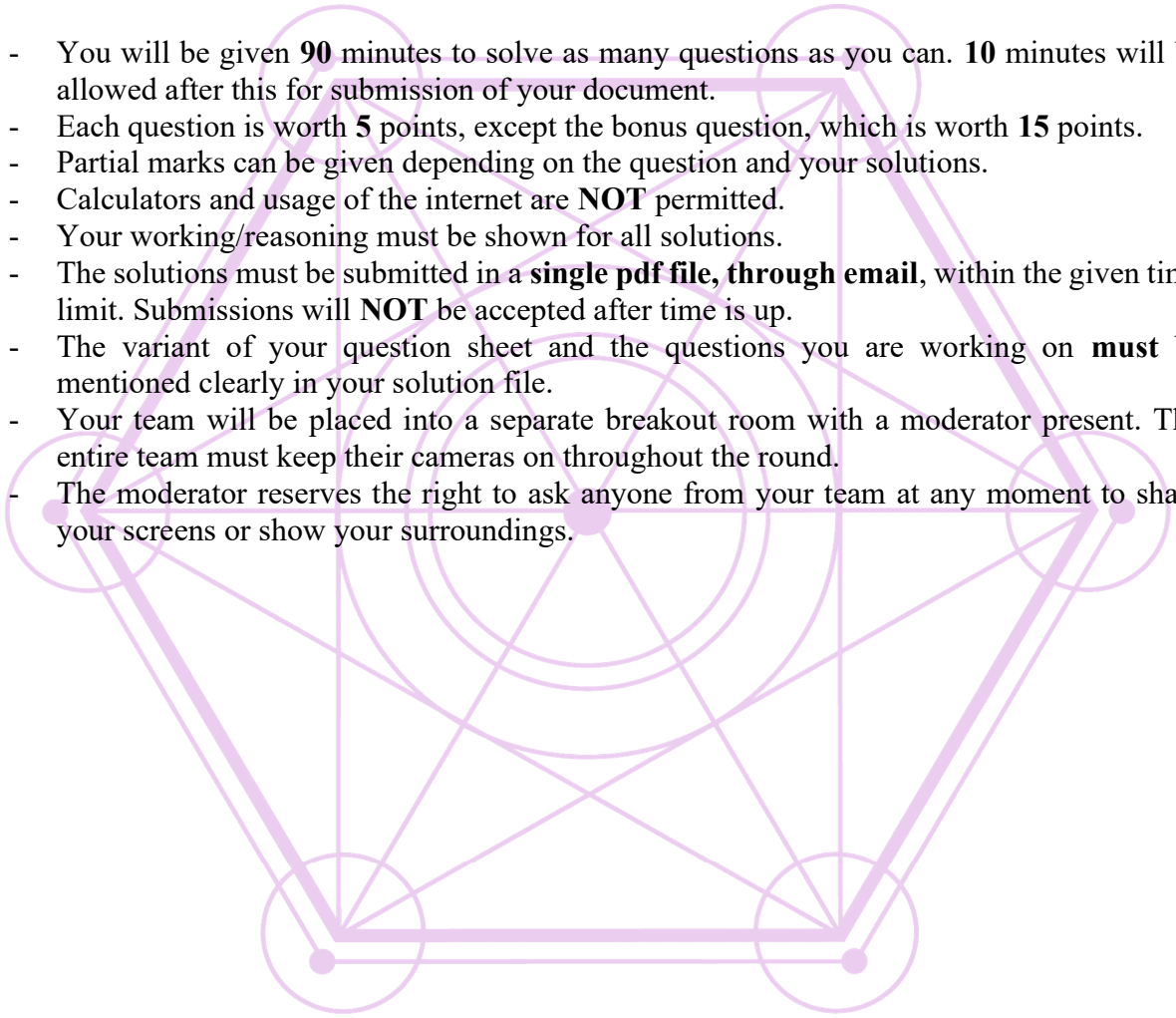
Math Gauge Round 2

Rules:

- You will be given **90** minutes to solve as many questions as you can. **10** minutes will be allowed after this for submission of your document.
- Each question is worth **5** points, except the bonus question, which is worth **15** points.
- Partial marks can be given depending on the question and your solutions.
- Calculators and usage of the internet are **NOT** permitted.
- Your working/reasoning must be shown for all solutions.
- The solutions must be submitted in a **single pdf file, through email**, within the given time limit. Submissions will **NOT** be accepted after time is up.
- The variant of your question sheet and the questions you are working on **must** be mentioned clearly in your solution file.
- Your team will be placed into a separate breakout room with a moderator present. The entire team must keep their cameras on throughout the round.
- The moderator reserves the right to ask anyone from your team at any moment to share your screens or show your surroundings.



