INTEGRATION

1. Basic integration
2. Given $\frac{dy}{dx}$ = find y

Example:

Integrate $2x+5$ w.r.t to x

written in proper notation:

integral sign

$$\int \underbrace{(2x+5)}_{\text{expression to integrate}} \underbrace{dx}_{\text{w.r.t } x}$$

can also be written as

$$\int (2x)\,dx + (5)\,dx$$

How to integrate:

$2x+5$

$$\int (2x+5)\,dx$$

$$= \frac{2x^{1+1}}{2} + \frac{5x^{0+1}}{1}$$

$$= x^2 + 5x + c$$

constant of integration

$8-3x$

$$\int (8-3x)\,dx$$

$$= \frac{8x^{0+1}}{1} - \frac{3x^{1+1}}{2}$$

$$= 8x - \frac{3}{2}x^2 + c$$

$(x+1)(2x+1)$

$= 2x^2 + x + 2x + 1$

$= 2x^2 + 3x + 1$

$$\int (2x^2+3x+1)\,dx$$

$$= \frac{2x^{2+1}}{3} + \frac{3x^{1+1}}{2} + \frac{1x^{0+1}}{1} + c$$

$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + x + c$$

$$\frac{(x^2+1)}{x^2}$$

$$\int \left(\frac{x^2}{x^2} + \frac{1}{x^2}\right) dx$$

$$= \int (1 + x^{-2})\,dx$$

$$= \frac{1x^{0+1}}{1} + \frac{x^{-2+1}}{-1} + c$$

$$= x - \frac{1}{x} + c$$

$\left(x + \frac{1}{x}\right)^2$

$\int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$= \frac{x^{2+1}}{3} + \frac{2x^{0+1}}{1} + \frac{x^{-2+1}}{-1} + c$

$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$

$(3x-5)^4\, dx$

If there's a linear expression inside the brackets, then there's a shortcut

$\int (3x-5)^4\, dx$

1. Add 1 to the power
2. Divide by that power
3. Divide by the derivative of the expression inside the brackets

$\int (3x-5)^4\, dx$

power increased by 1

$= \frac{(3x-5)^5}{(5)(3)} + c$

derivative

incremented power

All the examples done so far are called indefinite integrals, due to the absence of a range and the presence of the integration constant c

A definite integral : Any integral that has limits

upper limit

$\int_1^2 (3x-5)^4\, dx \rightarrow \left[\frac{(3x-5)^5}{15}\right]_1^2$ ....→ no constant

lower limit

↳ Input upper limit (2)
Input lower limit (1)
Find difference

$$\frac{(3(2)-5)^5}{15} - \frac{(3(1)-5)^5}{15}$$

$$= \frac{(6-5)^5}{15} - \frac{(-2)^5}{15}$$

$$= \frac{1}{15} + \frac{32}{15}$$

$$= \frac{33}{15}$$ Definite integral

$$\int_1^2 (x^2 - 5x + 4)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_1^2$$

$$= \left(\frac{2^3}{3} - \frac{5(2)^2}{2} + 4(2)\right) - \left(\frac{1^3}{3} - \frac{5(1)^2}{2} + 4(1)\right)$$

$$= \left(\frac{8}{3} - 10 + 8\right) - \left(\frac{1}{3} - \frac{5}{2} + 4\right)$$

Given $\frac{dy}{dx}$ and a point on the curve, find the equation of the curve

Example

$\frac{dy}{dx} = 2x + 5$, $(1, 9) \rightarrow$ point on curve , find equation of curve

Step 1 : $\int (2x+5)\,dx$

Step 2 : $\frac{2x^2}{2} + 5x + c$

$$y = x^2 + 5x + c$$

Step 3 : Input given coordinates and evaluate c

$$9 = (1)^2 + 5(1) + c$$

$9 = 1 + 5 + C$

$9 - 6 = C$

$\underline{3 = C}$ ↳ value of c

Step 4: Final equation

$$y = x^2 + 5x + 3$$

Example:

Given: $\frac{dy}{dx} = (5x-4)^3$

$(1, 0) \rightarrow$ point on curve

Find y when $x = 1.2$

Step 1: $\int (5x-4)^3 \, dx$

Step 2: $\frac{(5x-4)^4}{(4)(5)} + C$

Step 3: $y = \frac{(5x-4)^4}{20} + C$

$$0 = \frac{(5(1)-4)^4}{20} + C$$

$$0 = \frac{(1)^4}{20} + C$$

$$-\frac{1}{20} = C$$

Step 4: $y = \frac{(5x-4)^4}{20} - \frac{1}{20}$

when $x = 1.2$

$$y = \frac{(5(1.2)-4))^4}{20} - \frac{1}{20}$$

$$= \frac{(6-4)^4}{20} - \frac{1}{20}$$

$$= \frac{2^4}{20} - \frac{1}{20}$$

$$= \frac{16}{20} - \frac{1}{20}$$

$$= \frac{15}{}$$

$$= \frac{\frac{20}{3}}{4}$$

Example

Given: $\frac{dy}{dx} = x^2(x-3)$

$(2, -6) \to$ point on curve

find equation of y

$$\int (x^3 - 3x^2)\,dx$$

$$= \frac{x^4}{4} - \frac{3x^3}{3} + c$$

$$y = \frac{x^4}{4} - x^3 + c$$

$$-6 = \frac{2^4}{4} - (2)^3 + c$$

$$-6 = \frac{16}{4} - 8 + c$$

$$-6 = 4 - 8 + c$$

$$-6 + 4 = c$$

$$-2 = c$$

$$\boxed{y = \frac{x^2}{4} - x^3 - 2}$$

$\hookrightarrow$ Equation of y

Slide 85 onwards

Q32.

$\frac{dy}{dx} = 4x + k$

t.p $= (-2, -1) \to \frac{dy}{dx} = 0$ here

Find

i) value of k

$$\left.\frac{dy}{dx}\right|_{x=-2} = 0$$

$$0 = 4(-2) + k$$

$$0 = -8 + k$$

$$\underline{8 = k}$$

$\hookrightarrow$ Ans

ii) coordinates where the curve meets the y-axis.

$$\int (4x + 8)\,dx$$

$$= \frac{4x^2}{} + 8x + c$$

$$2$$

$y = 2x^2 + 8x + c$

$-1 = 2(-2)^2 + 8(-2) + c$

$-1 = 8 - 16 + c$

$-1 = -8 + c$

$7 = c$

$y = 2x^2 + 8x + 7$

$y = 2(0) + 8(0) + 7$

$y = 7$

$(0, 7)$ ↳ Ans

Q35.

$\frac{dy}{dx} = kx - 5$

$(1, 0)$ and $(0, 6)$ → points on curve

$\int (kx - 5)\, dx$

$= \frac{kx^2}{2} - 5x + c$

$0 = \frac{k(1)^2}{2} - 5(1) + c$

$0 = \frac{k}{2} - 5 + 11$

$0 = \frac{k}{2} + 6$

$-6 = \frac{k}{2}$

$-12 = k$ ↳ Ans

$6 = \frac{k(0)^2}{2} - 5(1) + c$

$6 = -5 + c$

$11 = c$ ↳ Ans

HOMEWORK : Slide 85

Questions 2 – 17 → Basic integration

Questions ~~22~~, ~~27~~, ~~34~~, ~~38~~, ~~42~~, ~~45~~, ~~46~~, ~~48~~, ~~53~~

22. $\int_1^4 (6x - 3\sqrt{x})\, dx$

$= \left[\frac{6x^2}{2} - \frac{3x^{1.5}}{1.5}\right]_1^4$ ✗

$= [3(4)^2 - 2(3)(\sqrt{4})] - [3(1)^2 - 2(3)(\sqrt{1})]$

$= [(3)(16) - (6)(2)] - [3 - 6]$

$= (48 - 12) - (-3)$

$= 36 + 3$

$= 39$ → Ans

27. $(1,3) \rightarrow$ point on curve

$$\frac{dy}{dx} = 2x(2x^2 - 3)$$

$$= 4x^3 - 6x$$

$$\int (4x^3 - 6x)\,dx$$

$$= \frac{4x^4}{4} - \frac{6x^2}{2} + C$$

$$y = x^4 - 3x^2 + C$$

$y = x^4 - 3x^2 + 5 \rightarrow$ Ans (a)

$$3 = (1)^4 - 3(1)^2 + C$$
$$3 = 1 - 3 + C$$
$$3 = -2 + C$$
$$5 = C$$

b) Illegible

34. $\frac{dy}{dx} = 2x + k$

tangent at $(3,6)$ passes through origin

$$\text{Grad} = \frac{\Delta y}{\Delta x} = \frac{6}{3} = 2$$

a) value of k

$$2 = 2(3) + k$$
$$2 = 6 + k$$
$$-4 = k \longrightarrow \text{Ans (a)}$$

b) equation of the curve

$$\frac{dy}{dx} = 2x - 4$$

$$\int (2x - 4)\,dx$$

$$y = \frac{2x^2}{2} - 4x + C$$

$y = x^2 - 4x + 9 \rightarrow$ Ans (b)

$$y = x^2 - 4x + C$$

$$6 = (3)^2 - 4(3) + C$$
$$6 = 9 - 12 + C$$
$$6 = -3 + C$$

$9 = c$

38. $\frac{dy}{dx} = 15x^2 - 12x$

$(1, 3) \rightarrow$ point on curve

$\int (15x^2 - 12x)\, dx$

$= \frac{15x^3}{3} - \frac{12x^2}{2} + c$

$y = 5x^3 - 6x^2 + c$

$y = 5x^3 - 6x^2 + 4 \rightarrow$ Ans

$3 = 5(1)^3 - 6(1) + c$

$3 = 5 - 6 + c$

$4 = c$

42. $\int_1^4 (x^2 - 4 + 4x^{-2})\, dx$

$= \left[\frac{x^3}{3} - 4x + \frac{4x^{-1}}{-1} \right]_1^4$

$= \left[\frac{4^3}{3} - 4(4) - 4\sqrt{4} \right] - \left[\frac{1^3}{3} - 4(1) - 4\sqrt{1} \right]$

