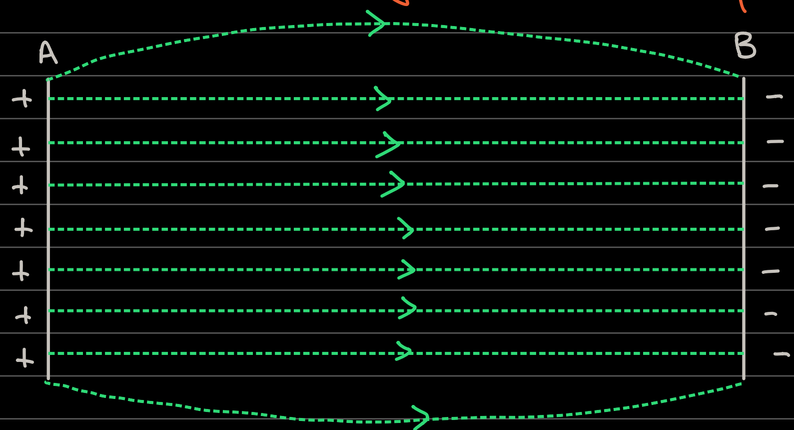
ELECTRIC FIELD : ELECTRICITY

- An electric field is a region around any charged particle where other point charges, if placed, will experience either an attractive or a repulsive force

- Electric fields can be classified as uniform or non uniform

1. Uniform Electric Fields
   are those in which field lines are parallel and equidistant from each other, as shown below.

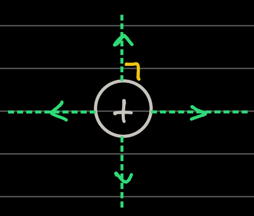
Note: The direction of an electric field is from the positive (+) region to the negative (-) region, or from a region of high potential to a region of lower potential.



- In an eletric field such as this, regardless of where the charge is placed, it experiences the same force

2. Non-uniform electric fields
   are those in which the field lines are neither equidistant nor parallel.

Examples:

1.


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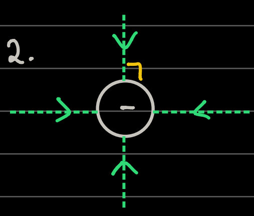

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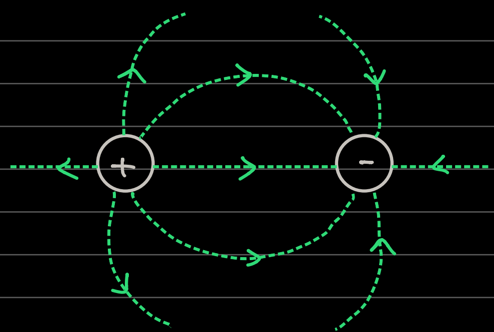

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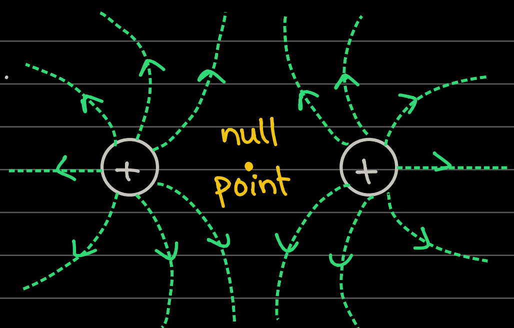

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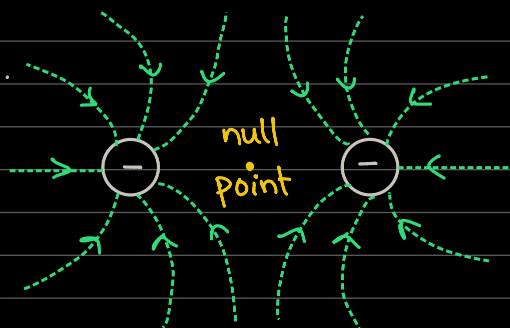
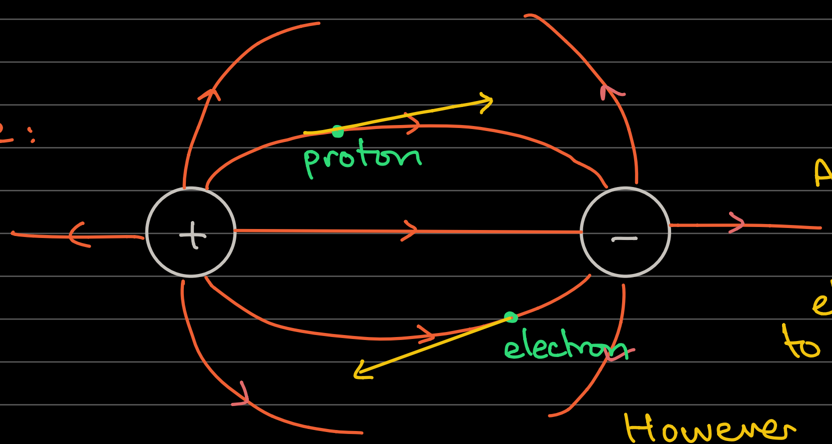