Variant 1

Q1) The positive divisors of 12 (other than itself) are 1, 2, 3, 4, and 6. Their sum, 1 + 2 + 3 + 4 + 6, is greater than 12. An abundant number is a number for which the sum of its positive divisors (other than itself) is greater than the number itself. This means that 12 is an abundant number. Which of the following is also an abundant number?
(A) 8 (B) 10 (C) 14 (D) 18 (E) 22

Done

Q2) The value of $\sqrt{36 * \sqrt{16}}$ is

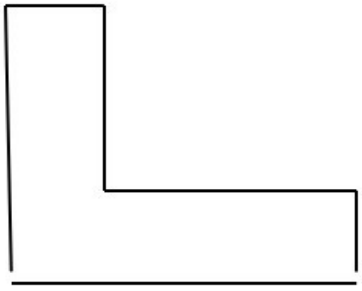
Done

Q3) Ali has 20 football cards, and Hamza has 44 baseball cards. They agree to trade such that Hamza gives Ali 2 baseball cards for every card Ali gives to Hamza. After how many such trades will Ali and Hamza each have an equal number of cards?

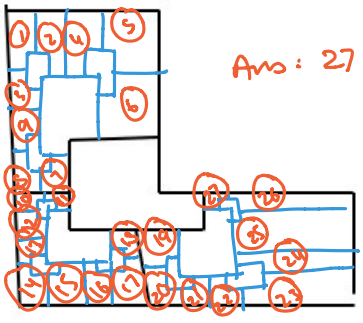
Done

Q4) An L shape is made by adjoining three congruent squares. The L is subdivided into four smaller L shapes, as shown. Each of the resulting L's is subdivided in this same way. After the third round of subdivisions, how many L's of the smallest size are there?
(Images given below are not drawn to scale)

Done



Original L



After 1 round

Q5) Assume that x is any even number. Prove that the squared and cubed values of x are also even.

Done

Q6) Determine all possible values for the area of a right-angled triangle with one side length equal to 60 and with the property that its side lengths form an arithmetic sequence.

Done

Q7) Amir and Bushra play a card game. Amir starts with a hand of 6 cards: 2 red, 2 yellow and 2 green. Bushra starts with a hand of 4 cards: 2 purple and 2 white. Amir plays first. Amir and Bushra alternate turns. On each turn, the current player chooses one of their own cards at random and places it on the table. The cards remain on the table for the rest of the game. A player wins and the game ends when they have placed two cards of the same colour on the table. Determine the probability that Amir wins the game.

Done

Q8) The positive integers 34 and 80 have exactly two positive common divisors, namely 1 and 2. How many positive integers n with $1 \leq n \leq 30$ have the property that n and 80 have exactly two positive common divisors?

Done Raza

Q9) Caleb bought 300 pieces of candy A for $5.00, and x pieces of candy B for $7.00. She calculated that the average price of all of the candy that she purchased was $1.50 per 100 pieces. What is the value of x?

$12.00

$\frac{12}{x} = 1.50$ $\quad \frac{12}{1.5} = x \quad 8 = x$

800 total − 300 for $5 = 500 for $7

Ans: 500

Q10) In the diagram, all rows, columns, and diagonals have the same sum. What is the value of x?
(A) 12 (B) 13 (C) 16 (D) 17 (E) 18

14	19	12 a
d	15	b
x	11	16 c

$19+15+11 = 45$
$14+19+a = 45$
$a = 45-14-19$
$a = 12$
$14+15+c = 45$
$c = 45-14-15$
$c = 16$

$16+11+x = 45$
$x = 45-16-11$
Ans 10 → $x = 18$

Q11) What is the maximum value of the integer n, for which $n^{200} < 5^{300}$?

Done

Q12) The integer numbers x and y satisfy 2x = 5y. Only one of the following can be x + y. Which is it?

a) 2011
b) 2010
c) 2009
d) 2008
e) 2007

Done

Q13) If b is divisible by a and c is divisible by b, then show that c is divisible by a.

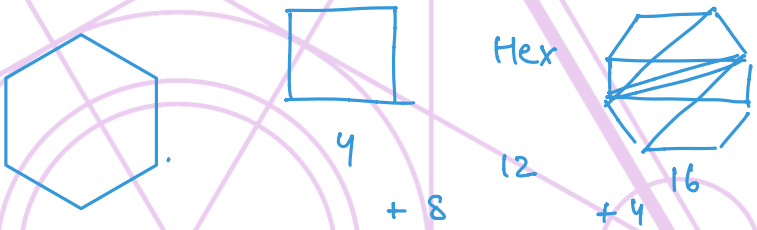
Done

Q14) $x^3+y^3+z^3=55$?
Find the the list of combinations of the values of x, y and z to solve this.

