SERIES : APGP PRACTICE Qs.

From Worksheets (NG), Slide 191 onwards

Homework Questions to attempt :
~~80~~, 82, ~~12~~, ~~39~~, ~~41~~, ~~43~~, ~~45~~, ~~55~~, ~~57~~, ~~63~~, ~~71~~, ~~74~~

12. i) $a = 250000 \rightarrow$ in yr. 2000
$r = 1.05$

2000	1
01	2
02	3
03	4
⋮	⋮

Yr 2008 = Term 9

$T_n = ar^{n-1}$
$T_9 = (250000)(1.05^8)$

$T_9 = 369363$
$\approx 369000 \rightarrow$ Ans (i)
Profit in yr. 2008

ii) 2009 → 10^{th} term

$$S_{10} = \frac{a(1-r^n)}{1-r}$$
$$S_{10} = \frac{250000(1-1.05^{10})}{1-1.05}$$

$= 3144473$
$\approx 3140000 \rightarrow$ Ans ii)
Total profit in 2000–2009 inclusive

iii) $a = 250000$
$d = +D$

$$S_n = \frac{n}{2}(a+(n-1)d)$$
$$S_{10} = 5(250000 + 9D)$$
$$= 1250000 + 45D$$

$$3144473 = 1250000 + 45D$$
$$3144473 - 1250000 = 45D$$
$$\frac{1894473}{45} = \frac{45D}{45}$$
$$42099 = D$$
$42100 \approx D \rightarrow$ Ans iii)
Value of D.

39 a) $a = 161$, $T_2 = 154$ } $-7 = d$ $S_m = 0$, find m

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_m = \frac{m}{2}[2(161) + (m-1)(-7)]$$

$$0 = \frac{m}{2}[322 - 7m + 7]$$

$$0 = \frac{m}{2}[329 - 7m]$$

$$0 = \frac{329m}{2} - \frac{7m^2}{2}$$

$$0 = 329m - 7m^2$$

$$0 = 47m - m^2$$

$$0 = m^2 - 47m$$

$$0 = m(m-47)$$

$\times$ $m = 0$ or $m = 47$ ✓

↳ Ans a) value of m

b) $S_n < 0.9 S_\infty$

$$\frac{a(1-r^n)}{1-r} < 0.9\left(\frac{a}{1-r}\right)$$

$$\left(\frac{a}{1-r}\right)(1-r^n) < 0.9\left(\frac{a}{1-r}\right)$$

$$1 - r^n < 0.9$$

$$1 - 0.9 < r^n$$

$$0.1 < r^n$$

or $r^n > 0.1$ → shown

Ans b)

41. a) $a = 100$, $S_\infty = 2000$

$$S_\infty = \frac{a}{1-r}$$

$$2000 = \frac{100}{1-r}$$

$$1 - r = \frac{100}{2000}$$

$$1 - r = 0.05$$

$$1 - 0.05 = r$$

$$0.95 = r$$

$$ar = T_2$$

$$(100)(0.95) = T_2$$

$$95 = T_2$$

↳ Ans a) value of T_2

b) i) Arithmetic P.

$T_3 = 90 \longrightarrow$ ① $a + 2d = 90$

$T_5 = 80$

$a = 90 - 2d$

$a = 90 - 10$

$a = 80$

↳ ② $a + 4d = 80$

$90 - 2d + 4d = 80$

$90 + 2d = 80$

$2d = 80 - 90$

$d = -5$

$\therefore$ $a = 80$ → Ans b) i)

$d = -5$

ii) $S_m = S_{m+1}$

↳ $\frac{n}{2}[2a + (n-1)d] = \frac{n+1}{2}[2a + (n+1-1)d]$

$n[2a + (n-1)(-5)] = n+1[2a + n(-5)]$

$n[2(80) - 5n + 5] = n+1[2(80) - 5n]$

$n(165 - 5n) = (n+1)(160 - 5n)$

$165n - \cancel{5n^2} = 160n - \cancel{5n^2} + 160 - 5n$

$0 = -10n + 160$

$10n = 160$

$n = 16$

$\therefore$ $m = 16$ → Ans b) ii)

iii) $S_n = 0$

$\frac{n}{2}[2(80) + (n-1)(-5)] = 0$

$\frac{n}{2}[160 - 5n + 5] = 0$

$\frac{n}{2}[165 - 5n] = 0$

$\frac{165n - 5n^2}{2} = 0$

$5n^2 - 165n = 0$

$n^2 - 33n = 0$

$n(n - 33) = 0$

$n = 0$ ✗ or $n = 33$ ✓ → Ans b) iii)

