the following equation:

$d\sin\theta = n\lambda$ where λ = wavelength
θ = angle through which the light diffracts
n = the number of order (i.e. the order n of the "second bright fringe" is 2)
d = a constant called "grating spacing", which refers to the space b/w. two slits
↳ value provided by manufacturer



Example 1:

Q. A diffraction grating is provided where grating spacing $d = 2\times10^{-6}$ m and $\lambda = 450$ nm

i) Calculate the angle of the first order.

$$d\sin\theta = n\lambda$$
$$(2\times10^{-6})\sin\theta = (1)(450\times10^{-9})$$
$$\theta = \sin^{-1}\left(\frac{450\times10^{-9}}{2\times10^{-6}}\right)$$
$$\theta = 13.0°$$

ii) Calculate the angle of the second order

$$(2\times10^{-6})\sin\theta = (2)(450\times10^{-9})$$
$$\theta = 26.7°$$

iii) Calculate the angle of the third order

$$(2 \times 10^{-6}) \sin\theta = (3)(450 \times 10^{-9})$$
$$\theta = 42.5°$$

iv) Calculate the angle of the fourth order

$$(2 \times 10^{-6}) \sin\theta = (4)(450 \times 10^{-9})$$
$$\theta = 64.2°$$

v) Calculate the angle of the fifth order

$$(2 \times 10^{-6}) \sin\theta = (5)(450 \times 10^{-9})$$
θ = error

- Hence, it could be said that the 4^{th} is the maximum / highest possible order

- Total number of fringes (considering both above and below the 0^{th} order)

$$= 1 + 2(4)$$
$$= 1 + 8$$
$$= 9$$

- The general formula for obtaining the maximum order on one side (of the 0^{th} order) is as follows:

$\theta < 90°$ $\quad\quad$ $\sin\theta < 1$

↳ because max possible value of $\sin\theta$ is 1, at $\sin 90°$

$$d\sin\theta = n\lambda$$
$$\sin\theta = \frac{n\lambda}{d}$$

since $\sin\theta < 1$

$$\boxed{\frac{n\lambda}{d} < 1} \rightarrow \text{general formula}$$

From previous example:

$$\frac{n(450 \times 10^{-9})}{2 \times 10^{-6}} < 1$$
$$n < 4.44$$

↳ and as such, the max. order will be the greatest integer value less than 4.4

Example 2:

Q. $d = 2 \times 10^{-6}$

$\lambda = 590nm$

i) Angle for 1st order

$$d\sin\theta = n\lambda$$
$$(2\times10^{-6})\sin\theta = (1)(590\times10^{-9})$$
$$\theta = 17^\circ$$

(ii) Angle for 2nd order

$$(2\times10^{-6})\sin\theta = (2)(590\times10^{-9})$$
$$\theta = 36.2^\circ$$

iii) Angle for 3rd order

$$(2\times10^{-6})\sin\theta = (3)(590\times10^{-9})$$
$$\theta = 64.2^\circ$$

iv) Angle for 4th order

$$(2\times10^{-6})\sin\theta = (4)(590\times10^{-9})$$
$$\theta = \text{error}$$

v) Determine the maximum order no.

$$\frac{n\lambda}{d} < 1$$
$$\frac{n(590\times10^{-9})}{2\times10^{-6}} < 1$$
$$n < 3.39$$
$$\max n = 3$$

Note: when comparing example 1 and example 2 above, the only difference is that the wavelength of the light in example 2 is greater than that in example 1

↳ and as a result, the maximum order no. is smaller in example 2 than in example 1, and this makes sense because:

$$\frac{n\lambda}{d} < 1$$
$$n\lambda < d$$
$$\boxed{\downarrow n < \frac{d}{\lambda \uparrow}}$$

Example 3:

Q. For the two wavelengths (example 1 & example 2), calculate the angular separation.

i) in the 1st order

$$\underbrace{17}_{\lambda = 450} - \underbrace{13}_{\lambda = 590} = 4^\circ$$

n, d = same

(ii) in the 2nd order

$$\underbrace{36.2}_{\lambda = 450} - \underbrace{26.7}_{\lambda = 590} = 9.5^\circ$$

n, d = same

iii) in the 3rd order

$64.2 - 42.5 = 21.7°$

iv) in the 4th order

This cannot be calculated as there is no fourth order for $\lambda = 590nm$

Angular separation refers to the difference in the angle for the two different wavelengths in the same order

- From example 3, it can be concluded that the angular separation increases as the order number increases and hence, the maximum angular separation will always arise at the highest possible order number.

