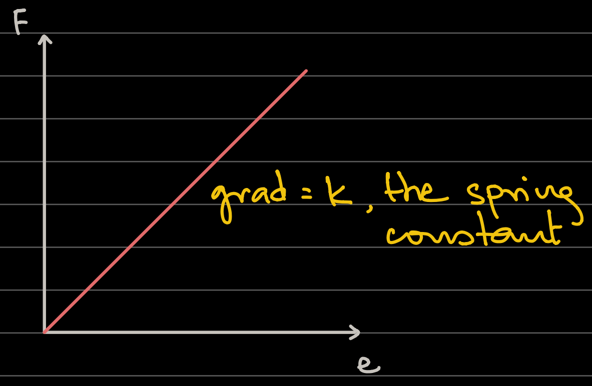
SPRINGS: DEFORMATION OF SOLIDS

Hooke's Law

The extension of a spring is directly proportional to the force that is applied on it, but only until the limit of proportionality is reached.



$F \propto e$

$F = ke$ where k is a constant known as the spring constant or the force constant

"k", the spring constant is defined as $k = \frac{F}{e}$, that is, force per unit extension

↳ SI units : Nm^{-1}

Converting the Spring Constant into the SI units

ie. $k = 50\,Ncm^{-1}$

$= \frac{50\,N}{1\,cm}$

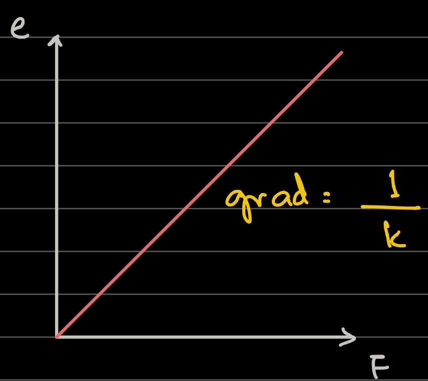
$= \frac{50\,N}{0.01\,m}$

$= 5000\,Nm^{-1}$

In short, to go from Ncm^{-1} to Nm^{-1}...

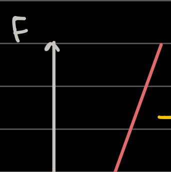
$Ncm^{-1} \times 100 \longrightarrow Nm^{-1}$

Note: When the axes of the force-extension graph are switched, the gradient now represents the inverse of the spring constant, that is, $\frac{1}{k}$



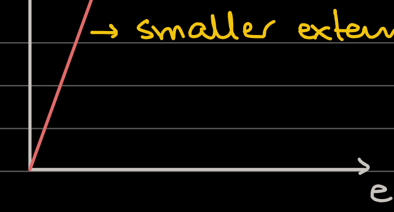
Determining the nature of a spring based on the spring constant:

Example:



Rigid Spring

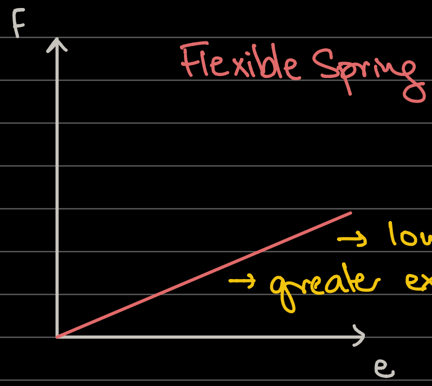
→ steep gradient, high k value



→ smaller extension per unit force

When the gradient of the force-extension curve is high, or when the k value is greater, it means that the spring is more rigid and hence, a lesser extension occurs for every unit of force applied.

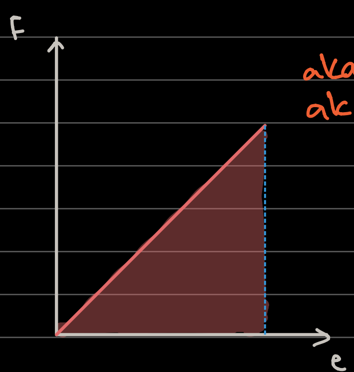
Example:



→ low grad, lower value of
→ greater extension per unit force applied

Concept of "Energy Stored by a Spring"

- Whenever a spring is either stretched or compressed, it stores energy.
- This energy is called "Elastic Potential Energy", "Strain Energy", or "Work Done by the spring"
- The value for this energy can be obtained from the area between the graph and the extension axis.
  ↳ ie. area under the force-extension graph



Elastic Potiential Energy
aka Strain Energy
aka Work Done by the Spring:

