

Doing differentiation questions:

↳ To find the exact gradient at a point on a curve

Example: curve $\rightarrow y = 3x^2 - 6x - 4$, find gradient of the tangent at $x = 3$

Step 1 → Differentiate

$$[3x^2] - [6x^1] - 4x^0$$

Multiply the power of the variable with its respective co-efficient then subtract 1 from the power.

$$\text{Therefore: } \frac{dy}{dx} \Big| = 6x^1 - 6x^0 - 0 \\ \frac{dy}{dx} \Big| = 6x - 6$$

Step 2 → Input the x value into $\frac{dy}{dx}$ to find the gradient of the tangent at that point on the curve.

$$\begin{aligned} x &= 3 \\ &= 6(3) - 6 \\ &= 18 - 6 \\ &= 12 \rightarrow \text{gradient of tangent at } x = 3. \end{aligned}$$

Step 3 → Find corresponding y value of given x value to find a point through which the tangent passes, so that its equation can be formed

$$\begin{aligned} x &= 3 \\ y &= 3x^2 - 6x + 4 \\ &= 3(3^2) - 6(3) + 4 \\ &= 3(9) - 18 + 4 \\ &= 27 - 14 \\ &= 13 \end{aligned} \quad (3, 13)$$

Step 4 → Use the point and the gradient of the tangent to find its equation

point $\Rightarrow (3, 13)$, slope $\Rightarrow (12)$

$$\begin{aligned} y &= 12x + c \\ 13 &= 12(3) + c \\ 13 &= 36 + c \\ -23 &= c \end{aligned}$$

∴, the equation of the tangent $\rightarrow y = 12x - 23$

Another Example:

$$y = x^2 - x\sqrt{x} + \frac{8}{x^2}, \text{ Find the equation of the tangent at } x = 3$$

$$y = x^2 - x^1 \cdot x^{\frac{1}{2}} + 8x^{-2}$$

$\frac{dy}{dx}$ is the gradient function, plug in any value of x and you get

$$= \boxed{1}x^2 - \boxed{-1}x^{\frac{3}{2}} + \boxed{18}x^{-2}$$

$$\frac{dy}{dx} = 2x - \frac{3}{2}x^{\frac{1}{2}} - 16x^{-3}$$

the gradient of the tangent
at that point on the curve.

$$\text{for } x = 3 \rightarrow \left. \frac{dy}{dx} \right|_{x=3} = 2x - \frac{3}{2}x^{\frac{1}{2}} - 16x^{-3} = 2(3) - \frac{3}{2}\sqrt{3} - 16\left(\frac{1}{3^3}\right)$$

↳ Bla bla you get the point
↳ crunch the numbers later

How and when to use the chain rule:

- Chain rule is used in problems where the equation of the curve looks something like

$$y = (x^2 + 2)^5, \text{ i.e. a whole raised to a power.}$$

Chain rule is essentially a shortcut instead of needing to expand the entire thing

How it works.

- ① Take the whole power and multiply by the coefficient. This is the new coefficient
- ② Subtract 1 from the whole power. This is the new whole power
- ③ Differentiate whatever was inside the brackets, and write the result in the rightmost position as a multiplied value.

Example:

$$y = \boxed{1} (\underbrace{x^2 + 2}_x)^{\cancel{5}-1}$$

$$\frac{dy}{dx} = \cancel{2x}$$

Therefore,

$$\frac{dy}{dx} = 5(x^2 + 2)^4(2x) \rightarrow \text{now this is the gradient function. Input any value of } x \text{ to get the value of the gradient of the tangent at that point on the curve.}$$

HOMEWORK → Ex. 15.1 Q1-7 , Ex. 15.2 Q1-4

Exercise 15.1

1. Differentiate the following with respect to x (where a and b are constants).

- | | |
|--|---------------------------------------|
| (a) $3x^2 + 4x - 5$ | (b) $x^4 - 7x^3 + 6x$ |
| (c) $2x^3 + 5x^2 - 4x + 9$ | (d) $4x + \frac{2}{x}$ |
| (e) $9x^2 - \frac{3}{x^2}$ | (f) $\frac{6}{x^3} - \frac{1}{x} + 3$ |
| (g) $3a + bx^2$ | (h) $5x^2 + \frac{4}{x} - 2$ |
| (i) $3x + 2\sqrt{x} - 3$ | (j) $8x^2 + 3x - \sqrt{x}$ |
| (k) $2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 6x + 8$ | (l) $6x\sqrt{x} - 6\sqrt{x}$ |
| (m) $4x^2\sqrt{x} - \frac{6}{\sqrt{x}}$ | (n) $ax - \frac{b}{x}$ |

2. Differentiate the following with respect to x .

- | | | |
|----------------------------|-------------------------------------|--------------------------------------|
| (a) $\frac{2x^2 + 4x}{x}$ | (b) $\frac{x^2 - 6x + 4}{x}$ | (c) $\frac{4x^3 - 5x - 3}{2x}$ |
| (d) $\frac{x^2 + 4}{2x^2}$ | (e) $\frac{3x^2 + x - 1}{\sqrt{x}}$ | (f) $\frac{6x^2 - \sqrt{x} + 2}{2x}$ |

3. Find $\frac{dy}{dx}$ for the following functions of x .

- | | | |
|--------------------------|---------------------------------|--|
| (a) $(x + 1)(2x - 1)$ | (b) $x(\sqrt{x} - 2)$ | (c) $(1 + \sqrt{x})(1 - \sqrt{x})$ |
| (d) $4x^2(3 - \sqrt{x})$ | (e) $\frac{(2x + 1)(x - 2)}{x}$ | (f) $\frac{(1 - x)(4x - 1)}{\sqrt{x}}$ |

