Math Homework: Q~~31~~, ~~32~~, ~~36~~, ~~37~~, ~~38~~ from slide 100 onwards.

Q31. $f(x) = 2x+1$
$g(x) = x^2 - 2$

i)a) $f(g(x)) = 2(x^2-2)+1$
$= 2x^2 - 4 + 1$
$fg(x) = 2x^2 - 3$

i)b) $gf(x) = (2x+1)^2 - 2$
$= 4x^2 + 4x + 1 - 2$
$gf(x) = 4x^2 + 4x - 1$

ii) $2a^2 - 3 = 4a^2 + 4a - 1$
$-2 = 2a^2 + 4a$
$0 = 2a^2 + 4a + 2$
$= 2a^2 + 2a + 2a + 2$
$= 2a(a+1) + 2(a+1)$
$= (2a+2)(a+1)$
$\therefore,\ a = -1$

iii) $b^2 - 2 = b$
$0 = b^2 - b - 2$
$= b^2 + 1b - 2b - 2$
$= b(b+1) - 2(b+1)$
$= (b-2)(b+1)$
$= b = 2$ or $b = -1$
Since $b \neq a$, then the value of b must be 2
$\therefore,\ b = 2$

iv) $f^{-1}(x) \Rightarrow y = 2x+1$
$\frac{y-1}{2} = x$
$f^{-1}(x) = \frac{x-1}{2}$

$f^{-1}(g(x)) = \frac{x^2-2-1}{2}$
$f^{-1}g(x) = \frac{x^2-3}{2}$

v) $h(x) = x^2 - 2,\ x \leq 0$

$h^{-1}(x) \Rightarrow y = x^2 - 2$

$y + 2 = x^2$

$\pm\sqrt{y+2} = x$ → keep (-) since we used the left side of the graph.

$\therefore,\ h^{-1}(x) = -\sqrt{x+2}$

Q32. $f(x) = \frac{x+3}{2x-1},\ x \in \mathbb{R},\ x \neq \frac{1}{2}$

i) Show that $ff(x)$ ⇒ Essentially show that $f(x)$ is a self inverse

$$ff(x) = \frac{\left(\frac{x+3}{2x-1}\right)+3}{2\left(\frac{x+3}{2x-1}\right)-1}$$

ii) Hence, $f^{-1}(x) = \frac{x+3}{2x-1}$

$$= \frac{\dfrac{x+3+3(2x-1)}{2x-1}}{\dfrac{2x+6-2x+1}{2x-1}}$$

since $f(x)$ is a self inverse function, $f^{-1}(x) = f(x)$.

$$= \frac{\dfrac{x+\cancel{3}+6x-\cancel{3}}{2x-1}}{\dfrac{7}{2x-1}}$$

$$= \frac{\dfrac{x+6x}{2x-1}}{\dfrac{7}{2x-1}}$$

$$= \frac{x+6x}{2x-1} \times \frac{2x-1}{7}$$

$$= \frac{(x+6x)\cancel{(2x-1)}}{\cancel{(2x-1)}(7)}$$

$$= \frac{x+6x}{7}$$

$$= \frac{7x}{7}$$

$= x \rightarrow$ Shown

Q36. $f(x) = 2x^2 - 8x + 10, \quad 0 \leq x \leq 2$

$g(x) = x \quad 0 \leq x \leq 10$

i) Complete square for $f(x)$

$$\begin{aligned}
f(x) &= 2x^2 - 8x + 10 \\
&= 2[x^2 - 8x + (-4)^2 - (-4)^2] + 10 \\
&= 2[(x-4)^2 - 16] + 10 \\
&= 2(x-4)^2 - 32 + 10
\end{aligned}$$