$= \left(\frac{64}{3} - 16 - 8 \right) - \left(\frac{1}{3} - 4 - 4 \right)$

$= \frac{64}{3} - \frac{48}{3} - \frac{24}{3} - \frac{1}{3} + \frac{24}{3}$ ✗

$= \frac{-9}{3} + \frac{24}{3}$

$= \frac{15}{3}$

$= 5 \longrightarrow$ Ans

46. $\frac{dy}{dx} = 2x - 5$

$P(4, -2) \rightarrow$ point on curve

a) equation of normal at P

$2(4) - 5$

$= 8 - 5$

$= 3 \rightarrow$ grad at P

$m_\perp \times 3 = -1$

$m_\perp = -\frac{1}{3} \rightarrow$ grad. of normal

$y = \frac{-x}{3} + c$

$-2 = \frac{-4}{3} + c$

$-\frac{6}{3} + \frac{4}{3} = c$

$-\frac{2}{3} = c$

$y = \frac{-x-2}{3}$ → Ans (a)

b) equation of curve

$\int (2x-5)\,dx$

$= \frac{2x^2}{2} - 5x + c$

$y = x^2 - 5x + c$

$-2 = (4)^2 - 5(4) + c$

$-2 = 16 - 20 + c$

$-2 = -4 + c$

$2 = c$

$y = x^2 - 5x + 2$ → Ans (b)

48. $\frac{dy}{dx} = \frac{3}{4} - kx$

$\left(\frac{3}{4} - k(-1)\right) \times \left(\frac{3}{4} - k(1)\right) = -1$

$\left(\frac{3}{4} + k\right)\left(\frac{3}{4} - k\right) = -1$

$\frac{9}{16} - k^2 + 1 = 0$

$-k^2 + \frac{9}{16} + \frac{16}{16} = 0$

$-k^2 + \frac{25}{16} = 0$

$k^2 = \frac{25}{16}$

$\boxed{k = \frac{5}{4}}$ → Ans

(4, 0) → point on curve

$\int \left(\frac{3}{4} - \frac{5x}{4}\right) dx$

$= \frac{3x}{4} - \frac{5x^2}{8} + c$

$y = \frac{3}{4}x - \frac{5}{8}x^2 + 7$ → Ans

$$0 = \frac{12}{4} - \frac{5(4)^2}{8} + c$$

$$0 = 3 - 10 + c$$

$$0 = -7 + c$$

$$7 = c$$

53. (x, y) $\quad \frac{dy}{dx} = 1 + \frac{1}{2x^2}$

$y = 3x + 1$ at P

i) Coordinates of P

$\frac{dy}{dx}$ returns gradient

$$1 + \frac{1}{2x^2} = 3$$

$$\frac{1}{2x^2} = 2$$

$$1 = 4x^2$$

$$\frac{1}{4} = x^2$$

$$\frac{1}{2} = x$$

$$y = 3x + 1$$

$$y = 3\left(\frac{1}{2}\right) + 1$$

$$y = \frac{3}{2} + 1$$

$$y = \frac{5}{2}$$

$P\left(\frac{1}{2}, \frac{5}{2}\right) \rightarrow$ Ans (i)

*

ii) $\int \left(1 + \frac{1}{2}(x^{-2})\right) dx$

$$= 1x + \frac{1 \; x^{-1}}{2(-1)} + c$$

$$= x - \frac{1}{2x} + c$$

$$\frac{5}{2} = \frac{1}{2} - \frac{1(2)}{2(1)} + c$$

$$\frac{5}{2} = \frac{1}{2} - 1 + c$$

$$\frac{6}{2} = c$$

$$y = x - \frac{1}{2x} + 3$$

$3 = c$

Q18.

$$\int_2^8 f(x)\,dx = 24$$

a) $\int_2^8 3f(x)\,dx$

$= 3\int_2^8 f(x)\,dx$

$= 3 \times 24$

$= 72 \rightarrow$ Ans (a)

b) $\int_2^8 [f(x) - 3]\,dx$

$= \int_2^8 f(x)\,dx - \int_2^8 (3x)\,dx$

$= 24 - 3[8 - 2]$

$= 24 - 3(8) - (-3(2))$

$= 24 - 24 + 6$

$= 6$

19. $\int_0^3 g(x)\,dx = 8$

a) $\int_0^3 2g(x)\,dx$

$= 2\int_0^3 g(x)\,dx$

$= 2 \times 18$

$= 16 \rightarrow$ Ans (a)

b) $\int_0^3 [g(x) + 1]\,dx$

$= \int_0^3 g(x)\,dx + \int_0^3 dx$

$= 8 + [x]_0^3$

$= 8 + [3 - 0]$

$= 11$

c) $\int_3^0 g(x)\,dx$

$= -\int_0^3 g(x)\,dx$

$= -8$

24. $\int_1^5 h(x)\,dx = 4$

a) $\int_1^5 2h(x)\,dx$

$= 2\int_1^5 h(x)\,dx$

$= 2 \times 4$

$= 8$

b) $\int_1^5 [h(x) + 3]\,dx$

$= \int_1^5 h(x)\,dx + 3\int_1^5 dx$

$= 4 + 3[x]_1^5$

$= 4 + 3[5-1]$

$= 4 + 12$
$= 16$ //

c) $\int_5^1 h(x)\,dx$

$= -\int_1^5 h(x)\,dx$

$= -4$ //

$\int_1^5 [h(x) + kx]\,dx$

$= \int_1^5 h(x) + \int_1^5 (kx)\,dx$

$28 = 4 + \frac{k[x^2]_1^5}{2}$

$2(24) = k[x^2]_1^5$

$48 = k[25-1]$

$48 = 24k$

$2 = k \longrightarrow$ Ans

26. $\int_0^2 f(x)\,dx = 5$

$\int_2^3 f(x)\,dx = 5$

a) $\int_0^3 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$

$= 5 + 5$
$= 10$

b) $\int_0^2 f(x)\,dx + \int_3^2 f(x)\,dx$

$= 5 + (-5)$
$= 0$

c) $\int_0^2 [4f(x) + 2]\,dx$

$4\int^2 f(x)\,dx + \int^2 (2)\,dx$

$= (4 \times 5) + 2[x]_0^2$

$= 20 + 2[2-0]$

$= 20 + 4$

$= 24$

28. $\int_2^5 f(x)\,dx = 12$

a) $\int_5^2 f(x)\,dx$

$= -\int_2^5 f(x)\,dx$

$= -12$

b) $\int_2^5 3f(x)\,dx$

$= 3\int_2^5 f(x)\,dx$

$= 3 \times 12$

$= 36$

c) $\int_2^4 [f(x)+4]\,dx + \int_4^5 f(x)\,dx$

$= \underline{\int_2^4 f(x)\,dx} + 4\int_2^4 dx + \underline{\int_4^5 f(x)\,dx}$

$= 12 + 4[x]_2^4$

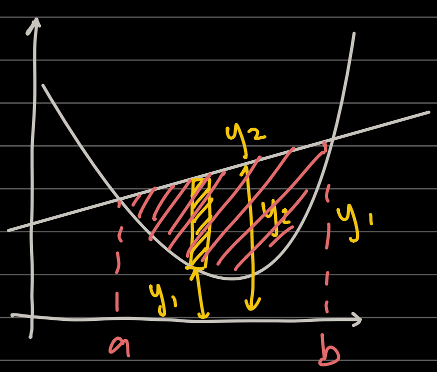
$= 12 + 4[4-2]$

$= 12 + 4[2]$

$= 12 + 8$

$= 20$

AREA UNDER CURVE



$$A = \int_a^b (y_2 - y_1)\,dx$$

$$\frac{dy}{dx} = x^{-1} + 4x^0$$

Slide 87 Worksheets (NG)

1. curve $\rightarrow$ $y = (x-2)^2$

line $\rightarrow$ $\frac{9-4}{5} = \frac{5}{5}$

$\rightarrow y = x + 4$

$$= \int_0^5 (x+4)\,dx - \int_0^5 (x-2)^2\,dx$$

$$= \left[\frac{(x+4)^2}{2} - \frac{(x-2)^3}{3}\right]_0^5$$

$$= \left(\frac{9^2}{2} - \frac{3^3}{3}\right) - \left(\frac{4^2}{2} - \left(\frac{-8}{3}\right)\right)$$

$$= \left(\frac{81}{2} - \frac{27}{3}\right) - \left(\frac{16}{2} + \frac{8}{3}\right)$$

