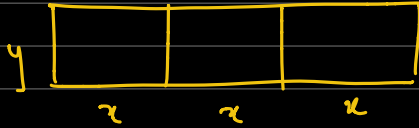# APPLIED DIFFERENTIATION

From Slide 73 worksheets (NG) Q2,3,4,5

2.

$$4y + 6x = 600$$
$$4y = 600 - 6x$$
$$y = \frac{600 - 6x}{4}$$

$$A = 3xy$$
$$A = 3x\frac{(600 - 6x)}{4}$$
$$A = \frac{1800x - 18x^2}{4}$$
$$A = 450x - 4.5x^2$$

$$\frac{dA}{dx} = 450 - 9x$$

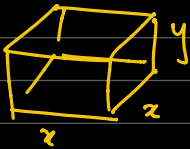$$\left.\frac{d^2A}{dx^2}\right| = -9$$

$$x = 50$$
$\hookrightarrow$ Hence, max

$$0 = 450 - 9x$$
$$9x = 450$$
$$x = 50 \longrightarrow \underline{Ans}$$

3.

$$x^2y = 32$$
$$y = \frac{32}{x^2}$$

$$P = 2x + 2y$$
$$P = 2x + 2\left(\frac{32}{x^2}\right)$$
$$P = 2x + \frac{64}{x^2}$$
$$P = 2x + 64x^{-2}$$

$$\left.\frac{d^2P}{dx^2}\right| = \frac{384}{x^3} = \frac{384}{64}$$

$$\Big|_{x=4} = 6$$
$\hookrightarrow$ min point
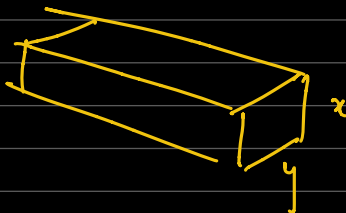
$$\frac{dP}{dx} = 2 - \frac{128}{x^3}$$

$$0 = 2 - \frac{128}{x^3}$$
$$\frac{128}{x^3} = 2$$
$$64 = x^3$$
$$4 = x \longrightarrow \underline{Ans}$$

4.

$$S = 5x^2y$$
$$S = 5x^2(60 - x)$$
$$S = 300x^2 - 5x^3$$

$$120 = 2(x + y)$$
$$60 = x + y$$
$$60 - x = y$$

$$\frac{dS}{dx} = 600x - 15x^2 \qquad \frac{d^2S}{dx^2} = 600 - 30x$$

$$= 600 - 30(40)$$
$$0 = 600x - 15x^2 \qquad = 600 - 1200$$
$$15x^2 = 600x \qquad = -600$$
$$15x = 600$$
$$x = 40 \longrightarrow \underline{Ans} \qquad \hookrightarrow Max$$

5.

$$2x \qquad x$$

$$4(2x) + 4(x) + 4h = 360$$
$$8x + 4x + 4h = 360$$
$$12x + 4h = 360$$
$$\frac{4h}{4} = \frac{360 - 12x}{4}$$
$$h = 90 - 3x$$

$$V = 2x^2(90 - 3x)$$
$$V = 180x^2 - 6x^3 \longrightarrow shown$$

$$\frac{dV}{dx} = 360x - 18x^2$$

$$\left.\frac{d^2V}{dx^2}\right| = 360 - 36x \quad = 360 - 36(20)$$
$$18x^2 = 360x \qquad\qquad = negative$$
$$18x = 360$$
$$x = 20 \qquad\qquad x = 20 \qquad \hookrightarrow max$$

$$h = 90 - 3(20)$$
$$= 90 - 60 \qquad\qquad Dimensions = 20 \times 40 \times 30$$
$$= 30 \qquad\qquad\qquad \hookrightarrow \underline{Ans}$$

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29.
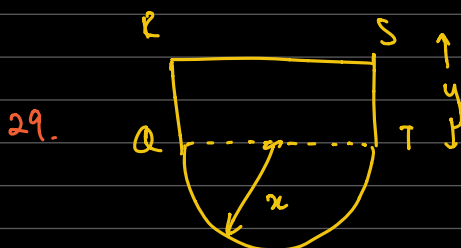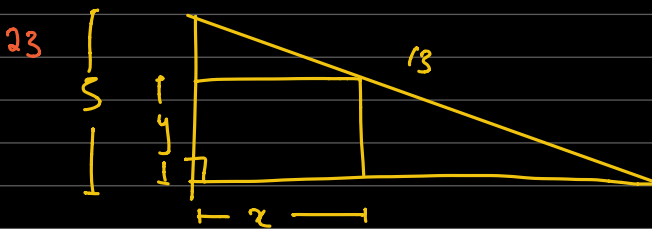
$$100 = 2x + 2y + \pi x$$

