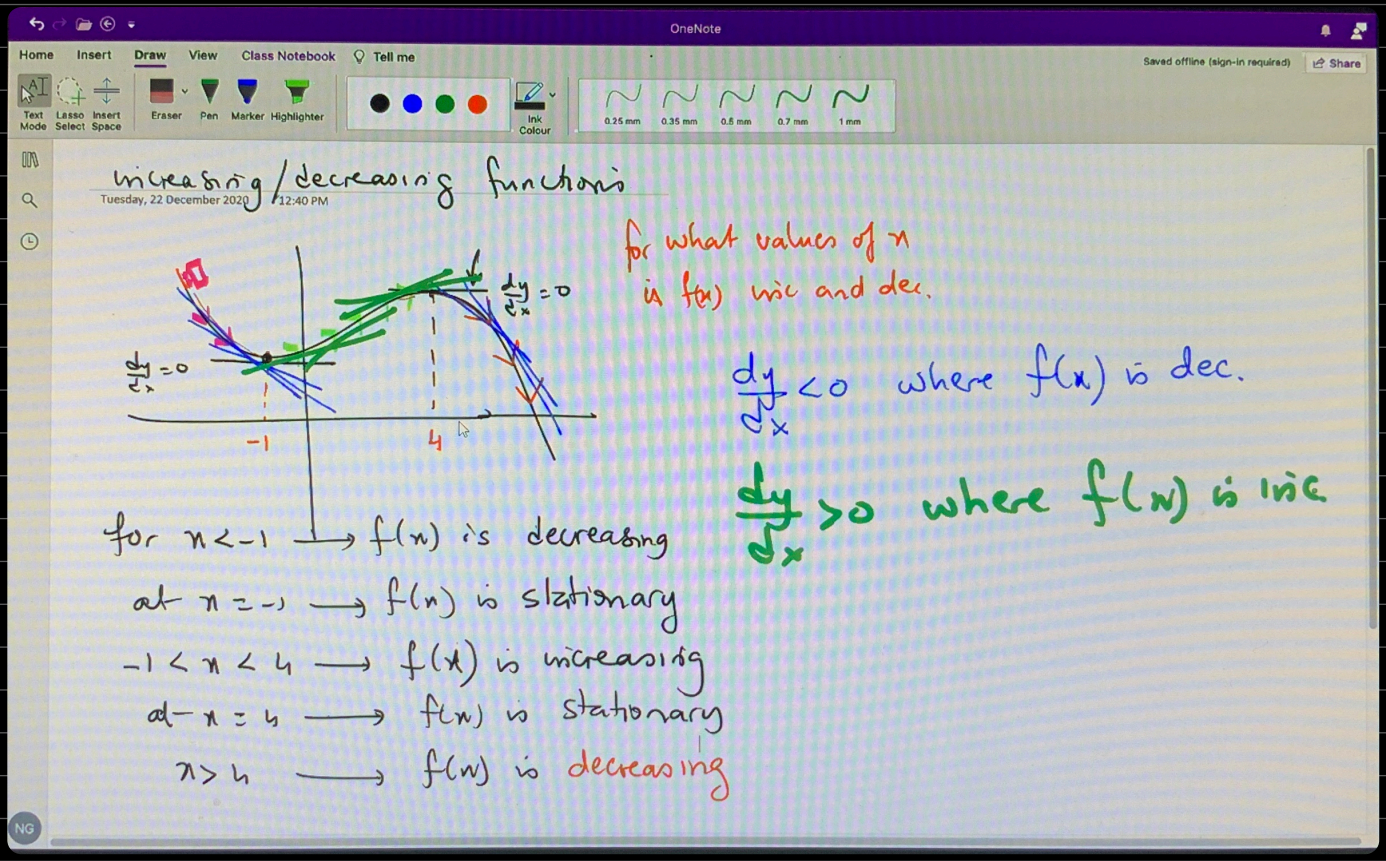
INCREASING / DECREASING FUNCTIONS



4. $y = 1 - x + 2x^2 - x^3$ is increasing

$$\frac{dy}{dx} = -1 + 4x - 3x^2$$

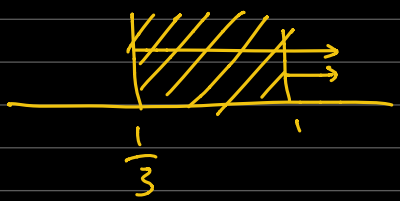
$$-3x^2 + 4x - 1 > 0$$

$$3x^2 - 4x + 1 < 0$$

$$3x^2 - 3x - x + 1 < 0$$

$$3x(x-1) - 1(x-1) < 0$$

$$(3x-1)(x-1) < 0$$



$$\frac{1}{3} < x < 1$$

5. $y = 2x^3 + 3x^2 - 12x + 4$

$\frac{dy}{dx} = 6x^2 + 6x - 12$

$6x^2 + 6x - 12 < 0$

$6x^2 + 12x - 6x - 12 < 0$

$6x(x+2) - 6(x+2) < 0$

$(6x - 6)(x+2) < 0$

$x = 1 \quad x = -2$

$-2 < x < 1$

$\begin{matrix} -2 \\ 1 \end{matrix}$] → stationary points

6. $s = 4 - 3t + 2t^2$

$\frac{ds}{dt} = -3 + 4t$

$= 4t - 3$

$4t - 3 < 0$

$4t < 3$

$t < \frac{3}{4}$

14. $y = x^3 + x^2 + 5x + 6$ is always increasing

$\frac{dy}{dx} = 3x^2 + 2x + 5$ → grad. function

↳ a is positive, so it's a u-shaped curve.

For all gradient values to be positive, this u-shaped curve must lie entirely above the x-axis

↳ whether it actually does or doesn't is

determined by the discriminant

$b^2 - 4ac$

$2^2 - 4(3)(5)$

$4 - 4(15)$

$4 - 60$

-54 → Since the discriminant is negative, it proves that the graph is a u-shaped curve not crossing the x-axis

↳ Hence, all possible values are positive

15. $y = x^3 + 9x^2 + 3x - 1$ is always increasing, find range of possible values.

$\frac{dy}{dx} = 3x^2 + 2ax + 3$

↳ a is positive → u-shaped

For all values to be positive, graph must lie entirely above the x-axis

↳ ∴, discriminant < 0

$b^2 - 4ac < 0$

$(2a)^2 - 4(3)(3) < 0$

$4a^2 - 4(9) < 0$

$4a^2 - 36 < 0$

$4a^2 < 36$

$a^2 < 9$

$a < +3$

$a > -3$

$-3 < a < 3$ → Ans