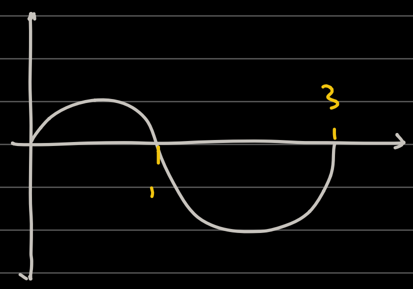
$$= 31.5 - \left(\frac{48}{6} + \frac{16}{6}\right)$$

$$= 31.5 - \frac{64}{6}$$

$$= 31.5 - 10.67$$

$$= 20.83$$

Q2. $y = x(x-1)(x-3)$



$$A = \cancel{\int_1^3 0\,dx} - \int_1^3 (x(x-1)(x-3))\,dx$$

$$x(x^2 - 4x + 3)$$

$$x^3 - 4x^2 + 3x$$

$$A = -\int_1^3 (x^3 - 4x^2 + 3x)\,dx$$

$$= -\left[\frac{x^4}{4} - \frac{4}{3}x^3 + \frac{3x^2}{2}\right]_1^3$$

$$= -\left[\frac{3^2}{4} - \frac{4}{3}(3)^3 + \frac{3(3)^2}{2}\right] - \left[\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right]$$

$$= -\left(\frac{9}{4} - \frac{4}{3}(27) + \frac{27}{2}\right) - \left(\frac{3}{12} - \frac{16}{12} + \frac{18}{12}\right)$$

$$= -\left(\frac{9}{4} - \frac{108}{3} + \frac{27}{2}\right) - \left(\frac{5}{12}\right)$$

$$= -\left(\frac{27}{12} - \frac{432}{12} + \frac{162}{12}\right) - \frac{5}{12}$$

$$=$$

Q3. $y = 2x$, $y = x(4-x)$

$$2x = x(4-x)$$
$$2x = 4x - x^2$$
$$0 = x^2 - 4x + 2x$$
$$0 = x^2 - 2x$$

$$A = \int_0^2 (2x - (4x - x^2))\,dx$$

$$= \int_0^2 (2x - 4x + x^2)\,dx$$

$0 = x(x-2)$

$x = 0 \qquad x = 2$

$$= \int_0^2 (x^2 - 2x)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}}\right]_0^2$$

$$= \left[\frac{2^3}{3} - 2^2\right] - 0$$

$$= \left(\frac{8}{3} - 4\right)$$

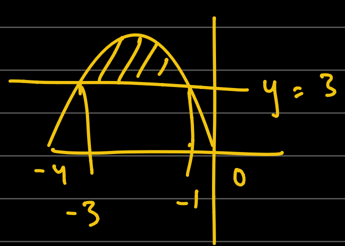
$$= \frac{8}{3} - \frac{12}{3}$$

$$= \left|-\frac{4}{3}\right|$$

$$= \frac{4}{3}$$

Q4. $y = -x(x+4) \qquad y = 3$

↳ $x = -4$ or $x = 0$



$3 = -x^2 - 4x$

$= x^2 + 4x + 3$

$= x^2 + 3x + 1x + 3$

$= x(x+3) + 1(x+3)$

$= (x+1)(x+3)$

$x = -1$ or $x = -3$

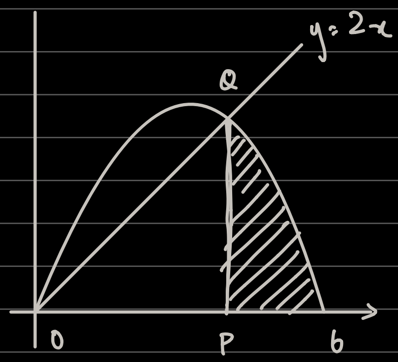
$$A = \int_{-3}^{-1} (-x^2 - 4x - 3)\,dx$$

$$= \left[-\frac{x^3}{3} - \frac{4x^2}{2} - 3x\right]_{-3}^{-1}$$

$$= \left(\frac{1}{3} - \frac{4}{2} + 3\right) - \left(\frac{-(-3)^3}{3} - \frac{4(-3)^2}{2} - 3(-3)\right)$$

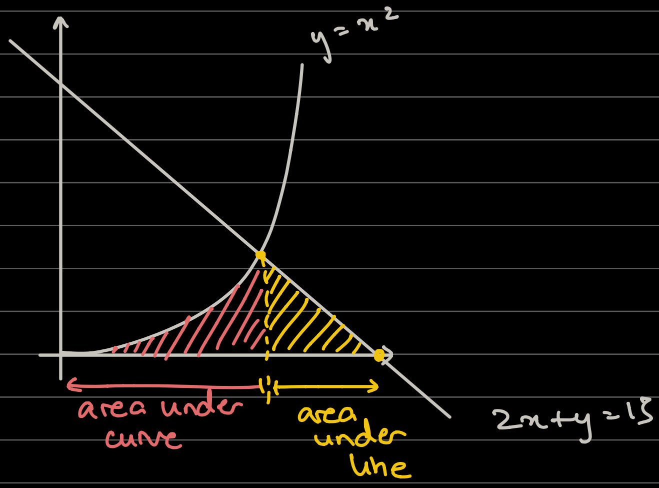
$=$

5.



6.

7.



HW: 9, 14, 15

Q9. $y = \frac{1}{3}x^2$

$y = \sqrt{3x}$

$(0,0) \quad (3,3)$

$$\int_0^3 \left(\sqrt{3}\,x^{\frac{1}{2}} - \frac{x^2}{3}\right) dx$$

$$\int_0^3 \frac{(\sqrt{3})(2)\,x^{\frac{3}{2}}}{3} - \frac{x^3}{9}$$

Q.13. a) $\int_1^2 (10x^{-2})\,dx$

$= \left[-\frac{10}{x}\right]_1^2$

$= \left(-\frac{10}{2}\right) - \left(-\frac{10}{1}\right)$

$= -\frac{10}{2} + \frac{10}{1}$

$= \frac{10}{2}$

$= 5 \rightarrow$ Ans(a)

b) $\int_2^3 (10x^{-2})\,dx$

$= \left[-\frac{10}{x}\right]_2^3$

$= -\frac{10}{3} - \left(-\frac{10}{2}\right)$

$= -\frac{20}{6} + \frac{30}{6}$

$= \frac{10}{6} \div 2 = \frac{10}{12}$

$\frac{10}{12} = -\frac{10}{p} - \left(-\frac{10}{2}\right)$

$\frac{10}{12} = \frac{10}{2} - \frac{10}{p}$

$\frac{10}{p} = \frac{10}{2} - \frac{10}{12}$

$= \frac{60}{12} - \frac{10}{12}$ X reattempt

$\frac{10}{p} = \frac{50}{12}$

$\frac{120}{50} = p$

17. $y = 9 - x^2$

$y = x^2 - 3x$

$\int_0^3$

$x^2 - 3x = 0$

$x^2 = 3x$

$x = 3$

18.

20. $y = (x-1)^2$
$y = x + 1$

from $x = 1$ to $x = 3$

messed up upper vs. lower

$$\int_1^3 (x^2 - 2x + 1 - (x+1))\,dx$$

$$= \left[\frac{x^3}{3} - \frac{2x^2}{2} + \cancel{x} - \frac{x^2}{2} - \cancel{x} \right]_1^3$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} \right]_1^3$$

$$= \left(\frac{3^3}{3} - \frac{3(3)^2}{2} \right) - \left(\frac{1}{3} - \frac{3}{2} \right)$$

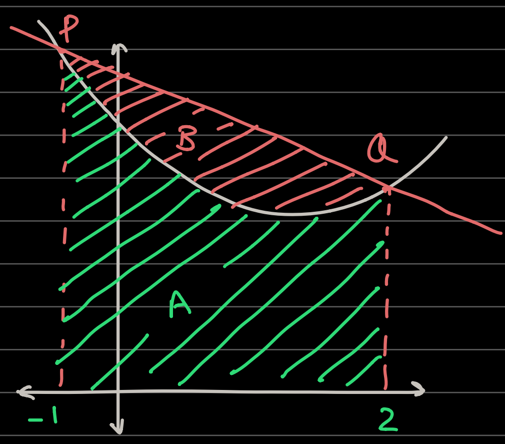
$$= 9 - \frac{27}{2} - \frac{1}{3} + \frac{3}{2}$$

$$= 9 - \frac{24}{2} - \frac{1}{3}$$

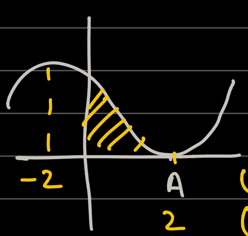
× reattempt

$$= \frac{54}{6} - \frac{72}{6} - \frac{2}{6}$$

Q21.