43. i)

Model A → Arithmetic $d = 1000$

1	2	3	4	5
1000	2000	3000		
50	100	150		

$50 \xrightarrow{+50} 100 \xrightarrow{+50} 150$

$d = 50$ for charity

$$S_{40} = \frac{40}{2}[2(50) + 39(50)]$$
$$= 20[100 + 1950]$$
$$= 20(2050)$$
$$S_{40} = 41000$$

∴, the total amount donated to charity after 40 days is \$41000 (if Model A is used).

ii) $a = 1000$
$r = 1.1$

1	2	3	4
1000	1100	1210	1331
50	55	60.5	66.55

$\times 1.1$ $\times 1.1$

$r = 1.1$ of charity

$$S_{40} = \frac{50(1 - 1.1^{40})}{1 - 1.1}$$

$$S_{40} = 22000$$

Ans. ii) Total donated to charity after 40 days if Model B is used

45. a) $l = 4a$
$S_6 = 360$

$360 = 3(4a + a)$
$120 = 5a$
$24^\circ = a$

P of a sector $= 2r + r\theta$
$= 2(5) + 5(24)$
$= 10 + 120$
$= 130\text{cm}$

↳ Ans a) Perimeter of sector

b) i) $a = 2k+3$

$T_2 = k+6 \longrightarrow$ ① $(2k+3)r = k+6$

$T_3 = k \longrightarrow$ ② $(2k+3)r^2 = k$

① $r = \dfrac{k+6}{2k+3}$

② $\dfrac{\cancel{(2k+3)}(k+6)^2}{(2k+3)^{\cancel{2}\,1}} = k$

$k^2 + 12k + 36 = k(2k+3)$

$k^2 + 12k + 36 = 2k^2 + 3k$

$0 = k^2 - 9k - 36$

$0 = k^2 - 12k + 3k - 36$

$0 = k(k-12) + 3(k-12)$

$0 = (k+3)(k-12)$

$k = -3$ or $k = 12$

$\times$ since all terms are positive

↳ Ans b) i)

ii) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{2k+3}{1-\frac{2}{3}}$

$= \dfrac{27}{\frac{1}{3}}$

$= 27 \times 3$

$= 81$

↳ Ans b) ii)

Sum to convergence

$r = \dfrac{k+6}{2k+3} = \dfrac{12+6}{24+3}$

$= \dfrac{18}{27}$

$= \dfrac{2}{3}$

55a) Arithmetic Progression

$a = 1$ Show that $S_{10} = a - b\sin^2 x$

$T_2 = \cos^2 x$

$T_2 = 1 - \sin^2 x$

$a \longrightarrow T_2$

$1 \longrightarrow 1 - \sin^2 x$

$d = -\sin^2 x$

$S_{10} = \frac{10}{2}\left[2(1) + 9(-\sin^2 x)\right]$

$= 5[2 - 9\sin^2 x]$

$= 10 - 45\sin^2 x$

↳ Shown
Ans (a)

$a = 10$
$b = 45$

b) $a = 1$

$T_2 = \frac{1}{3}\tan^2\theta$, $0 < \theta < \frac{\pi}{2}$ ie. θ is acute

i) ... for which the progression is convergent, ie, the common ratio is between -1 and 1 exclusive

$1 \times r = \frac{1}{3}\tan^2\theta$

$r = \frac{1}{3}\tan^2\theta$

$\sqrt{3r} = \sqrt{\tan^2\theta}$

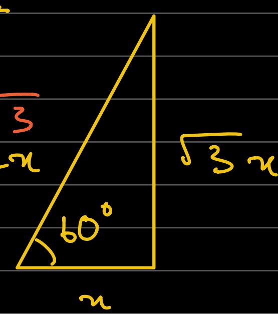
$\sqrt{3r} = \tan\theta$

$\frac{1}{3}\tan^2\theta < 1$

$\tan^2\theta < 3$

$\tan\theta < \sqrt{3}$

opposite
$>\sqrt{3}$ adj.
$\therefore \sqrt{3}$
$+ \searrow <\sqrt{3}$
$2x$

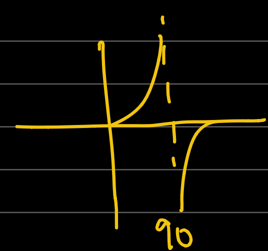


$\frac{1}{3}\tan^2\theta > -1$

$\tan^2\theta > -3$

$\tan\theta > \sqrt{-3} \rightarrow$ No solution

$\tan\theta < \sqrt{3}$



$0 < \theta < \frac{\pi}{30}$ or 60°

$\therefore$, $0 < \theta < \frac{\pi}{3}$ → Ans b)i)

