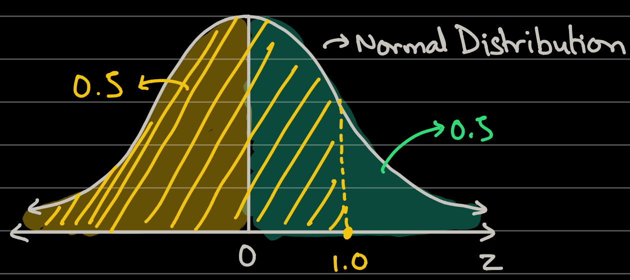
NORMAL DISTRIBUTION : PROBABILITY AND STATISTICS



- the curve is bell shaped
- All above the horizontal axis (z)
- Goes from $-\infty$ to $+\infty$
- The area under it is 1
- The curve is symmetric about 0

2nd decimal place — 3rd decimal place — 1st decimal place

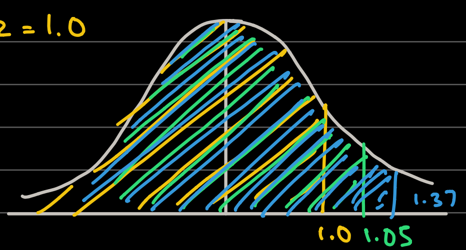
z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

If I want to find the ▨ area, have to use the table

Table will be used to find the areas to the left of a positive z value

↳ that area represents the probability

ie. $z = 1.0$



$P(z < 1.0) = 0.8413$

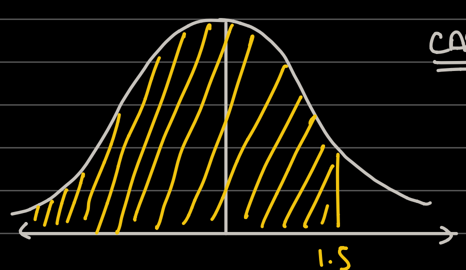
$P(z < 1.05) = 0.8531$

$P(z < 1.37) = 0.9147$

From the table:

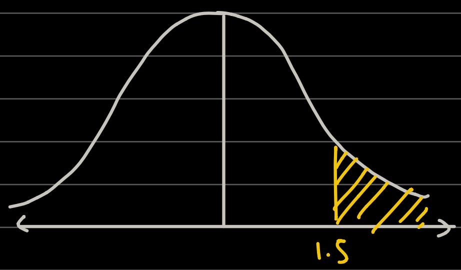
$$
\begin{array}{rr}
P(X < 1.234) = & 0.8907 \\
& + \quad 7 \\
\hline
& 0.8914
\end{array}
$$

<u>CASE 1</u>



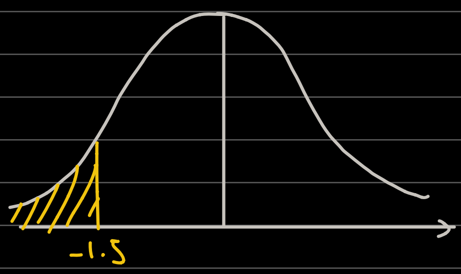
$P(Z < 1.5) = 0.9332$

CASE 2



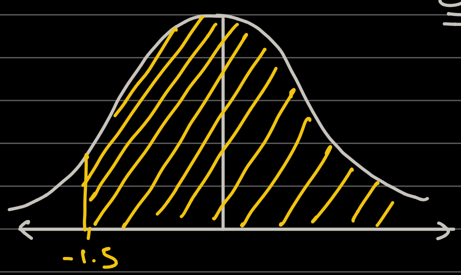
$$P(Z > 1.5) = 1 - P(Z < 1.5)$$
$$= 1 - 0.9332$$
$$= 0.0668$$

CASE 3



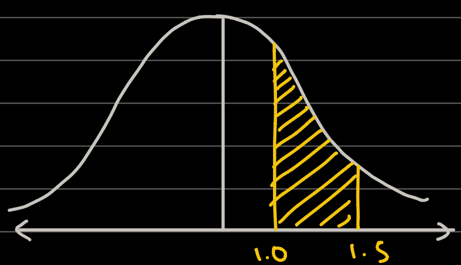
$$P(Z < -1.5) = P(Z > 1.5)$$
$$= 1 - P(Z < 1.5)$$
$$= 1 - 0.9332$$
$$= 0.0668$$

CASE 4



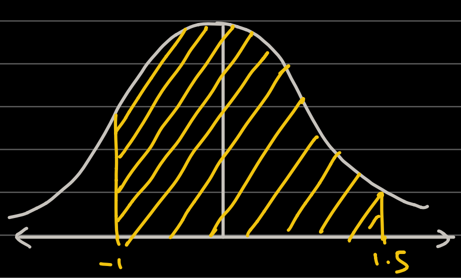
$$P(Z > -1.5) = P(Z < 1.5)$$
$$= 0.9332$$