42. a)



$y = x^3 - 12x + k$

$(x, 0)$
$(2, 0)$

$$\frac{dy}{dx} = 3x^2 - 12$$

$0 = 3x^2 - 12$
$12 = 3x^2$
$4 = x^2$
$\pm 2 = x$

$0 = 2^3 - 12(2) + k$
$0 = 8 - 24 + k$
$16 = k$ ↳ value of k

Ans (a)

b) Area of shaded region

$$\int_0^2 (x^3 - 12x + k)\,dx$$

$$= \left[\frac{x^3}{4} - \frac{12x^2}{2} + kx\right]_0^2$$

$$= \frac{2^3}{4} - 6(2)^2 + 16(2) - 0$$

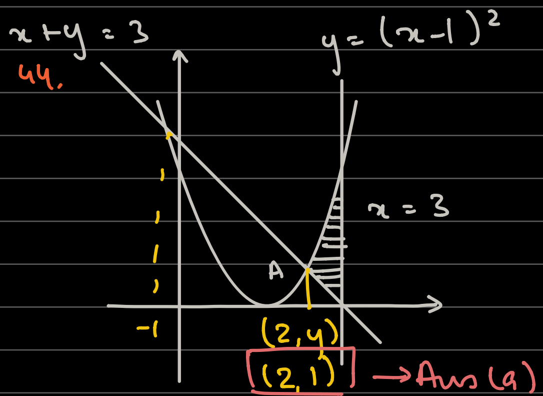
$$= \frac{8}{4} - 24 + 32$$

$$= 2 - 24 + 32$$

$$= 2 + 8$$

$$\boxed{= 10} \rightarrow \text{Ans (b)}$$

44.



a) A

$$(x-1)^2 = 3 - x$$

$$x^2 - 2x + 1 = 3 - x$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad \text{or} \quad x = -1$$

$$x + y = 3$$

$$2 + y = 3$$

$$y = 1$$

b) Area of shaded region

UB : $(x-1)^2$

LB = $3 - x$

Range = $2 \rightarrow 3$

$$\int_2^3 [x^2 - 2x + 1 - (3 - x)]\,dx$$

$$x^2 - 2x + 1 - 3 + x$$

$$x^2 - x - 2$$

$$\int_2^3 (x^2 - x - 2)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3$$

$$= \left[\frac{3^3}{3} - \frac{3^2}{2} - 2(3)\right] - \left[\frac{2^3}{3} - \frac{2^2}{2} - 2(2)\right]$$

$$= 9 - \frac{9}{2} - 6 - \left(\frac{8}{3} - 2 - 4\right)$$

$$= 3 - \frac{9}{2} - \left(\frac{8}{3} - 6\right)$$

$$= 3 - \frac{9}{2} - \frac{8}{3} + 6$$

$$= 9 - \frac{27}{6} - \frac{16}{6}$$

$$= 9 - \frac{11}{6}$$

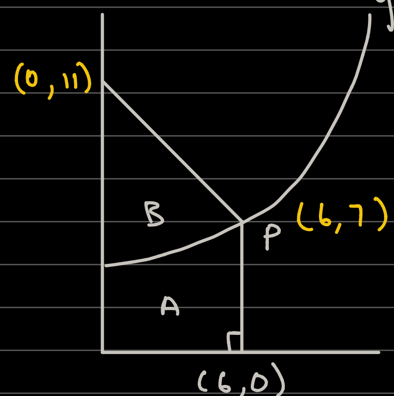
$$= 9 - 1\frac{5}{6}$$

$$= 7\frac{1}{6}$$

$$= \boxed{\frac{43}{6}} \rightarrow \text{Ans (b)}$$

50.

$y = \frac{x^2}{9} + \frac{x}{6} + k$, k constant



$A_A = 23u^2$

$$\int_0^6 \left(\frac{1}{9}(x^2) + \frac{1}{6}(x) + k\right) dx$$

$$= \left[\frac{x^3}{27} + \frac{x^2}{12} + kx\right]_0^6$$

$$23 = \frac{6^3}{27} + \frac{6^2}{12} + k(6)$$

$$= \frac{216}{27} + \frac{36}{12} + 6k$$

$$23 = 8 + 3 + 6k$$

$$23 - 11 = 6k$$

$$\frac{12}{6} = \frac{6k}{6}$$

$$2 = k \rightarrow \text{shown}$$

a) Q

$$\left.\frac{dy}{dx}\right|_{x=6} = \frac{2}{9}x + \frac{1}{6} = \frac{12}{9} + \frac{1}{6}$$

$$= \frac{4}{3} + \frac{1}{6}$$

$$= \frac{8}{6} + \frac{1}{6}$$

$$= \frac{9}{6}$$

$$= \frac{3}{2} \rightarrow m$$

$$m_{\perp} \times \frac{3}{2} = -1$$

$$m = -\frac{2}{3}$$

$$y = \frac{-2}{3}x + c$$

$$7 = \frac{-2(6)}{3} + c$$

$$7 = \frac{-12}{3} + c$$

$$7 + 4 = c$$

$$11 = c$$

$$P \rightarrow y = \frac{x^2}{9} + \frac{x}{6} + 2$$

$$y = \frac{6^2}{9} + \frac{6}{6} + 2$$

$$= 4 + 1 + 2$$

$$= 7$$

$$(6, 7) \rightarrow P$$

$$y = 11 - \frac{2}{3}x$$

$$y = 11 - 0$$

$$\boxed{y = 11} \rightarrow \text{Ans (a)}$$

b) Area of B

$$A_{A+B} = \frac{(11+7)6}{2}$$

$$= \frac{18 \times 6}{2}$$

$$= \frac{108}{2}$$

$$= 54u^2$$

$$54u^2 - 23u^2$$

$$\boxed{= 31u^2} \rightarrow \text{Ans (b)}$$

Slide 87 ~~Q1-7~~, ~~9~~, 14, 15, 13, 17, 18, 20, 21, 42, 44, 50
10, 11, 12, 16, 19, 22-41

1. Eq. of line:

$m = \frac{9-4}{5-0} = \frac{5}{5} = 1$

$y = x + c$

$[y = x+4] \rightarrow$ line equation : UB

LB : $y = (x-2)^2$
$= x^2 - 4x + 2$

$$\int_0^5 (x + 4 - (x^2 - 4x + 2))dx$$

$$= \int_0^5 (5x - x^2 + 2)\,dx$$

$$= \left[\frac{5x^2}{2} - \frac{x^3}{3} + 2x\right]_0^5$$

$$= \frac{5(5)^2}{2} - \frac{5^3}{3} + 2(5)$$

$$= \frac{125}{2} - \frac{125}{3} + 10$$

$$= \frac{375}{6} - \frac{250}{6} + 10$$

$$= \frac{125}{6} + 10$$

$$= \frac{125}{6} + \frac{60}{6}$$

$$\boxed{= \frac{185}{6}} \rightarrow \text{Ans 1.}$$

2. $y = x(x-1)(x-3) \longrightarrow$ roots : $x = 0$ or $x = 1$ or $x = 3$

$y = x(x^2 - 3x - 1x + 3)$

$y = x(x^2 - 4x + 3)$

$y = x^3 - 4x^2 + 3x \longrightarrow$ lower bound

$y = 0 \rightarrow$ upper bound

$$\int_1^3 (0 - (x^3 - 4x^2 + 3x))\,dx$$

$$\int_1^3 (0 - x^3 + 4x^2 - 3x)\,dx$$

$$\int_1^3 (-x^3 + 4x^2 - 3x)\,dx$$

$$= \left[-\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} \right]_1^3$$

$$= -\frac{3^4}{4} + \frac{4(3)^3}{3} - \frac{3(3)^2}{2} - \left(\frac{-1}{4} + \frac{4}{3} - \frac{3}{2} \right)$$

$$= -\frac{81}{4} + \frac{108}{3} - \frac{27}{2} - \left(-\frac{5}{12} \right)$$

$$= \frac{432}{12} - \frac{243}{12} - \frac{162}{12} + \frac{5}{12}$$

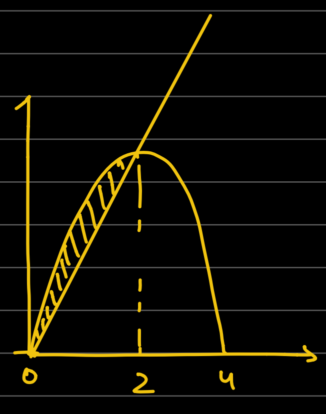
$$= \frac{32}{12}$$

$$= 2\frac{8}{12}$$

$$\boxed{= 2\frac{2}{3}} \rightarrow \text{Ans 2.}$$

3. $y = x(4-x) \rightarrow UB$

$y = 2x \rightarrow LB$



$2x = x(4-x)$

$2x = 4x - x^2$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0$ or $x = 2$

$$\int_0^2 (4x - x^2 - 2x)\,dx$$

$$\int_0^2 (2x - x^2)\,dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^2$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= 2^2 - \frac{2^3}{3}$$

$$= 4 - \frac{8}{3}$$

$$= \frac{12}{3} - \frac{8}{3}$$

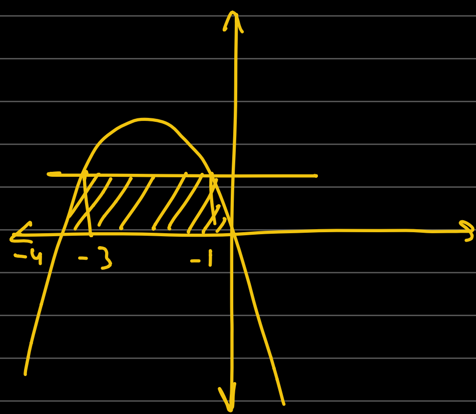
$$= \frac{4}{3}$$

$$\boxed{= 1\tfrac{1}{3}\ u^2} \rightarrow \text{Ans } 3.$$

4. $y = -x(x+4) \rightarrow$ UB $\longrightarrow$ $0 = -x^2 - 4x$

$y = 3 \rightarrow$ UB

$$0 = x^2 + 4x$$
$$0 = x(x+4)$$



$$3 = -x^2 - 4x$$
$$0 = x^2 + 4x + 3$$
$$0 = x^2 + 3x + x + 3$$
$$0 = x(x+3) + 1(x+3)$$
$$0 = (x+1)(x+3)$$