Possible values of θ

ii) $S_\infty = \dfrac{a}{1-r}$

$$\frac{\pi}{3} = \frac{1}{1-\frac{1}{3}\tan^2\theta}$$

$$\left(1-\frac{1}{3}\tan^2\theta\right)\left(\frac{\pi}{3}\right) = 3$$

$$\frac{\pi}{3} - \frac{\pi}{9}\tan^2\theta = 3$$

$$\frac{\pi}{3} - 3 = \frac{\pi}{9}\tan^2\theta$$

$$3\pi - 3 = \pi\tan^2\theta$$

$$\sqrt{\frac{3\pi-3}{\pi}} = \sqrt{\tan^2\theta}$$

$$\sqrt{\frac{3\pi-3}{\pi}} = \tan\theta$$

check it

$$2.045 = \tan\theta$$

$$1.116 = \theta$$

$$1.12 = \theta \rightarrow$$ Ans b) ii) Value of θ

57. a) $S_n = n^2 + 8n$

$$\frac{n}{2}[2a + (n-1)d] = n^2 + 8n$$

$$\frac{n}{2}[2a + dn - d] = n^2 + 8n$$

$$na + \frac{dn^2}{2} - \frac{dn}{2} = n^2 + 8n$$

$$na - \frac{dn}{2} + \frac{dn^2}{2} = n^2 + 8n$$

$$na - \frac{dn}{2} = 8n \qquad \frac{dn^2}{2} = n^2$$

$$\cancel{n}\left(a - \frac{d}{2}\right) = 8\cancel{n} \qquad d\cancel{n^2} = 2\cancel{n^2}$$

$$a - \frac{d}{2} = 8 \qquad d = 2$$

$a - \frac{2}{2} = 8$

$a - 1 = 8$ $\quad\therefore,\ a = 9 \rightarrow$ Ans a)

$a = 9$ $\quad d = 2$

b) Geometric Progression

$a - 9 = T_2$ $\quad$ Calculate a

$S_3 - a = 30$

All terms are positive

$a - 9 = ar$ (1)

$a - ar = 9$

$a(1-r) = 9$

$a = \frac{9}{1-r}$

$a = \frac{9}{1-\frac{2}{3}}$

$a = 9 \times 3$

$a = 27$

Ans b)

(2) $a + ar + ar^2 - a = 30$

$ar + ar^2 = 30$

$a(r + r^2) = 30$

$\frac{9(r+r^2)}{1-r} = 30$

$9r + 9r^2 = 30 - 30r$

$3r + 3r^2 = 10 - 10r$

$3r^2 + 13r - 10 = 0$

$3r^2 + 15r - 2r - 10 = 0$

$3r(r+5) - 2(r+5) = 0$

$(3r-2)(r+5) = 0$

$r = \frac{2}{3}$ $\quad\quad$ $r = -5$

× since all terms are positive

63 a) $T_2 = 24 \longrightarrow ar = 24$

$T_4 = 13.5$ $\quad r = \frac{24}{a} \longrightarrow r = \frac{24}{32} = \frac{3}{4}$

i) $ar^3 = 13.5$

$\frac{\cancel{a}24^3}{a^{\cancel{3}}{}^2} = 13.5$

$24^3 = 13.5a^2$

$\sqrt{\frac{13824}{13.5}} = a$

$32 = a \rightarrow$ Ans a) i)

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{32}{1-\frac{3}{4}}$

$= 32 \times 4$

$S_\infty = 128$

Ans a) ii)

Sum to convergence

b) $a = 3$, $T_2 = 5$ $\quad +2 = d$

$S_n = 360$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$2(360) = n[2(3)+(n-1)(2)]$