- For identical charges, the null point is in the centre
- However, for non-identical charges, the null point is further away from the larger charge and closer towards the small charge

Note:



A proton, when placed in an electric field, moves in the direction of the electric field at a tangent to the field line.

However, an electron, when placed in an electric, moves in the opposite direction to the electric field at a tangent to the field line.

↪ Given in exams

Important Prerequisites in the Electricity chapter:

mass of proton : 1.66×10^{-27} kg

charge of proton : 1.6×10^{-19} C

mass of electron : 9.11×10^{-31} kg

charge of electron : 1.6×10^{-19} C

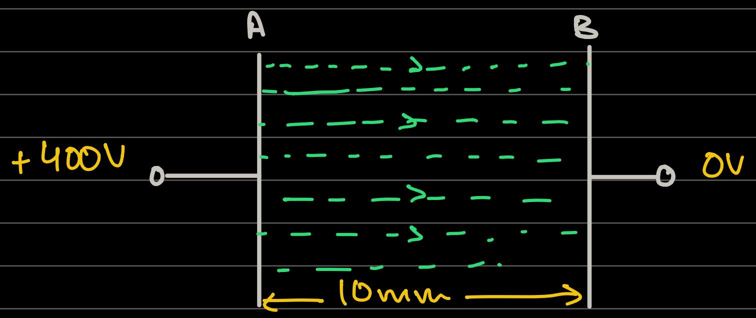
CALCULATING ELECTRIC FIELD STRENGTH

- Denoted by the symbol "E"
- Can be calculated using two formulae
  - The first of these formulae, shown below, applies only to a uniform electric field

Formula #1 $\left[E = \frac{V}{D} \right]$ ⟶ where V = voltage / potential difference (↪ in Volts)

D = Distance b/w

charged plates
↳ in metres



↳ Hence, the electric field strength in this particular case would be:

$$E = \frac{400}{10 \times 10^{-3}}$$

$$E = 4 \times 10^{4}\ Vm^{-1}$$

Formula #2
- This formula is used to define electric field strength
- It is a "universal" formula, meaning that it can be used for both uniform and non-uniform electric fields

Electric field strength is defined as the force per unit of positive charge

Hence:

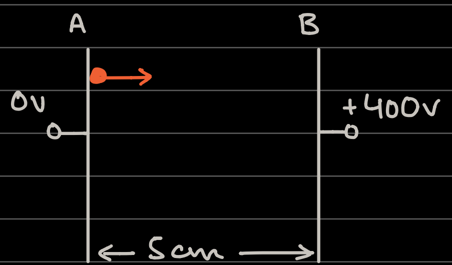
$$\left[E = \frac{F}{q}\right] \quad \text{or} \quad \left[Eq = F\right]$$

where E = Electric field strength
F = Force
q = Unit of positive charge

General units: NC^{-1}

APPLICATION OF THESE FORMULAE:

Example:



Q. An electron, initially at rest at A, travels from A to B.

i) Calculate the electric field strength

$$E = \frac{V}{d} = \frac{400}{0.05} = \boxed{8000\ Vm^{-1}}$$

Remember:
mass of electron → 9.11×10^{-31} kg
charge (q) of electron → -1.6×10^{-19} C

ii) Calculate the force exerted by the electric field onto the electron
↳ magnitude of

