GEOMETRIC PROGRESSION

↳ Progression in which there is a common ratio instead of a common difference

$\times 3 \quad \times 3 \quad \times 3 \rightarrow$ Common Ratio

ie. 2, 6, 18, 54

IMPORTANT FORMULAS:

value of the n^{th} term:

$T_n = a \times r^{n-1}$ where a = first term
r = common ratio
n = term number

Sum of first n terms:

$$S_n = \frac{a(1-r^n)}{1-r}$$

when the common ratio r is such that $-1 < r < 1$, then the sum to convergence / sum to infinity can be given by the following formula.

$$S_\infty = \frac{a}{1-r}$$

GP questions to attempt

From Worksheets (NG) Slide 85 onwards
~~22b~~, ~~23b~~, ~~24b~~, ~~25b~~, 21b ??, ~~29c~~, ~~30b~~, ~~35b~~, ~~38a~~, 39c, 41b

22b. $T_1 = x+5$
$T_2 = x+1$
$T_3 = x$

i) value of x

$r^2(x+5) = x$
$r(x+5) = x+1$
$r = \frac{x+1}{x+5}$

$\left(\frac{x+1}{x+5}\right)^2 (x+5) = x$

$\frac{(x+1)^2 \times \cancel{(x+5)}}{(x+5)^{\cancel{2}\,1}} = x$

$\frac{x^2+2x+1}{x+5} = x$

$\cancel{x^2} + 2x + 1 = \cancel{x^2} + 5x$

$\frac{1}{3} = \frac{3x}{3}$

$\frac{1}{3} = x$

↳ Ans i) Value of x

ii) The common ratio

$r = \frac{x+1}{x+5}$

$r = \frac{\frac{1}{3}+1}{\frac{1}{3}+5}$

$r = \frac{4/3}{16/3}$

$= \frac{\cancel{4}^{1}}{3} \times \frac{3}{\cancel{16}_4}$

$= \frac{3}{12}$

$= \frac{1}{4}$

↳ Ans(ii) value of r

iii) Sum to infinity

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{16/3}{1-1/3}$

$= \frac{^{8}\cancel{16}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}}$

$= 8$

↳ Ans (iii) → Sum to convergence / infinity

↱ Re-Attempt

23b. $a = a$ Calculate:

$r = +ve$ i) The value of a

$T_2 + T_3 = \frac{10a}{9}$ ii) Sum to infinity

$S_4 = 65$

$S_4 = \frac{a(1-r^4)}{1-r} \longrightarrow 65 = \frac{a(1-r^4)}{1-r}$

$\frac{65(1-r)}{(1-r^4)} = a$

$T_2 = a \times r$

$T_3 = a \times r^2$

$ar + ar^2 = \frac{10a}{9}$

$a(r + r^2) = \frac{10a}{9}$

$r + r^2 = \frac{10}{9}$

$9r^2 + 9r - 10 = 0$ -90

$9r^2 + 15r - 6r - 10 = 0$ $10 \quad 9$

$3r(3r+5) - 2(3r+5) = 0$ $45 \quad 2$

$(3r-2)(3r+5) = 0$ $6 \quad 15$

$r = \frac{2}{3}$ $r = \frac{-5}{3} \to \times$

↓

$r = +ve$

$S_4 = \frac{a(1-r^4)}{1-r}$

$65 = \frac{a(1-(2/3)^4)}{1-2/3}$

$65 \times 3 = a(1 - 16/81)$

$195 = \frac{65a}{81}$

$\frac{195 \times 81}{65} = a$

$243 = a$ → Ans (i) Value of a

ii) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{243}{\frac{1}{3}}$

$S_\infty = 243 \times 3$

$S_\infty = 729$ → Ans (ii) Sum to infinity

24b1. $T_2 = 24$ Calculate:

$T_5 = \frac{8}{9}$ i) value of r

ii) Value of a

iii) Sum to infinity.

