

Q8 from Exercise A

Q. Solve the following,  $-180 \leq \theta < 180$

a)  $\sin(2\theta + 30) = -0.5$

Step 1: fix the range

↳ Gotta make  $-180 \leq \theta < 180$  look like  $2\theta + 30$

$$\begin{aligned} & -180 \leq \theta < 180 \\ \hookrightarrow & -330 \leq \underline{2\theta + 30} < 390 \end{aligned}$$

Step 2: Make a substitution

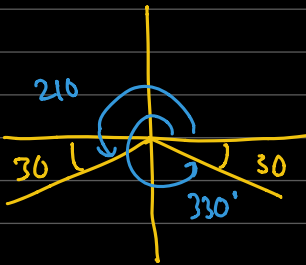
$$\begin{aligned} x &= 2\theta + 30 \\ -330 &\leq x < 390 \end{aligned}$$

$$\sin x = -0.5$$

and then just solve normally

Step 3: find the basic angle and place in appropriate quadrants

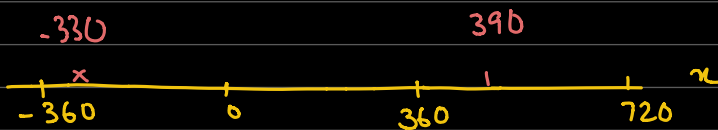
$$\begin{aligned} x &= \sin^{-1}(0.5) \\ x &= 30^\circ \end{aligned}$$



Step 4: find  $\theta$

ie. if it was  $-410 \leq x < 360$

$-720$  would be the starting point



Starting point:  $-360$   
(multiple of 360 before the lower end of the range)

Add both angles to starting point separately.

$$-360 + 210 = -150$$

$$-360 + 330 = -30$$

→ In range

And then add both angles to 0

$$0 + 210 = 210$$

$$0 + 330 = 330$$

And then add both angles to 360

$$360 + 210 = 570$$

$$360 + 330 = 690$$

→ Out of the range

Last step; find  $\theta$

equated all in-range values you just summed to  $2\theta + 30$ , or whatever you substituted  $x$  with.

↳ These will be the final answers, and they will be within the very first range that was initially given in the question

Q. Solve  $\tan(50 - 2\theta) = 2$  for  $-180 \leq \theta \leq 180$

fix range

$$360 + 50 \geq 50 - 2\theta \geq -360 + 50$$
$$410 \geq 50 - 2\theta \geq -310 \rightarrow \text{range fixed.}$$

$$\Downarrow$$
$$-310 \leq 50 - 2\theta \leq 410$$

substitute

$$x = 50 - 2\theta$$

$$-310 \leq x \leq 410$$

$$\tan x = 2$$

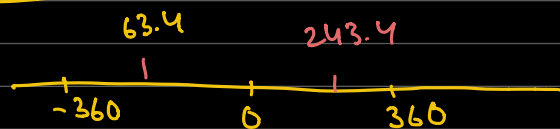
$$x = \tan^{-1}(2)$$

$$x = 63.4^\circ$$



$$x = 63.4, 243.4$$

points



Add to -360

$$-360 + 63.4 = -296.6$$

$$-360 + 243.4 = -116.6$$

Add to 0

$$0 + 63.4 = 63.4$$

$$0 + 243.4 = 243.4$$

→ In range

Add to 360

$$360 + 63.4 = 723.4$$

$$360 + 243.4 = 603.4$$

→ out of range

replace

$$\textcircled{1} -296.6 = 50 - 2\theta$$

$$173.3 = \theta$$

$$\textcircled{2} -116.6 = 50 - 2\theta$$

$$83.3 = \theta$$

$$\textcircled{3} 63.4 = 50 - 2\theta$$

$$-6.7 = \theta$$

$$\textcircled{4} 243.4 = 50 - 2\theta$$

$$-96.7 = \theta$$

→ All values are within the initial range  
 $180 \leq \theta \leq 180$

$$\text{Q. } \cos(2\theta - 40) = -0.4$$

$$-360 \leq \theta \leq 360$$

fix range

$$(-360 \times 2) - 40 \leq 2\theta - 40 \leq (360 \times 2) - 40$$

$$-760 \leq 2\theta - 40 \leq 680$$

subst.

$$x = 2\theta - 40$$

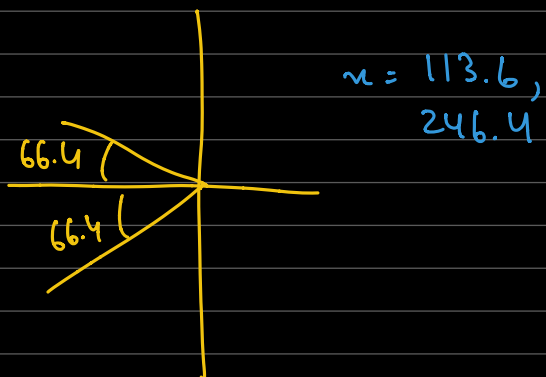
$$-760 \leq x \leq 680$$

$$\cos x = -0.4$$

Alpha

$$\alpha = \cos^{-1}(0.4)$$

$$\alpha = 66.4$$



Range



-1080	-966.4	x
	-833.6	x

-720	-606.4	} →
	-473.6	

-360	-246.4	}
	-113.6	

0	113.6
	246.4

360	473.6	}
	606.4	