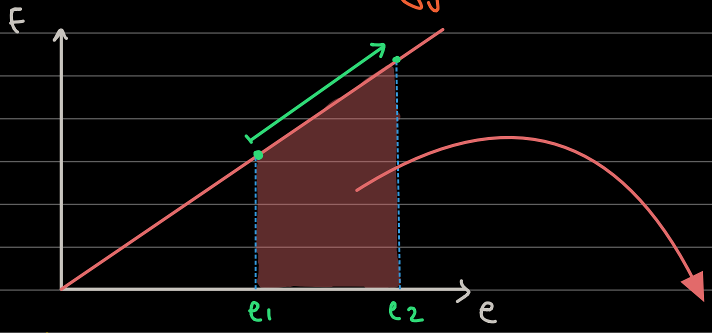
$$EPE = \frac{1}{2} F \cdot e$$

$F \downarrow (ke)$

$$= \frac{1}{2} ke^2$$

Concept of "Additional Strain Energy"

- Consider a material which has undergone an initial extension e_1 when a force of F_1 was applied
- If the force increases to F_2 and the corresponding extension is represented by e_2, then the energy stored ONLY during the SECOND STAGE is termed as "Additional Strain Energy"



- Additional Strain Energy: $\frac{1}{2}F_2e_2 - \frac{1}{2}F_1e_1$

Since $F = ke$

$F_1 = ke_1$, $F_2 = ke_2$

$$\frac{1}{2}ke_2^2 - \frac{1}{2}ke_1^2$$

$$\boxed{= \frac{1}{2}k\left[e_2^2 - e_1^2\right]} = \text{Additional Strain Energy}$$

Example:

Q. Spring $k = 40\,N\,cm^{-1} \rightarrow 4000\,N\,m^{-1}$

$e_1 = 4cm \rightarrow e_2 = 6cm$

$0.04m$ $\quad$ $0.06m$

Additional Strain Energy $= \frac{1}{2}(4000)\left[0.06^2 - 0.04^2\right]$

$= 4J$

Example:

Spring $k = 30\,N\,cm^{-1} \rightarrow 3000\,N\,m^{-1}$

$e_1 = 7cm \longrightarrow e_2 = 5cm$

$0.07m$ $\quad$ $0.05m$

Can we still use the term additional strain energy?

Yes, although keep in mind that your answer will turn out to be negative.

What is the significance of the negative answer?

Energy was lost / given out by the material

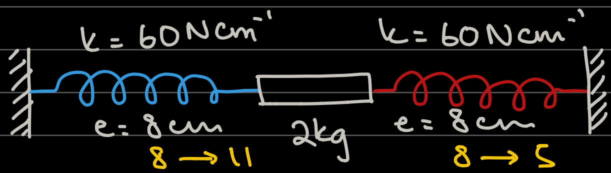
Calculate the Additional Strain Energy

Additional Strain Energy $= \frac{1}{2}(3000)\left[0.05^2 - 0.07^2\right]$

$= -3J \rightarrow$ Energy Released

Example:

Q. i)



The mass is pulled 3 cm to the right and then released. Calculate the total change in elastic potential energy.

$\frac{1}{2}(6000)(0.11^2 - 0.08^2)$ $\quad$ $\frac{1}{2}(6000)(0.05^2 - 0.08^2)$

$= 17.1J$ gained by blue $\quad$ $= -11.7J$ lost by red

$17.1 - 11.7$

$= 5.4J \rightarrow$ Net change in EPE

ii) Given that all of this energy is converted into the KE of the block, calculate the initial speed with which this block begins to move

$\frac{1}{2}mv^2 = 5.4$

$\frac{1}{2}(2)v^2 = 5.4$

$v = 2.3ms^{-1}$

iii) Describe the motion of this object.

Oscillatory motion about it's mean position (ie. simple harmonic motion)

Another Example:

Q. Spring P $\quad$ Spring Q $\quad$ Calculate the ratio of:

F $\quad$ F

k $\quad$ 3k

↳ $e = \frac{F}{k}$ $\quad$ ↳ $e = \frac{F}{3k}$

$\frac{\text{Strain energy in P}}{\text{Strain energy in Q}}$

$\frac{\text{EPE in P}}{\text{EPE in P}} = \frac{\cancel{\frac{1}{2}}Fe_1}{\cancel{\frac{1}{2}}Fe_2} = \frac{e_1}{e_2} = \frac{F/k}{F/3k} = \frac{3}{1}$

∴, ratio $\longrightarrow$ Strain energy in P : Strain energy in Q

3 : 1

Behaviour of Springs Beyond the Limit of Proportionality

- Hooke's law is only applicable till the limit of proportionality, therefore, graph will no longer be a straight line.

