Doing differentiation questions:
↳ to find the exact gradient at a point on a curve

Example: curve → $y = 3x^2 - 6x - 4$, find gradient of the tangent at $x = 3$

Step 1 → Differentiate

$$\boxed{3}x^{2} - \boxed{6}x^{1} - 4x^{0} \qquad {}^{2-1} \quad {}^{1-1}$$

Multiply the power of the variable with it's respective co-efficient then subtract 1 from the power.

Therefore: $\left.\dfrac{dy}{dx}\right| = 6x^1 - 6x^0 - 0$
$= 6x - 6$

Step 2 → Input the x value into $\frac{dy}{dx}$ to find the gradient of the tangent at that point on the curve.

$x = 3$
$= 6(3) - 6$
$= 18 - 6$
$= 12$ → gradient of tangent at $x = 3$.

Step 3 → Find corresponding y value of given x value to find a point through which the tangent passes, so that it's equation can be formed

$x = 3$
$y = 3x^2 - 6x + 4$
$= 3(3^2) - 6(3) + 4$
$= 3(9) - 18 + 4$
$= 27 - 14$
$= 13$

$(3, 13)$

Step 4 → Use the point and the gradient of the tangent to find it's equation

point → $(3, 13)$, slope → (12)

$y = 12x + c$
$13 = 12(3) + c$
$13 = 36 + c$
$-23 = c$

∴, the equation of the tangent → $y = 12x - 23$

Another Example:

$y = x^2 - x\sqrt{x} + \dfrac{8}{x^2}$, Find the equation of the tangent at $x = 3$

$y = x^2 - x^1 \cdot x^{\frac{1}{2}} + 8x^{-2}$

* $\frac{dy}{dx}$ is the gradient function, plug in any value of x and you get

$= \boxed{1}x^{2} \boxed{-1}x^{\frac{3}{2}} + \boxed{8}x^{-2}$

the gradient of the tangent at that point on the curve.

$$\frac{dy}{dx} = 2x - \frac{3}{2}x^{\frac{1}{2}} - 16x^{-3}$$

for $x = 3$ $\longrightarrow$ $\left.\frac{dy}{dx}\right|_{x=3} = 2x - \frac{3}{2}x^{\frac{1}{2}} - 16x^{-3}$ $= 2(3) - \frac{3}{2}\sqrt{3} - 16\left(\frac{1}{3^3}\right)$

↳ Bla bla you get the point
↳ crunch the numbers later

How and when to use the chain rule:

- Chain rule is used in problems where the equation of the curve looks something like

$$y = (x^2 + 2)^5$$, ie. a whole raised to a power.

Chain rule is essentially a shortcut instead of needing to expand the entire thing

How it works.

1. Take the whole power and multiply by the coefficient. This is the new coefficient
2. Subtract 1 from the whole power. This is the new whole power
3. Differentiate whatever was inside the brackets, and write the result in the rightmost position as a multiplied value.

Example:

$$y = \boxed{1}(\underbrace{1x^{2} + 2}_{\frac{dy}{dx} = 2x})^{5}$$

$\times$ -1

Therefore,

$$\frac{dy}{dx} = 5(x^2 + 2)^4(2x)$$ → now this is the gradient function. Input any value of x to get the value of the gradient of the tangent at that point on the curve.

HOMEWORK ⟶ Ex. 15.1 Q1-7 , Ex. 15.2 Q1-4

Exercise 15.1

1. Differentiate the following with respect to x (where a and b are constants).

(a) $3x^2 + 4x - 1$ (b) $x^4 - 7x^2 + 6x$

(c) $2x^3 + 5x^2 - 4x + 9$ (d) $4x + \frac{2}{x}$

(e) $9x^2 - \frac{3}{x^2}$ (f) $\frac{6}{x^3} - \frac{1}{x} + 3$

(g) $3a + bx^2$ (h) $5x^2 + \frac{4}{x} - 2$

(i) $3x + 2\sqrt{x} - 3$ (j) $8x^2 + 3x - \sqrt{x}$

(k) $2x^{\frac{5}{2}} - 4x^{\frac{3}{2}} - 6x + 8$ (l) $6x\sqrt{x} - 6\sqrt{x}$

(m) $4x^2\sqrt{x} - \frac{6}{\sqrt{x}}$ (n) $ax - \frac{b}{x}$

2. Differentiate the following with respect to x.

(a) $\frac{2x^2 + 4x}{x}$ (b) $\frac{x^2 - 6x + 4}{x}$ (c) $\frac{4x^3 - 5x - 3}{2x}$

