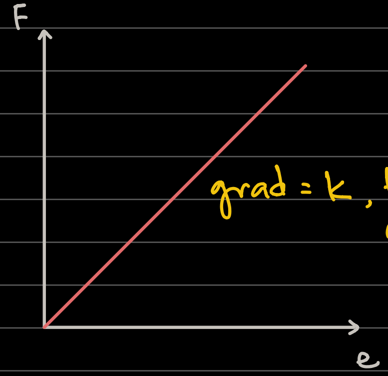
DEFORMATION OF SOLIDS

Hooke's Law

The extension of a spring is directly proportional to the force that is applied on it, but only until the limit of proportionality is reached.



$F \propto e$

$F = ke$ where k is a constant known as the spring constant or the force constant

"k", the spring constant is defined as $k = \frac{F}{e}$, that is, force per unit extension

↳ SI units : Nm^{-1}

Converting the Spring Constant into the SI units

ie. $k = 50\,Ncm^{-1}$

$= \frac{50\,N}{1\,cm}$

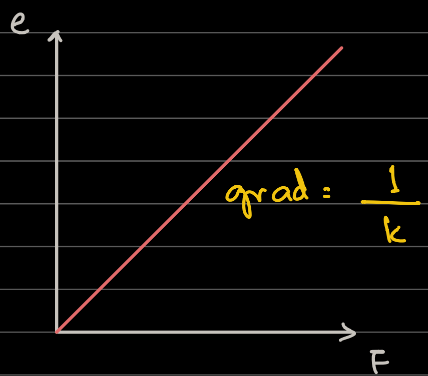
$= \frac{50\,N}{0.01\,m}$

$= 5000\,Nm^{-1}$

In short, to go from Ncm^{-1} to Nm^{-1}...

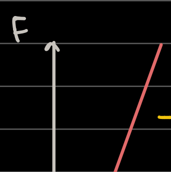
$Ncm^{-1} \times 100 \longrightarrow Nm^{-1}$

Note: When the axes of the force-extension graph are switched, the gradient now represents the inverse of the spring constant, that is, $\frac{1}{k}$



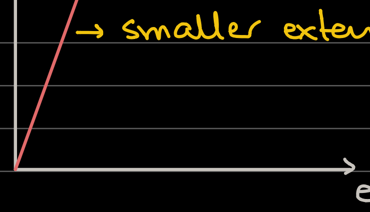
Determining the nature of a spring based on the spring constant :

Example :



Rigid Spring

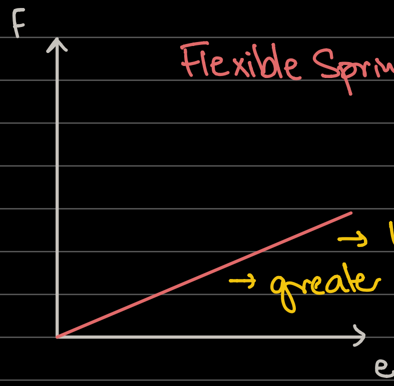
→ steep gradient, high k value



→ smaller extension per unit force

When the gradient of the force-extension curve is high, or when the k value is greater, it means that the spring is more rigid and hence, a lesser extension occurs for every unit of force applied.

Example:

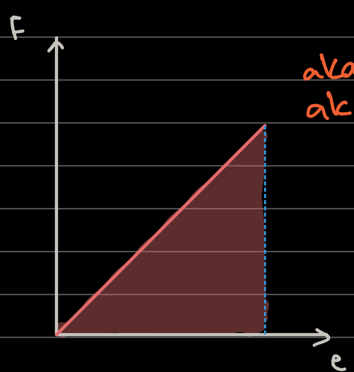


Flexible Spring

→ low grad, lower value of
→ greate extension per unit force applied

Concept of "Energy Stored by a Spring"

- Whenever a spring is either stretched or compressed, it stores energy.
- This energy is called "Elastic Potential Energy", "Strain Energy", or "Work Done by the spring"
- The value for this energy can be obtained from the area between the graph and the extension axis.
  ↳ ie. area under the force-extension graph



