ARITHMETIC PROGRESSION

→ The first term of a progression is called "a"

Example 2 , 6 , 10
(+4 → , +4 →)

The n^{th} term is denoted by T_n
↳ term number

Here, 4 is the common difference, denoted as "d"

The n^{th} term, or T_n, is given by

[1] $T_n = a + (n-1)d$

ie. $T_n = 2 + (4-1)(4)$
$= 2 + 3 \times 4$
$= 14$ → The 4^{th} term / where $n = 4$

S_n refers to the sum of the first n values in a progression
It is given by:

[2] $S_n = \frac{n}{2}[2a + (n-1)d]$

ie. $S_n = \frac{4}{2}[2(2) + (4-1)(4)]$
$= 2[4 + 3 \times 4]$
$= 2[16]$
$= 32$ → Sum of the first four terms

[3.]
Alternate Sum Formula → $S_n = \frac{n}{2}[a + l]$ where l = last term

From the 7- Series ApGp worksheet

4. $a = -10$
$T_{15} = 11$
$l = 41$

$T_n = a + (n-1)d$, $S_n = \frac{n}{2}[2a + (n-1)d]$, $S_n = \frac{n}{2}[a + l]$

Have to first find the term number (n) associated with the last term. using $T_n = a + (n-1)d$

$41 = -10 + (n-1)d$ → $\frac{51}{n-1} = d$

$11 = -10 + 14d$

$\frac{21}{14} = d$ → $\frac{51}{n-1} = \frac{21}{14}$

$$1 + \frac{(14)(51)}{21} = n$$

$35 = n \rightarrow$ term number of the last term

Now we can use the alternate sum formula:

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{35}{2}[-10 + 41]$$

$$= \frac{35}{2} \times 31$$

$$= 542.5 \rightarrow \text{Ans} \rightarrow \text{Sum of all the terms of the progression}$$

16a) Sum of all integers between 100 and 400 that are divisible by 7

First term in the progression in this case would be the smallest integer greater than 100 that is divisible by 7 ~~101~~, ~~102~~, ~~103~~, ~~104~~, 105

And similarly, the last term would be the greatest integer smaller than 400 that's divisible by 7. ~~395~~, ~~396~~, ~~397~~, ~~398~~, 399

And 7 would be d, the common difference.

So... $a = 105$, $l = 399$, $d = 7$

$$T_n = a + (n-1)d$$

$$399 = 105 + (n-1)7$$

$$399 = 105 + 7n - 7$$

$$\frac{399 - 105 + 7}{7} = n$$

$$43 = n \longrightarrow S_n = \frac{n}{2}(a + l)$$

$$= \frac{43}{2}(105 + 399)$$

$$= 10836 \rightarrow \text{Ans}$$

→ Do Q22(a).

23. a) $a = 0.79$, $l = 1.24$, $d = 0.025$

$$T_n = a + (n-1)d$$

$$1.24 = 0.79 + (n-1)0.025$$

i) No. of slabs = 19 slabs

$$\frac{1.24 - 0.79}{0.025} + 1 = n$$

$$19 = n \rightarrow \text{Ans}$$

ii) Length of path.

$S_{19} = \frac{19}{2}(0.79 + 1.24)$

$S_{19} = 19.285 \rightarrow$ Ans

iii) $a = 2.16$, $l = 3.24$

$S_n = \frac{19}{2}[2.16 + 3.24]$

$S_n = 51.3$

∴, total cost = £ 51.30 → Ans

24a) i) $S_4 = 3T_4$

$\frac{4}{2}[2a + (4-1)d] = 3(a + (4-1)d)$

$2(2a + 3d) = 3(a + 3d)$

$4a + 6d = 3a + 9d$

$a = 3d \rightarrow$ Ans (i)

(ii) $T_{18} = 3$

$a + (18-1)d = 3$

$3d + 17d = 3$

$20d = 3$

$d = \frac{3}{20}$

$a = \frac{9}{20}$

$S_{16} = \frac{16}{2}[2a + (16-1)d]$

$= 8[2(\frac{9}{20}) + 15(\frac{3}{20})]$

$= 8(\frac{18}{20} + \frac{45}{20})$

$= 8(\frac{63}{20})$

$= 25.2 \rightarrow$ Ans

25.a) $T_5 = 22$

$a + (5-1)d = 22$

$a + 4d = 22$

$a = 22 - 4d$

$a = 22 - 4(2.5)$

$a = 22 - 10$

$a = 12 \rightarrow$ Ans.