4. Find the value of $f'(x)$ at the given value of x .

- | | |
|------------------------------------|---------------------------------------|
| (a) $f(x) = 3x^2 - 2x - 4, x = 2$ | (b) $f(x) = 6x - \frac{3}{x}, x = -1$ |
| (c) $f(x) = 3x - 4\sqrt{x}, x = 4$ | (d) $f(x) = (x - 4)(x + 3), x = 3$ |

DIFFERENTIATION**Worksheet**

5. Calculate the gradient of the tangent to the curve at the given point.
- (a) $y = 4x^2 - 6x + 1$, $(2, 5)$ (b) $y = \frac{6-4x}{x}$, $x = -2$
 (c) $y = \sqrt{x}(2-x)$, $x = 9$ (d) $y = \frac{(x+1)(2x-3)}{x}$, $x = -1$
6. Calculate the gradient(s) of the curve at the point(s) where y is given.
- (a) $y = x^2 - 2x$, $y = -1$ (b) $y = 2x^2 + 3x$, $y = 2$
 (c) $y = \frac{x-9}{x}$, $y = 4$ (d) $y = \frac{x^2+4}{x^2}$, $y = 5$
7. Calculate the gradient(s) of the curve at the point(s) where it crosses the given line.
- (a) $y = 2x^2 - 5x + 1$, y-axis (b) $y = \frac{x-4}{x}$, x-axis
 (c) $y = 2x^2 - 8$, x-axis (d) $y = \frac{x+2}{x}$, $y = x$
8. Find the coordinates of the point on the curve $y = x^3 - 3x^2 + 6x + 2$ at which the gradient is 3.
9. The curve $y = ax^2 + \frac{b}{x}$ has gradients 2 and -1 at $x = 1$ and $x = 4$ respectively. Find the value of a and of b .
10. The gradient of the tangent to the curve $y = ax^3 + bx$ at the point $(2, -4)$ is 6. Calculate the values of the constants a and b .
11. Given that the gradient of the curve $y = \frac{a}{x} + bx^2$ at the point $P(3, -15)$ is -13. Find the value of a and of b . Show that the tangent to the curve at the point where $x = 1$ has the same gradient as that at P .
12. The tangent to the curve $y = \frac{a}{x} + bx$ at $(1, 3)$ is parallel to the line $y = 2x + 1$. Calculate the value of a and of b .
13. Given that $f(x) = px^2 + qx$ and that $f(2) = -2$ and $f'(2) = 3$, calculate the value of p and of q .
14. The equation of a curve is $y = 9x + \frac{1}{x}$. Find
 (a) the gradient of the curve where $x = 2$,
 (b) the coordinates of the points where the tangent is horizontal.
15. Given the curve $y = x^3 - 3x^2 - 9x + 11$, find $\frac{dy}{dx}$. Hence obtain
 (a) the x -coordinates of the points where the gradient is 15,
 (b) the coordinates of the points where the gradient is zero.
16. The curve $y = x^2 + 2x$ has gradient 3 at the point (a, b) . Find the value of a and of b .
17. The curve $y = ax + \frac{3}{x}$ has gradient 1 at $x = 2$. Find the value of a and the x -coordinate of another point at which the gradient is 1.

Exercise 15.2

WORKSHEET

1. Differentiate the following w.r.t. x .

- (a) $(x+2)^3$ (b) $(2x-1)^4$ (c) $(\frac{1}{4}x+2)^3$
 (d) $(1-4x)^6$ (e) $(2-3x^2)^4$ (f) $(1-x+x^2)^3$

2. Differentiate the following w.r.t. x .

- (a) $\frac{3}{(3-4x)^2}$ (b) $\frac{4}{(2x+7)^2}$ (c) $\frac{6}{(2-x)^2}$ (d) $\frac{2}{(6x^2+5)^2}$

3. Differentiate the following w.r.t. x .

- (a) $\sqrt{2x-3}$ (b) $\sqrt{6-2x}$ (c) $\sqrt{x^2-2}$
 (d) $\sqrt{5-3x^2}$ (e) $\sqrt{x^2-x+1}$ (f) $\sqrt{x^2+2x+2}$

4. Differentiate the following w.r.t. x .

- (a) $(2-\sqrt{x})^4$ (b) $\frac{1}{\left(1-\frac{1}{x}\right)^2}$ (c) $\frac{1}{2(3x-2)^2}$
 (d) $2(\sqrt{x}+2)^{\frac{1}{2}}$ (e) $\left(x-\frac{1}{x}\right)^3$ (f) $(\sqrt{x}+2x)^4$

5. Find $\frac{dy}{dx}$ and the gradient of the curve at the given value of x .

- (a) $y = (3x-1)^4$, $x = 1$ (b) $y = \sqrt{5-2x}$, $x = \frac{1}{2}$
 (c) $y = \frac{1}{2x-3}$, $y = 1$ (d) $y = (4x-5)^3$, $y = 27$

6. Calculate the coordinates of the point on the curve $y = (1-x)^4$ at which the gradient is -4 .

7. Calculate the coordinates of the point on the curve $y = \sqrt{x^2-2x+5}$ at which $\frac{dy}{dx} = 0$.

8. The curve $y = (a-x)^3$ has gradient $-\frac{1}{3}$ at $x = 2$. Find the possible values of a .

9. Find $\frac{dy}{dx}$ and calculate the gradient of the tangent to the curve

- (a) $y = (x^2-2x-4)^3$ at the point where $x = -1$,
 (b) $y = \frac{1}{\sqrt{1+x}}$ at the point where $x = 3$.