(d) $\frac{x^2 + 4}{2x^2}$ (e) $\frac{3x^2 + x - 1}{\sqrt{x}}$ (f) $\frac{6x^2 - \sqrt{x} + 2}{2x}$

3. Find $\frac{dy}{dx}$ for the following functions of x.

(a) $(x + 1)(2x - 1)$ (b) $x(\sqrt{x} - 2)$ (c) $(1 + \sqrt{x})(1 - \sqrt{x})$

(d) $4x^2(3 - \sqrt{x})$ (e) $\frac{(2x + 1)(x - 2)}{x}$ (f) $\frac{(1 - x)(4x - 1)}{\sqrt{x}}$

4. Find the value of $f'(x)$ at the given value of x.

(a) $f(x) = 3x^2 - 2x - 4,\ x = 2$ (b) $f(x) = 6x - \frac{3}{x},\ x = -1$

(c) $f(x) = 3x - 4\sqrt{x},\ x = 4$ (d) $f(x) = (x - 4)(x + 3),\ x = 3$

5. Calculate the gradient of the tangent to the curve at the given point.
   (a) $y = 4x^2 - 6x + 1$, $(2, 5)$
   (b) $y = \frac{6 - 4x}{x}$, $x = -2$
   (c) $y = \sqrt{x}(2 - x)$, $x = 9$
   (d) $y = \frac{(x + 1)(2x - 3)}{x}$, $x = -1$

6. Calculate the gradient(s) of the curve at the point(s) where y is given.
   (a) $y = x^2 - 2x$, $y = -1$
   (b) $y = 2x^2 + 3x$, $y = 2$
   (c) $y = \frac{x - 9}{x}$, $y = 4$
   (d) $y = \frac{x^2 + 4}{x^2}$, $y = 5$

7. Calculate the gradient(s) of the curve at the point(s) where it crosses the given line.
   (a) $y = 2x^2 - 5x + 1$, y-axis
   (b) $y = \frac{x - 4}{x}$, x-axis
   (c) $y = 2x^2 - 8$, x-axis
   (d) $y = \frac{x + 2}{x}$, $y = x$

8. Find the coordinates of the point on the curve $y = x^3 - 3x^2 + 6x + 2$ at which the gradient is 3.

9. The curve $y = ax^2 + \frac{b}{x}$ has gradients 2 and -1 at $x = 1$ and $x = 4$ respectively. Find the value of a and of b.

10. The gradient of the tangent to the curve $y = ax^3 + bx$ at the point $(2, -4)$ is 6. Calculate the values of the constants a and b.

11. Given that the gradient of the curve $y = \frac{a}{x} + bx^2$ at the point $P(3, -15)$ is -13. Find the value of a and of b. Show that the tangent to the curve at the point where $x = 1$ has the same gradient as that at P.

12. The tangent to the curve $y = \frac{a}{x} + bx$ at $(1, 3)$ is parallel to the line $y = 2x + 1$. Calculate the value of a and of b.

13. Given that $f(x) = px^3 + qx$ and that $f(2) = -2$ and $f'(2) = 3$, calculate the value of p and of q.

14. The equation of a curve is $y = 9x + \frac{1}{x}$. Find
    (a) the gradient of the curve where $x = 2$,
    (b) the coordinates of the points where the tangent is horizontal.

15. Given the curve $y = x^3 - 3x^2 - 9x + 11$, find $\frac{dy}{dx}$. Hence obtain
    (a) the x-coordinates of the points where the gradient is 15,
    (b) the coordinates of the points where the gradient is zero.