$x = -1$ or $x = -3$

$$\int_{-3}^{-1} (3 - (x^2 + 4x))\,dx$$

$$= \int_{-3}^{-1} (3 - x^2 - 4x)\,dx$$

$$= \left[3x - \frac{x^3}{3} - \frac{4x^2}{2} \right]_{-3}^{-1}$$

$$= 3(-1) - \frac{(-1)^3}{3} - 2(-1)^2 - \left(3(-3) - \frac{(-3)^3}{3} - \frac{4(-3)^2}{2} \right)$$

$$= -3 + \frac{1}{3} - 2 - (-9 + 9 - 18)$$

$$= -5 + \frac{1}{3} + 18$$

$$= 13\frac{1}{3}$$

5. $y = x(5-x) \longrightarrow$ roots : $x = 0$ or $x = 5$

$y = 2x$

$2x = 5x - x^2$

$0 = x^2 - 5x + 2x$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0$ or $\underbrace{x = 3}_{x_Q}$

$y = 2(3)$

$y = 6$

[$\therefore$, $Q = (3, 6)$] $\rightarrow$ Ans 5.

Area under curve from $x = 3$ to $x = 5$

$$\int_3^5 (5x - x^2)\,dx$$

$$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_3^5$$

$$= \frac{5(5)^2}{2} - \frac{5^3}{3} - \left(\frac{5(3)^2}{2} - \frac{3^3}{3}\right)$$

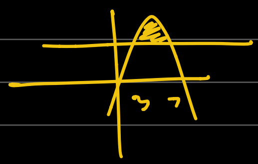
$$= \frac{125}{2} - \frac{125}{3} - \left(\frac{45}{2} - 9\right)$$

$$= \frac{375}{6} - \frac{250}{6} - \frac{27}{2}$$

$$= \frac{125}{6} - \frac{81}{6}$$

$$= \frac{44}{6}$$

$$\boxed{= 7\tfrac{1}{3}}$$ $\rightarrow$ Ans 5.



6. $y = 10x - x^2$ $\rightarrow$ UB $\longrightarrow$ $y = x(10-x)$

$\hookrightarrow$ roots : $x = 0$ or $x = 10$

$y = 21 \rightarrow$ LB

$21 = 10x - x^2$

$0 = x^2 - 10x + 21$

$= x^2 - 3x - 7x + 21$

$= x(x-3) - 7(x-3)$

$= (x-7)(x-3)$

$x = 7$ or $x = 3$

$$\int_3^7 (10x - x^2 - 21)\,dx$$

$$= \left[5x^2 - \frac{x^3}{3} - 21x \right]_3^7$$

$$= 5(7)^2 - \frac{7^3}{3} - 21(7) - \left(5(3)^2 - \frac{3^3}{3} - 21(3) \right)$$

$$= 245 - \frac{343}{3} - 147 - (45 - 9 - 63)$$

$$= 245 - 114\tfrac{1}{3} - 147 + 27$$

$$= 272 - 261\tfrac{1}{3}$$

$$\boxed{= 10\tfrac{2}{3}} \longrightarrow \text{Ans 6.}$$

7. $y = x^2$
$y = 15 - 2x$

a) Find P and Q

$15 - 2x = x^2$
$= x^2 + 2x - 15$
$= x^2 + 5x - 3x - 15$
$= x(x+5) - 3(x+5)$
$0 = (x-3)(x+5)$

$x = 3$ or $x = -5$

$\downarrow$

$y = 15 - 2x$
$y = 15 - 2(3)$
$y = 15 - 6$
$y = 9$ $\qquad$ $P(3, 9) \rightarrow$ Ans (a)

$15 - 2x = 0$
$15 = 2x$
$7.5 = x$ $\qquad$ $Q\left(\frac{15}{2}, 0\right) \rightarrow$ Ans (a)

b) Area of the shaded region = Area under curve from $O \rightarrow P$ + Area under line from $P \rightarrow Q$

$$\int_0^3 (x^2)\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^3$$

$$= \frac{3^3}{3}$$

$$= 9$$

$$\frac{(7.5-3)\times 9}{2}$$

$$= \frac{4.5 \times 9}{2}$$

$$= \frac{40.5}{2}$$

$$= 20.25$$

$$9 + 20.25 = \boxed{29\tfrac{1}{4}} \rightarrow \text{Ans (b)}$$

8. $y = 16 - x^2 \rightarrow UB$

$0 = 16 - x^2$

$x^2 = 16$

$x = \pm 4$

$y = 16 - 0$

$y = 16$

$P(4, 0)$ $\qquad Q(0, 16)$

$$m = \frac{-16}{4} = -4$$

$y = -4x + c$

$y = 16 - 4x \rightarrow LB$

Area under line from 0 to 4

$$\frac{4 \times 16}{2} = 32 \rightarrow A$$

Area btw. line and curve

$$\int_0^4 (4x - x^2)\,dx$$

$$= \left[2x^2 - \frac{x^3}{3} \right]_0^4$$

$$= 2(16) - \frac{64}{3}$$

$$= 32 - 21\frac{1}{3}$$

$$= 10\frac{2}{3}$$

Ratio $\rightarrow$ A : B

$32 : \frac{32}{3}$

$\boxed{3 : 1} \rightarrow$ Ans (8)

9. $y = \frac{1}{3}x^2 \rightarrow$ LB

$y^2 = 3x \longrightarrow$ UB

$y = \sqrt{3x}$

$y = (3x)^{\frac{1}{2}}$

$$\int_0^3 \left((3x)^{\frac{1}{2}} - \frac{x^2}{3} \right) dx$$

$$= \left[\frac{2\sqrt{3}(x)^{\frac{3}{2}}}{3} - \frac{x^3}{9} \right]_0^3$$

$$= \left[\frac{2\cancel{\sqrt{3}} \times x \times \cancel{\sqrt{3}}}{\cancel{3}} - 3 \right]$$

$$= [2x - 3]_0^3$$

$$= 6 - 3$$

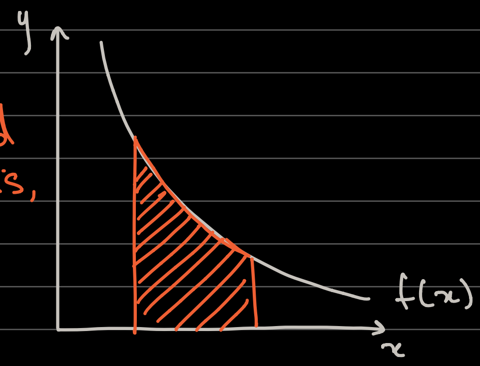
$\boxed{= 3} \longrightarrow$ Ans 9.

APPLICATION OF INTEGRATION IN VOLUMES

Slide 90

Example :

If the shaded region is rotated about the x-axis, what would the volume of the resulting solid be ?



For any prism, the volume is base area × height

If base area is continuously changing (like in the graph above), it needs to be accounted for and multiplied by an infinitesimally small height (dx)

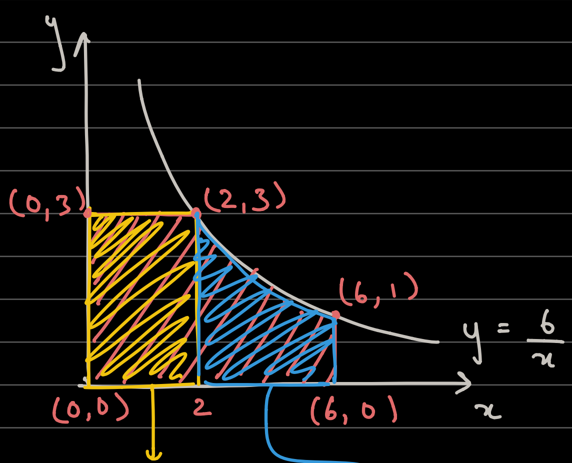
For the above example, at any value of x:

Base Area is given by → πr^2
$= \pi(f(x))^2$

and when multiplied by infinitesimally small height :

$$V = \int_a^b (\pi(f(x))^2)\, dx$$

Q1.