$720 = n[6+2n-2]$

$720 = n[4+2n]$

$720 = 4n+2n^2$

$0 = 2n^2+4n-720$

$0 = n^2+2n-360$

$0 = n^2+20n-18n-360$

$0 = n(n+20)-18(n+20)$

$= (n-18)(n+20)$

Ans b) $\leftarrow n = 18$ $\qquad n = -20$ ✗

71.a) Arithmetic

$S_n = 2n^2+8n$

$\frac{n}{2}[2a+(n-1)d] = 2n^2+8n$

$\frac{n}{2}[2a+dn-d] = 2n^2+8n$

$na+\frac{dn^2}{2}-\frac{dn}{2} = 2n^2+8n$

$\frac{d}{2}n^2+n\left(a-\frac{d}{2}\right) = 2n^2+8n$

$\frac{d}{2} = 2$

$d = 4$

$a-\frac{d}{2} = 8$

$a-\frac{4}{2} = 8$

$a = 10$

$\therefore$, $a = 10$, $d = 4$ $\longrightarrow$ Ans a)

b) $a = 64$, $T_2 = 48$

$64r = 48$

$r = \frac{48}{64}$

T_2 GP $= T_9$ AP

① $48 = a+8d \longrightarrow 48 = 64+8d$

② $36 = a+(n-1)d$

$\frac{48-64}{8} = d$

$-2 = d$

$r = \frac{6}{8}$

$r = \frac{3}{4}$

$T_3 = 36$

$36 = 64 + (n-1)(-2)$

$36 = 64 - 2n + 2$

$36 - 66 = -2n$

$2n = 30$

$n = 15 \longrightarrow$ Ans b)

74.a) $a = 300$
$d = +12$

i) $T_n = 540$

ii) S_{26}

$T_n = a + (n-1)d$

$540 = 300 + (n-1)(12)$

$240 = 12n - 12$

$\frac{252}{12} = \frac{12n}{12}$

$21 = n \longrightarrow$ Ans a) i)

$S_{26} = 13[2a + 25(12)]$

$= 13[600 + 300]$

$= 13[900]$

$= 11700$ Seconds $\longrightarrow 195$ min $\longrightarrow 3$ hours 15 min

↳ Ans a) ii)

b) $T_2 = 48$, $T_3 = 32$ } $\times \frac{2}{3} = r$

$\frac{2}{3}a = 48$

$a = 48 \times \frac{3}{2}$

$a = 72$

$S_\infty = \frac{a}{1-r}$

$= \frac{72}{1/3}$

$= 72 \times 3$

$S_\infty = 216 \longrightarrow$ Ans b)

Sum to convergence

80

$r \neq 1$

a	ar	ar^2
a	$a+8d$	$a+20d$
8		

i) $ar = a + 8d$

① $8r = 8 + 8d$

$8(r-1) = 8d$

$r - 1 = d$

96

② $8r^2 = 8 + 20d$

$8r^2 = 8 + 20(r-1)$

$8r^2 = 8 + 20r - 20$

$8r^2 = 20r - 12$

$8r^2 - 20r + 12 = 0$

$$8r^2 - 8r - 12r + 12 = 0$$

$$8r(r-1) - 12(r-1) = 0$$

$$(8r-12)(r-1) = 0$$

$r = \frac{12}{8}$ ↳ $r \neq 1$ so ✗

$r = 1.5$

↳ Ans i)

ii) GP → $r = 1.5$

$ar^3 = 8(1.5)^3$

$T_4 = 27$

↳ Ans ii) GP

AP → $r - 1 = d$

$1.5 - 1 = d$

$0.5 = d$

$T_4 = a + 3d$

$= 8 + 3(0.5)$

$= 8 + 1.5$

$= 9.5$

↳ Ans ii) AP

$d = -4$

82 $a = 36$, $T_2 = 32$ $\frac{32}{36} = \frac{16}{18} = \frac{8}{9}$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{36}{1 - \frac{8}{9}}$$

$$= \frac{36}{\frac{1}{9}}$$

$$= 36 \times 9$$

$= 324$ → Ans (i)

$a = 36$

$S_n = 0$

$d = -4$

$S_n = \frac{n}{2}[2(36) + (n-1)(-4)]$

$0 = \frac{n}{2}[72 - 4n + 4]$

$$0 = \frac{n}{2}[76 - 4n]$$

$$0 = 38n - 2n^2$$

$$0 = 19n - n^2$$

$$0 = n^2 - 19n$$

$$0 = n(n-19)$$

$n \neq 0$ so $n = 19 \rightarrow$ Ans ii)