$$2y = 100 - 2x - \pi x$$
$$y = \frac{100 - 2x - \pi x}{2}$$

$$A = \frac{\pi x^2}{2} + 2xy$$

$$= \frac{\pi x^2}{2} + 2x\left(\frac{100 - 2x - \pi x}{2}\right)$$

$$= \frac{\pi x^2}{2} + 100x - 2x^2 - \pi x^2$$

$$A = 100x - 2x^2 - \frac{\pi x^2}{2} \rightarrow \text{shown}$$

$$\frac{dA}{dx} = 100 - 4x - \pi x$$

$$0 = 100 - 4x - \pi x$$
$$4x + \pi x = 100$$
$$(4 + \pi)x = 100$$
$$x = \frac{100}{\pi + 4}$$
$$x = 14$$

$$A = 100x - 2x^2 - \frac{\pi x^2}{2}$$
$$= 100(14) - 2(14)^2 - \frac{\pi(14)^2}{2}$$
$$= 1400 - 392 - 308$$
$$= 700 \rightarrow \text{shown}$$

34. a)

$5x$

$5x^2\sqrt{5}$

$10x$

$\sqrt{25x^2 + 100x^2} = \sqrt{125x^2}$

$= \sqrt{5 \times 5 \times 5 \times x^2}$

$= 5x\sqrt{5}$

$$\frac{5x\sqrt{5} \times 10x}{2} = \frac{50x^2\sqrt{5}}{2} \rightarrow \text{Ans}$$

b) $h = \frac{60 - 13x^2}{2x}$

$$1200 = \frac{4(50x^2\sqrt{5})}{2} + 10xh$$

HW: 34, 35, 20 – 29,
Slide 79, Q1 – 9

FROM Slide 79:

1. $L = \dfrac{4xy^2}{h}$ $h = 30$ $x + y = 15$

                                  $y = 15 - x$

$= \dfrac{4(x)(15-x)^2}{30}$

$= \dfrac{4(x)(225 - 30x + x^2)}{30}$

$= \dfrac{4(225x - 30x^2 + x^3)}{30}$

$= 36x - 4x^2 + \dfrac{4}{30}x^3$

$\dfrac{dL}{dx} = 30 - 8x + \dfrac{12}{30}x^2$

$0 = \dfrac{2x^2 - 8x + 30}{3}$

$0 = 2x^2 - 40x + 150$

$0 = x^2 - 20x + 75$

$0 = x^2 - 15x - 5x + 75$

$0 = x(x-15) - 5(x-15)$

$x = 5$ or $x = 15$

$\left.\dfrac{d^2L}{dx^2}\right|_{x=5} = \dfrac{4}{5}x - 8 = \dfrac{4(5)}{5} - 8$

$= 4 - 8$

$= -4$

$\hookrightarrow$ max point

$L = \dfrac{4x(15-x)^2}{30}$

$L = \dfrac{4(5)(15-5)^2}{30}$

$= \dfrac{2(10)^2}{3}$

$= \dfrac{2 \times 100}{3}$

$= 66.67 \longrightarrow$ Ans

2.
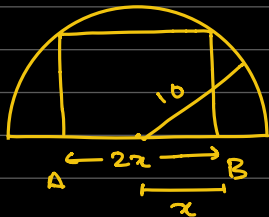
i) $\to \dfrac{10^2 - x^2}{\sqrt{100 - x^2}}$

$(A)^2 = (2x \times \sqrt{100 - x^2})^2$

$A^2 = 4x^2(100 - x^2) \to$ shown

$A^2 = 400x^2 - 4x^4$

$\dfrac{dA^2}{dx} = 800x - 16x^3$

$0 = 800x - 16x^3$

$16x^3 = 800x$
$16x^2 = 800$
$x^2 = 50$
$x = 7.07 \to$ Ans

$A^2 = 4x^2(100 - x^2)$
$A^2 = 4(50)(100 - 50)$
$A^2 = 200 \times 50$
$A^2 = 10000$
$A = 100 \longrightarrow$ Ans

3.

$V = \pi r^2 h + \dfrac{2}{3}\pi r^3$

$45\pi = \pi\left(r^2 h + \dfrac{2}{3}r^3\right)$

$45 = r^2 h + \dfrac{2}{3}r^3 \to$ shown

$A = 2\pi rh + \pi r^2 + 2\pi r^2$
$\quad = 2\pi rh + 3\pi r^2$

$\quad = 2\pi r\left(\dfrac{45}{r^2} - \dfrac{2}{3}r\right) + 3\pi r^2$

$\quad = \dfrac{90\pi}{r} - \dfrac{4\pi r^2}{3} + 3\pi r^2$

$\quad = \dfrac{90\pi}{r} - \dfrac{4\pi r^3 + 9\pi r^2}{3}$

$\dfrac{45 - \dfrac{2}{3}r^3}{r^2} = h$

$\dfrac{45}{r^2} - \dfrac{2r^3}{3r^2} = h$

$\dfrac{45}{r^2} - \dfrac{2}{3}r = h$

$$A = \frac{90\pi}{r} + \frac{5\pi r^2}{3} \qquad \rightarrow \text{shown}$$

4.