16. The curve $y = x^2 + 2x$ has gradient 3 at the point (a, b). Find the value of a and of b.

17. The curve $y = ax + \frac{3}{x}$ has gradient 1 at $x = 2$. Find the value of a and the x-coordinate of another point at which the gradient is 1.

Exercise 15.2

1. Differentiate the following w.r.t. x.
   (a) $(x+2)^5$ (b) $(2x-1)^4$ (c) $(\frac{1}{4}x+2)^5$
   (d) $(1-4x)^{10}$ (e) $(2-3x^2)^4$ (f) $(1-x+x^2)^3$

2. Differentiate the following w.r.t. x.
   (a) $\frac{3}{(3-4x)^3}$ (b) $\frac{4}{(2x+7)}$ (c) $\frac{6}{(2-x)^2}$ (d) $\frac{2}{(6x^2+5)}$

3. Differentiate the following w.r.t. x.
   (a) $\sqrt{2x-3}$ (b) $\sqrt{6-2x}$ (c) $\sqrt{x^2-2}$
   (d) $\sqrt{5-3x^2}$ (e) $\sqrt{x^2-x+1}$ (f) $\sqrt{x^2+2x+2}$

4. Differentiate the following w.r.t. x
   (a) $(2-\sqrt{x})^6$ (b) $\frac{1}{\left(1-\frac{1}{x}\right)^3}$ (c) $\frac{1}{2(3x-2)^2}$
   (d) $2\left(\sqrt{x}+2\right)^{\frac{1}{2}}$ (e) $\left(x-\frac{1}{x}\right)^3$ (f) $\left(\sqrt{x}+2x\right)^4$

5. Find $\frac{dy}{dx}$ and the gradient of the curve at the given value of x.
   (a) $y=(3x-1)^4, x=1$ (b) $y=\sqrt{5-2x}, x=\frac{1}{2}$
   (c) $y=\frac{1}{2x-3}, y=1$ (d) $y=(4x-5)^3, y=27$

6. Calculate the coordinates of the point on the curve $y=(1-x)^4$ at which the gradient is -4.

7. Calculate the coordinates of the point on the curve $y=\sqrt{x^2-2x+5}$ at which $\frac{dy}{dx}=0$.

8. The curve $y=(a-x)^3$ has gradient $-\frac{1}{3}$ at $x=2$. Find the possible values of a.

9. Find $\frac{dy}{dx}$ and calculate the gradient of the tangent to the curve
   (a) $y=(x^2-2x-4)^3$ at the point where $x=-1$.
   (b) $y=\frac{1}{\sqrt{1+x}}$ at the point where $x=3$.

*10. Given that $f(x)=\sqrt{1+\sqrt{x}}$, where $x \geqslant 0$, show that $f'(x)=\frac{1}{4\sqrt{x+x\sqrt{x}}}$.

Exercise 15.2 (ANSWERS)

1. (a) $5(x+2)^4$ (b) $8(2x-1)^3$ (c) $\frac{5}{4}\left(\frac{1}{4}x+2\right)^4$
   (d) $-40(1-4x)^9$ (e) $-24x(2-3x^2)^3$ (f) $3(2x-1)(1-x+x^2)^2$

2. (a) $\frac{36}{(3-4x)^4}$ (b) $-\frac{8}{(2x+7)^2}$ (c) $\frac{12}{(2-x)^3}$ (d) $\frac{-24x}{(6x^2+5)^2}$

3. (a) $\frac{1}{\sqrt{2x-3}}$ (b) $-\frac{1}{\sqrt{6-2x}}$ (c) $\frac{x}{\sqrt{x^2-2}}$
   (d) $-\frac{3x}{\sqrt{5-3x^2}}$ (e) $\frac{2x-1}{2\sqrt{x^2-x+1}}$ (f) $\frac{x+1}{\sqrt{x^2+2x+2}}$