$\pi 3^2 \times 2$

$= 18\pi$

$$\int_2^6 \left(\pi\left(\frac{6}{x}\right)^2\right) dx$$

$$= \left[-\frac{36\pi}{x}\right]_2^6$$

$$= \frac{-36\pi}{6} - \left(-\frac{36\pi}{2}\right)$$

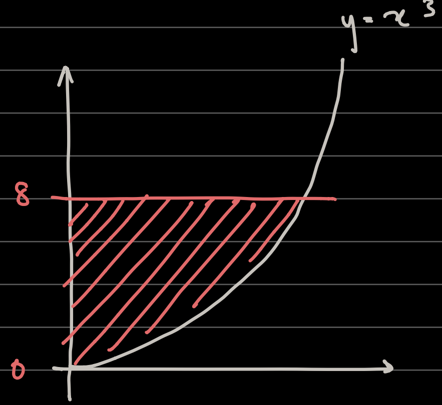
$$18\pi + 12\pi = 30\pi$$

Ans
Volume of the resulting solid

$$= -6\pi + 18\pi$$

$$= 12\pi$$

Q2



$$\int_0^8 \left(\pi (y^{\frac{1}{3}})^2\right) dy$$

$$= \left[\frac{3\pi y^{\frac{5}{3}}}{5}\right]_0^8$$

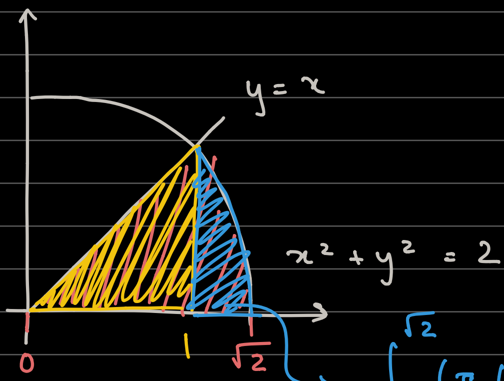
$$= \left[\frac{3\pi y \times (\sqrt[3]{y})^2}{5}\right]_0^8$$

$$= \frac{3\pi(8) \times (\sqrt[3]{8})^2}{5}$$

$$= \frac{24\pi \times 4}{5}$$

$$\boxed{= 19.2\pi} \longrightarrow \text{Ans}$$

Q3.



$$x^2 + x^2 = 2$$
$$2x^2 = 2$$
$$x^2 = 1$$
$$x = \pm 1$$

$$x^2 + y^2 = 2$$
$$y^2 = 2 - x^2$$

$$\int_1^{\sqrt{2}} \left(\pi(2 - x^2)\right) dx$$

$$= \pi \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}$$

$$= \pi \left[2\sqrt{2} - \frac{(\sqrt{2})^3}{3} - \left(2 - \frac{1}{3}\right)\right]$$

$$\int_0^1 (\pi x^2)\, dx$$

$$= \left[\frac{\pi x^3}{3}\right]_0^1$$

$= \frac{\pi}{3}$

$$= \pi \left[2^{\frac{3}{2}} - \frac{2^{\frac{3}{2}}}{3} - \frac{5}{3} \right]$$

$=$

$$\frac{\pi}{3} + \pi \left[\frac{2(2^{\frac{3}{2}})}{3} - \frac{5}{3} \right]$$

$$= \pi \left[\frac{1}{3} + \frac{2^{\frac{5}{2}}}{3} - \frac{5}{3} \right]$$

$$= \pi \left[\frac{(\sqrt{2})^5}{3} - \frac{4}{3} \right]$$

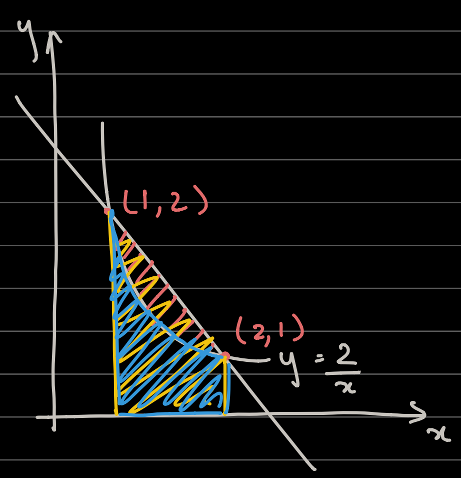
$$= \pi \left[\frac{(2^2)(\sqrt{2}) - 4}{3} \right]$$

$$= \pi \left[\frac{4\sqrt{2} - 4}{3} \right]$$

$$= \pi \left[\frac{4(\sqrt{2} - 1)}{3} \right]$$

$$= \frac{4(\sqrt{2} - 1)\pi}{3} \rightarrow \text{Ans 3.}$$

Q4.



$$m = \frac{1-2}{1} = -1$$

$$y = -x + c$$
$$2 = -1 + c$$
$$3 = c$$

$$y = 3 - x \rightarrow \text{line}$$

$$\int_1^2 (\pi (3-x)^2)\, dx$$

$$= \int_1^2 \pi (9 - 6x + x^2)\, dx$$

$$= \pi \left[9x - 3x^2 + \frac{x^3}{3} \right]_1^2$$

$$= \pi \left[18 - 12 + \frac{8}{3} - \left(9 - 3 + \frac{1}{3}\right) \right]$$

$$\int_1^2 \pi\left(\frac{2}{x}\right)^2 dx$$

$$= \int_1^2 \frac{4\pi}{x^2}\, dx$$

$$= \left[-\frac{4\pi}{x}\right]_1^2$$

$$= 4\pi\left[-\frac{1}{x}\right]_1^2$$

$$= 4\pi\left[-\frac{1}{2} - \left(-\frac{1}{1}\right)\right]$$

$$= 4\pi\left[\frac{1}{2}\right]$$

$$= 2\pi$$

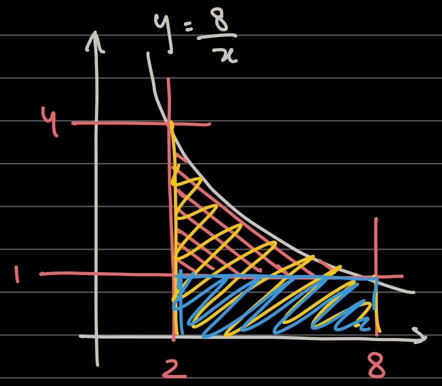
$$= \pi\left[8\frac{2}{3} - 6\frac{1}{3}\right]$$

$$= \pi\, 2\frac{1}{3}$$

$$= \frac{7\pi}{3}$$

$$\frac{7\pi}{3} - 2\pi = \frac{\pi}{3} \rightarrow \text{Ans}$$

Q10.



$$\pi 1^2 \times 6$$
$$= 6\pi$$

$$24\pi - 6\pi = 18\pi\ u^3$$

↳ Ans

$$\int_2^8 \pi\left(\frac{8}{x}\right)^2 dx$$

$$= \int_2^8 \frac{64\pi}{x^2}\, dx$$

$$= 64\pi\left[-\frac{1}{x}\right]_2^8$$

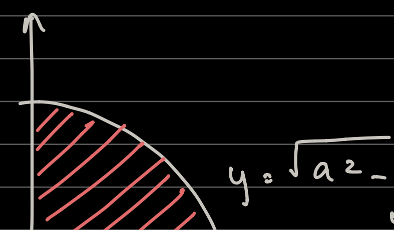
$$= 64\pi\left[-\frac{1}{8} - \left(-\frac{1}{2}\right)\right]$$

$$= 64\pi\left[-\frac{1}{8} + \frac{4}{8}\right]$$

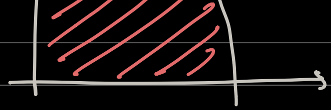
$$= 64\pi\left[\frac{3}{8}\right]$$

$$= 24\pi$$

Q11.



$$y^2 = a^2 - x^2$$



$$\int_0^a \pi(a^2 - x^2)\,dx$$

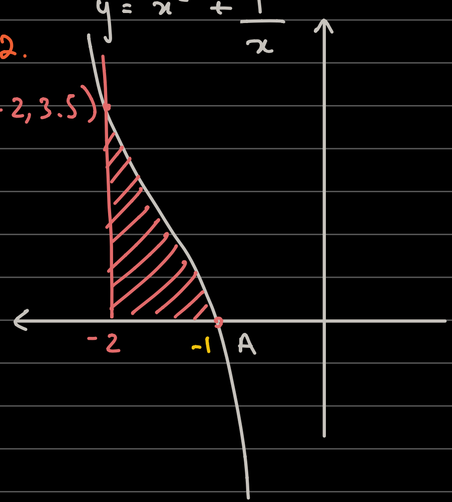
$$= \pi\left[a^2x - \frac{x^3}{3}\right]_0^a$$

$$= \pi\left[a^3 - \frac{a^3}{3}\right]$$

$$= \pi\left[\frac{2a^3}{3}\right]$$

$$= \frac{2a^3\pi}{3}$$

Q12.



$$0 = x^2 + \frac{1}{x}$$

$$0 = x\left(x + \frac{1}{x^2}\right)$$

$x = 0$ or $x + \frac{1}{x^2} = 0$

$$x = -\frac{1}{x^2}$$

$$x^3 = -1$$

$A(-1, 0)$ $\quad x = -1$

$$\int_{-2}^{-1} \left(\pi\left(x^2 + \frac{1}{x}\right)^2\right)dx$$

$$= \int_{-2}^{-1} \left(\pi\left(x^4 + 2x + \frac{1}{x^2}\right)\right)dx$$

$$= \pi\left[\frac{x^5}{5} + x^2 - \frac{1}{x}\right]_{-2}^{-1}$$

$$= \pi\left[\frac{-1}{5} + 1 + 1 - \left(-\frac{32}{5} + 4 + \frac{1}{2}\right)\right]$$

$$= \pi\left[\frac{9}{5} - (4 - 6.4 + 0.5)\right]$$

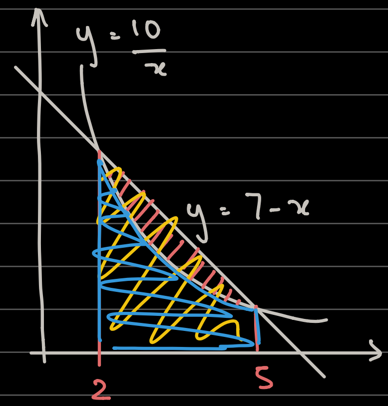
$= \pi\,(1.8 - (-1.9))$

$= \pi\,(1.8 + 1.9)$

$= 3.7\pi$

$\boxed{= 11.6 \text{ unit}^3} \rightarrow$ Ans

Q16.



