S15 / 11 / Q8

$f(x) = 5 + 3\cos(\frac{1}{2}x)$ for $0 \leq x \leq 2\pi$

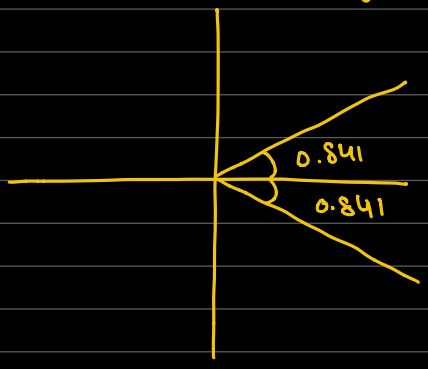
(i) $7 = 5 + 3\cos(\frac{1}{2}x)$

$\frac{2}{3} = \cos(\frac{1}{2}x)$

$\alpha = \cos^{-1}(\frac{2}{3})$

$\alpha = 0.841$

$\theta = 0.841, 5.442, 7.12$



$\theta = \frac{1}{2}x$

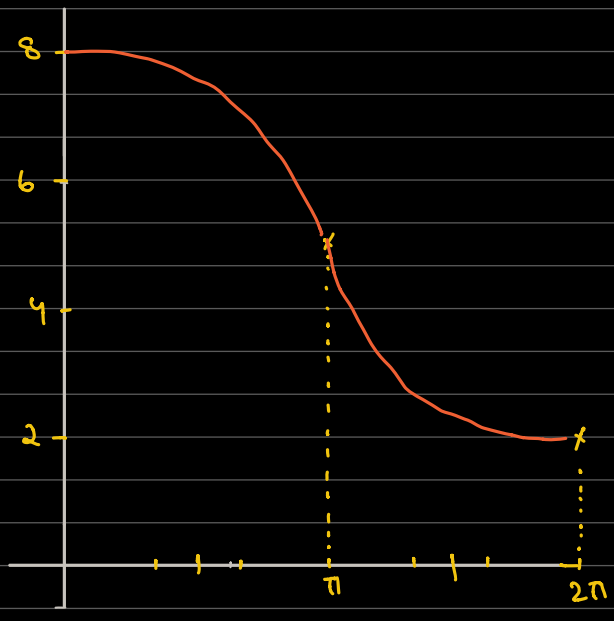
$2\theta = x \rightarrow x$ must be less than $2\pi = 6.28$

$2(0.841) = 1.682$ ✓

$2(5.442) =$ out of range

$\therefore x = 1.68$

(ii) $y = 3\cos(\frac{1}{2}x) + 5$



(iii) $f(x)$ has an inverse because for the specified domain ($0 \leq x \leq 2\pi$), it is a one-to-one function.

(iv) $y = 5 + 3\cos(\frac{1}{2}x)$

$\frac{y-5}{3} = \cos(\frac{1}{2}x)$

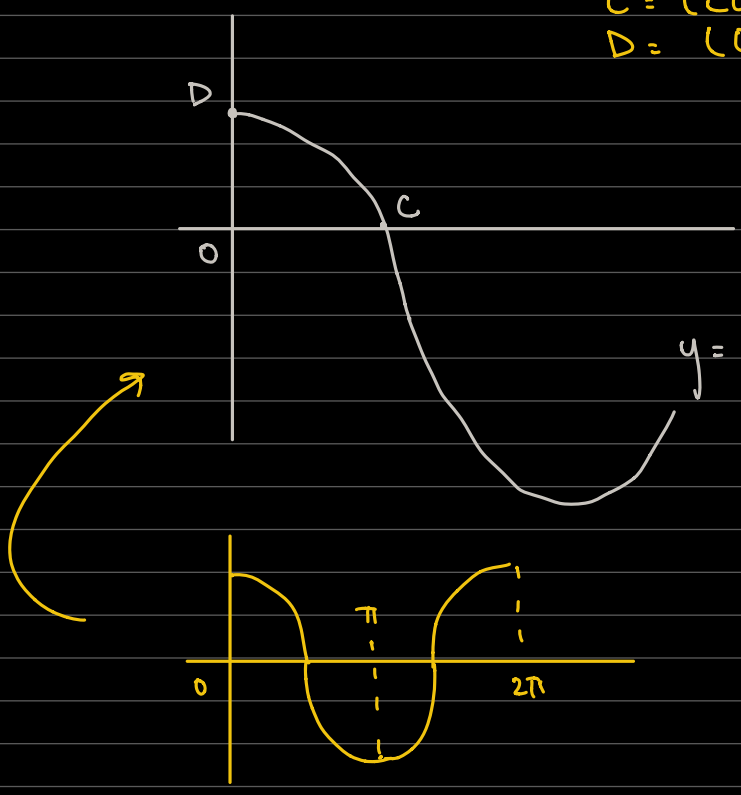
$\therefore f^{-1}(x) = 2\cos^{-1}(\frac{x-5}{3})$