*10. Given that $f(x) = \sqrt{1+\sqrt{x}}$, where $x \geq 0$, show that $f'(x) = \frac{1}{4\sqrt{x+x\sqrt{x}}}$.

Exercise 15.2 (ANSWERS)

1. (a) $5(x+2)^4$ (b) $8(2x-1)^3$ (c) $\frac{5}{4}\left(\frac{1}{4}x+2\right)^4$
 (d) $-40(1-4x)^6$ (e) $-24x(2-3x^2)^3$ (f) $3(2x-1)(1-x+x^2)^2$
2. (a) $\frac{36}{(3-4x)^4}$ (b) $-\frac{8}{(2x+7)^2}$ (c) $\frac{12}{(2-x)^3}$ (d) $\frac{-24x}{(6x^2+5)^2}$
3. (a) $\frac{1}{\sqrt{2x-3}}$ (b) $-\frac{1}{\sqrt{6-2x}}$ (c) $\frac{x}{\sqrt{x^2-2}}$
 (d) $-\frac{3x}{\sqrt{5-3x^2}}$ (e) $\frac{2x-1}{2\sqrt{x^2-x+1}}$ (f) $\frac{x+1}{\sqrt{x^2+2x+2}}$
4. (a) $-\frac{3(2-\sqrt{x})^2}{\sqrt{x}}$ (b) $\frac{-3}{x^2\left(1-\frac{1}{x}\right)^4}$ (c) $-\frac{3}{(3x-2)^4}$
 (d) $\frac{1}{2\sqrt{x}(\sqrt{x}+2)^2}$ (e) $3\left(x-\frac{1}{x}\right)^2\left(1+\frac{1}{x^2}\right)$ (f) $4(\sqrt{x}+2x)^3\left(\frac{1}{2\sqrt{x}}+2\right)$

5. (a) 96 (b) $-\frac{1}{2}$ (c) -2 (d) 108

6. (0, 1) 7. (1, 2) 8. $a = \frac{7}{3}$ or $\frac{5}{3}$ 9. (a) -12 (b) $-\frac{1}{16}$

15 Exercise 15.1

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|--|---|---|-------------------------------|
| 1. (a) $6x + 4$ | (b) $4x^3 - 14x + 6$ | (c) $6x^2 + 10x - 4$ | (d) $4 - \frac{2}{x^2}$ |
| (e) $18x + \frac{6}{x^3}$ | (f) $-\frac{18}{x^4} + \frac{1}{x^2}$ | (g) $2bx$ | (h) $10x - \frac{4}{x^2}$ |
| (i) $3 + \frac{1}{\sqrt{x}}$ | (j) $16x + 3 - \frac{1}{2\sqrt{x}}$ | (k) $5x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 6$ | |
| (l) $9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ | (m) $10x^{\frac{3}{2}} + 3x^{-\frac{3}{2}}$ | (n) $a + \frac{b}{x^2}$ | |
| 2. (a) 2 | (b) $1 - \frac{4}{x^2}$ | (c) $4x + \frac{3}{2x^2}$ | (d) $-\frac{4}{x^3}$ |
| (e) $\frac{9}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ | (f) $3 + \frac{1}{4}x^{-\frac{3}{2}} - x^{-2}$ | | |
| 3. (a) $4x + 1$ | (b) $\frac{3}{2}\sqrt{x} - 2$ | (c) -1 | (d) $24x - 10x^{\frac{1}{2}}$ |
| (e) $2 + \frac{2}{x^2}$ | (f) $-6x^{\frac{1}{2}} + \frac{5}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ | | |
| 4. (a) 10 | (b) 9 | (c) 2 | (d) 5 |
| 5. (a) 10 | (b) $-1\frac{1}{2}$ | (c) $-4\frac{1}{6}$ | (d) 5 |
| 6. (a) 0 | (b) -5, 5 | (c) 1 | (d) -8, 8 |

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|--|--|--|------------------------|
| 7. (a) -5 | (b) $\frac{1}{4}$ | (c) -8, 8 | (d) -2, $-\frac{1}{2}$ |
| 8. (1, 6) | | 9. $a = -\frac{1}{7}, b = -\frac{16}{7}$ | 10. $a = 1, b = -6$ |
| 11. $a = 9, b = -2$ | | 12. $a = \frac{1}{2}, b = \frac{5}{2}$ | 13. $p = 2, q = -5$ |
| 14. (a) $8\frac{3}{4}$ | (b) $\left(-\frac{1}{3}, -6\right), \left(\frac{1}{3}, 6\right)$ | 15. (a) -2, 4 | (b) (-1, 16), (3, -16) |
| 16. $a = \frac{1}{2}, b = \frac{5}{4}$ | | 17. $a = \frac{7}{4}; -2$ | |

Exercise 15.1 Q1.

1. Differentiate with respect to x (a and b are constants)

a) $3x^2 + 4x - 1$

$$\frac{dy}{dx} = 6x + 4$$

b) $x^4 - 7x^2 + 6x$

$$\frac{dy}{dx} = 4x^3 - 14x + 6$$

c) $2x^3 + 5x^2 - 4x + 9$

$$\frac{dy}{dx} = 6x^2 + 10x - 4$$

d) $4x + \frac{2}{x}$

$$= 4x + 2x^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= 4 - 2x^{-2} \\ &= 4 - \frac{2}{x^2}\end{aligned}$$

e) $9x^2 - \frac{3}{x^2}$

$$= 9x^2 - 3x^{-2}$$

$$\begin{aligned}\frac{dy}{dx} &= 18x + 6x^{-3} \\ &= 18x + \frac{6}{x^3}\end{aligned}$$