$\int_2^5 \pi\left(\frac{10}{x}\right)^2 dx$

$= 100\pi\left[-\frac{1}{x}\right]_2^5$

$= 100\pi\left[-\frac{1}{5} - \left(\frac{-1}{2}\right)\right]$

$= 100\pi\left[-\frac{2}{10} + \frac{5}{10}\right]$

$= 100\pi\left(\frac{3}{10}\right)$

$= 30\pi$

$\int_2^5 \pi(7-x)^2\,dx$

$= \int_2^5 \pi(49 - 14x + x^2)\,dx$

$= \pi\left[49x - 7x^2 + \frac{x^3}{3}\right]_2^5$

$= \pi\left[49(5) - 7(25) + \frac{125}{3} - \left(49(2) - 7(4) + \frac{8}{3}\right)\right]$

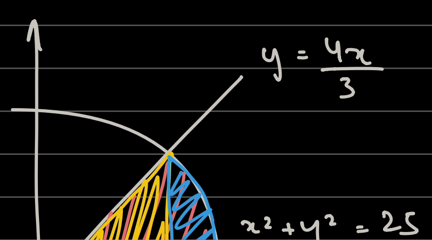
$= \pi\left[245 - 175 + 41\frac{2}{3} - \left(98 - 28 + 2\frac{2}{3}\right)\right]$

$= \pi\left[111\frac{2}{3} - 72\frac{2}{3}\right]$

$= 39\pi$

$39\pi - 30\pi = 9\pi \rightarrow$ Ans

Q19.



$$x^2 + \left(\frac{4x}{3}\right)^2 = 25$$

$$x^2 + \frac{16x^2}{9} = 25$$

$$9x^2 + 16x^2 = 225$$

$$25x^2 = 225$$

$$x^2 = 9$$

$$x = \pm 3$$

$$\int_0^3 \pi\left(\frac{4x}{3}\right)^2 dx$$

$$= \int_0^3 \frac{16\pi}{9} \cdot x^2 \, dx$$

$$= \frac{16\pi}{9}\left[\frac{x^3}{3}\right]_0^3$$

$$= 16\pi$$

$$\int_3^5 \pi(25 - x^2)\, dx$$

$$= \pi\left[25x - \frac{x^3}{3}\right]_3^5$$

$$= \pi\left[125 - \frac{125}{3} - (75 - 9)\right]$$

$$= \pi\left[\frac{250}{3} - 66\right]$$

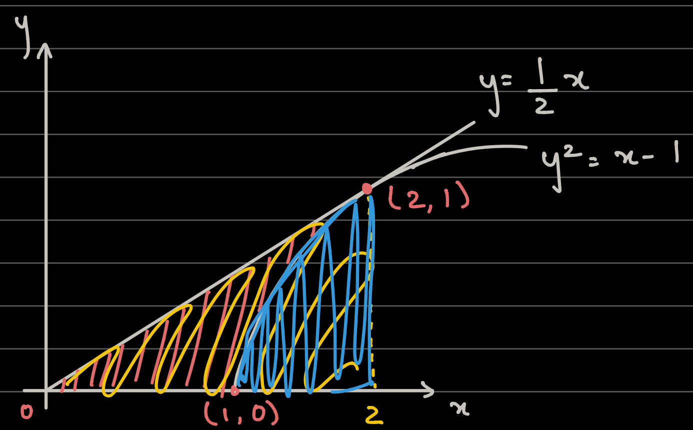
$$= \pi\left[\frac{250}{3} - \frac{198}{3}\right]$$

$$= \frac{52\pi}{3}$$

$$16\pi + 17\tfrac{1}{3}\pi = 33\tfrac{1}{3}\pi$$

Ans

Q23.



$$\int_0^2 \pi\left(\frac{x}{2}\right)^2 dx$$

$$= \frac{\pi}{4} \cdot \left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{\pi}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{3}$$

$$\int_1^2 \pi(x - 1)\, dx$$

$$= \pi\left[\frac{x^2}{2} - x\right]_1^2$$

$$= \pi\left[2 - 2 - \left(\frac{1}{2} - 1\right)\right]$$

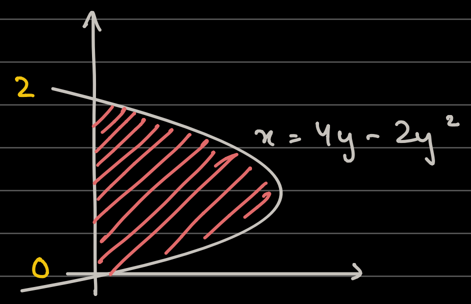
$= \frac{2\pi}{3}$

$= \pi\left[0 - \left(-\frac{1}{2}\right)\right]$

$= \frac{1}{2}\pi$

$\frac{4}{6}\pi - \frac{3}{6}\pi = \frac{\pi}{6} \rightarrow$ Ans

Q5.



$0 = 4y - 2y^2$

$0 = y(4 - 2y)$

$y = 0$ or $4 - 2y = 0$

$2 = y$

$$\int_0^2 \pi(4y - 2y^2)^2 \, dy$$

$$= \int_0^2 \pi(16y^2 - 16y^3 + 4y^4) \, dy$$

$$= \pi\left[\frac{16y^3}{3} - \frac{16y^4}{4} + \frac{4y^5}{5}\right]_0^2$$

$$= \pi\left[\frac{16(2)^3}{3} - \frac{16(2)^4}{4} + \frac{4(2)^5}{5}\right]$$

$$= \pi\left[42\tfrac{2}{3} - 64 + 25.6\right]$$

$$= \pi\left[67.6 + \frac{2}{3} - 64\right]$$

$$= \pi\left(3\tfrac{3}{5} + \frac{2}{3}\right)$$

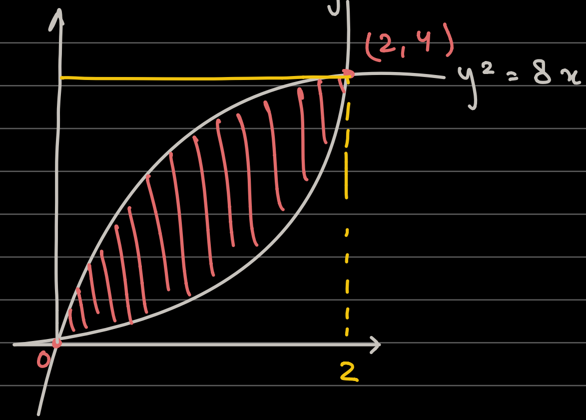
$$= \pi\left(3\tfrac{9}{15} + \frac{10}{15}\right)$$

$$= \pi\left(3\tfrac{19}{15}\right)$$

$$\boxed{= 4\tfrac{4}{15}\pi} \rightarrow \text{Ans}$$

$y = x^2$

Q7.



$$\int_0^4 \pi y \, dy$$

$$= \pi \left[\frac{y^2}{2}\right]_0^4$$

$$= \pi \frac{16}{2}$$

$$= 8\pi$$

$$\int_0^4 \pi \left(\frac{y^2}{8}\right)^2 dy$$

$$= \frac{\pi}{64}\left[\frac{y^5}{5}\right]_0^4$$

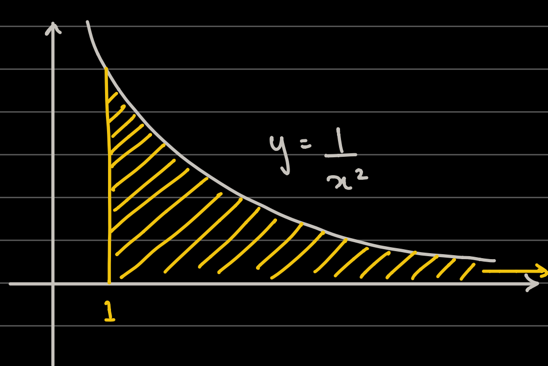
$$= \frac{\pi}{64}\left[\frac{1024}{5}\right]$$

$$= \frac{16\pi}{5}$$

$$8\pi - 3.2\pi$$

$$\boxed{= 4.8\pi} \longrightarrow \text{Ans}$$

IMPROPER INTEGRALS



→ area continues to accumulate to infinity because the graph never touches the x-axis