$T_n = a \times r^{n-1}$

$T_2 = ar$

① $\frac{24}{r} = a$

$T_5 = a \times r^4$

② $\frac{8}{9} = \frac{24}{r} \times r^4$

ii) $\frac{24}{r} = a$

$\frac{24}{\frac{1}{3}} = a$

$24 \times 3 = a$

iii) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{72}{1-\frac{1}{3}}$

$S_\infty = \frac{72^3 \times 3}{2}$

$$\frac{8}{9} = 24r^3$$

$$\sqrt[3]{\frac{8}{9 \times 24}} = r$$

$$\frac{1}{3} = r \rightarrow \text{Ans (i)}$$

Value of r

$72 = a$

↳ Ans ii

Value of a

$S_\infty = 108$

↳ Ans iii

Sum to infinity

24b2.

	1		2		3
	72	→	24	→	8
		÷3		÷3	
2nd GP	5184	→	576	→	64
		÷9		÷9	

Point to note:

If the entire GP is raised to a certain value (ie. squared, cubed, etc.), then the resulting common difference will also be raised to the same power.

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{5184}{1 - \frac{1}{9}}$$

$$= \frac{5184 \times 9}{8}$$

$$= 5832 \rightarrow \text{Ans 2i)}$$

Sum to infinity of 2nd GP

Calculate a

25b. $T_3 = 36$

$S_3 = 76 \longrightarrow S_3 = \frac{a(1-r^3)}{1-r}$

$T_1 = a$

$T_2 = ar$

$T_3 = ar^2$

$ar^2 = 36$

$a = \frac{36}{r^2}$

$76 = a + ar + ar^2$

$76 = a(1 + r + r^2)$

$76 = \frac{36}{r^2}(1 + r + r^2)$

$76r^2 = 36(1 + r + r^2)$

$76r^2 = 36 + 36r + 36r^2$

$40r^2 = 36 + 36r$

$0 = \frac{40r^2 - 36r - 36}{4}$

$0 = 10r^2 - 9r - 9$

$0 = 10r^2 - 15r + 6r - 9$

$0 = 5r(2r-3) + 3(2r-3)$

$0 = (5r+3)(2r-3)$

$r = -\frac{3}{5}$ ↳ x

$r = \frac{3}{2}$

$r = 1.5$

$T_3 = ar^2$

$36 = a\left(\frac{3}{2}\right)^2$

$36 = \frac{9a}{4}$

$\frac{36 \times 4}{9} = a$

$16 = a$

↳ Ans (b)

length of shortest side in cm.

21b. $T_2 + 2 = a$

$T_2 + T_3 = \frac{5}{3}$

i) Calculate:
possible values of a and r

ii) Given that all terms are +ve, calculate:
Sum to infinity

$T_2 + 2 = a$

$ar + 2 = a$

$\frac{2}{1-r} = a$

Can't complete question
because this. value is not legible

$ar + ar^2 = \frac{5}{3}$

$a(r + r^2) = \frac{5}{3}$

$3r + 3r^2 = \frac{5(1-r)}{2}$

$6r + 6r^2 = 5 - 5r$

$11r + 6r^2 = 5$

$0 = 6r^2 + 11r - 5$

$=$

-30	
1	-30
2	-15
3	-10
5	-6
6	-5

29c. $a = 12r$

$S_\infty = 4$

Calculate:
T_3

$S_\infty = \frac{a}{1-r}$

$4 = \frac{12r}{1-r}$

$4 - 4r = 12r$

$4 = 16r$

$\frac{1}{4} = r$

$a = 12(\frac{1}{4})$

$a = 3$

$T_3 = ar^2$

$= 3(\frac{1}{4})^2$

$= \frac{3}{16}$ → Ans
↳ Value of T_3

30b. $T_1 = x + 6$

$T_2 = x$

$T_3 = x - 3$

Calculate:
i) Value of x
ii) T_5

① $r(x+6) = x$

② $r(x) = x - 3$

$xr = x - 3$

$r = \frac{x-3}{x}$

$\left(\frac{x-3}{x}\right)(x+6) = x$

ii) $a = x + 6$

$= 6 + 6$

$a = 12$

$r(x+6) = x$

$r\,12 = 6$

$r = 0.5$

$T_5 = ar^4$

$= 12 \times (\frac{1}{2})^4$

$= \frac{12}{16}$

$= \frac{3}{4}$ → Ans ii
Value of T_5.