$E = \frac{F}{q}$ OR $Eq = F$

$(8000)(-1.6\times10^{-19}) = F$

$\boxed{1.28 \times 10^{-15}\,N} = F$ → Ans. Magnitude of force

iii) Calculate the work done in moving the electron from A to B

The Mechanics method:
(Requires intermediate values such as force to be calculated)

$w.d = F \times d$

$= (1.28 \times 10^{-15})(0.05)$

$w.d = \boxed{6.4 \times 10^{-17}\,J}$ → Ans Work Done

The (direct) "electrical" method:
(Doesn't require intermediate values to be calculated. All required values are given in the question)

$\text{Voltage} = \frac{w.d}{\text{charge}}$ $Vq = W$ where W = Work Done, V = Voltage, q = charge of particle

$V = \frac{W}{q}$

$Vq = W$

$(400)(1.6\times10^{-19}) = W$

$\boxed{6.4\times10^{-17}} = W$

↳ Ans Work Done

iv) Calculate the acceleration of the electron

$F = ma$

$\frac{F}{m} = a$

$\frac{1.28\times10^{-15}}{9.11\times10^{-31}} = a$

$\boxed{1.41 \times 10^{15}\,ms^{-2}} = a$

↳ Ans Acceleration of electron

v) Calculate the final speed of the electron as it reaches plate B.

Mechanics method:

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(1.41\times10^{15})(0.05)$

$v = \boxed{1.19 \times 10^{7}\,ms^{-1}}$ → Ans Final speed of electron

Electrical method:

$$\text{voltage} = \frac{\text{w.d}}{\text{charge}}$$

$$V = \frac{\frac{1}{2}mv^2}{q}$$

$$\left[\frac{2Vq}{m}\right]^{\frac{1}{2}} = v$$

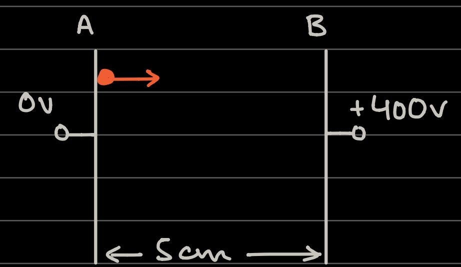
$$\boxed{1.19 \times 10^7 \, ms^{-1}} = v$$

↳ Ans final speed of electron

$$\sqrt{\frac{2Vq}{m}} = v$$

where V = voltage
v = velocity of particle
q = charge of particle
m = mass of particle

Other ways of using the direct velocity formula ($\sqrt{\frac{2Vq}{m}} = v$)



Q. Calculate the speed when the electron is midway between the plates

Total potential difference = 400V

Potential Difference mid-way = 200V

$$\sqrt{\frac{2Vq}{m}} = v$$

$$\sqrt{\frac{2(200)(1.6 \times 10^{-19})}{9.11 \times 10^{-31}}} = v$$

$$8.38 \times 10^6 = v$$

Q. Calculate the speed when the electron has travelled 3/4 of the way

Potential Difference at 3/4 = 300

$$\sqrt{\frac{2Vq}{m}} = v$$

$$\sqrt{\frac{2(300)(1.6 \times 10^{-19})}{9.11 \times 10^{-31}}} = v$$

$$1.03 \times 10^7 \, ms^{-1} = v$$

Q. Four charges: ${}^{23}_{11}Na^+$, ${}^{40}_{20}Ca^{2+}$, ${}^{4}_{2}\alpha^{2+}$, e^{-1}

All four charges are released simultaneously. Which charge reaches the opposite plate with the largest velocity?

$$v = \sqrt{\frac{2qV}{m}}$$

→ Velocity depends on voltage, charge, and mass

Assume that the particles are placed in identical electric fields

Hence, $v \propto \sqrt{\frac{q}{m}}$ ∴, the particle with the greatest $q:m$ ratio will have the greatest velocity

$\frac{q}{m}$ ratios :

Sodium : $\frac{1}{23}$ Calcium : $\frac{2}{40}$

alpha (Helium nucleus) : $\frac{2}{4}$ electron : $\frac{1}{\frac{1}{1840}} = \frac{1840}{1}$ → longest $q:m$ ratio, hence greatest velocity