- The graph will start bending towards the extension axis which indicates that after the limit of proportionality is exceeded, a small force produces a much larger extension

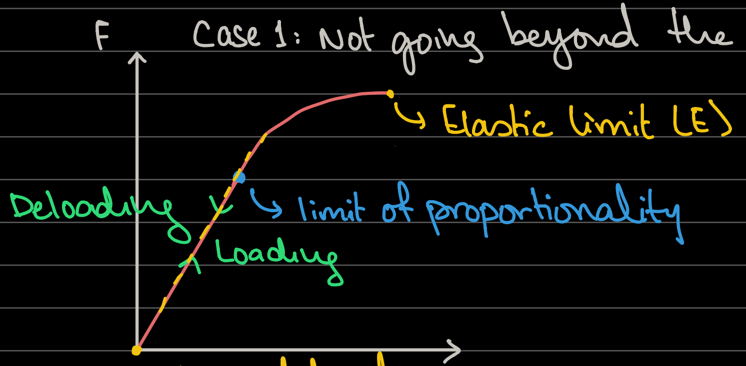
- If the material is stretched further, a point known as Elastic Limit (E) is reached

↳ Elastic limit is the the furthermost point until which the material exhibits ELASTIC DEFORMATION, ie. it regains it's true size once the force is removed

- But if the material goes beyond the Elastic Limit (E), it experiences a permanent extension known as PLASTIC DEFORMATION

↳ This term implies that even if the force is removed, the material is no longer able to achieve it's true size.

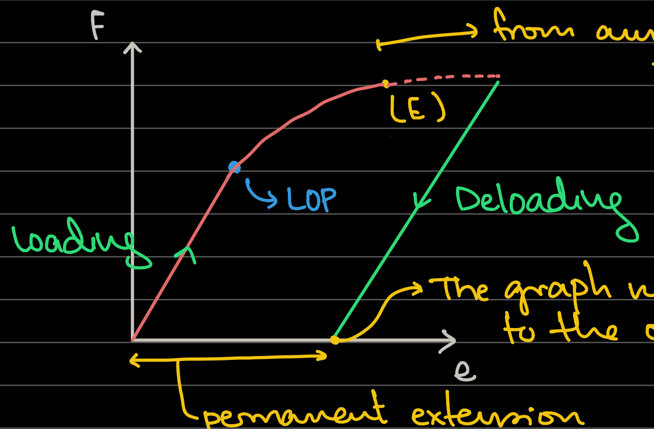
Case 1: Not going beyond the elastic limit (ELASTIC DEFORMATION)



↳ Upon deloading, the graph returns to this point, hence

→ "Loading" and "de-loading" follows the same pattern, because there is no permanent extension

Case 2: Going beyond the elastic limit (PLASTIC DEFORMATION)

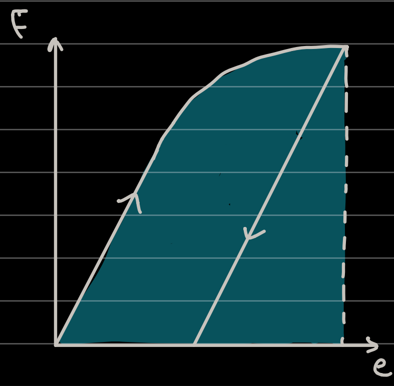


→ from any point here onwards, spring no longer returns to its original shape

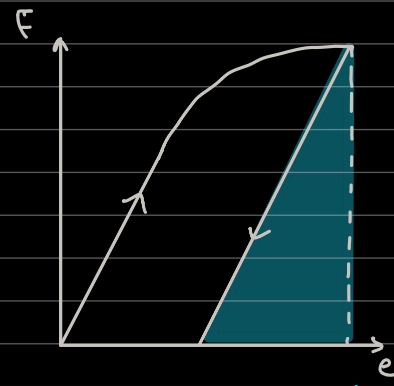
→ The graph no longer returns to the original point, and thus it can be said that the deloading pattern no longer "follows the same path" as the loading graph/curve

Loading Curve

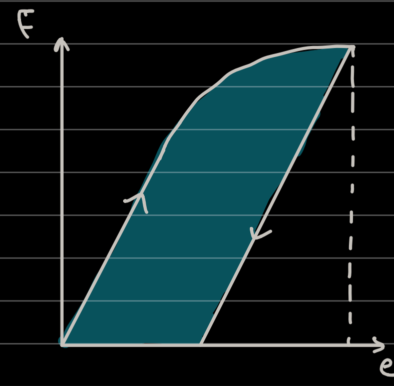
Deloading Curve



Energy stored in the material during loading



Energy recovered during deloading



Energy that cannot be recovered during a complete cycle of loading / deloading.