f) $\frac{6}{x^3} - \frac{1}{x} + 3$

$$= 6x^{-3} - 1x^{-1} + 3$$

$$\begin{aligned}\frac{dy}{dx} &= -18x^{-4} + 1x^{-2} \\ &= \frac{1}{x^2} - \frac{18}{x^4}\end{aligned}$$

g) $3a + bx^2$

$$\frac{dy}{dx} = 2bx$$

h) $5x^2 + \frac{4}{x} - 2$

$$= 5x^2 + 4x^{-1} - 2$$

$$\begin{aligned}\frac{dy}{dx} &= 10x - 4x^{-2} \\ &= 10x - \frac{4}{x^2}\end{aligned}$$

i) $3x + 2\sqrt{x} - 3$

$$= 3x + 2x^{\frac{1}{2}} - 3$$

$$\begin{aligned}\frac{dy}{dx} &= 3 + 1x^{-\frac{1}{2}} \\ &= 3 + \frac{1}{\sqrt{x}}\end{aligned}$$

j) $8x^2 + 3x - \sqrt{x}$

$$= 8x^2 + 3x - 1x^{\frac{1}{2}}$$

$$\begin{aligned}\frac{dy}{dx} &= 8x + 3 - \frac{1}{2}x^{-\frac{1}{2}} \\ &= 8x + 3 - \frac{1}{2\sqrt{x}}\end{aligned}$$

k) $2x^{\frac{5}{2}} - 4x^{\frac{3}{2}} - 6x + 8$

$$\frac{dy}{dx} = 5x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 6$$

l) $6x\sqrt{x} - 6\sqrt{x}$

$$= 6x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$$

$$\begin{aligned}\frac{dy}{dx} &= 9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \\ &= 9\sqrt{x} - \frac{3}{\sqrt{x}}\end{aligned}$$

m) $4x^2\sqrt{x} - \frac{6}{\sqrt{x}}$

$$\begin{aligned}\frac{dy}{dx} &= 10x^{\frac{5}{2}} - 6x^{-\frac{1}{2}} \\ &= 10x^{\frac{3}{2}} + \frac{3}{x^{\frac{3}{2}}}\end{aligned}$$

n) $ax - \frac{b}{x}$

$$= ax - bx^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= a + bx^{-2} \\ &= a + \frac{b}{x^2}\end{aligned}$$

Q2. Differentiate with respect to x .

$$a) \frac{2x^2 + 4x}{x}$$

$$= \frac{2x^2}{x} + \frac{4x}{x}$$

$$= 2x + 4$$

$$\frac{dy}{dx} = 2$$

$$b) \frac{x^2 - 6x + 4}{x}$$

$$= \frac{x^2}{x} - \frac{6x}{x} + \frac{4}{x}$$

$$= x - 6 + 4x^{-1}$$

$$\frac{dy}{dx} = 1 - 4x^{-2}$$

$$= 1 - \frac{4}{x^2}$$

$$c) \frac{4x^3 - 5x - 3}{2x}$$

$$= 2x^2 - \frac{5}{2} - \frac{3}{2x}$$

$$= 2x^2 - \frac{5}{2} - \frac{3}{2}x^{-1}$$

$$\frac{dy}{dx} = 4x + \frac{3}{2}x^{-2}$$

$$= 4x + \frac{3}{2x^2}$$

$$d) \frac{x^2 + 4}{2x^2}$$

$$= \frac{1}{2} + 2x^{-2}$$

$$\frac{dy}{dx} = -4x^{-3}$$

$$= -\frac{4}{x^3}$$

$$e) \frac{3x^2 + x - 1}{\sqrt{x}}$$

$$= 3x^2 x^{-\frac{1}{2}} + x \cdot x^{-\frac{1}{2}} - 1 x^{-\frac{1}{2}}$$

$$= 3x^{\frac{3}{2}} + x^{\frac{1}{2}} - 1 x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

$$= \frac{9}{2}\sqrt{x} + \frac{1}{2\sqrt{x}} + \frac{1}{6\sqrt{x^3}}$$

$$f) \frac{6x^2 - \sqrt{x} + 2}{2x}$$

$$= \frac{6x^2}{2x} - \frac{\sqrt{x}}{2x} + \frac{2}{2x}$$

$$= 3x - \frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$$

$$\frac{dy}{dx} = 3 + \frac{1}{4}x^{-\frac{3}{2}} - 1x^{-2}$$

$$= 3 + \frac{1}{4\sqrt{x^3}} - \frac{1}{x^2}$$

Q3. Find $\frac{dy}{dx}$ for the following functions of x .

$$a) (x+1)(2x-1)$$

$$= 2x^2 - x + 2x - 1$$

$$= 2x^2 + x - 1$$

$$b) x(\sqrt{x} - 2)$$

$$= x \cdot x^{\frac{1}{2}} - 2x$$

$$= x^{\frac{3}{2}} - 2x$$

$$c) (1+\sqrt{x})(1-\sqrt{x})$$

$$= 1 - x$$

$$\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = 4x + 1$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2$$

$$d) \frac{dy}{dx} = 4x^2(3 - \sqrt{x}) \\ = 12x^2 - 4x^{\frac{5}{2}}$$

$$e) \frac{(2x+1)(x-2)}{x} \\ = \frac{2x^2 - 4x + x - 2}{x} \\ = 2x - 3 - 2x^{-1}$$

$$f) \frac{(1-x)(4x-1)}{\sqrt{x}}$$

$$= \frac{4x+1-4x^2+x}{\sqrt{x}} \\ = \frac{5x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + 1x^{-\frac{1}{2}} - 4x^{\frac{3}{2}}}{5x^{\frac{1}{2}} + 1x^{-\frac{1}{2}} - 4x^{\frac{3}{2}}} - 4x^2 \cdot x^{\frac{1}{2}} \\ = \frac{5}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} - 6x^{\frac{1}{2}} \\ = \frac{5}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}} - 6\sqrt{x}$$