$(x-3)(x+6) = x^2$

$\cancel{x^2} + 6x - 3x - 18 = \cancel{x^2}$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6 \rightarrow$ Ans(i) Value of x

35b. $r = +ve$ Calculate r

$T_2 + T_3 = \frac{24a}{25}$

$ar + ar^2 = \frac{24}{25}a$

$a(r+r^2) = \frac{24}{25}a$

$r + r^2 = \frac{24}{25}$

-600

$30 \quad -20$

$40 \quad -15$

$25r + 25r^2 = 24$

$0 = 25r^2 + 25r - 24$

$0 = 25r^2 + 40r - 15r - 24$

$0 = 5r(5r+8) - 3(5r+8)$

$0 = (5r-3)(5r+8)$

$r = \frac{3}{5} \qquad r = -\frac{8}{5} \rightarrow \times$

$\downarrow$

$r = +ve$

$\therefore, r = \frac{3}{5} \rightarrow$ Ans(i) Value of r

ii) $S_\infty = 250$

$\frac{a}{1-r} = 250$

$\frac{a}{\frac{2}{5}} = 250$

$\frac{5a}{2} = 250$

$5a = 500$

$a = 100 \rightarrow$ Ans ii Value of a given that

38a. $T_2 = -80$ Calculate

$T_5 = 1250$ i) Value of a & r

ii) S_7

① $ar = -80$

② $ar^4 = 1250$

$a = \frac{-80}{r}$ $\longrightarrow$ $\frac{-80}{r} r^4 = 1250$

$-80r^3 = 1250$

$r^3 = \frac{1250}{-80}$

$r = \sqrt[3]{-15.625}$

$r = -2.5$ $\rightarrow$ Ans i) a) Value of r

$ar = -80$

$a(-2.5) = -80$

$a = \frac{-80}{-2.5}$

$a = 32$

↳ Ans i) b) Value of a

ii) $S_7 = \frac{a(1-r^7)}{1-r}$

$= \frac{32(1-(-2.5)^7)}{1-(-2.5)}$

$= \frac{32 \times 611.35}{3.5}$

$S_7 = 5589.5$

$= 5590$ $\rightarrow$ Ans ii Sum of first 7 term

Re-attempting 23b.

$r = +ve$

$T_2 + T_3 = \frac{10a}{9}$

$ar + ar^2 = \frac{10a}{9}$

$r + r^2 = \frac{10}{9}$

$9r + 9r^2 = 10$

$0 = 9r^2 + 9r - 10$

$= 9r^2 + 15r - 6r - 10$

$= 3r(3r+5) - 2(3r+5)$

$= (3r-2)(3r+5)$

Ans (i) Value of r $\leftarrow$ $r = \frac{2}{3}$ ↓ $r = +ve$

$r = \frac{-5}{3}$ $\rightarrow \times$

$S_4 = 65$

$\frac{a(1-r^4)}{1-r} = 65$

$a(1-(\frac{2}{3})^4) = 65(\frac{1}{3})$

$a(1 - \frac{16}{81}) = \frac{65}{3}$

$\frac{65a}{81} = \frac{65}{3}$

$a = 27$

↳ Ans (ii) value of a

$S_\infty = \frac{a}{1-r}$

$= \frac{27}{\frac{1}{3}}$

$= 27 \times 3$

$S_\infty = 81$

↳ Ans iii Sum to infinity

41b. $a = 10$

$r = 1.2$

$S_n = 800$ where n is the last term in the progression

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$800 = \frac{a(1-(1.2)^n)}{1-1.2}$$

$$(800)(-0.2) = a - a(1.2)^n$$

$$-160 = 10 - 10(1.2)^n$$

$$-170 = -10(1.2)^n$$

$$17 = 1.2^n$$

$$\ln 17 = \ln 1.2^n$$

$$\ln 17 = n \times \ln 1.2$$

$$\frac{\ln 17}{\ln 1.2} = n$$

$$15.53 = n$$

↳ Round up to 16 → Ans b.