Q15) How many nonempty subsets of {1, 2, 3…, 8} exist in which the sum of the largest element and the smallest element is 9? Done

Q16) How many right-angled triangles can be formed by joining three vertices of a given regular 14-gon?

a) 42
b) 84
c) 88
d) 98
e) 168

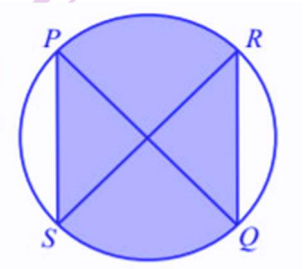


Q17) Express 1.41414141… as a fraction. (Hint: Start by letting x=1.41414141… and work your way from there.)

Done

Q18) In the diagram, P Q and RS are diameters of a circle with radius 4. If P Q and RS are perpendicular, what is the area of the shaded region?

Done



BONUS QUESTION:
Suppose that α and β are the two positive roots of the equation $x^2 - \sqrt{15}\, x^{log_{15} x} = 0$. Determine the value of αβ.

Done Ans. 15^2, working shown below

Q5:- $\chi = 2n$

$x^2 = (2n)^2 = 4n^2 = 2(2n^2) = 2m$ ✓

$x^3 = (2n)^3 = 8n^3 = 2(4n^3) = 2k$ ✓

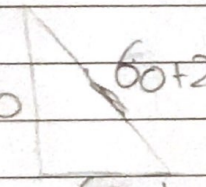
↳ shown

Q1) $1+2+3+6+9 = 21 > 18$ (D)

Q2) $\sqrt{36 \times 4} = \sqrt{144} = 12$

Q3) $\frac{20+44}{2} = 32$

Q6)



$\frac{1}{2} \times 60 \times 80$

$= 2400\ u^2$

$(60+2d)^2 = (60+d)^2 + 60^2$

$3600 + 2400d + 4d^2 = 3600 + 120d + d^2 + 3600$

$3d^2 + 120d - 3600 = 0$

$d^2 + 40d - 1200 = 0$

$d^2 - 20d + 60d - 1200 = 0$

$d(d-20) + 60(d-20) = 0$

$d = -60,\ d = 20$

Q12) $2x = 5y$

$x = \frac{5}{2}y$

$\frac{5}{2}y + y = \frac{7}{2}y = 2009 \Rightarrow y = 287 \times 2$

$7 \overline{)2009}$ (287)

14

60

56

49

49

(C)

Date: _____

Q11) $n^{2^{100}} < 5^{3^{100}}$

$(n^2)^{100} < (5^3)^{100}$

$\Leftrightarrow n^2 < 5^3$

$n^2 < 125$

$\boxed{n = 11}$

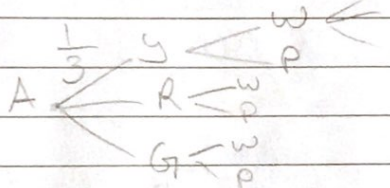
Q13) $a = c \cdot p$ where p and q are integers.

$c = b \cdot q$

$a = (b \cdot q) \cdot p$ since $q \cdot p$ is also an integer, b divides a.

$a = b \cdot (q \cdot p)$

Q17)



$\frac{1}{3} \times 1 \times \frac{1}{5} + \frac{1}{3} \times 1 \times \frac{4}{5} \times \frac{2}{3} \times \frac{2}{4}$

(Y, P, Y) + (Y, P, R, W, Y or R)

$\frac{1}{5} + \frac{4}{15} = \boxed{\frac{7}{15}}$

Q14) $x^3 + y^3 + z^3 = 55 = 64 - 8 - 1 = 4^3 - 2^3 - 1^3 \rightarrow 3!$

$= 27 + 27 + 1 = 3^3 + 3^3 + 1^3 \rightarrow \frac{3!}{2!}$

$(x, y, z) = (4, -2, -1), (4, -1, -2), (-2, 4, -1), (-2, -1, 4)$
$(-1, -2, 4), (-1, 4, -2)$
$(3, 3, 1), (3, 1, 3), (1, 3, 3)$