$f(x) = 2(x-4)^2 - 22$, $a = 2$, $b = -4$, $c = -22$

ii) Range of $f(x)$

$f(0) = 10$, $f(2) = 2(2)^2 - 8(2) + 10$

$= 2(4) - 16 + 10$

$= 8 - 16 + 10$

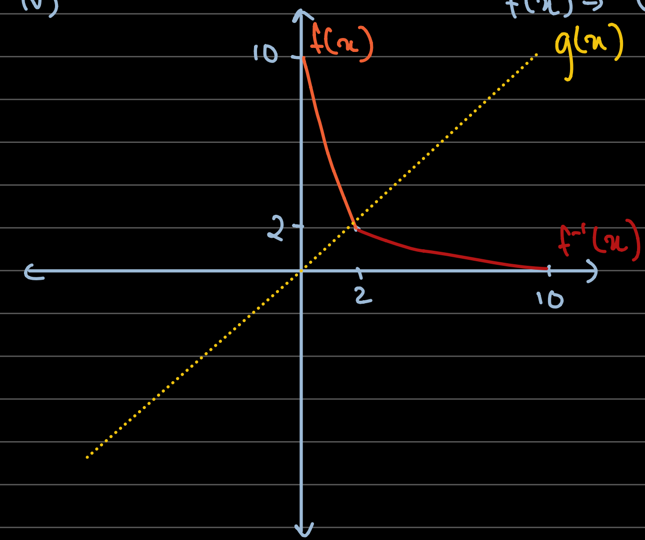
$f(2) = 2$

$\therefore$, $2 \leq f(x) \leq 10$

iii) Domain of $f^{-1}(x)$

$= 2 \leq x \leq 10$

iv) $f(x) \Rightarrow (h,k) \Rightarrow (4,-22)$



$f^{-1}(x)$ is a reflection of $f(x)$ across $g(x)$.

v) $f^{-1}(x) \Rightarrow y = 2(x-4)^2 - 22$

$$4 \pm \sqrt{\frac{y+22}{2}} = x$$

↳ Use (−) since we kept the left side

$$\therefore, \quad 4 - \sqrt{\frac{x+22}{2}} = f^{-1}(x)$$

Q37. $f(x) = 3x + a, \; x \in \mathbb{R}$
$g(x) = b - 2x, \; x \in \mathbb{R}$
$\left.\begin{matrix} a \\ b \end{matrix}\right\}$ constants

i) $ff(2) = 10$

$$3(3x+a) + a = 10$$
$$9x + 3a + a = 10$$
$$9x + 4a = 10$$
$$9(2) + 4a = 10$$
$$4a = 10 - 18$$
$$4a = -8$$
$$a = -2$$

$g^{-1}(x) \Rightarrow y = b - 2x$

$$x = \frac{b-y}{2}$$
$$g^{-1}(x) = \frac{b-x}{2}$$
$$g^{-1}(2) = 3$$
$$\frac{b-2}{2} = 3$$
$$b - 2 = 6$$
$$b = 8$$

$\therefore, \; a = -2 \;\&\; b = 8$

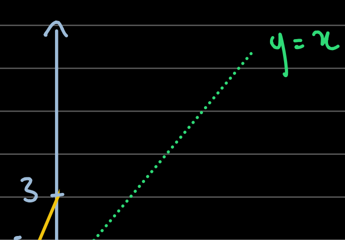
Q38. $f(x) = 2x + 3, \quad x \leq 0$
$g(x) = x^2 - 6x, \; x \leq 3$

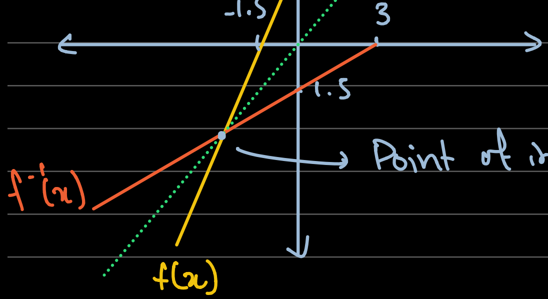
i) $f^{-1}(x) \Rightarrow y = 2x + 3$

$$\frac{y-3}{2} = x$$
$$f^{-1}(x) = \frac{x-3}{2}$$

$$f(x) = f^{-1}(x)$$
$$2x + 3 = \frac{x-3}{2}$$
$$4x + 6 = x - 3$$
$$3x = -9$$
$$x = -3$$

ii)





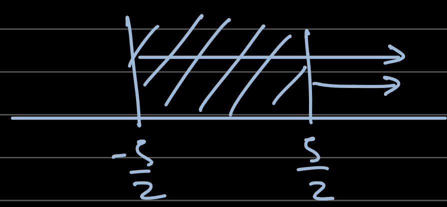
Point of intersection $=(-3,-3)$

iii) $gf(x) \le 16$

$$(2x+3)^2 - 6(2x+3) \le 16$$

$$4x^2 + 12x + 9 - 12x - 18 \le 16$$
$$4x^2 - 9 - 16 \le 0$$
$$4x^2 - 25 \le 0$$
$$4x^2 - 10x + 10x - 25 \le 0$$
$$2x(2x-5) + 5(2x-5) \le 0$$
$$(2x+5)(2x-5) \le 0$$

$$x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2}$$



$\therefore$, $-\frac{5}{2} \le x \le \frac{5}{2}$