Q. For the same charges, calculate which particle will have the greatest momentum.

$p = mv$

$p = m \times \sqrt{\frac{2Vq}{m}}$

$p = \sqrt{2Vqm}$ ⟶ hence, momentum depends on voltage, charge, and mass

$p \propto \sqrt{q \cdot m}$

Calculating $q \cdot m$

Sodium : 1×23 Calcium : 2×40 ⟶ greatest momentum

alpha : 2×4 electron : $1 \times \frac{1}{1840}$

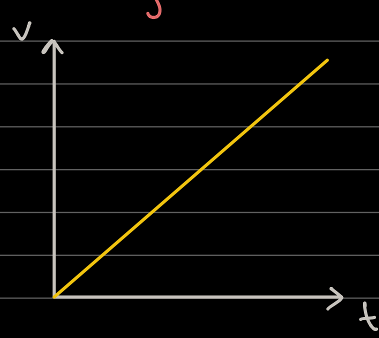
Q. The distance between the two plates A and B is now doubled (from 5cm to 10cm) Suggest what happens to the final velocity of the electron if it now travels from one plate to the other.

Velocity is independent of the distance, it is only dependent on voltage, charge, and mass.

∴, the final velocity remains unchanged

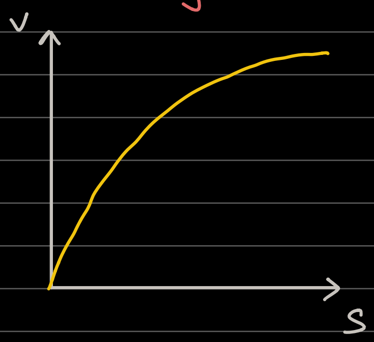
Q. Sketch the following graphs for the electron moving from one plate to the other

i) Velocity - time



Since electric field is uniform, F, and hence, a, remains constant → ∴, constant grad.

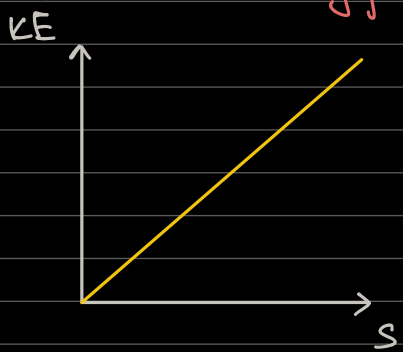
ii) Velocity - distance



$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2as$$
$$v^2 = 2as$$

$$v^2 \propto s$$

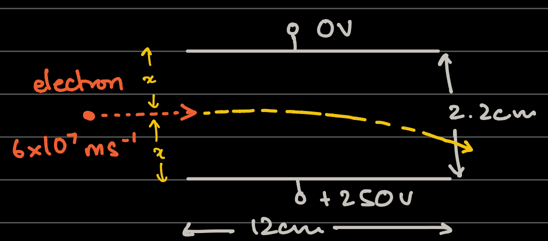
iii) Kinetic Energy - distance



$$K.E = w.d$$
$$w.d = F \times s$$
$$K.E = F.s$$

$$KE \propto s$$

Example :



i) Calculate the electric field strength

$$E = \frac{V}{d} = \frac{250}{0.022} = 1.1 \times 10^4 \ Vm^{-1}$$

ii) Calculate the force on the electron

$F = Eq = (1.1 \times 10^4)(1.6 \times 10^{-19})$

$F = 1.8 \times 10^{-15}$ N (downwards)

iii) Calculate the acceleration of the electron

$F = ma \rightarrow 1.8 \times 10^{-15} = (9.11 \times 10^{-31})\,a$

$a = 2 \times 10^{15} \ ms^{-1}$ (downwards)

iv) Calculate the time taken for the electron to travel between the plates

Horizontally : $u = 6.0 \times 10^7 \ ms^{-1}$

$a = 0$

$s = 12cm \ (0.12 m)$

Distance = speed × time

time = $\frac{\text{Distance}}{\text{speed}}$

$t = \frac{0.12}{6 \times 10^7} = 2 \times 10^{-9}\,s$

v) Calculate the vertical distance fallen as it reaches the opposite end

$S_v = ut + \frac{1}{2}at^2$

$S_v = 0 + \frac{1}{2}(2 \times 10^{15})(2 \times 10^{-9})^2$

$S_v = 4 \times 10^{-3}\,m$

$= 0.004\,m$

$= 0.4\,cm$

vertical displacement from initial position is 0.4cm

vi) Comment on whether this electron strikes the bottom plate or leaves from the other side?