CASE 5



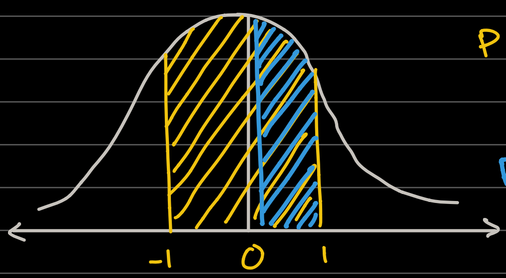
$$P(1.0 < Z < 1.5) = P(Z < 1.5) - P(Z < 1)$$
$$= 0.9332 - 0.8413$$
$$= 0.0919$$

CASE 6



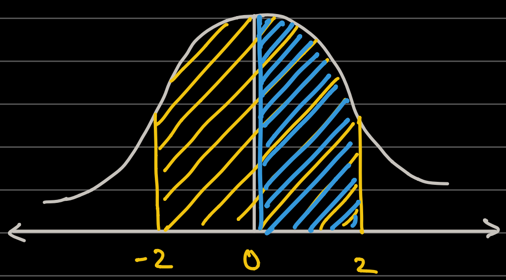
$$P(-1 < Z < 1.5) = P(Z < 1.5) - P(Z < -1)$$
$$= P(Z < 1.5) - P(Z > 1)$$
$$= P(Z < 1.5) - (1 - P(Z < 1))$$
$$= 0.9332 - (1 - 0.8413)$$
$$= 0.7745$$

SOME VALUES TO REMEMBER



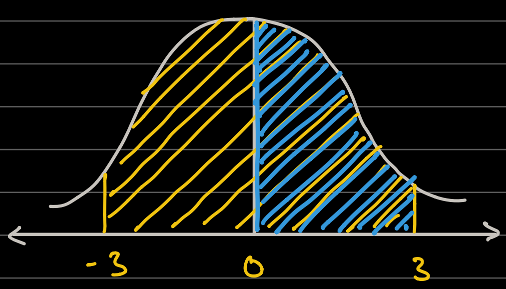
$P(-1 < z < 1) = 0.6826$
$= 68.26\%$

$P(0 < z < 1) \approx 34\%$



$P(-2 < z < 2) = 0.9544$
$= 95.44\%$

$P(0 < z < 2) \approx 48\%$



$P(-3 < z < 3) = 0.9972$
$= 99.72\%$

$P(0 < z < 3) \approx 49\%$

Normal Distribution is a continuous distribution

ie. X is the amount of juice in a carton (ml)
or ie. X is the height / weight of a person

For example:

Let's say a company says that a juice box can have X juice in ml. The probability in this case would be a normal distribution, because volume of liquid is a continuous quantity.

in this case, $X \sim N(\mu, \sigma^2)$

where μ = mean
σ^2 = variance

Let's say a company says that on average, each juice box has 60 ml of juice, with a standard deviation σ of 3 ml and hence, a variance (σ^2) of 9 ml.

we would represent that in the following manner:

$X \sim N(60, 3^2)$ where μ = 60ml
s.d = 3ml
σ^2 = 9 m

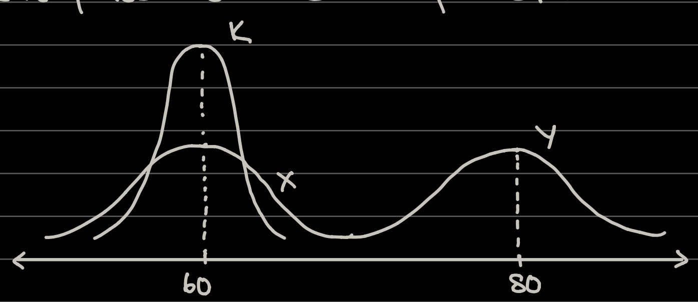
Another company says that they have 80ml juice per box, and a variance of 3^2 ml.

$Y \sim N(80, 3^2)$

A third company says that they have 60 ml of juice / box, and a variance of 1^2 ml.

$K \sim N(60, 1^2)$

If we were to place the three companies on the same axis.



The function for a normal distribution graph is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)}$$

However, we dont have to deal with it.

To go from x to z → which is what we used earlier to get those probabilities

$$z = \frac{x-\mu}{\sigma}$$

Let's say a manager in company x wants to find the probability that x, the juice / box in ml, is $\leq$ 63ml.

so $P(x \leq 63)$

we have to convert x to z

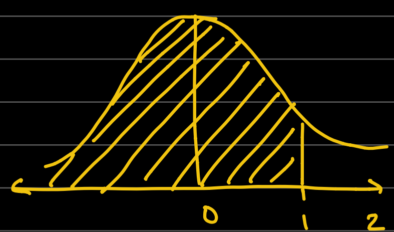
$$P(x \leq 63) = P\left(z \leq \frac{63-60}{3}\right)$$

$$= P(z \leq 1)$$

$$= 0.8413$$

$$= \underline{\underline{84.13\%}}$$

↳ the chance that a juice box contains $\leq$ 63 ml of juice

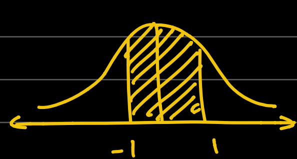


$X \sim N(\mu, \sigma^2)$ $\quad$ $Z \sim N(0, 1)$

"z" is the no. of standard deviations the observed value is from the mean.