$\cos^{-1}\left(\frac{y-5}{3}\right) = \frac{1}{2}x$

$2\cos^{-1}\left(\frac{y-5}{3}\right) = x$

S15/13/Q7b.

b.



$C = (\cos^{-1}c, 0)$

$D = (0, d)$

$0 = a\cos(x) - b.$

$0 = a\cos(\cos^{-1}(c)) - b$

$0 = ac - b$

$\frac{b}{a} = c \rightarrow$ Ans 1

$y = a\cos(x) - b$

$d = a\cos(0) - b$

$d = a - b \rightarrow$ Ans 2

$\therefore c = \frac{b}{a}$

$d = a - b$

S16/11/Q11

$f(x) = 4\sin x - 1, \quad -\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$

a) Range of f $\rightarrow -5 \leq f(x) \leq 4$ x $-5 \leq f(x) \leq 3$

b) $0 = 4\sin x - 1$

$1 = 4\sin x$

$\frac{1}{4} = \sin x$

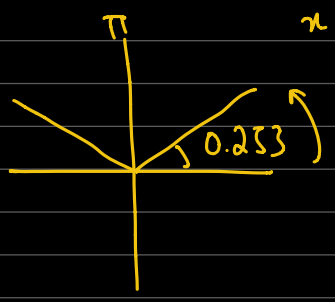
$\alpha = \sin^{-1}\left(\frac{1}{4}\right)$

$y = 4\sin(0) - 1$

$y = -1$

Ans 2 $\rightarrow (0, -1)$

$\alpha = 0.253$



$x = 0.253$

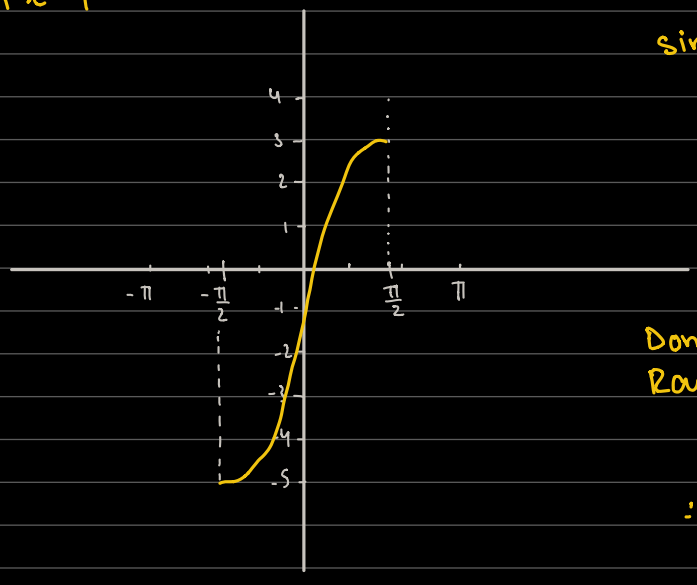
∴, coordinates where the graph intersects the axes:

1 → (0.253, 0)

2 → (0, −1)

Ans 1 → (0.253, 0)

iii) $y = 4\sin x - 1$



(iv) $y = 4\sin x - 1$

$\sin^{-1}\left(\frac{y+1}{4}\right) = x$

∴, $f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{4}\right)$

	$f(x)$	$f^{-1}(x)$
Domain	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-5 \le x \le 3$
Range	$-5 \le f \le 3$	$-\frac{\pi}{2} \le f^{-1}(x) \le \frac{\pi}{2}$

∴, for $f^{-1}(x)$ ...