$S_8 = \frac{8}{2}[2a + (n-1)d]$

$= 4[2(22-4d) + 7d]$

$166 = 8(22-4d) + 28d$

$166 = 176 - 32d + 28d$

$166 = 176 - 4d$

$166 - 176 = -4d$

$2.5 = d$

26.b) $l = 3a$

$S_4 = 360$

$S_4 = \frac{4}{2}[a + 3a]$

$360 = 2(4a)$

$\frac{360}{8} = \frac{8a}{8}$

$45 = a$

$135 = l$

a , b , c , l

45 , x , y , 135

30 ↗ ↗

45 75 105 135 → Ans

28a). $S_{10} = T_{10}$

$\frac{10}{2}[2a + 9d] = a + 9d$

$10a + 45d = a + 9d$

$9a = -36d$

$a = -4d$

$T_5 = a + 4d$

$= -4d + 4d$

$T_5 = 0 \rightarrow$ shown

29.a) $T_{40} = -54 \longrightarrow a + 39d = -54 \longrightarrow$

$S_{15} + S_{30} = 0$ $\quad a = -54 - 39d$

↳ $\frac{15}{2}[2a + 14d] + \frac{30}{2}[2a + 29d] = 0$

$15a + 105d + 30a + 435d = 0$

$45a + 540d = 0$

$45(-54 - 39d) + 540d = 0$

$-2430 - 1755d + 540d = 0$

$-2430 - 1215d = 0$

$-2430 = 1215d$

$-2 = d \rightarrow$ Ans.

$a + 39(-2) = -54$

$a - 78 = -54$

$a = -54 + 78$

$a = 24 \rightarrow$ Ans

Arithmetic Progression Questions to attempt:

From the 7series APGP worksheet:

4, 16a

From worksheets (NG) Slide 58 onwards

22a, 23a, 24a, 25a, 26b, 27a, 28a, 29a, 19a, 20a, 21a, 17a, 18a

5a, 9a, 12a, 13a, 15b, 8d, 30a, 31a+b, 34a+b, 35a, 41a+c, 39a, 38c, 37a

22.a).

AP formulas:

$T_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[a + l]$

$a = 2$

$S_8 = 58$

$S_n = 325$ where n is the number of terms in the AP

i) $58 = \frac{8}{2}[2(2) + (8-1)d]$

$58 = 4[4 + 7d]$

$58 = 16 + 28d$

$\frac{42}{28} = \frac{28d}{28}$

$1.5 = d \rightarrow$ Ans

ii) $S_n = 325$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$325 = \frac{n}{2}[2(2) + (n-1)(1.5)]$

$325 = \frac{n}{2}[4 + 1.5n - 1.5]$

$325 = \frac{n}{2}(2.5 + 1.5n)$

$650 = 2.5n + 1.5n^2$

$0 = 1.5n^2 + 2.5n - 650$

$= 3n^2 + 5n - 1300$

$= 3n^2 - 60n + 65n - 1300$

$= 3n(n-20) + 65(n-20)$

$= (3n + 65)(n-20)$

-3900: 100×39, 150×26, 130×30

75×52, 60×65

$n = \frac{-65}{3}$ ✗

$n = 20$ ↳ Ans

iii) Last term

$T_{20} = 2 + (20-1)(1.5)$

$= 2 + (19)1.5$

$= 2 + 28.5$

$T_{20} = 30.5 \rightarrow$ Ans

27a) $T_8 = 150$

$T_{53} = -30$

i) First term and common difference

$T_n = a + (n-1)d$

$T_8 = a + 7d$

$150 = a + 7d$

$T_{53} = a + 52d$

$-30 = a + 52d$

$150 - 7d = a$

$150 - 7(4) = a$

$150 - 28 = a$

$122 = a$

↳ First term

$-30 = 150 - 7d + 52d$

$-30 = 150 + 45d$

$\frac{-180}{45} = \frac{45d}{45}$

$-4 = d \rightarrow$ common Difference

∴, first term → 122
common difference → -4
→ Ans

19a) $S_{10} = 3(S_5)$

$S_{10} = 5[2a + 9d] = 3(S_5) = 3\left(\frac{5}{2}[2a + 4d]\right)$

$= 3\left(\frac{10a + 20d}{2}\right)$

$= 3(5a + 10d)$

$10a + 45d = 15a + 30d$

$15d = 5a$

$3d = a$

$T_{10} : T_5$

$T_{10} = a + 9d$
$= 3d + 9d$
$= 12d$

$T_5 = a + 4d$
$= 3d + 4d$
$= 7d$

$12d : 7d \rightarrow$ 12:7 ratio
↳ Ans

$T_5 = 0.14$

$\frac{7d}{7} = \frac{0.14}{7}$

$d = 0.02$

$3d = a = 0.06$

$S_{200} = 100[2(0.06) + (199 \times 0.02)]$
$= 100[0.12 + 3.98]$
$= 100 \times 4.1$
$= 410 \rightarrow$ Ans