No. of days required

37b i) $a = 10000$

$r = 0.9$

$$T_n < 2000$$

$$ar^{n-1} < 2000$$

$$r^{n-1} < \frac{2000}{10000}$$

$$r^{n-1} < 0.2$$

$$\ln r^{n-1} < \ln 0.2$$

$$(n-1)(\ln r) < \ln 0.2$$

$$n-1 < \frac{\ln 0.2}{\ln 0.9}$$

$$n-1 < 15.28$$

$n < 16.28$ → Rounded up to 17 because a bit after the 16th year is completed is when it falls below 2000

ii) 95 96 97 98 99 00 01 02 03 04

$$S_{10} = \frac{a(1-0.9^{10})}{1-0.9}$$

$$= \frac{10000(1-0.9^{10})}{0.1}$$

$= 65132$

≈ 65100 → Total paid from '95 to '04 inclusive

iii) $$S_{20} = \frac{10000(1-0.9^{20})}{0.1}$$

$= 87842$ ⟶ $87842 - 65132$

$= 22710$ → Sum of the next 10 yrs.

iv) $S_n > 90000$

$$\frac{10000(1-0.9^n)}{0.1} > 90000$$

$$1-0.9^n > \frac{9000}{10000}$$

$$1-0.9^n > 0.9$$

$$1-0.9 > 0.9^n$$

$$0.1 > 0.9^n$$

$$\ln 0.1 > \ln 0.9^n$$

$$\frac{\ln 0.1}{\ln 0.9} > \frac{n \ln 0.9}{\ln 0.9}$$

$$\frac{\ln 0.1}{\ln 0.9} > n$$

$21.85 > n$ → Round up to $n = 22$

↳ Ans. no. of years

33b. $a = 8$

$a, \quad a+4d, \quad a+20d$

① $ar = a + 4d$

$8r = 8 + 4d$

$$r = \frac{8+4d}{8}$$

② $r(a+4d) = a + 20d$

$r(8+4d) = 8 + 20d$

$$\frac{8+4d(8+4d)}{8} = 8 + 20d$$

$(8+4d)^2 = 64 + 160d$

$16d^2 + 64d + \cancel{64} = \cancel{64} + 160d$

$16d^2 - 96d = 0$

$4d^2 - 24d = 0$

$d^2 - 6d = 0$

$d(d-6)$

NS ↵ ↳ $d = 6$

44.a) $a = 1.3$

$l = 5.7$

$S_n = 42$ where n is the number of term

$$S_n = \frac{n}{2}(a+l)$$

$$42 = \frac{n}{2}(1.3+5.7)$$

$$\frac{84}{7} = \frac{7n}{7}$$

$12 = n$ → Ans a

No. of sides

b) $a = 25$

$S_3 = 61$

.... But this is better :

$$S_3 = \frac{a(1-r^3)}{1-r}$$

$$61 = \frac{25(1-r^3)}{1-r}$$

$$61 = \frac{25-25r^3}{1-r}$$

$$61 - 61r = 25 - 25r^3$$

$$36 = 61 - 25r^3$$

$$= 25r^3 - 61r + 36$$

↳ This is fine...

$$a + ar + ar^2 = 61$$

$$25 + 25r + 25r^2 = 61$$

c) $S_n = 480$

$a = 35$

$r = 0.95$

Calculate :

i) T_{10}

ii) value of last term number

$$480 = \frac{35(1-0.95^n)}{1-0.95}$$

$$480 = \frac{35(1-0.95^n)}{0.05}$$

$$\frac{24}{35} = 1 - 0.95^n$$

$$0.95^n = 1 - \frac{24}{35}$$

$$n \ln(0.95) = \ln(0.314)$$

$$n = \frac{\ln(0.314)}{\ln(0.95)}$$

$$n = 22.58$$

$n \approx 23$ ↳ 23^{rd} day → Ans (ii)

$$T_{10} = ar^9$$

$$= 35 \times 0.95^9$$

$$= 22.05$$

$= 22\text{km}$ → Ans (i)