2. What's the probability that the juice box contains juice b/w. 57 and 63 ml

$$\frac{57-60}{3} = \frac{-3}{3} = -1$$



$$P(z \leq 1)$$
$$= 0.8413$$
$$P(z \geq -1) = P(z \geq 1)$$
$$= 1 - 0.8413$$
$$= 0.1587$$

$0.8413 - 0.1587$
$= 0.6826$
$= 68.26\%$ chance

Normal distribution questions from questions :

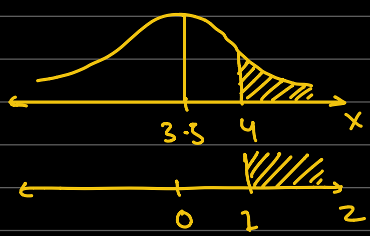
Exercise G

1. Avg of 3.5g / cup

$\sigma = 0.5$
$\mu = 3.5$

Find the probability that

a) > 4g



$X \sim N(\mu, \sigma^2)$

$X \sim N(3.5, 0.25)$

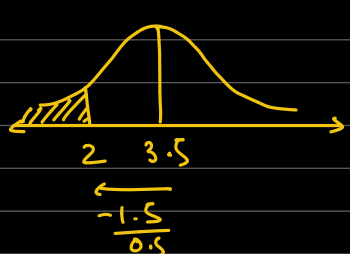
$Z \sim N(0, 1)$

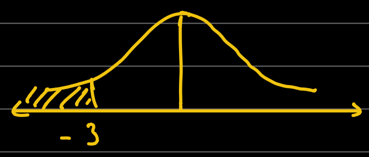
$P(Z > 1) = 1 - P(Z < 1)$
$= 1 - 0.8413$
$= \underline{0.1587}$ → Ans

b) > 3g

$P(Z > -1) = P(Z < 1)$
$= \underline{0.8413}$ → Ans

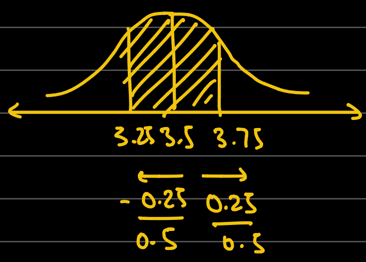
c) < 2g





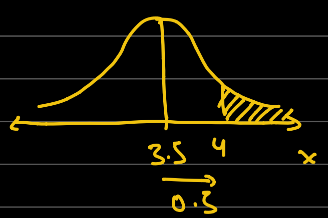
$P(z < -3) = 1 - P(z < 3)$
$= 1 - 0.9986$
$= 0.0014$ → Ans

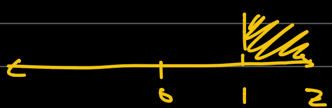
d) $3.25 < x < 3.75$



$P(-0.5 < z < 0.5) = P(z<0.5) - (1 - P(z<0.5))$
$= P(z<0.5) - 1 + P(z<0.5)$
$= 2P(z<0.5) - 1$
$= 2(0.6915) - 1$
$= 0.383$ → Ans

e) How many cups will likely overflow if 4g cups are used for the next 1000 drinks



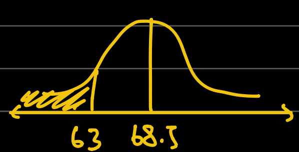


$P(z>1) = 1 - P(z<1)$
$= 1 - 0.8413$
$= 0.1587$

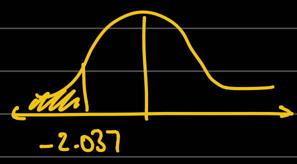
$0.1587 \times 1000 = 158.7$
$= 159$ cups → Ans

2. $\mu = 68.5$
$\sigma = 2.7$

a) < 63.0 kg



$\frac{63 - 68.5}{2.7} = -2.037 = z$

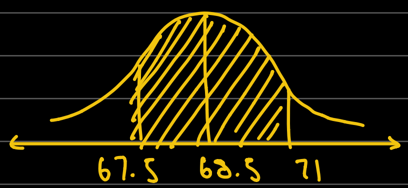


$P(z < -0.2037) = 1 - P(z < 2.037)$

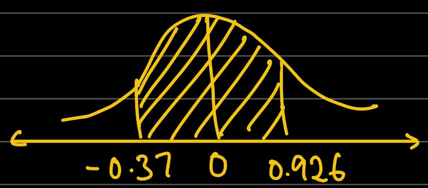
$= 1 - (0.9788 + 3 = 0.9791)$

$= \underline{0.0209} \to$ Ans

b) $67.5 < x < 71.0$ leg inclusive



$-0.370 \quad \frac{1}{2.7} \qquad \frac{2.5}{2.7} = 0.926$



$P(-0.37 < z < 0.926) = P(z < 0.926) - (1 - P(z < 0.37))$

$= 0.8212 + 15 = 0.8227$

$= 0.8227 - (1 - 0.6443)$

$= 0.467$

$0.467 \times 100 = 467$ kids