Q4. Find the value of $f'(x)$ at the given value of x .

$$a) f(x) = 3x^2 - 2x - 4, \quad x = 2$$

$$b) f(x) = 6x - \frac{3}{x}, \quad x = -1$$

$$\left. \frac{dy}{dx} \right|_{x=2} = 6x - 2 \\ = 6(2) - 2 \\ = 12 - 2 \\ = 10 \rightarrow \text{Ans.}$$

$$\left. \frac{dy}{dx} \right|_{x=-1} = 6x - 3x^{-1} \\ = 6 + 3(-1)^{-2} \\ = 6 + \frac{3}{(-1)^2} \\ = 9 \rightarrow \text{Ans.}$$

$$c) f(x) = 3x - 4\sqrt{x}, \quad x = 4 \quad d) f(x) = (x-4)(x+3), \quad x = 3$$

$$\left. \frac{dy}{dx} \right|_{x=4} = 3x - 4x^{\frac{1}{2}} \\ = 3 - 2x^{-\frac{1}{2}} \\ = 3 - \frac{2}{\sqrt{4}} \\ = 2$$

$$\left. \frac{dy}{dx} \right|_{x=3} = x^2 - x - 12 \\ = 2x - 1 \\ = 2(3) - 1 \\ = 6 - 1 \\ = 5 \rightarrow \text{Ans.}$$

Q5. Calculate the gradient of the tangent to the curve at the given point.

$$a) y = 4x^2 - 6x + 1, \quad (2, 5)$$

$$b) y = \frac{6-4x}{x}, \quad x = -2$$

$$\left. \frac{dy}{dx} \right|_{x=2} = 8x - 6 \\ = 16 - 16 \\ = 10 \rightarrow \text{grad.}$$

$$\left. \frac{dy}{dx} \right|_{x=-2} = 6x^{-1} - 4 \\ = -6x^{-2} \\ = \frac{-6}{(-2)^2} \\ = \frac{-6}{4} \\ = -\frac{3}{2}$$

$$c) y = \sqrt{x}(2-x), \quad x = 9$$

$$\left. \frac{dy}{dx} \right|_{x=9} = 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \\ = x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} \\ = \frac{1}{\sqrt{9}} - \frac{3}{2}\sqrt{9} \\ = 1 - \frac{9}{2}$$

$$\begin{aligned} x &= 9 \\ &= -\frac{3}{6} \end{aligned}$$

d) $y = \frac{(x+1)(2x-3)}{x}, x = -1$

$$\begin{aligned} &= \frac{2x^2 - 3x + 2x - 3}{x} \\ &= 2x - 1 - 3x^{-1} \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{x=-1} = 2 + 3x^{-2} = 2 + \frac{3}{(-1)^2} = 2 + 3 = 5 \rightarrow \text{Ans.}$$

Q6. Calculate the gradient(s) of the curve at the point(s) where y is given

a) $y = x^2 - 2x, y = -1$

$$\begin{aligned} -1 &= x^2 - 2x \\ 0 &= x^2 - 2x + 1 \\ &= x^2 - x - x + 1 \\ &= x(x-1) - 1(x-1) \\ &= (x-1)(x-1) \\ &= x = 1 \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{x=1} = 2x - 2 = 2(1) - 2 = 2 - 2 = 0 \rightarrow \text{Ans.}$$

b) $y = 2x^2 + 3x, y = 2$

$$\begin{aligned} 2 &= 2x^2 + 3x \\ 0 &= 2x^2 + 3x - 2 \\ &= 2x^2 + 4x - x - 2 \\ &= 2x(x+2) - 1(x+2) \\ &= (2x-1)(x+2) \\ x &= \frac{1}{2} \text{ or } x = -2 \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=\frac{1}{2} \\ x=-2}} = 4x + 3 = 4\left(\frac{1}{2}\right) + 3 = 5 \rightarrow \text{Ans.}$$

$$= 4(-2) + 3 = -8 + 3 = -5 \rightarrow \text{Ans.}$$

c) $y = \frac{x-9}{x}, y = 4$

$$4 = \frac{x-9}{x}$$

$$\begin{aligned} 4x &= x - 9 \\ 3x &= -9 \\ x &= -3 \end{aligned}$$

$$\frac{dx}{dy} = \frac{9x^{-2}}{\frac{9}{x^2}}$$

$$\begin{aligned} x &= -3 \rightarrow \frac{9}{(-3)^2} = \frac{9}{9} = 1 \rightarrow \text{Ans.} \end{aligned}$$

d) $y = \frac{x^2 + 4}{x^2}$, $y = 5$

$$5 = \frac{x^2 + 4}{x^2}$$

$$5x^2 = x^2 + 4$$

$$0 = 4x^2 - 4$$

$$= 4x^2 - 4x + 4x - 4$$

$$= 4x(x-1) + 4(x-1)$$

$$= (4x+4)(x-1)$$

$$x = -1 \text{ or } x = 1$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=-1 \\ x=1}} = \begin{aligned} & -8x^{-3} = -\frac{8}{x^3} \\ & = -\frac{8}{(-1)^3} \\ & = 8 \rightarrow \underline{\text{Ans.}} \\ & = -\frac{8}{(1)^3} \\ & = -8 \rightarrow \underline{\text{Ans.}} \end{aligned}$$