43.a) $a = 50$

$T_4 = 10.8$

Calculate :

i) T_5

ii) S_∞

iii) First term number where term value < 0.25

$$T_4 = ar^3$$

$$10.8 = 50r^3$$

$$\sqrt[3]{\frac{10.8}{50}} = r$$

$$0.6 = r$$

$$T_5 = ar^4$$

$$= 50 \times 0.6^4$$

$= 6.48$ → Ans (i)

ii) $S_\infty = \dfrac{a}{1-r}$

$$= \frac{50}{}$$

(iii) $T_n < 0.25$

$$50r^n < 0.25$$

$$r^n < 0.25$$

$$1-0.6$$

$$= \frac{50}{0.4}$$

$$= 125$$ → Ans (ii)

$$\frac{\quad}{50}$$

$$n \times \ln r < \ln(0.005)$$

$$n < \frac{\ln(0.005)}{\ln(0.6)}$$

$$n < 10.37$$

↳ Round up to 11

↳ Ans (iii)

b.

From the new APGP worksheet (The one that begins with S15/11/Q7)

① S15/11/Q7

a) $T_3 = \frac{1}{3}$

$T_4 = \frac{2}{9}$

$\frac{1}{3} \xrightarrow[\times 3]{\times 2} \frac{2}{9}$

$\frac{2}{3} = r$

$\frac{1}{3} = a\left(\frac{2}{3}\right)^2$

$\frac{1}{3} = a\frac{4}{9}$

$\frac{1}{\cancel{3}_1} \times \frac{\cancel{9}^3}{4} = a$

$\frac{3}{4} = a$

$S_\infty = \frac{a}{1-\frac{2}{3}}$

$= \frac{3/4}{1/3}$

$= \frac{3}{4} \times \frac{3}{1}$

$= \frac{9}{4}$ → Ans a

Sum to infinity

b) $S_5 = 360$
$l = 4a$

→ $S_n = \frac{n}{2}(a+l)$

$360 = \frac{5}{2}(a+4a)$

$720 = 5(5a)$

$\frac{720}{25} = \frac{25a}{25}$

$28.8 = a$

$l = 4(28.8)$

$l = 115.2$

↓

Ans b

Angle of largest angle

② S15/12/Q8

a) $a = 56$
$T_2 = 53$ } -3
$l = -22$

$T_n = a + (n-1)d$

$-22 = 56 + (n-1)(-3)$

$-78 = -3n + 3$

$-78 = 3 - 3n$

$\frac{3n}{3} = \frac{81}{3}$

$n = 27$

$S_{27} = \frac{27}{2}(a + l)$

$= 13.5(56 - 22)$

$= 13.5(34)$

$S_{27} = 459$

↳ Ans a

Sum of all the terms in the progression

b) $a = 2k+6$

$T_2 = 2k$

$T_3 = k+2$

(i) value of k

$T_n = ar^{n-1}$

$2k = (2k+6)r$

$\frac{2k}{2k+6} = r$

$k+2 = (2k+6)r^2$

$k+2 = (2k+6)\left(\frac{2k}{2k+6}\right)^2$

$k+2 = (\cancel{2k+6})(2k)^2$

$(2k+6)^{2}$

$$k+2 = \frac{4k^2}{2k+6}$$

$$(k+2)(2k+6) = 4k^2$$

$$2k^2+10k+12 = 4k^2$$

$$0 = 2k^2-10k-12$$

$$0 = 2k^2-12k+2k-12$$

$$= 2k(k-6)+2(k-6)$$