OR Energy dissipated as heat

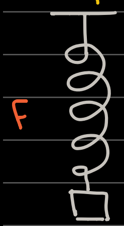
OR Energy lost, etc.

~ Next sub-topic begins on next page ~

↓

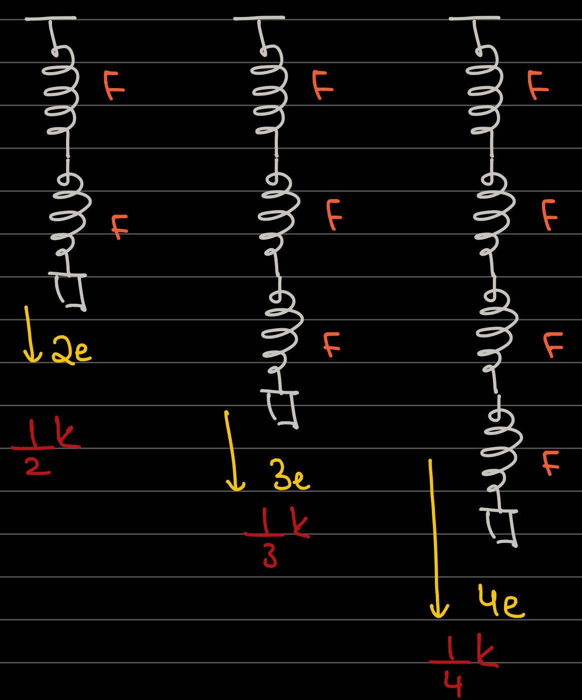
ARRANGEMENT OF SPRINGS : IN PARALLEL / SERIES

→ A spring by itself



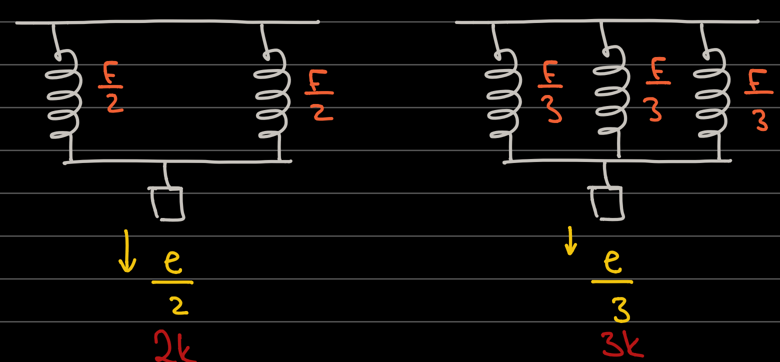
Spring Constant = k

SERIES:



- All springs experience the same amount of force
- Total extension in series is always the <u>SUM</u> of the individual extensions