domain = $-5 \le x \le 3$

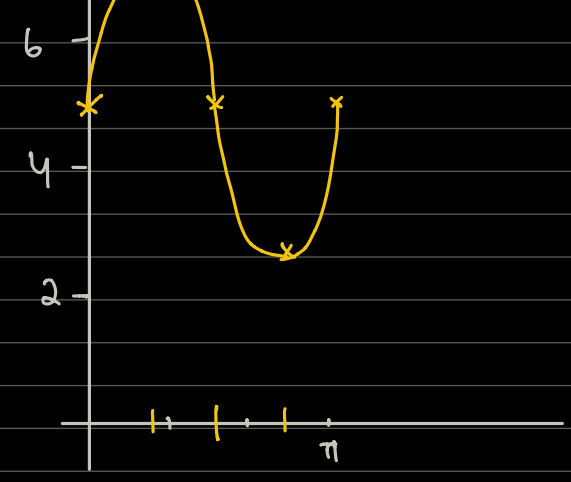
range = $-\frac{\pi}{2} \le f^{-1}(x) \le \frac{\pi}{2}$

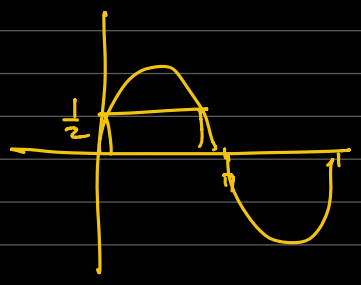
W16/12/Q10

$f(x) = 5 - 2\sin 2x, \quad 0 \le x \le \pi$

i) Range of f → $3 \le f(x) \le 7$

ii) $y = 5 - 2\sin 2x$



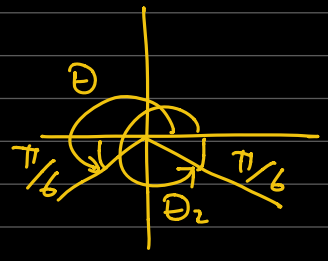


(iii) $6 = 5 - 2\sin 2x$

$-\frac{1}{2} = \sin 2x \qquad \theta = 2x$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = \frac{\pi}{6}$



$\frac{1}{2}\theta$ must be $< \pi$

$\theta = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$

$\frac{7}{6}\pi, \quad \frac{11\pi}{6}$

$\downarrow \times\frac{1}{2} \qquad \downarrow \times\frac{1}{2}$

$\frac{7}{12}\pi \qquad \frac{11}{12}\pi$

↳ Ans ↙

$\therefore, x = \frac{7}{12}\pi$

(iv) largest value of $k = \frac{\pi}{4}$

(v) $y = 5 - 2\sin 2x$

$\frac{5-y}{2} = \sin 2x$

$\sin^{-1}\left(\frac{5-y}{2}\right) = 2x$

$\frac{1}{2}\sin^{-1}\left(\frac{5-y}{2}\right) = x$

$\therefore, g^{-1}(x) = \frac{1}{2}\sin^{-1}\left(\frac{5-x}{2}\right)$

S17 / 12 / Q10

$f(x) = 3\tan\left(\frac{1}{2}x\right) - 2 \qquad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

i) $3\tan(\frac{1}{2}x) - 2 + 4 = 0$

$3\tan(\frac{x}{2}) = -2$

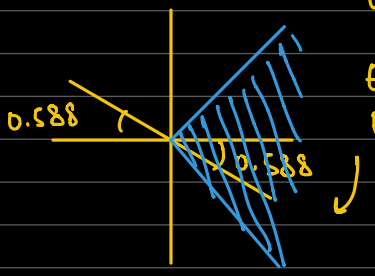
$\tan(\frac{x}{2}) = -\frac{2}{3}$

$\theta = \frac{x}{2}$

$2\theta = x$

$\alpha = \tan^{-1}(\frac{2}{3})$

$\alpha = 0.588$



$\theta = 2.55$,

$\theta = -0.588$

2θ should be less than $\frac{\pi}{2} = 1.571$

θ should be less than $\frac{\pi}{4} = 0.785$.