$$= (2k+2)(k-6)$$

$$= k=-1 \qquad k=6$$

$\times$ ✓ ↳ $k = +ve$

↳ Ans b)i) Positive value of k

ii) Sum to infinity

$$S_\infty = \frac{a}{1-r}$$

$$r = \frac{2k}{2k+6} = \frac{12}{18} = \frac{2}{3}$$

↳ $$S_\infty = \frac{18}{1-2/3} = \frac{18}{\frac{1}{3}} = 18\times 3$$

$S_\infty = 54 \rightarrow$ Ans b)ii) Sum to infinity

③ S15/13/Q9

a) $a = -2222$

$d = 17$

$$T_n > 0$$

$$a+(n-1)d > 0$$

$$-2222+17n-17 > 0$$

$$\frac{17n}{17} > \frac{2239}{17}$$

$$n > 131.7$$

↳ Round up to 132^{nd} term

$$T_{132} = a+131d$$

$$= -2222+131(17)$$

$$T_{132} = 5$$

↳ Ans a) Value of first positive term

b) $a = \sqrt{3}$

$T_2 = 2\cos\theta \qquad 0 < \theta < \pi$

Find the set of values of θ for which the progression is convergent

↳ ie, find the range of values such that $-1 < r < 1$

$$T_2 = ar$$

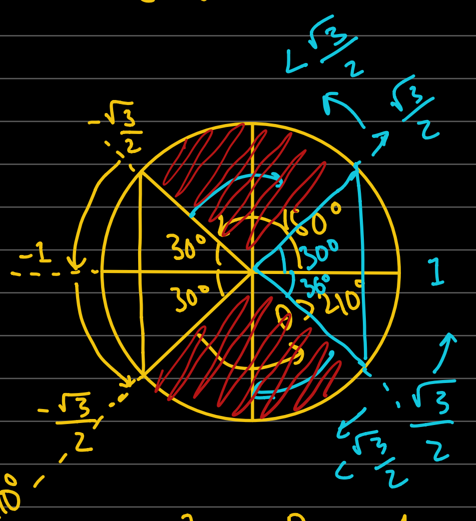
$$2\cos\theta = \sqrt{3}\, r$$

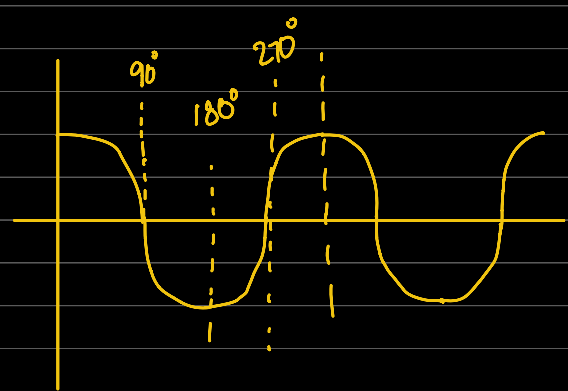
$$\frac{2\cos\theta}{\sqrt{3}} = r$$

$$\frac{2\cos\theta}{\sqrt{3}} > -1$$

$$\cos\theta > -\frac{\sqrt{3}}{2}$$

$$\theta > 210^\circ$$





$$\frac{2}{\sqrt{3}}\cos\theta < 1$$

$$\cos\theta < \sqrt{3}$$

$30^\circ < \theta < 150^\circ$

↳ Ans b Range of values for

2

which progression is convergent

(4) W15 / 11 / Q8

$a = 4x$

$T_2 = x^2$

$r = \frac{1}{4}x$

i) Arithmetic

$d = 12$

$T_2 = a + d$

$x^2 = 4x + 12$

$0 = x^2 - 4x - 12$

$= x^2 - 6x + 2x - 12$

$= x(x-6) + 2(x-6)$

$= (x+2)(x-6)$

$= x = -2 \quad x = 6$

$T_2 = 4x + 12$
$= -8 + 12$
$= 4$

$T_2 = 4x + 12$
$= 24 + 12$
$= 36$

Ans(i)

ii) Geometric

$S_\infty = 8$

$\frac{a}{1-r} = 8$

$\frac{4x}{1 - \frac{1}{4}x} = 8$

$4x = 8(1 - \frac{1}{4}x)$

$4x = 8 - 2x$

$6x = 8$

$x = \frac{4}{3}$

$T_3 = ar^2$

$= 4(\frac{4}{3})(\frac{1}{4} \times \frac{4}{3})^2$

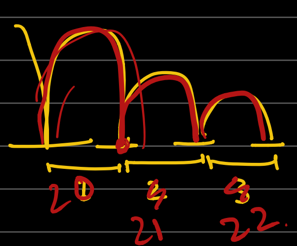
$= \frac{16}{3} \times (\frac{1}{3})^2$

$= \frac{16}{3} \times \frac{1}{9}$

$T_3 = \frac{16}{27}$ Ans(ii)