4. (a) $-\frac{3(2-\sqrt{x})^5}{\sqrt{x}}$ (b) $\frac{-3}{x^2\left(1-\frac{1}{x}\right)^4}$ (c) $-\frac{3}{(3x-2)^3}$
   (d) $\frac{1}{2\sqrt{x}\left(\sqrt{x}+2\right)}$ (e) $3\left(x-\frac{1}{x}\right)^2\left(1+\frac{1}{x^2}\right)$ (f) $4\left(\sqrt{x}+2x\right)^3\left(\frac{1}{2\sqrt{x}}+2\right)$

5. (a) 96 (b) $-\frac{1}{2}$ (c) -2 (d) 108

6. (0, 1) 7. (1, 2) 8. $a=\frac{7}{3}$ or $\frac{5}{3}$ 9. (a) -12 (b) $-\frac{1}{16}$

15 Exercise 15.1

1. (a) $6x + 4$ (b) $4x^3 - 14x + 6$ (c) $6x^2 + 10x - 4$ (d) $4 - \frac{2}{x^2}$

   (e) $18x + \frac{6}{x^3}$ (f) $-\frac{18}{x^4} + \frac{1}{x^2}$ (g) $2bx$ (h) $10x - \frac{4}{x^2}$

   (i) $3 + \frac{1}{\sqrt{x}}$ (j) $16x + 3 - \frac{1}{2\sqrt{x}}$ (k) $5x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 6$

   (l) $9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ (m) $10x^{\frac{3}{2}} + 3x^{-\frac{3}{2}}$ (n) $a + \frac{b}{x^2}$

2. (a) 2 (b) $1 - \frac{4}{x^2}$ (c) $4x + \frac{3}{2x^2}$ (d) $-\frac{4}{x^3}$

   (e) $\frac{9}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ (f) $3 + \frac{1}{4}x^{-\frac{3}{2}} - x^{-2}$

3. (a) $4x + 1$ (b) $\frac{3}{2}\sqrt{x} - 2$ (c) -1 (d) $24x - 10x^{\frac{2}{3}}$

   (e) $2 + \frac{2}{x^2}$ (f) $-6x^{\frac{1}{2}} + \frac{5}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

4. (a) 10 (b) 9 (c) 2 (d) 5

5. (a) 10 (b) $-1\frac{1}{2}$ (c) $-4\frac{1}{6}$ (d) 5

6. (a) 0 (b) $-5, 5$ (c) 1 (d) $-8, 8$

7. (a) -5 (b) $\frac{1}{4}$ (c) $-8, 8$ (d) $-2, -\frac{1}{2}$

8. $(1, 6)$

9. $a = -\frac{1}{7}, b = -\frac{16}{7}$

10. $a = 1, b = -6$

11. $a = 9, b = -2$

12. $a = \frac{1}{2}, b = \frac{5}{2}$

13. $p = 2, q = -5$

14. (a) $8\frac{3}{4}$ (b) $\left(-\frac{1}{3}, -6\right), \left(\frac{1}{3}, 6\right)$

15. (a) $-2, 4$ (b) $(-1, 16), (3, -16)$

16. $a = \frac{1}{2}, b = \frac{5}{4}$

17. $a = \frac{7}{4}; -2$

Exercise 15.1 Q1.