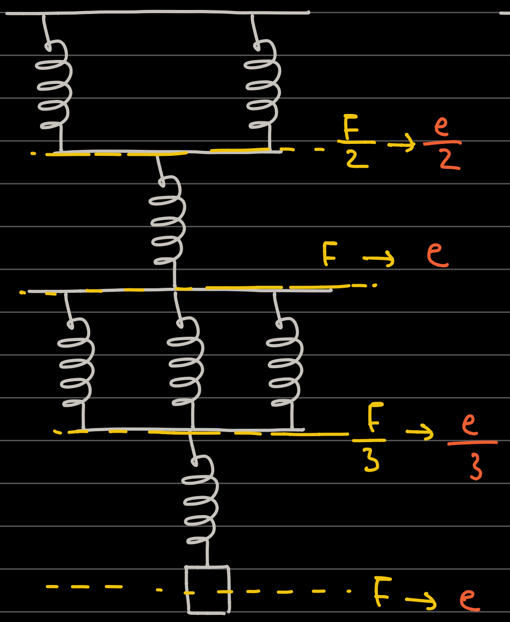
PARALLEL

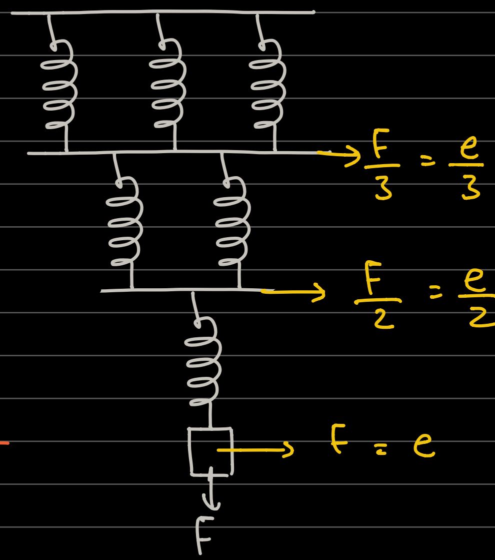


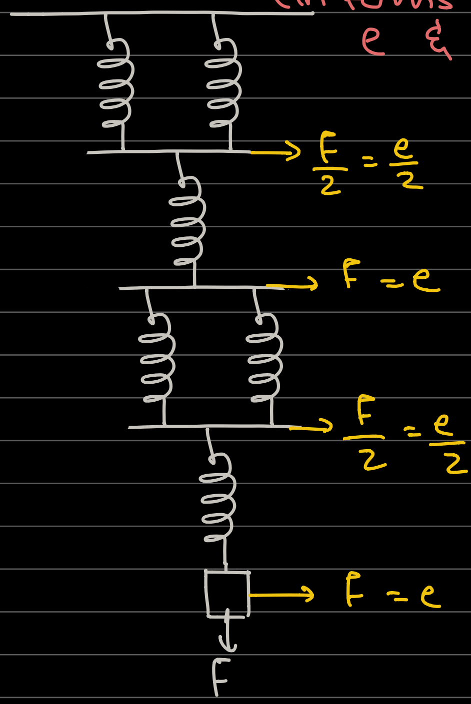
- In parallel arrangements, the force gets divided equally among the strings
- The resulting extension is the same as the extension of one spring if the (divided) force was applied on it individually.

Note → $F = ke$, k and e are inversely proportional, a doubled extension value results in the k value being halved for the entire system, for example.
↳ Remember this when dealing with series / parallel combinations

Examples: Find the total extension and combined spring constant. (in terms of e & k)







$\frac{e}{2} + e + \frac{e}{3} + e = \frac{17e}{6}$

$k_c = \frac{6e}{17}$

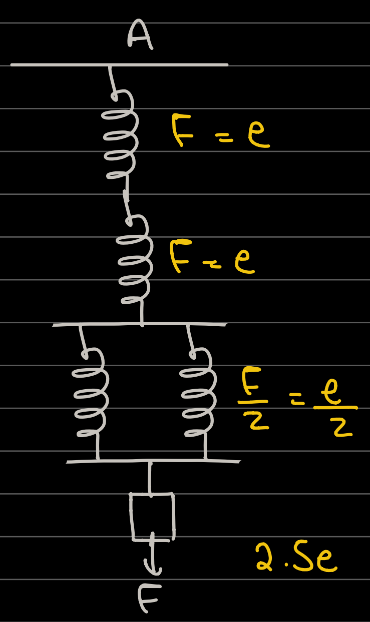
$\frac{e}{3} + \frac{e}{2} + e = \frac{11e}{6}$

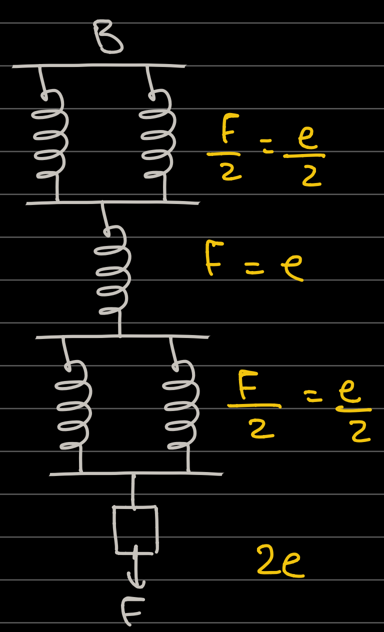
$k_c = \frac{6e}{11}$

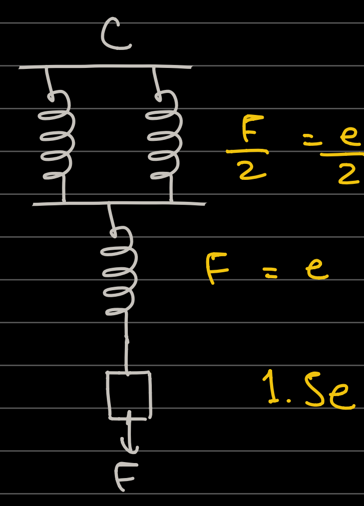
$\frac{e}{2} + e + \frac{e}{2} + e = 3e$