(5) W15 / 13 / Q6

i) a) Model A $\rightarrow$ Arithmetic



$a = 1.92m$

$d = 0.08m$

$S_{20} = \frac{n}{2}[2a + 19d]$

$= 10[2(1.92) + 19(-0.08)]$

$= 10[3.84 - 1.52]$

$= 10(2.32)$

$= 23.2$

↳ Ans i) a)

b) Geometric

$a = 1.92$
$r = 0.96$

$$S_{20} = \frac{a(1-r^{20})}{1-r}$$
$$= \frac{1.92(1-0.96^{20})}{1-0.96}$$
$$= 26.78$$
$$= 26.8$$

↳ Ans i) b)

ii) $$S_\infty = \frac{a}{1-r}$$
$$= \frac{1.92}{1-0.96}$$
$$= \frac{1.92}{0.04}$$
$$= 48m$$

↳ Ans ii

⑥ S16 / 11 / Q9

a) $a = 50$
$T_3 = 32$

$ar^2 = 32$
$50r^2 = 32$
$$r^2 = \frac{32}{50}$$
$$r = \sqrt{\frac{32}{50}}$$
$r = 0.8$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{50}{1-0.8}$$
$$= \frac{50}{0.2}$$
$= 250 \rightarrow$ Ans(a)

b) $d = 3\cos x - 2\sin x$

$a + 2d = T_3$
$2\sin x + 2(3\cos x - 2\sin x) = \sin x + 2\cos x$
$2\sin x + 6\cos x - 4\sin x = \sin x + 2\cos x$
$6\cos x - 2\sin x = \sin x + 2\cos x$
$$\frac{4\cos x}{3\cos x} = \frac{3\sin x}{3\cos x}$$
$$\frac{4}{3} = \tan x \rightarrow \text{shown}$$

Ans b) i)

$d = 3\cos(53.1) - 2\sin(53.1)$
$= 1.80 - 1.60$
$= 0.2$

$a = 2\sin x$
$= 2\sin(53.1)$
$= 1.599$
$= 1.60$

$$\tan x = \frac{4}{3}$$
$$x = \tan^{-1}\left(\frac{4}{3}\right)$$
$x = 53.1°$

$S_{20} = 10[2(1.6) + 19(0.2)]$
$= 10[3.2 + 3.8]$
$= 10[7]$
$= 70$

↳ Ans b) ii)

7) S16/12/Q9

i) a) $a = 10$
$d = 2$

$T_{30} = a + (n-1)d$
$= 10 + 29(2)$
$= 10 + 58$
$= 68$ litres
↳ Ans i) a)

b) $S_n > 2000$
$\frac{n}{2}[20 + 2(n-1)] > 2000$
$\frac{n[20 + 2n - 2]}{2} > 2000$
$\frac{n[18 + 2n]}{2} > 2000$
$9n + n^2 > 2000$
$n^2 + 9n - 2000 > 0$

-2000
$20 \cdot 100$
$40 \cdot 50$
$80 \cdot 25$
$400 \cdot 5$

$n = 40.447$
↳ During the 41st day
↳ Ans i) b)

ii) $a = 10$
$r = 1.1$

$S_{30} = \frac{a(1 - 1.1^{29})}{1 - 1.1}$
$= \frac{10(1 - 1.1^{29})}{1 - 1.1}$
$= 1486.3$ litres wasted
$= 513.69$ litres left

$\frac{513.69}{2000} \times 100 = 25.68$
$= 25.7\%$ → Ans (ii)

8) S16/13/Q4

AP GP
$a = a = 3$
$T_3 = T_2$ ⟶ $a + 2d = ar$
$3 + 2d = 3r$
$d = \frac{3r - 3}{2}$
$T_{13} = T_3$
↳ $a + 12d = ar^2$
$3 + 12d = 3r^2$