1. Differentiate with respect to x (a and b are constants)

a) $3x^2 + 4x - 1$

$\frac{dy}{dx} = 6x + 4$

b) $x^4 - 7x^2 + 6x$

$\frac{dy}{dx} = 4x^3 - 14x + 6$

c) $2x^3 + 5x^2 - 4x + 9$

$\frac{dy}{dx} = 6x^2 + 10x - 4$

d) $4x + \frac{2}{x}$

$= 4x + 2x^{-1}$

$\frac{dy}{dx} = 4 - 2x^{-2}$

$= 4 - \frac{2}{x^2}$

e) $9x^2 - \frac{3}{x^2}$

$= 9x^2 - 3x^{-2}$

$\frac{dy}{dx} = 18x + 6x^{-3}$

$= 18x + \frac{6}{x^3}$

f) $\frac{6}{x^3} - \frac{1}{x} + 3$

$= 6x^{-3} - 1x^{-1} + 3$

$\frac{dy}{dx} = -18x^{-4} + 1x^{-2}$

$= \frac{1}{x^2} - \frac{18}{x^4}$

g) $3a + bx^2$

$\frac{dy}{dx} = 2bx$

h) $5x^2 + \frac{4}{x} - 2$

$= 5x^2 + 4x^{-1} - 2$

$\frac{dy}{dx} = 10x - 4x^{-2}$

$= 10x - \frac{4}{x^2}$

i) $3x + 2\sqrt{x} - 3$

$= 3x + 2x^{\frac{1}{2}} - 3$

$\frac{dy}{dx} = 3 + 1x^{-\frac{1}{2}}$

$= 3 + \frac{1}{\sqrt{x}}$

j) $8x^2 + 3x - \sqrt{x}$

$= 8x^2 + 3x - 1x^{\frac{1}{2}}$

$\frac{dy}{dx} = 8x + 3 - \frac{1}{2}x^{-\frac{1}{2}}$

$= 8x + 3 - \frac{1}{2\sqrt{x}}$

k) $2x^{\frac{5}{2}} - 4x^{\frac{3}{2}} - 6x + 8$

$\frac{dy}{dx} = 5x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 6$

l) $6x\sqrt{x} - 6\sqrt{x}$

$= 6x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$

$\frac{dy}{dx} = 9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$

$= 9\sqrt{x} - \frac{3}{\sqrt{x}}$

m) $4x^2\sqrt{x} - \frac{6}{\sqrt{x}}$

$= 4x^{\frac{5}{2}} - 6x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 10x^{\frac{3}{2}} + 3x^{-\frac{3}{2}}$

$= 10x^{\frac{3}{2}} + \frac{3}{x^{\frac{3}{2}}}$

n) $ax - \frac{b}{x}$

$= ax - bx^{-1}$

$\frac{dy}{dx} = a + bx^{-2}$

$= a + \frac{b}{x^2}$

Q2. Differentiate with respect to x.

a) $\dfrac{2x^2+4x}{x}$

$= \dfrac{2x^2}{x} + \dfrac{4x}{x}$

$= 2x + 4$

$\dfrac{dy}{dx} = 2$

b) $\dfrac{x^2-6x+4}{x}$

$= \dfrac{x^2}{x} - \dfrac{6x}{x} + \dfrac{4}{x}$

$= x - 6 + 4x^{-1}$

$\dfrac{dy}{dx} = 1 - 4x^{-2}$

$= 1 - \dfrac{4}{x^2}$

c) $\dfrac{4x^3-5x-3}{2x}$

$= 2x^2 - \dfrac{5}{2} - \dfrac{3}{2x}$

$= 2x^2 - \dfrac{5}{2} - \dfrac{3}{2}x^{-1}$

$\dfrac{dy}{dx} = 4x + \dfrac{3}{2}x^{-2}$

$= 4x + \dfrac{3}{2x^2}$

d) $\dfrac{x^2+4}{2x^2}$

$= \dfrac{1}{2} + 2x^{-2}$

$\dfrac{dy}{dx} = -4x^{-3}$

$= \dfrac{-4}{x^3}$

e) $\dfrac{3x^2+x-1}{\sqrt{x}}$

$= 3x^2x^{-\frac{1}{2}} + x.x^{-\frac{1}{2}} - 1x^{-\frac{1}{2}}$

$= 3x^{\frac{3}{2}} + x^{\frac{1}{2}} - 1x^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{9}{2}x^{\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{3}{2}}$

$= \dfrac{9}{2}\sqrt{x} + \dfrac{1}{2\sqrt{x}} + \dfrac{1}{6\sqrt{x^3}}$

f) $\dfrac{6x^2-\sqrt{x}+2}{2x}$

$= \dfrac{6x^2}{2x} - \dfrac{\sqrt{x}}{2x} + \dfrac{2}{2x}$

$= 3x - \dfrac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

$\dfrac{dy}{dx} = 3 + \dfrac{1}{4}x^{-\frac{3}{2}} - 1x^{-2}$

$= 3 + \dfrac{1}{4\sqrt{x^3}} - \dfrac{1}{x^2}$

Q3. Find $\dfrac{dy}{dx}$ for the following functions of x.

