GEOMETRIC DISTRIBUTION

↳ The third type of distribution after normal and binomial

- Geometric distribution is a discrete distribution
- Random variable X is a discrete random variable
- Just like in binomial, the trials are independent and the probability of "success" is constant.

$$X \sim Geo(\underbrace{p}_{\text{the probability of success}})$$

and X = the number of trials needed to obtain the first successful outcome

Example: Eduardo calls his friend. $\underbrace{0.3}_{p\,=\,\text{probability of success}}$ chance the friend picks up.

$X \sim Geo(0.3)$ where X is the number of calls made until his friend answers phone call

Note: In geometric dist. $X \neq 0$ and the number of trials (theoretically) can be infinite (however, its practically unlikely).

ie. Probability that he answers on the second call

$$P(X=2) = 0.7 \times 0.3$$
$$= 0.21$$

... on the fifth call

$$P(X=5) = 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3$$
$$= 0.072$$

.... on the n^{th} call

$$P(X=n) = (0.7)^{n-1} \times 0.3$$

Hence, we can make a general formula for a geometrical distribution:

$$P(X=n) = (1-p)^{n-1} \times p$$

Note: However, if there's a limit to the number of trials, the situation will be a little different

Let's assume that Eduardo stops calling after the 3rd unreceived call.

Probability that friend picks up second call

$P(X=2) = 0.7 \times 0.3$
$= 0.21$

" " " " " third call

$P(X=3) = 0.7^2 \times 0.3$
$= 0.147$

" " " " " fourth call

$P(X=4) = 0$ because the fourth call doesn't exist

Probability that Eduardo doesn't talk to his friend.

All three calls unanswered : 0.7^3
$= 0.343$

Probability that Eduardo talks to his friend:

$1 - (0.7)^3$
$= 1 - 0.343$
$= 0.657$

or

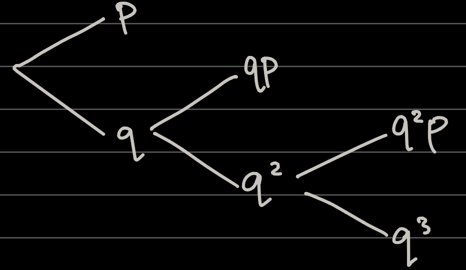
$P(1) + P(2) + P(3)$
$0.3 + (0.7 \times 0.3) + (0.7^2 \times 0.3)$
$= 0.3 + 0.21 + 0.147$
$= 0.657$

Note: In probability distributions of DRVs, the mode is the variable with the highest probability

↳ In geometric distributions, the mode is always $X=1$, that is, the first trial

In general terms:
* Limited to 3 trials here

p = success
q = failure



$p + qp + q^2p + q^3 = 1$

$p + qp + q^2p = 1 - q^3$

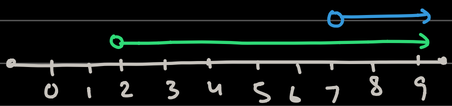
$P(X=1) + P(X=2) + P(X=3) = 1 - q^3$

$P(X \leq 3) = 1 - q^3$

For an r number of max trials ← $\boxed{P(X \leq r) = 1 - q^r}$

$$P(X > r) = 1 - P(X \leq r)$$
$$= 1 - (1 - q^r)$$
$$\boxed{P(X>r) = q^r}$$

↳ Probability that there is not a single success



lets say $a = 2$
$b = 5$
$a+b = 7$

$X \sim Geo(p)$

$$P(X > a+b \mid X > a) \longrightarrow \frac{P(X > a+b \cap X > a)}{P(X > a)}$$

"|" means "given"

this is the same as

$$\frac{P(X > a+b)}{P(X > a)}$$

$$= \frac{q^{a+b}}{q^a} = q^b = P(X > b)$$

Hence, we can say that:

$$\boxed{P(X > a+b \mid X > a) = P(X > b) = q^b}$$

↳ Remember this

As a summary, you should remember the following:

$$P(X \leq n) = 1 - q^n$$

$$P(X > n) = q^n$$

$$P(X > a+b \mid X > a) = P(X > b) = q^b$$

PRACTICE QUESTIONS

Q. Jack is playing a board game in which he needs to throw a 6. Find the probability that ...

a) 4 attempts are needed to obtain a 6

$p = \frac{1}{6} \quad q = \frac{5}{6}$ $\qquad X \sim Geo\left(\frac{1}{6}\right)$

$P(X=4) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6}$

$= 0.0965 \rightarrow \underline{\underline{Ans}}$

b) at least two attempts are needed

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X = 1)$

$= 1 - \left(\frac{1}{6}\right)$

$= \frac{5}{6} \rightarrow \underline{\underline{Ans}}$

c) he throws a 6 in ≤ 3 attempts

$P(X \leq 3) = 1 - P(X > 3)$ $\dashrightarrow$ remember : $P(X > r) = q^r$

$= 1 - \left(\frac{5}{6}\right)^3$

$= 0.421 \rightarrow \underline{\underline{Ans}}$

d) he needs > 3 attempts

$P(X > 3) = q^3$

$= \left(\frac{5}{6}\right)^3$