Q7. Calculate the gradient(s) of the curve at the point(s) where it crosses the given line

a) $y = 2x^2 - 5x + 1$, y -axis
 $\hookrightarrow x = 0$

$$\left. \frac{dy}{dx} \right|_{\substack{x=0}} = \begin{aligned} & 4x - 5 = 4(0) - 5 \\ & = -5 \rightarrow \underline{\text{Ans.}} \end{aligned}$$

b) $y = \frac{x-4}{x}$, x -axis
 $\hookrightarrow y = 0$

$$\begin{aligned} 0 &= \frac{x-4}{x} \\ 4 &= x \\ \left. \frac{dy}{dx} \right|_{\substack{x=4}} &= \begin{aligned} & \frac{4}{x^2} = \frac{4}{4^2} \\ & = \frac{4}{16} \\ & = \frac{1}{4} \rightarrow \underline{\text{Ans.}} \end{aligned} \end{aligned}$$

c) $y = 2x^2 - 8$, x -axis
 $\hookrightarrow y = 0$

$$\begin{aligned} 0 &= 2x^2 - 8 \\ &= 2x^2 + 4x - 4x - 8 \\ &= 2x(x+2) - 4(x+2) \\ &= (2x-4)(x+2) \\ x &= 2 \text{ or } x = -2 \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=2 \\ x=-2}} = \begin{aligned} & 4x = 4(2) \\ & = 8 \rightarrow \underline{\text{Ans.}} \\ & = 4(-2) \\ & = -8 \rightarrow \underline{\text{Ans.}} \end{aligned}$$

d) $y = \frac{x+2}{x}$, $y = x$

$$x = \frac{x+2}{x}$$

$$x^2 = x + 2$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=-1}} = \begin{aligned} & -\frac{2}{x^2} = -\frac{2}{(-1)^2} \\ & = -2 \rightarrow \underline{\text{Ans.}} \end{aligned}$$

$$\begin{aligned}
 0 &= x^2 - x - 2 \\
 &= x^2 - 2x + x - 2 \\
 &= x(x-2) + 1(x-2) \\
 &= (x+1)(x-2) \\
 x &= -1 \quad \text{or} \quad x = 2
 \end{aligned}
 \quad \left| \begin{array}{l} x = 2 \\ x = -1 \end{array} \right. \quad \begin{aligned}
 &= \frac{-2}{(2)^2} \\
 &= -\frac{1}{2} \rightarrow \underline{\text{Ans.}}
 \end{aligned}$$

Exercise 15.2 — Chain Rule. — Q1-4

Q1. Differentiate the following with respect to x .

a) $(x+2)^5$ b) $(2x-1)^4$ c) $(\frac{1}{4}x+2)^5$

$$\begin{aligned}
 \frac{dy}{dx} &= 5(x+2)^4(1) \\
 \frac{dy}{dx} &= 5(x+2)^4 \rightarrow \underline{\text{Ans.}}
 \end{aligned}
 \quad \begin{aligned}
 \frac{dy}{dx} &= 4(2x-1)^3(2) \\
 \frac{dy}{dx} &= 5(\frac{1}{4}x+2)^4(\frac{1}{4})
 \end{aligned}$$

d) $(1-4x)^{10}$ e) $(2-3x^2)^4$

$$\begin{aligned}
 \frac{dy}{dx} &= 10(1-4x)^9(-4) \\
 \frac{dy}{dx} &= 4(2-3x^2)^3(-6x)
 \end{aligned}$$

f) $(1-x+x^2)^3$

$$\frac{dy}{dx} = 3(1-x+x^2)^2(2x-1)$$

Q2. Differentiate the following with respect to x

a) $\frac{3}{(3-4x)^3}$

$$= 3(3-4x)^{-3}$$

$$\frac{dy}{dx} = -9(3-4x)^{-4}(-4)$$

b) $\frac{4}{(2x+7)}$

$$\begin{aligned}
 &= 4(2x+7)^{-1} \\
 &= -4(2x+7)^{-2}(2)
 \end{aligned}$$

c) $\frac{6}{(2-x)^2}$

$$\begin{aligned}
 &= 6(2-x)^{-2} \\
 \frac{dy}{dx} &= -12(2-x)^{-3}(-1)
 \end{aligned}$$

d) $\frac{2}{(6x^2+5)}$

$$\begin{aligned}
 &= 2(6x^2+5)^{-1} \\
 \frac{dy}{dx} &= -2(6x^2+5)^{-2}(12x)
 \end{aligned}$$

Q3. Differentiate the following with respect to x .

a) $\sqrt{2x-3}$

$$\frac{dy}{dx} = \frac{1}{2}(2x-3)^{\frac{1}{2}}(2)$$

b) $\sqrt{6-2x}$
 $= (6-2x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(6-2x)^{-\frac{1}{2}}(-2)$$

$$\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = 2$$

$$c) \frac{\sqrt{x^2 - 2}}{(x^2 - 2)^{\frac{1}{2}}} = \frac{1}{(x^2 - 2)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 - 2)^{-\frac{1}{2}}(2x)$$

$$d) \frac{\sqrt{5 - 3x^2}}{(5 - 3x^2)^{\frac{1}{2}}} = \frac{1}{(5 - 3x^2)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{2}(5 - 3x^2)^{-\frac{1}{2}}(-6x)$$

$$e) \frac{\sqrt{x^2 - x + 1}}{(x^2 - x + 1)^{\frac{1}{2}}} = \frac{1}{(x^2 - x + 1)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 - x + 1)^{-\frac{1}{2}}(2x - 1)$$

$$f) \frac{\sqrt{x^2 + 2x + 2}}{(x^2 + 2x + 2)^{\frac{1}{2}}} = \frac{1}{(x^2 + 2x + 2)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 2x + 2)^{-\frac{1}{2}}(2x + 2)$$