a) $(x+1)(2x-1)$

$= 2x^2 - x + 2x - 1$

$= 2x^2 + x - 1$

$\dfrac{dy}{dx} = 4x + 1$

b) $x(\sqrt{x}-2)$

$= x \cdot x^{\frac{1}{2}} - 2x$

$= x^{\frac{3}{2}} - 2x$

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} - 2$

c) $(1+\sqrt{x})(1-\sqrt{x})$

$= 1 - x$

$\dfrac{dy}{dx} = -1$

d) $4x^2(3-\sqrt{x})$

$= 12x^2 - 4x^{\frac{5}{2}}$

$\frac{dy}{dx} = 24x - 10x^{\frac{3}{2}}$

e) $\frac{(2x+1)(x-2)}{x}$

$= \frac{2x^2 - 4x + x - 2}{x}$

$= 2x - 3 - 2x^{-1}$

$\frac{dy}{dx} = 2 + 2x^{-2}$

$= 2 + \frac{2}{x^2}$

f) $\frac{(1-x)(4x-1)}{\sqrt{x}}$

$= \frac{4x + 1 - 4x^2 + x}{\sqrt{x}}$

$= 5x^1 \cdot x^{-\frac{1}{2}} + 1x^{-\frac{1}{2}} - 4x^2 \cdot x^{-\frac{1}{2}}$

$= 5x^{\frac{1}{2}} + 1x^{-\frac{1}{2}} - 4x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{5}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} - 6x^{\frac{1}{2}}$

$= \frac{5}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}} - 6\sqrt{x}$

Q4. Find the value of $f'(x)$ at the given value of x.

a) $f(x) = 3x^2 - 2x - 4, \quad x = 2$

$\left.\frac{dy}{dx}\right|_{x=2} = 6x - 2 = 6(2) - 2$

$= 12 - 2$

$= 10 \longrightarrow$ Ans.

b) $f(x) = 6x - \frac{3}{x}, \quad x = -1$

$= 6x - 3x^{-1}$

$\left.\frac{dy}{dx}\right|_{x=-1} = 6 + 3x^{-2} = 6 + \frac{3}{(-1)^2}$

$= 9 \rightarrow$ Ans.

c) $f(x) = 3x - 4\sqrt{x}, \quad x = 4$

$= 3x - 4x^{\frac{1}{2}}$

$\left.\frac{dy}{dx}\right|_{x=4} = 3 - 2x^{-\frac{1}{2}} = 3 - \frac{2}{\sqrt{4}}$

$= 2$

d) $f(x) = (x-4)(x+3), \quad x = 3$

$= x^2 - x - 12$

$\left.\frac{dy}{dx}\right|_{x=3} = 2x - 1 = 2(3) - 1$

$= 6 - 1$

$= 5 \rightarrow$ Ans.

Q5. Calculate the gradient of the tangent to the curve at the given point.

a) $y = 4x^2 - 6x + 1, \quad (2, 5)$

$\left.\frac{dy}{dx}\right|_{x=2} = 8x - 6 = 8(2) - 6$

$= 16 - 16$

$= 10 \rightarrow$ grad.

b) $y = \frac{6 - 4x}{x}, \quad x = -2$

$= 6x^{-1} - 4$

$\left.\frac{dy}{dx}\right|_{x=-2} = -6x^{-2} = \frac{-6}{x^2} = \frac{-6}{(-2)^2}$

$= \frac{-6}{4}$

$= -\frac{3}{2}$

c) $y = \sqrt{x}(2 - x), \quad x = 9$

$= 2x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$\left.\frac{dy}{dx}\right| = x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{1}{\sqrt{9}} - \frac{3}{2}\sqrt{9}$

$= 1 - 9$

$x = 9$

$= -\frac{25}{6}$

d) $y = \frac{(x+1)(2x-3)}{x}$, $x = -1$

$= \frac{2x^2 - 3x + 2x - 3}{x}$

$= 2x - 1 - 3x^{-1}$

$\left.\frac{dy}{dx}\right|_{x=-1} = 2 + 3x^{-2} = 2 + \frac{3}{(-1)^2}$

$= 2 + 3$

$= 5 \rightarrow$ Ans.