$k_c = \frac{1k}{3}$

Example Question







Q. Arrange the above diagrams in ascending order of extension

Ans. C → B → A

SPRING COMBINATIONS WITH NON-IDENTICAL SPRING

$k = 2\,Ncm^{-1}$ $\quad \frac{12}{2} = 6\,cm$

$F = 12N$

$k = 3\,Ncm^{-1}$ $\quad \frac{12}{3} = 4\,cm$

$F = 12N$

$k = 6\,Ncm^{-1}$ $\quad \frac{12}{6} = 2\,cm$

$F = 12N$

$F = 12N$

Total extension $= 6 + 4 + 2$

$= 12cm$

Combined Spring Constant (k_c):

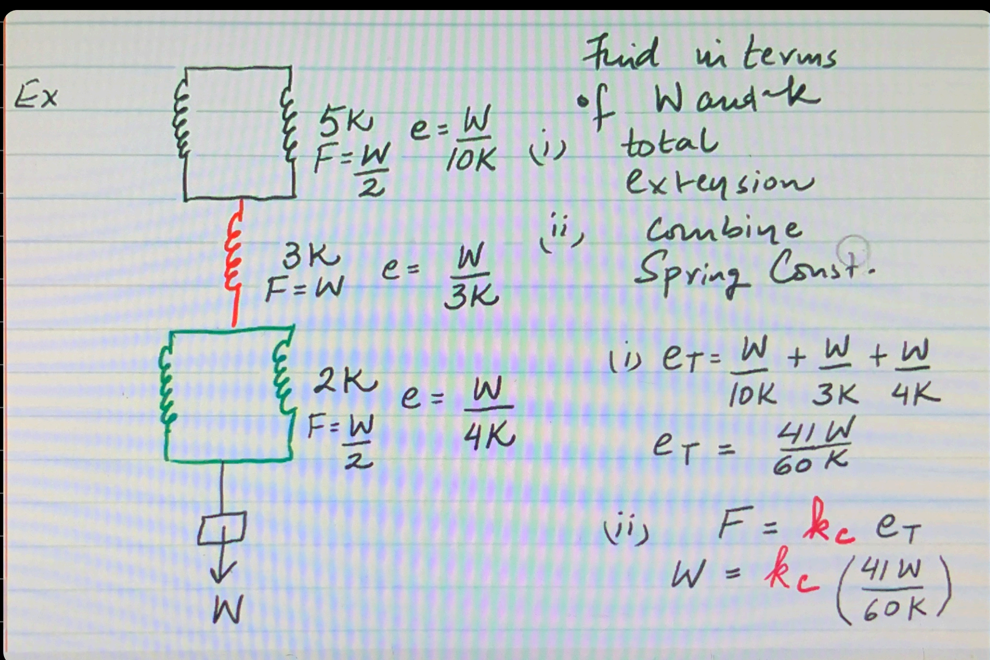
$F = k_c e_c$ where e_c is the combined extension

$12 = k_c 12$

$1 = k_c$

Combined Spring Constant $= 1\,Ncm^{-1}$

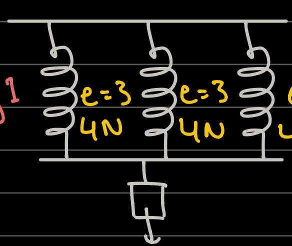
Example:



Example

Q. In fig 1, each spring extends by 3cm when a force of 12N is applied on the system.

Fig 1



$e=3$ $e=3$ $e=3$

4N 4N 4N

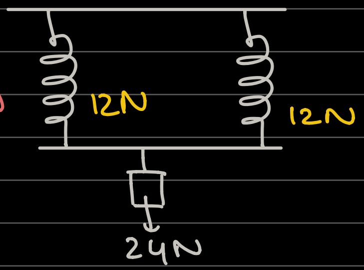
12N

$F = ke$ $\qquad \frac{4}{3} = k$

$4 = k3$

The middle spring is now removed and weight is changed to 24N (fig 2). Calculate the new extension.

Fig



$F = ke$

$12 = \frac{4}{3}e$

$9 = e$

$\therefore$, New extension $= 9cm$.