Q4. Differentiate with respect to x

$$a) (2 - \sqrt{x})^6 \\ = (2 - x^{\frac{1}{2}})^6$$

$$b) \frac{1}{\left[1 - \frac{1}{x}\right]^3}$$

$$\frac{dy}{dx} = 6(2 - \sqrt{x})^5 \left(-\frac{1}{2}\right)$$

$$= 1\left(1 - \frac{1}{x}\right)^{-3}$$

$$\frac{dy}{dx} = -3\left(1 - \frac{1}{x}\right)^{-4} \left(\frac{1}{x^2}\right)$$

$$c) \frac{1}{2(3x-2)^2} \\ = \frac{1}{2}(3x-2)^{-2}$$

$$\frac{dy}{dx} = -1(3x-2)^{-3}(3)$$

$$d) 2(\sqrt{x}+2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 1(\sqrt{x}+2)^{-\frac{1}{2}} \left(\frac{1}{2}\right)$$

$$e) \left[x - \frac{1}{x}\right]^3 \\ = (x - x^{-1})^3$$

$$\frac{dy}{dx} = 3(x - x^{-1})^2(2)$$

$$f) (\sqrt{x} + 2x)^4$$

$$\frac{dy}{dx} = 4(\sqrt{x} + 2x)^3 \left(2\frac{1}{2}\right)$$

→ From Add Math Ho Soo Thong slide 347
Homework 15th Dec - Ex. 15.1 Q8 - Q17 onwards

8. Coordinates on the curve $y = x^3 - 3x^2 + 6x + 2$ where the gradient is 3.

$$y = x^3 - 3x^2 + 6x + 2$$

$$\frac{dy}{dx} = 3x^2 - 6x + 6 \rightarrow \text{gradient function}$$

$$3 = 3x^2 - 6x + 6$$

$$0 = 3x^2 - 6x + 3$$

$$= 3x^2 - 3x - 3x + 3$$

$$= 3x(x-1) - 3(x-1)$$

$$= (3x-3)(x-1)$$

$$x = 1$$

$$\begin{aligned}y &= (1)^3 - 3(1)^2 + 6(1) + 2 \\&= 1 - 3 + 6 + 2 \\&= 6\end{aligned}$$

coordinates: $(1, 6) \rightarrow \underline{\underline{\text{Ans}}}$

$$9. y = ax^2 + \frac{b}{x} \quad \left. \frac{dy}{dx} \right|_{x=1} = 2 \quad \left. \frac{dy}{dx} \right|_{x=4} = -1$$

$$= ax^2 + bx^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= 2ax - bx^{-2} \\&= 2ax - \frac{b}{x^2}\end{aligned}$$

$$\underline{x=1} \quad 2 = 2a(1) - \frac{b}{1}$$

$$2 = 2a - b$$

$$\frac{2+b}{2} = a$$

$$2 - \frac{16}{7} = 2a$$

$$2 - 2 \frac{2}{7} = 2a$$

$$-\frac{2}{7} = 2a$$

$$-\frac{1}{7} = a \rightarrow \underline{\underline{\text{Ans}}}$$

$$\underline{x=4} \quad -1 = 2a(4) - \frac{b}{4^2}$$

$$-1 = 8a - \frac{b}{16}$$

$$-1 = 8(\frac{2+b}{2}) - \frac{b}{16}$$

$$-1 = 4(2+b) - \frac{b}{16}$$

$$-1 = 8 + 4b - \frac{b}{16}$$

$$16(-1) = 64b - b$$

$$16 = \frac{63b}{-9}$$

$$\frac{16}{-7} = \frac{-7b}{-7}$$

$$-\frac{16}{7} = b \rightarrow \underline{\underline{\text{Ans}}}$$

$$10. \quad y = ax^3 + bx$$

$$\frac{dy}{dx} = 3ax^2 + b$$

$$x=2 \quad 6 = 3a(2)^2 + b$$

$$6 = 3a(4) + b$$

$$6 = 12a + b$$

$$6 - 12a = b \quad \rightarrow \quad 6 - 12(1) = b$$

$$b - 12 = b$$

$$-6 = b$$

$$-4 = a(2)^3 + b(2)$$

$$-4 = 8a + 2b$$

$$-4 = 8a + 2(6 - 12a)$$

$$-4 = 8a + 12 - 24a$$

$$-4 = -16a + 12$$

$$16a = 12 + 4$$

$$16a = 16$$

$$a = 1 \rightarrow \underline{\underline{Ans}}$$

\downarrow
Ans

$$11. \quad y = \frac{a}{x} + bx^2$$

$$= ax^{-1} + bx^2$$

$$\frac{dy}{dx} = -ax^{-2} + 2bx$$

$$\text{grad at } P \quad \frac{-13}{\cancel{x}} = \frac{-a}{\cancel{x}^2} + 2b(3)$$