Q6. Calculate the gradient(s) of the curve at the point(s) where y is given

a) $y = x^2 - 2x$, $y = -1$

$-1 = x^2 - 2x$

$0 = x^2 - 2x + 1$

$= x^2 - x - x + 1$

$= x(x-1) - 1(x-1)$

$= (x-1)(x-1)$

$= x = 1$

$\left.\frac{dy}{dx}\right|_{x=1} = 2x - 2 = 2(1) - 2$

$= 2 - 2$

$= 0 \rightarrow$ Ans.

b) $y = 2x^2 + 3x$, $y = 2$

$2 = 2x^2 + 3x$

$0 = 2x^2 + 3x - 2$

$= 2x^2 + 4x - x - 2$

$= 2x(x+2) - 1(x+2)$

$= (2x-1)(x+2)$

$x = \frac{1}{2}$ or $x = -2$

$\frac{dy}{dx} = 4x + 3$

$x = \frac{1}{2}$: $= 4(\frac{1}{2}) + 3$

$= 5 \rightarrow$ Ans.

$x = -2$: $= 4(-2) + 3$

$= -5 \rightarrow$ Ans.

c) $y = \frac{x-9}{x}$, $y = 4$

$4 = \frac{x-9}{x}$

$4x = x - 9$

$3x = -9$

$x = -3$

$\frac{dx}{dy} = 9x^{-2}$

$= \frac{9}{x^2}$

$x = -3 \rightarrow \frac{9}{(-3)^2}$

$= \frac{9}{9}$

$= 1 \rightarrow$ Ans.

d) $\frac{x^2+4}{x^2}$, $y = 5$

$5 = \frac{x^2+4}{x^2}$

$5x^2 = x^2 + 4$

$0 = 4x^2 - 4$

$= 4x^2 - 4x + 4x - 4$

$= 4x(x-1) + 4(x-1)$

$= (4x+4)(x-1)$

$x = -1$ or $x = 1$

$\left.\frac{dy}{dx}\right|_{x=-1} = -8x^{-3} = \frac{-8}{x^3} = \frac{-8}{(-1)^3} = 8 \rightarrow$ Ans.

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{-8}{(1)^3} = -8 \rightarrow$ Ans

Q7. Calculate the gradient(s) of the curve at the point(s) where it crosses the given line

a) $y = 2x^2 - 5x + 1$, y-axis

↳ $x = 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 4x - 5 = 4(0) - 5$

$= -5 \longrightarrow$ Ans.

b) $y = \frac{x-4}{x}$, x-axis

↳ $y = 0$

$0 = \frac{x-4}{x}$

$4 = x$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{4}{x^2} = \frac{4}{4^2}$

$= \frac{4}{16}$

$= \frac{1}{4} \rightarrow$ Ans.

c) $y = 2x^2 - 8$, x-axis

↳ $y = 0$

$0 = 2x^2 - 8$

$= 2x^2 + 4x - 4x - 8$

$= 2x(x+2) - 4(x+2)$

$= (2x-4)(x+2)$

$x = 2$ or $x = -2$

$\left.\frac{dy}{dx}\right|_{x=2} = 4x = 4(2)$

$= 8 \rightarrow$ Ans.

$\left.\frac{dy}{dx}\right|_{x=-2} = 4(-2)$

$= -8 \rightarrow$ Ans.

d) $y = \frac{x+2}{x}$, $y = x$

$x = \frac{x+2}{x}$

$x^2 = x + 2$

$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{-2}{x^2} = \frac{-2}{(-1)^2}$

$= -2 \rightarrow$ Ans.