$\theta = -0.588 = \frac{x}{2}$

$-1.176 = x \rightarrow$ Ans (i)

(ii) $y = 3\tan(\frac{x}{2}) - 2$

$\frac{y+2}{3} = \tan(\frac{x}{2})$

$2\tan^{-1}(\frac{y+2}{3}) = x$

$\therefore, f^{-1}(x) = 2\tan^{-1}(\frac{x+2}{3})$

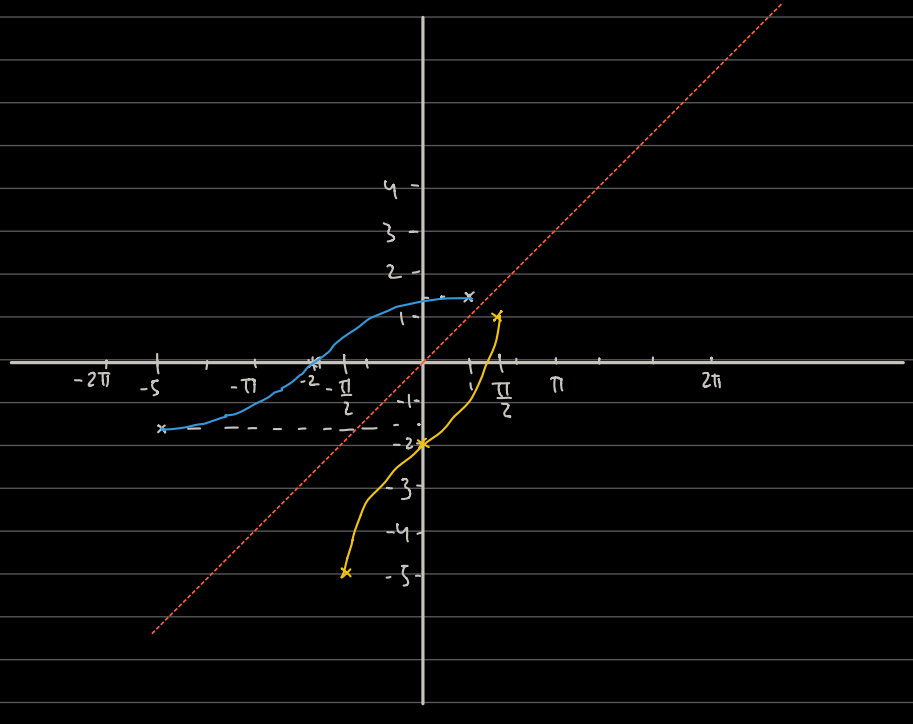
	$f(x)$	$f^{-1}(x)$
Domain	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$-5 \leq x \leq 1$
Range	$-5 \leq f \leq 1$	$-\frac{\pi}{2} \leq f^{-1}(x) \leq \frac{\pi}{2}$

$3\tan(\frac{\pi}{4}) - 2 = 1$

$3\tan(-\frac{\pi}{4}) - 2 = -5$

$\therefore$, domain of $f^{-1}(x) = -5 \leq x \leq 1$

(iii)



w17 / 11 / Q7

a) $y = a + b\sin x$

Amplitude $= \frac{3}{2} = 1.5 = b$ and the graph has been displaced 0.5 downwards

$\therefore$, $a = -0.5$ and $b = 1.5$

b)i) $(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$

$\sin\theta + \sin^2\theta - \sin\theta\cos\theta + 2\cos\theta + 2\sin\theta\cos\theta - 2\cos^2\theta = \sin\theta + \sin\theta\cos\theta$

$2\cos\theta + \sin^2\theta - 2\cos^2\theta = 0$

$2\cos\theta + 1 - \cos^2\theta - 2\cos^2\theta = 0$

$-3\cos^2\theta + 2\cos\theta + 1 = 0$

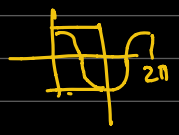
$0 = 3\cos^2\theta - 2\cos\theta - 1 \rightarrow$ shown

(ii) Solve $3\cos^2\theta - 2\cos\theta - 1 = 0$ for $-180° \leq \theta \leq 180°$

$3\cos^2\theta - 3\cos\theta + \cos\theta - 1 = 0$

$3\cos\theta(\cos\theta - 1) + 1(\cos\theta - 1) = 0$

$(3\cos\theta + 1)(\cos\theta - 1) = 0$

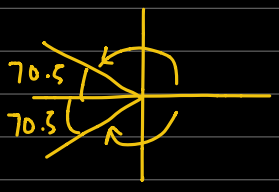


$\cos\theta = -\frac{1}{3}$ $\qquad$ $\cos\theta = 1$

$\theta = 0°$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$\alpha = 70.5°$