As such, the area under the graph becomes $\int_1^\infty \frac{1}{x^2} \, dx$

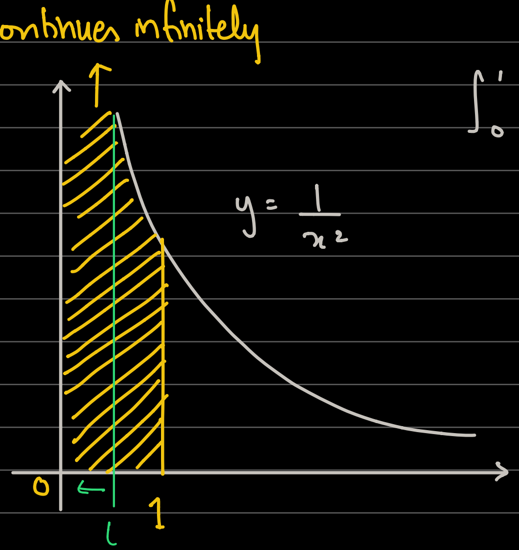
$$= \left[-\frac{1}{x} \right]_1^{\infty}$$

$$= -\frac{1}{\infty} - \left(-\frac{1}{1} \right)$$

$$= 1 - \frac{1}{\infty}$$ $\frac{1}{\infty}$ approaches 0

$$= 1 - 0$$

$= 1 \rightarrow$ the evaluated integral



$$\int_0^1 \frac{1}{x^2}\, dx$$

This is a different type of improper integral because it's not improper due to having ∞ as one of the limits, but instead due to there being a vertical asymptote / break in the graph at one of the limits

Replace the limit where the asymptote occurs with a variable l.

$$\int_l^1 \frac{1}{x^2}\, dx$$

$$= \left[-\frac{1}{x} \right]_l^1$$

$$= -\frac{1}{1} - \left(-\frac{1}{l} \right)$$

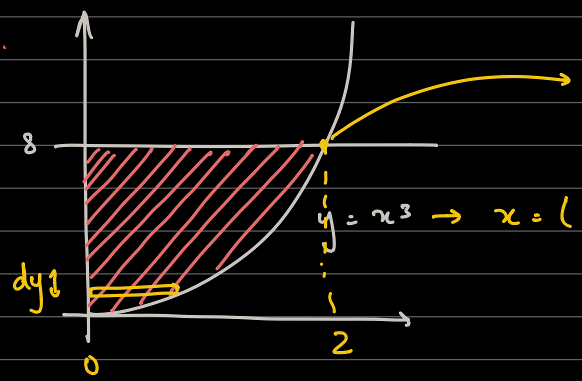
$$= \frac{1}{l} - 1$$ As $l \rightarrow 0$, $\frac{1}{l} \rightarrow \infty$

$$= \infty - 1$$

$= \infty \rightarrow$ No ans.

Classwork: Q ~~2~~, ~~13~~, 21, 22, 15, 29 from Worksheets NG Slide 90

2.



$8 = x^3$

$2 = x$

$y = x^3 \rightarrow x = (y)^{\frac{1}{3}}$

$$\int_0^8 \pi (y^{\frac{1}{3}})^2 \, dy$$

$$= \int_0^8 \pi y^{\frac{2}{3}} \, dy$$

$$= \pi \left[\frac{3y^{\frac{5}{3}}}{5} \right]_0^8$$

$$= \pi \left[\frac{3(\sqrt[3]{8})^5}{5} \right]$$

$$= \pi \left[\frac{3}{5} \cdot 32 \right]$$

$$= 19\frac{1}{5} \rightarrow \text{Ans}$$

13. curve: $x = y - y^2$

$0 = y - y^2$

$0 = (y)(1-y)$

$y = 0$ or $y = 1$

$$\int_0^1 \pi (y-y^2)^2 \, dy$$

$$= \int_0^1 \pi (y^2 - 2y^3 + y^4) \, dy$$

$$= \pi \left[\frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5} \right]_0^1$$

$$= \pi \left[\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right]$$

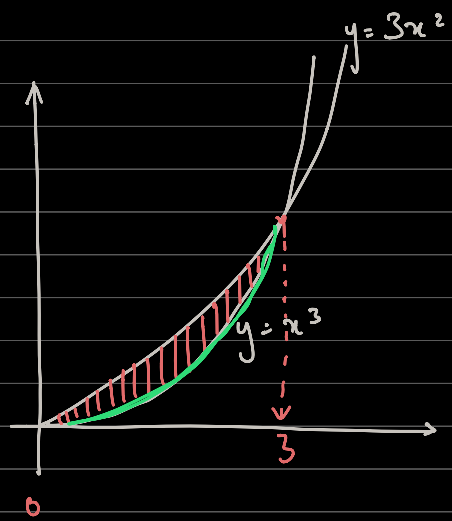
$$= \pi \left[\frac{20}{60} - \frac{30}{60} + \frac{12}{60} \right]$$

$$= \pi \left[\frac{2}{} \right]$$

60

$= \frac{\pi}{30}$ unit^3 $\longrightarrow$ Ans

46.



$3x^2 = x^3$

$3 = x$

$\int_0^3 \pi (3x^2)^2 \, dx$

$= \int_0^3 \pi \, 9x^4 \, dx$

$= 9\pi \int_0^3 x^4 \, dx$

$= 9\pi \left(\frac{x^5}{5} \right)_0^3$

$= 9\pi \left[\frac{3^5}{5} \right]$

$= 9\pi \left(\frac{243}{5} \right)$

$= 437.4\pi$

$\int_0^3 \pi (x^3)^2 \, dx$

$= \pi \int_0^3 x^6 \, dx$

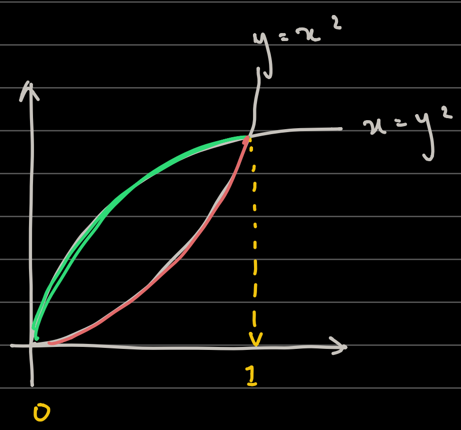
$= \pi \left[\frac{x^7}{7} \right]_0^3$

$= 312.4\pi$

$437.4\pi - 312.4\pi = 125\pi$

$= 392$ units^3 $\longrightarrow$ Ans

38.



$\int_0^1 \pi (x^{\frac{1}{2}})^2 \, dx$

$= \pi \int_0^1 x \, dx$

$= \pi \left(\frac{x^2}{2} \right]_0^1$

$= \frac{1}{2}\pi$

$\int_0^1 \pi (x^2)^2 \, dx$

$= \pi \int_0^1 x^4 \, dx$

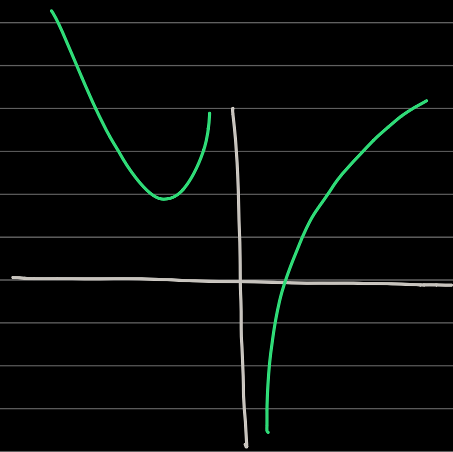
$= \pi \left[\frac{x^5}{5} \right]_0^1$

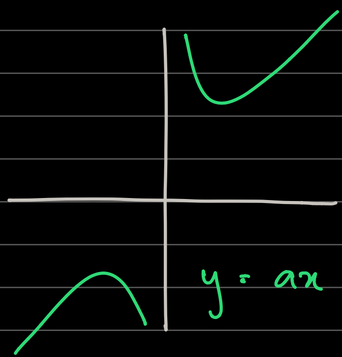
$= \frac{1}{5}\pi$

$\frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi}{10} - \frac{2\pi}{10} = \frac{3}{10}\pi \longrightarrow$ Ans

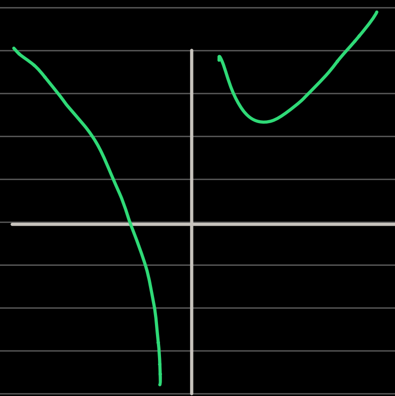
GRAPHS

$y = ax - \frac{b}{x}$





$y = ax + \frac{b}{x}$



29. $y = x^2 + \frac{16}{x}$

$$\int_1^2 \pi \left(x^2 + \frac{16}{x}\right)^2 dx$$

$$= \pi \int_1^2 \left(x^4 + 32x + \frac{16^2}{x^2}\right) dx$$

$$= \pi \int_1^2 \left(x^4 + 32x + \frac{256}{x^2}\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{32x^2}{2} - \frac{256}{x}\right]_1^2$$

$$= \pi \left[\frac{2^5}{5} + 16(4) - \frac{256}{2} - \left(\frac{1}{5} + 16 - 256\right)\right]$$

$$= \pi \left[\frac{32}{5} + 64 - 128 - \left(\frac{1}{5} - 240\right)\right.$$

$$= \pi \left[\frac{32}{5} - 64 - \frac{1}{5} + 240\right]$$

$$= \pi \left[\frac{31}{5} + 176\right]$$

$$= 182.2\pi \longrightarrow \text{Ans}$$