20a) $S_{25} = 200 \rightarrow 200 = \frac{25}{2}[2a + 24d]$

$S_3 = 4.2$

$200 = 25[a + 12d]$

$8 = a + 12d$

$8 = 1.4 - d + 12d$

$8 = 1.4 + 11d$

$\frac{6.6}{11} = \frac{11d}{11}$

$0.6 = d$

$4.2 = \frac{3}{2}(2a + 2d)$

$4.2 = 3[a + d]$

$1.4 = a + d$

$1.4 - d = a$

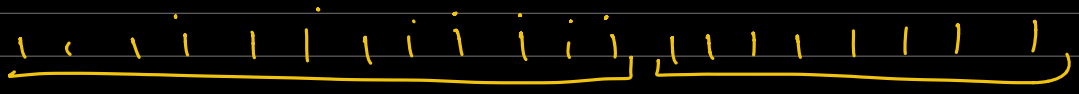
$1.4 - 0.6 = a$

$0.8 = a$

$T_{25} = a + 24d$
$= 0.8 + 24(0.6)$
$= 15.2\text{cm}$
↳ Ans

21a) $T_8 = 25$

$S_{20} - S_{12} = 404$



$T_8 = 25 = a + 7d$

$a + 7d = 25$

$a = 25 - 7d$

$a = 25 - 7(3)$

$a = 25 - 21$

$a = 4$

$10(2a + 19d) - 6(2a + 11d) = 404$

$20a + 190d - 12a - 66d = 404$

$8a + 124d = 404$

$8(25 - 7d) + 124d = 404$

$200 - 56d + 124d = 404$

$200 + 68d = 404$

$\frac{204}{68} = \frac{68d}{68}$

$3 = d$

$S_8 = 4[2(4) + 7(3)]$

$= 4[8 + 21]$

$S_8 = 4 \times 29$

$= 116 \rightarrow$ Ans

17a) $S_{15} = 2(S_{10})$

$S_{10} = 5[2(12) + 9d]$

$= 5[24 + 9d]$

$S_{10} = 120 + 45d$

$15[12 + 7d] = 240 + 90d$

$12 + 7d = 16 + 6d$

$d = 4$

$S_{15} = \frac{15}{2}[24 + 14d]$

$= 15[12 + 7d]$

$= 15[12 + 7(4)]$

$= 15[12 + 28]$

$= 600$

↳ Ans

18a) $S_8 = 24 \longrightarrow S_8 = 4[2a + 7d]$

$S_{18} = 90$

↓

$S_{18} = 9[2a + 17d]$

$90 = 9[2a + 17d]$

$10 = 2a + 17d$

$10 = 6 - 7d + 17d$

$10 = 6 + 10d$

$4 = 10d$

$0.4 = d$

$24 = 4[2a + 7d]$

$6 = 2a + 7d$

$6 = 2a + 7(0.4)$

$3.2 = 2a$

$1.6 = a$

$T_7 = a + 6d$

$= 1.6 + 6(0.4)$

$= 1.6 + 2.4$

$T_7 = 4 \longrightarrow$ Ans

5a) $a = 8$
$T_n = 52$ where n = largest term number
$S_n = 360$

$$S_n = \frac{n}{2}(a + l)$$
$$= \frac{n}{2}(8 + 52)$$
$$360 = \frac{n}{2}(60)$$
$$\frac{360}{30} = \frac{30n}{30}$$
$$12 = n \longrightarrow \underline{\text{Ans}}$$

9a) Already Done → the particle question

12a) $T_1 + T_{16} = 44$
$S_{18} = 3(S_{10}) \longrightarrow$

$$a + a + 15d = 44$$
$$2a + 15d = 44$$
$$2(1.5d) + 15d = 44$$
$$18d = 44$$
$$d = 2.44$$

$$9(2a + 17d) = 3[5(2a + 9d)]$$
$$3(2a + 17d) = 5(2a + 9d)$$
$$6a + 51d = 10a + 45d$$
$$\frac{6d}{4} = \frac{4a}{4}$$
$$\frac{3}{2}d = a$$
$$3.66 = a$$

13a) i) $a = 225$
$d = -12$

$$T_{10} = 225 + 9(-12)$$
$$= 225 - 108$$
$$= 117$$
$$= 1 \text{ min } 57 \text{ seconds}$$
↳ Ans(i)

ii) $S_{12} = 6[2(225) + 6(-12)]$
$= 6[450 - 72]$
$= 6(378)$
$= 2268$
$=$ 37 minutes 48 seconds
↳ Ans(ii)

15.b) → Illegible

8a) $S_n = n^2 + 3n$
$S_2 = 2^2 + 3(2)$
$S_2 = 4 + 6$
$S_2 = 10$

$S_3 = 3^2 + 3(3)$
$S_3 = 9 + 9$
$S_3 = 18$
$T_3 = 8$
$T_2 = 6$
$T_1 = 4$

$S_4 = 4^2 + 3(4)$
$= 16 + 12$
$= 28$
$T_4 = 10$

$\therefore$, $a = 4$, $d = 2$ $\rightarrow$ Ans

30a)