$0 = x^2 - x - 2$
$= x^2 - 2x + x - 2$
$= x(x-2) + 1(x-2)$
$= (x+1)(x-2)$
$x = -1$ or $x = 2$

$x = 2$

$= \frac{-2}{(2)^2}$
$= -\frac{1}{2} \rightarrow$ Ans.

Exercise 15.2 — Chain Rule. — Q1 – 4

Q1. Differentiate the following with respect to x.

a) $(x+2)^5$

$\frac{dy}{dx} = 5(x+2)^4(1)$
$= 5(x+2)^4 \rightarrow$ Ans.

b) $(2x-1)^4$

$\frac{dy}{dx} = 4(2x-1)^3(2)$

c) $(\frac{1}{4}x+2)^5$

$\frac{dy}{dx} = 5(\frac{1}{4}x+2)^4(\frac{1}{4})$

d) $(1-4x)^{10}$

$\frac{dy}{dx} = 10(1-4x)^9(-4)$

e) $(2-3x^2)^4$

$\frac{dy}{dx} = 4(2-3x^2)^3(-6x)$

f) $(1-x+x^2)^3$

$\frac{dy}{dx} = 3(1-x+x^2)^2(2x-1)$

Q2. Differentiate the following with respect to x

a) $\frac{3}{(3-4x)^3}$

$= 3(3-4x)^{-3}$

$\frac{dy}{dx} = -9(3-4x)^{-4}(-4)$

b) $\frac{4}{(2x+7)}$

$= 4(2x+7)^{-1}$
$= -4(2x+7)^{-2}(2)$

c) $\frac{6}{(2-x)^2}$

$= 6(2-x)^{-2}$
$\frac{dy}{dx} = -12(2-x)^{-3}(-1)$

d) $\frac{2}{(6x^2+5)}$

$= 2(6x^2+5)^{-1}$
$\frac{dy}{dx} = -2(6x^2+5)^{-2}(12x)$

Q3. Differentiate the following with respect to x.

a) $\sqrt{2x-3}$

$= (2x-3)^{\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}(2x-3)^{-\frac{1}{2}}(2)$

b) $\sqrt{6-2x}$

$= (6-2x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(6-2x)^{-\frac{1}{2}}(-2)$

$\frac{dy}{dx}$ 2 $\frac{dy}{dx}$ 2

c) $\sqrt{x^2-2}$
$= (x^2-2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^2-2)^{-\frac{1}{2}}(2x)$

d) $\sqrt{5-3x^2}$
$= (5-3x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(5-3x^2)^{-\frac{1}{2}}(-6x)$

e) $\sqrt{x^2-x+1}$
$= (x^2-x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^2-x+1)^{-\frac{1}{2}}(2x-1)$

f) $\sqrt{x^2+2x+2}$
$= (x^2+2x+2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^2+2x+2)^{-\frac{1}{2}}(2x+2)$

Q4. Differentiate with respect to x

a) $(2-\sqrt{x})^6$
$= (2-x^{\frac{1}{2}})^6$

$\frac{dy}{dx} = 6(2-\sqrt{x})^5\left(-\frac{1}{2}\right)$

b) $\frac{1}{\left[1-\frac{1}{x}\right]^3}$

$= 1\left(1-\frac{1}{x}\right)^{-3}$

$\frac{dy}{dx} = -3\left(1-\frac{1}{x}\right)^{-4}\left(\frac{1}{x^2}\right)$

c) $\frac{1}{2(3x-2)^2}$

$= \frac{1}{2}(3x-2)^{-2}$

$\frac{dy}{dx} = -1(3x-2)^{-3}(3)$

d) $2(\sqrt{x}+2)^{\frac{1}{2}}$

$\frac{dy}{dx} = 1(\sqrt{x}+2)^{-\frac{1}{2}}\left(\frac{1}{2}\right)$

e) $\left[x-\frac{1}{x}\right]^3$

$= (x-x^{-1})^3$

$\frac{dy}{dx} = 3(x-x^{-1})^2(2)$

f) $(\sqrt{x}+2x)^4$

$\frac{dy}{dx} = 4(\sqrt{x}+2x)^3\left(2\frac{1}{2}\right)$