Date:

bonus 9

$$x^2 = \sqrt{15}\, x^{\log_{15} x}$$

$$\frac{x^2}{x^{\log_{15} x}} = x^{\frac{1}{2}\log_x 15}$$

$$x^{2-\log_{15} x} = x^{\frac{1}{2}\log_x 15}$$

$$2 - \log_{15} x = \frac{1}{2}\log_x 15$$

$$2 - \log_{15} x = \frac{1}{2\log_{15} x}$$

$$4\log_{15} x - 2(\log_{15} x)^2 = 1$$

$$y = \log_{15} x$$

$$4y - 2y^2 = 1$$

$$2y^2 - 4y + 1 = 0$$

$$y = \frac{4 \pm \sqrt{4^2 - 2(4)}}{2(2)}$$

$$y = \frac{4 \pm 2\sqrt{2}}{4}$$

$$\frac{2+\sqrt{2}}{4} > 0, \quad \frac{2-\sqrt{2}}{4} > 0$$

$$\log_{15} \alpha = \frac{2+\sqrt{2}}{2}$$

$$\log_{15} \beta = \frac{2-\sqrt{2}}{2}$$

$$\log_{15} \alpha + \log_{15} \beta = \frac{2+\sqrt{2}}{2} + \frac{2-\sqrt{2}}{2}$$

$$\log_{15}(\alpha\beta) = 2$$

$$\boxed{\alpha\beta = 15^2}$$

28 1, (2), 3, 4, 5, (6), 7,

8, 9, 10, 11, 12, 13, (14)

(15), 16, 17, (18), 19, 20,

21, (22), 23, 24, (25/5), (26),

27, 28, 29, 30.

@11

$n^{200} < 5^{300}$ $\sqrt[100]{}$

$n^2 < 5^3$

$n < \sqrt[3]{125}$

$\sqrt{125} > 11$ $\boxed{n \geq 11}$

$125 > 121$

Q1 $18 = 1 \times 2 \times 3 \times 6 \times 9 \times 18$

$1+2+3+6+9 > 18$

$21 > 18$

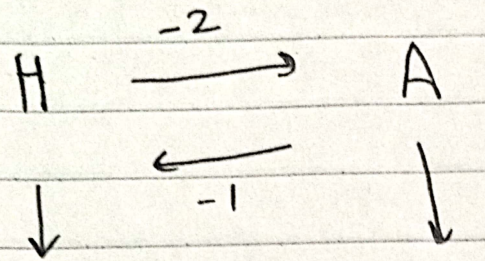
(D)

Q2 $\sqrt{36 * \sqrt{16}}$

$\sqrt{6 \times 6 \times 2 \times 2}$

$6 \times 2 = \boxed{12}$

23



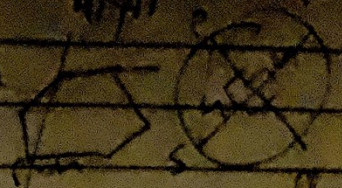
n = number of trades

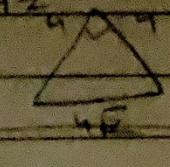
$20 + n = 44 - n$

$2n = 24$

$n = 12$

Date :





18) $A = \frac{\pi \cdot 4^2}{2} + (4\sqrt{2})^2$

$8\pi + 32$

17) $n = 1.414141\ldots$

$\frac{a}{b} = 1.414141\ldots$

$a = 1.414141\ldots$ — (b)

$0.\overline{41}$

$\frac{99}{99}$

$a = 41, b = 99 = \frac{140}{99}$

$= 16/2$

15) $b = ax$ where x & y are integers

$c = by$

$\frac{c}{y} = ax$

$c = axy$ $\therefore c = az$ where $z = xy$ i.e integer

hence c/a.

M. Kumga Products

Date :

Q15) $\{1+6\}$ $\{2+7\}$ $\{3+6\}$ $\{4+5\}$

$1+6(1)+6(5)+6(5)(4)+6(5)(4)(3)+6\ 1+$ $^4C_1+{}^4C_2+{}^4C_3+{}^4C_4$

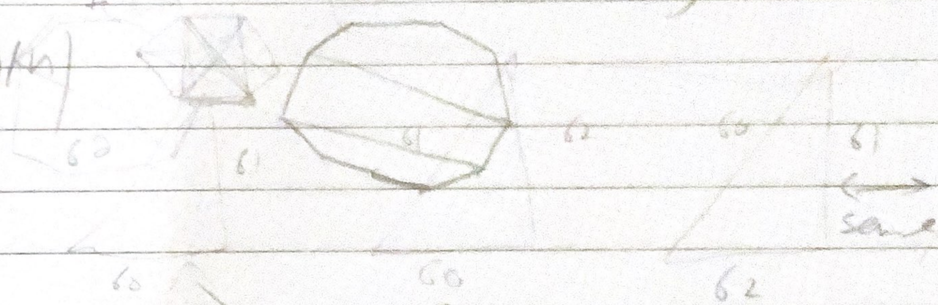
$1+{}^6C_1+{}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5+{}^6C_6$

$2^6+2^4+2^2+2^0$

$1+{}^2C_1+{}^2C_2$ $+1$

$64+16+4+1=85$ non empty set

6)



60(61) 60(62) (61)(62)

= 3661 3722 3782