$d \neq \frac{3(1) - 3}{2}$, hence $r = 1$ not possible

$3 + 12\left(\frac{3r-3}{2}\right) = 3r^2$
$3 + 18r - 18 = 3r^2$
$18r - 15 = 3r^2$
$= 3r^2 - 18r + 15$
$= 3r^2 - 15r - 3r + 15$
$= 3r(r-5) - 3(r-5)$
$= (3r-3)(r-5)$
$r = 1$ or $r = 5$

$d = \frac{3(5) - 3}{2}$
$d = \frac{15 - 3}{2}$
$d = \frac{12}{2}$
$d = 6$

∴, $d = 6$ and $r = 5$
↳ Ans

(9) w16/11/Q5

$S_2 = 50 \longrightarrow a + ar = 50$

$a(1+r) = 50$

$a = \frac{50}{1+r}$

$S_3 - a = 30$

$a + ar + ar^2 - a = 30$

$ar + ar^2 = 30$

$\frac{50r + 50r^2}{1+r} = 30$

$50r + 50r^2 = 30 + 30r$

$5r + 5r^2 = 3 + 3r$

$0 = 5r^2 + 2r - 3$

$= 5r^2 + 5r - 3r - 3$

$= 5r(r+1) - 3(r+1)$

$= (5r-3)(r+1)$

$r = \frac{3}{5}$ $\quad r = -1 \rightarrow \times$

$a = \frac{50}{1+0.6}$

$= \frac{50}{1.6}$

$= 31.25$

$S_\infty = \frac{a}{1-r}$

$= \frac{31.25}{0.4}$

$= 78.125$

$= 78.1 \longrightarrow$ Ans

(10) w16/12/Q8

a) $S_n = 3050$ km

$a = 200$

$d = -5$

$T_{15} = a + (n-1)d$

$= 200 + 14(-5)$

$= 200 - 70$

$= 130 \longrightarrow$ Ans a) i)

$3050 = \frac{n}{2}[2a + (n-1)d]$

$6100 = n[400 - 5n + 5]$

$6100 = n[405 - 5n]$

$6100 = 405n - 5n^2$

$0 = 5n^2 - 405n + 6100$

$0 = n^2 - 81n + 1220$

$0 = n^2 - 61n - 20n + 1220$

$= n(n-61) - 20(n-61)$

$= (n-20)(n-61)$

$n = 20$ $\quad n = 61$

1220

610 x 2

305 4

61 20

$S_{20} = 10[400 - 95]$

$= 10[305]$

$= 3050$

∴, he finishes the event on May 20th.

b) $T_3 = 8T_6 \longrightarrow \cancel{a}r^2 = 8\cancel{a}r^5$

$S_6 = 31.5$

$r^2 = 8r^5$

$1 = 8r^3$

$\frac{1}{8} = r^3$

$\frac{1}{2} = r$

$\frac{a(1-r^6)}{1-r} = 31.5$

$\frac{a\left(1-\left(\frac{1}{2}\right)^6\right)}{\frac{1}{2}} = 31.5$

$2a\left(1-\frac{1}{64}\right) = 31.5$

$\frac{2a \cdot 63}{64} = 31.5$

$\frac{126a}{126} = \frac{2016}{126}$

$a = 16 \rightarrow$ Ans b) i)

$S_\infty = \frac{a}{1-r}$

$= \frac{16}{0.5}$

$= 32 \rightarrow$ Ans b) ii)

W17 / 11 / Q3

a) $a_2 = 3a_1$ $\quad a_1 = a_1$

$r_1 = r_1$ $\quad r_2 = -2r_1$

$\frac{3a}{1-r} = \frac{a}{1-(-2r)}$

$3\cancel{a}(1+2r) = \cancel{a}(1-r)$

$3+6r = 1-r$

$2 = -7r$

$-\frac{2}{7} = r \longrightarrow$ Ans (a)

b) AP_1: $a = 15$, $T_2 = 19$ $\}$ $d = +4$

AP_2: $a = 420$, $T_2 = 415$ $\}$ $d = -5$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2a+(n-1)d]$

$2(15)+(n-1)(4) = 2(420)+(n-1)(-5)$

$30+4n-4 = 840-5n+5$

$26+4n = 845-5n$

$\frac{9n}{9} = \frac{819}{9}$

$n = 91 \longrightarrow$ Ans (b)