Example 4

Q. $\lambda_1 = 410nm$ $\lambda_2 = 590nm$
$d = 2 \times 10^{-6}$

i) Calculate the maximum angular separation

Since we know that a higher wavelength results in a smaller maximum order number, and that the maximum angular separation is calculated on the basis of the smaller of the two maximum order numbers resulting from two different wavelengths...

↳ we can just calculate the maximum order using $\lambda = 590nm$, the longer wavelength

$$\frac{n\lambda}{d} < 1$$

$$\frac{n(590 \times 10^{-9})}{2 \times 10^{-6}} < 1$$

$$n < 3.39$$

$$n < 3$$

↳ Maximum angular separation will occur at the 3rd order

For $\lambda = 410nm$

$$d\sin\theta = n\lambda$$

$$(2 \times 10^{-6})\sin\theta = (3)(410 \times 10^{-9})$$

$$\theta = 37.9°$$

For $\lambda = 510nm$

$$d\sin\theta = n\lambda$$

$$(2 \times 10^{-6})\sin\theta = (3)(590 \times 10^{-9})$$

$$\theta = 62.3°$$

Therefore, max angular separation: $62.3 - 37.9$
$= 24.4°$

NOTE : In some questions, the value for the grating spacing 'd' may not be explicitly stated.

Instead, it may be indicated as follows:

The diffraction grating contains 500 lines per mm or 5 slits per mm

i) Use this info to show / prove that the grating spacing, d, is 2×10^{-6} m.

500 lines per 1×10^{-3} m
1 line per x m

$$\frac{500}{1 \times 10^{-3}} = \frac{1}{x}$$

$$x = \frac{1 \times 10^{-3}}{500}$$

$$x = 2 \times 10^{-6} \longrightarrow \text{shown}$$

Example 5 :
Q. A diffraction grating has 650 lines per mm.

i) Calculate grating spacing 'd'

650 lines / 1×10^{-3} m
1 line / x m

$$x = \frac{1 \times 10^{-3}}{650}$$

$$x = 1.54 \times 10^{-6} \text{ m}$$

ii) Given that the 2nd order appears at 61° find λ

$$d \sin\theta = n\lambda$$

$$(1.54 \times 10^{-6}) \sin 61 = 2\lambda$$

$$\lambda = 673 \times 10^{-9}$$

$$\lambda = 673 \text{ nm}$$

iii) Which other wavelength, if any, from the visible light range, will overlap with the wavelength from (ii).

Visible light : 400nm – 700nm

For another wavelength to overlap after diffraction, it must be forming a bright fringe of some unknown order at the same angle as the 2nd order formed by the wavelength in (i) and (ii).

$$d \sin\theta = n\lambda$$

If new wavelength forms 1st order at 61° :

$$(1.54 \times 10^{-6}) \sin 61 = 1\lambda$$
$$\lambda = 1346 \text{ nm}$$ → new wavelength too long

Note: we skip 2nd order because plugging $n = 2$ into this equation would give us the wavelength in part (i) and (ii)

If new wavelength forms 3rd order at 61° :

$$(1.54 \times 10^{-6}) \sin 61 = 3\lambda$$
$$\lambda = 449 \text{ nm}$$ → new wavelength in range

If new wavelength forms 4th order at 61°

$$(1.54 \times 10^{-6}) \sin 61 = 4\lambda$$
$$\lambda = 337 \text{ nm}$$ → new wavelength below range

wavelength just decreases below range from this point onwards

Therefore, the only other wavelength in the visible light range is 449nm.