$$-13 = -\frac{a}{9} + 6b$$

$$-117 = -a + 54b$$

$$a = 117 + 54b$$

$$a = 117 + 54(-2)$$

$$a = 117 - 108$$

$$a = 9 \rightarrow \underline{\underline{Ans}}$$

$$-15 = \frac{a}{3} + b(3)^2$$

$$-45 = a + 27b$$

$$-45 = 117 + 54b + 27b$$

$$\frac{-162}{81} = \frac{81b}{81}$$

$$-2 = b \rightarrow \underline{\underline{Ans}}$$

At $x=1$, the gradient is = $\frac{-9}{1^2} + 2(-2)(1)$

$$= -9 - 4$$

$$= -13 \rightarrow \text{shown}$$

$$12. \quad y = \frac{a}{x} + bx$$

grad. of $y = \frac{2x+1}{x}$
 $m = 2$

$$\left. \frac{dy}{dx} \right| = -ax^{-2} + b$$

$$2 = -ax^{-2} + b \quad \text{at } x=1$$

$$2 = -\frac{a}{1^2} + b$$

$$2 = b - a$$

$$2 = 3 - a - a$$

$$2 = 3 - 2a$$

$$2a = 1$$

$$a = \frac{1}{2} \rightarrow \underline{\underline{\text{Ans}}}$$

$$y = \frac{a}{x} + bx$$

$$3 = \frac{a}{1} + b(1)$$

$$3 = a + b$$

$$3 - a = b$$

$$3 - \frac{1}{2} = b$$

$$\frac{5}{2} = b \rightarrow \underline{\underline{\text{Ans}}}$$

$$13. \quad f(x) = px^2 + qx$$

$$f(-2) = -2 \longrightarrow -2 = p(-2)^2 + q(-2)$$

$$\begin{cases} f'(2) = 3 \\ \left. \frac{dy}{dx} \right| = 2px + q \end{cases} \quad \begin{aligned} -2 &= 4p - 2q \quad ① \\ \frac{2q - 2}{4} &= p \quad \rightarrow \quad \underline{\underline{2(\frac{q}{2}) - 2}} = p \end{aligned}$$

$$\text{for } x=2, \quad 3 = 2p(2) + q$$

$$\frac{10}{3} - 2 = 4p$$

$$3 = 4p + q \quad ②$$

$$\frac{4}{3} = 4p$$

$$3 = 4\left(\frac{2q - 2}{4}\right) + q$$

$$\frac{4}{2} = p$$

$$3 = 2q - 2 + q$$

$$\frac{1}{3} = p$$

$$3 = 3q - 2$$

↪ Ans

$$\frac{5}{3} = 3q$$

$$\frac{5}{3} = q \rightarrow \underline{\underline{\text{Ans}}}$$

14. $y = 9x + \frac{1}{x}$ a) grad. of curve when $x = 2$

$$\left. \frac{dy}{dx} \right| = 9 - 1x^{-2}$$

$$= 9 - \frac{1}{x^2}$$

$$m = 9 - \frac{1}{2^2}$$

$$= 9 - \frac{1}{4}$$

$$= 8 \frac{3}{4}$$

$$m = \frac{35}{4} \rightarrow \underline{\text{Ans a)}$$

b) coordinates when $9 - \frac{1}{x^2} = 0$

$$9 - \frac{1}{x^2} = 0$$

$$9 = \frac{1}{x^2}$$

$$x^2 = \frac{1}{9}$$

$$x = \pm \frac{1}{3}$$

$$x = \frac{1}{3}$$

$$y = 9x + \frac{1}{x}$$

$$y = 9\left(\frac{1}{3}\right) + 3$$

$$y = 3 + 3$$

$$y = 6$$

$$(\frac{1}{3}, 6) \rightarrow \text{Ans b) 1.}$$

$$y = 9x + \frac{1}{x}$$

$$y = 9(-\frac{1}{3}) - 3$$

$$y = -3 - 3$$

$$y = -6$$

$$(-\frac{1}{3}, -6) \rightarrow \text{Ans b) 2}$$

15. $y = x^3 - 3x^2 - 9x + 11$

$$\left. \frac{dy}{dx} \right| = 3x^2 - 6x - 9$$

$$= 15$$

$$x = ?$$

$$15 = 3x^2 - 6x - 9$$

$$0 = 3x^2 - 6x - 24$$

$$0 = x^2 - 2x - 8$$

$$= x^2 - 4x + 2x - 8$$

$$= x(x-4) + 2(x-4)$$

$$= (x+2)(x-4)$$

$$x = -2 \quad \text{or} \quad x = 4$$

$$y = x^3 - 3x^2 - 9x + 11$$

$$= (-2)^3 - 3(-2)^2 - 9(-2) + 11$$

$$= -8 - 3(4) + 18 + 11$$

$$= -8 - 12 + 18 + 11$$

$$= -20 + 29$$

$$y = 9 \quad (-2, 9) \rightarrow \underline{\text{Ans}}$$

$$y = x^3 - 3x^2 - 9x + 11$$

$$= 4^3 - 3(4^2) - 9(4) + 11$$

$$= 64 - 3(16) - 36 + 11$$

$$= 64 - 48 - 36 + 11$$

$$y = -9$$

(4, -9) \rightarrow Ans

16. $y = x^2 + 2x$, $m=3$ at (a, b)

$$\left. \frac{dy}{dx} \right| = 2x + 2$$

$$3 = 2x + 2$$

$$1 = 2x$$

$$\frac{1}{2} = x$$

$$y = x^2 + 2x$$

$$= \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)$$
$$= \frac{1}{4} + 1$$

$$y = \frac{5}{4}$$

$$\left(\frac{1}{2}, \frac{5}{4}\right) \rightarrow \underline{\text{Ans}}$$

$$a = \frac{1}{2}, b = \frac{5}{4}$$

17. $y = ax + \frac{3}{x}$, $m=1$ at $x=2$

$$\left. \frac{dy}{dx} \right| = a - 3x^{-2}$$
$$1 = a - \frac{3}{(2)^2}$$
$$1 = a - \frac{3}{4}$$
$$\frac{7}{4} = a \rightarrow \underline{\text{Ans}}$$

$$1 = \frac{7}{4} - \frac{3}{x^2}$$

$$\frac{3}{x^2} = \frac{7}{4} - 1$$

$$\frac{3}{x^2} = \frac{3}{4}$$

$$x^2 = 4$$

$$x = \pm 2 \rightarrow x = 2 \rightarrow \underline{\text{Ans}}$$

↳ just take (+)
cuz we only need
one more point