$\theta = 109.5°, -109.5°$

$\therefore$, $\theta = -109.5°, 0°, 109.5° \rightarrow$ Ans

w17 / 12 / Q6

$f(x) = a + b\sin x$

a) i) $4 = a + b\sin\frac{\pi}{6}$ $\qquad$ $3 = a + b\sin\frac{\pi}{2}$

$4 - a = b\sin\frac{\pi}{6}$ $\qquad$ $3 = a + b$

$4 - a = \frac{b}{2}$ $\qquad$ $3 - b = a$

$8 - 2a = b$ $\qquad$ $3 - (8 - 2a) = a$

$8 - 2(5) = b$ $\qquad$ $3 - 8 + 2a = a$

$8 - 10 = b$ $\qquad$ $-5 + 2a = a$

$-2 = b \rightarrow$ Ans 2 $\qquad$ $a = 5 \rightarrow$ Ans 1

$\therefore$, $a = 5$ and $b = -2$

(ii) $ff(x) = 5 - 2\sin(5 - 2\sin\theta)$

$= 5 - 2\sin(5 - 0)$

$= 5 - 2\sin(5)$

$ff(0) = 6.92 \rightarrow$ Ans

b) $g(x) = c + d\sin x \rightarrow$ range $= -4 \leq g(x) \leq 10$

$\text{Amp} = \frac{14}{2} = 7 \rightarrow$ so $d = \pm 7$

And then the graph has been displaced 3 units upwards so $c = +3$ ✓

$\therefore, c = 3, d = \pm 7$

S18/12/Q10

i) $2\cos x + 3\sin x = 0$ for $0° \leq x \leq 360$

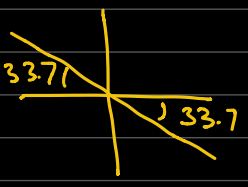
$3\sin x = -2\cos x$

$\frac{3\sin x}{\cos x} = -2$

$\tan x = -\frac{2}{3}$

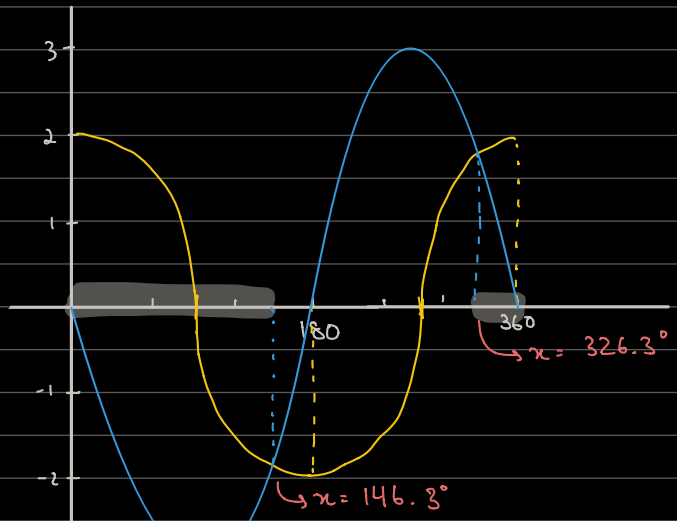
$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$

$\alpha = 33.7°$



$x = 146.3°, 326.3° \rightarrow$ Ans

ii) $y = 2\cos x$, $y = -3\sin x$ $\quad 0 \leq x \leq 360$



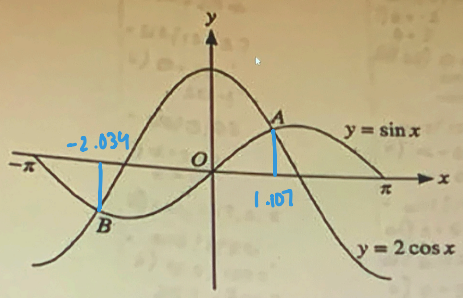
iii)

$\therefore$, ~~for~~ $x < 146.3°$ and $x > 326.3°$, $2\cos + 3\sin x > 0$ ✗

$0 \leq x < 146.3°$

$326.3° < x \leq 360°$

(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

$x = 1.107$

(i) Find the x-coordinate of A. [2]

$\sin x = 2\cos x$

(ii) Find the y-coordinate of B. [2]

$\tan x = 2$

$y = -0.895$

$\alpha = \tan^{-1}(2)$

$\alpha = 1.107$

$x = 1.107, -2.034$