It leaves from the other side because the distance between the particle and the bottom plate is 1.1 cm, whereas the ultimate vertical displacement is less than that, 0.4cm.

PROPERTIES OF ELECTRIC FIELDS

a) An electric field can apply force on a stationary charge as well as a charge already in motion

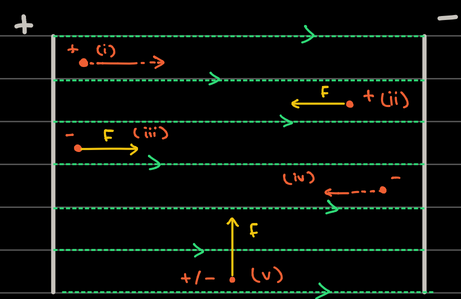
b) The force on a moving charge makes it perform "half projectile motion" or it travels a "parabolic" path

c) The direction of electric force is parallel to the electric field lines

INTERCONVERSION OF ENERGY IN AN ELECTRIC FIELD

- In mechanics, in the case of gravitational fields, kinetic energy was frequently converted into gravitational potential energy

- Similarly, under the influence of electric fields, kinetic energy is frequently converted into electric potential energy



(i)
- If a positive charge moves in the direction of the electric field, it's kinetic energy increases and therefore, in accordance with the law of conservation of energy, the electric potential energy will decrease

(ii)
- If a positive charge moves against the direction of the electric field, it's kinetic energy decreases. Hence, its electric potential energy increases

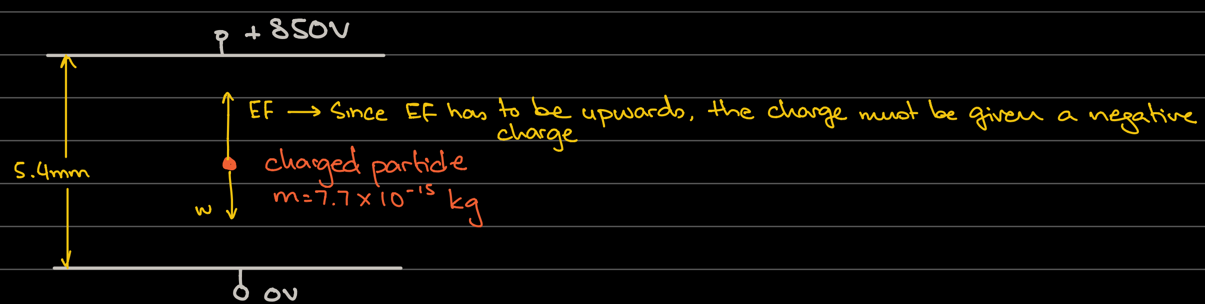
(iii)
- If a negative charge moves in the direction of the electric field, its kinetic energy decreases and it's electric potential energy increases

(iv)
- If a negative charge moves against the direction of the electric field, its kinetic energy increases and it's electric potential energy decreases

(v)
- If a charge (positive or negative) moves perpendicular to the direction of the electric field lines, then change in electric potential energy is zero

QUANTIZATION OF CHARGE

Example:



Q i) In order for the charge to stay stationary, predict, with reason, whether it should be given a positive or a negative charge

A. Weight applies a downward force, therefore electrical force must be upwards, hence, the particle must be negatively charged.

ii) Calculate the magnitude of this charge

w = Electrical force

$mg = Eq$

$mg = \left[\frac{V}{d}\right] q$

$$\boxed{\frac{mgd}{V} = q} \rightarrow \frac{(7.7\times10^{-15})(9.81)(0.0054)}{(850)} = 4.8\times10^{-19}C$$

This means that our charged particle has 3 more electrons than protons.

Charge of one electron : $1.6\times10^{-19}C$ —— $\times 3$ ——

↳ Any charge of any particle must be a multiple of this elementary charge

- This idea that charges always exist as integer/whole number multiples of the elementary charge (ie. the charge of an electron) is called the quantization of charged (ie. any charge can be defined as a "quantity" of elementary charges.

Q. Explain the meaning of the phrase "charges are quantized" ?

A. The term quantization means that charges exist as integer multiples of the elementary charge (ie. the charge of an electron ; $1.6\times10^{-19}C$)