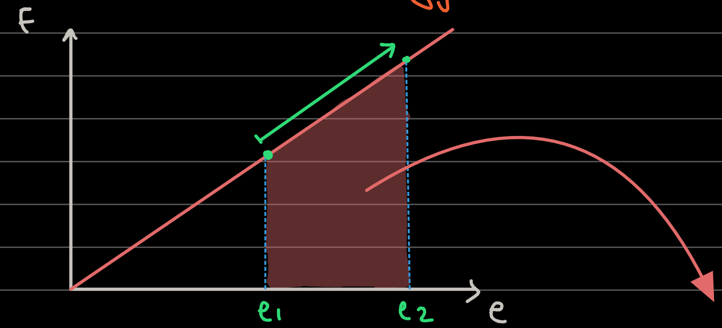
Elastic Potiential Energy
aka Strain Energy
aka Work Done by the Spring:

$$EPE = \frac{1}{2} \underbrace{F}_{(ke)} \cdot e$$

$$= \frac{1}{2} ke^2$$

Concept of "Additional Strain Energy"

- Consider a material which has undergone an initial extension e_1 when a force of F_1 was applied
- If the force increases to F_2 and the corresponding extension is represented by e_2, then the energy stored ONLY during the SECOND STAGE is termed as "Additional Strain Energy"



- Additional Strain Energy: $\frac{1}{2}F_2e_2 - \frac{1}{2}F_1e_1$

Since $F = ke$

$F_1 = ke_1$, $F_2 = ke_2$

$$\frac{1}{2}ke_2^2 - \frac{1}{2}ke_1^2$$

$$\left[= \frac{1}{2}k\left[e_2^2 - e_1^2\right] \right] = \text{Additional Strain Energy}$$

Example:

Q. Spring $k = 40\,\text{N cm}^{-1} \rightarrow 4000\,\text{N m}^{-1}$

$e_1 = 4\text{cm} \rightarrow e_2 = 6\text{cm}$

0.04m $\quad$ 0.06m

Additional Strain Energy $= \frac{1}{2}(4000)\left[0.06^2 - 0.04^2\right]$

$= 4\text{J}$

Example:

Spring $k = 30\,\text{N cm}^{-1} \rightarrow 3000\,\text{N m}^{-1}$

$e_1 = 7\text{cm} \longrightarrow e_2 = 5\text{cm}$

0.07m $\quad$ 0.05m

Can we still use the term additional strain energy?
Yes, although keep in mind that your answer will turn out to be negative.

What is the significance of the negative answer?
Energy was lost / given out by the material

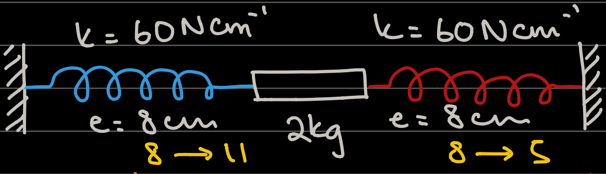
Calculate the Additional Strain Energy

Additional Strain Energy $= \frac{1}{2}(3000)\left[0.05^2 - 0.07^2\right]$

$= -3J \rightarrow$ Energy Released

Example:

Q. i)



The mass is pulled 3 cm to the right and then released. Calculate the total change in elastic potential energy.

$\frac{1}{2}(6000)(0.11^2 - 0.08^2)$

$= 17.1 J$ gained by blue

$\frac{1}{2}(6000)(0.05^2 - 0.08^2)$

$= -11.7 J$ lost by red

$17.1 - 11.7$

$= 5.4J \rightarrow$ Net change in EPE

ii) Given that all of this energy is converted into the KE of the block, calculate the initial speed with which this block begins to move

$\frac{1}{2}mv^2 = 5.4$

$\frac{1}{2}(2)v^2 = 5.4$

$v = 2.3 ms^{-1}$

iii) Describe the motion of this object.

Oscillatory motion about it's mean position