DYNAMICS

Newton's First Law (Law of inertia)

0 lvls. { If the resultant force is 0, that is, if forces are balanced, then an object at rest will stay at rest and objects in motion will continue moving at the same velocity. [Object maintains it's state]

Newton's Second Law

Force is equal to rate of change of momentum

[F = ma] → final result obtained from the second law

Force : rate of change of momentum

MOMENTUM

- Symbol p
- vector quantity (direction is important)
- Defined as a product of the mass and velocity of an object
- Formula [p = mv]
- Units : [$kgms^{-1}$] or [Ns]

From Newton's 2nd law = $F = \frac{\text{change in momentum}}{\text{time}}$

$$\left[F = \frac{\Delta p}{t}\right] \quad \text{OR} \quad [\underbrace{Ft}_{\text{Impulse}} = \Delta p]$$

Derivation of [f = ma] from the above formulas

$F = \frac{p_f - p_i}{t}$ where $p_f = mv$ and $p_i = mu$

$$F = \frac{mv - mu}{t}$$

$$F = \frac{m(v-u)}{t}$$

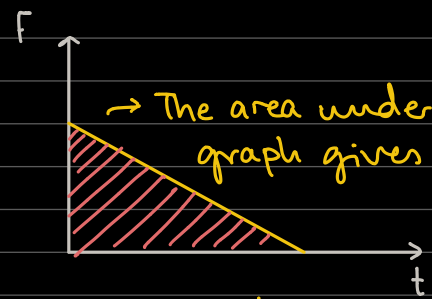
$$F = m\boxed{\frac{(v-u)}{t}}$$

$F = m\underline{a}$ → derived

Impulse : Impulse is defined as the product of the force and the time for which it acts

[Ft] or [Δp]

or [m(v−u)]

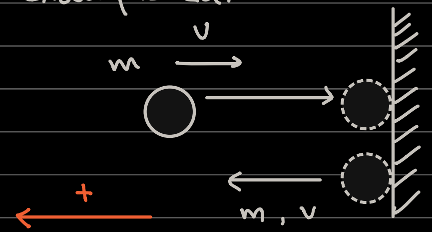


→ The area under a F vs. t graph gives Impulse

Units - Ns or $kgms^{-1}$

- How to calculate change in momentum (Δp)

Example 1a.



Q. Given that the ball rebounds elastically, calculate the change in momentum.

$$\Delta p = p_f - p_i$$
$$= mv - m(-v)$$
$$= mv + mv$$
$$\Delta p = 2mv$$

(taking left to be the positive direction)

Example 1b (same case as above)

Q. Calculate the Δp if the ball does not rebound / rebounds perfectly inelastically.

$$\Delta p = p_f - p_i$$
$$= mv - mv$$
$$= m(0) - m(-v)$$
$$\Delta p = mv$$

(taking left to be the positive direction

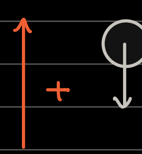
Example 1c (same case as above)

Q. Write down the range of Δp if ball rebounds <u>inelastically</u>.

Range : $mv < \Delta p < 2mv$ (taking left as the positive direction)

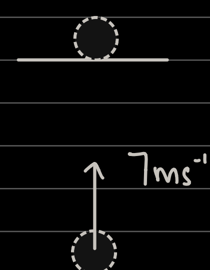
"Somewhere between stopping completely and rebounding perfectly elastically."

Example 2a.



Q1. Calculate the final velocity as the ball hits the ground.

$$v = u + at$$
$$v = 4.3 + (9.81)(1.51)$$
$$v = 19ms^{-1}$$



Q2. Given that it rebounds at a speed of $7ms^{-1}$, calculate the change in momentum during impact.

$$\Delta p = p_f - p_i$$
$$= mv - mv$$
$$= (1)(7) - (1)(-19)$$

(taking "up" to be the positive direction)

$= 7 + 19$

$\Delta p = 26 \text{ Ns}$

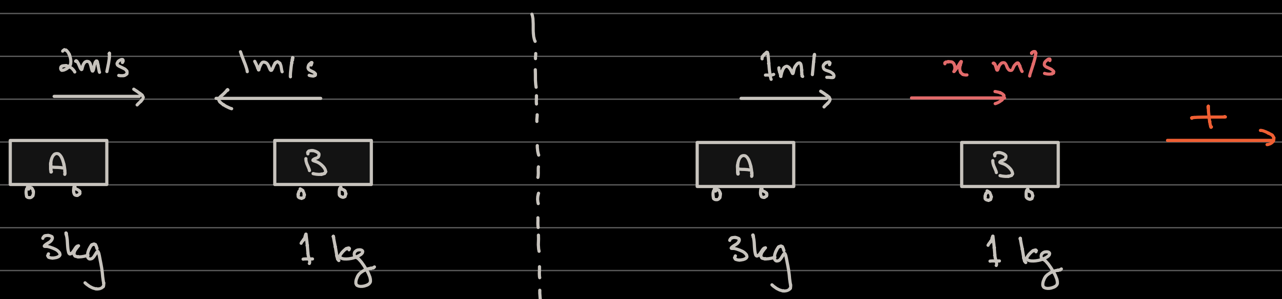
Q3. Time of impact is 12.5ms. Calculate the force exerted on the ground.

$$F = \frac{\Delta p}{t} = \frac{26}{0.0125 s} = 2080 N$$

PRINCIPLE / LAW OF CONSERVATION OF MOMENTUM

- According to this principle, the total momentum of the system always remains constant / conserved, provided that there is no external force acting on the system.
- This principle is also applicable for colliding bodies.
  - ↳ In case of collision, the total momentum of the system before collision must be equal to the total momentum of the system after the collision

Example



Before Collision | After collision.

Q. Apply the law of conservation to find x.

$$p \text{ Before collision} = p \text{ After collision}$$
$$mv + mv = mv + mv$$
$$(3)(2) + (1)(-1) = (3)(1) + (1)x$$
$$6 - 1 = 3 + x$$
$$2 = x$$

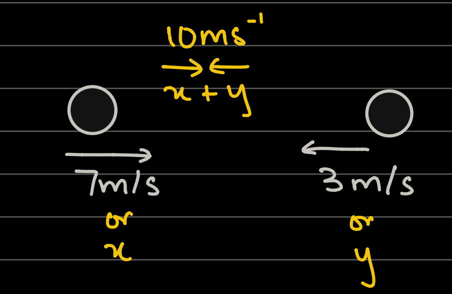
$\therefore, x = 2 m/s.$

- The nature of collision between two bodies can be classified either as
  1. elastic collision
  2. inelastic collision

Properties of elastic collisions:

- The momentum of the system remains conserved
- The kinetic energy of the system remains conserved
- The total energy of the system remains conserved
- The speed of approach before collision is equal to the speed of separation after the collision

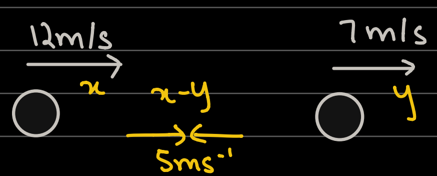
Examples:
(Approach)



Speed of approach b/w. these two

S.O.A = $10ms^{-1}$

S.O.A = $x+y$

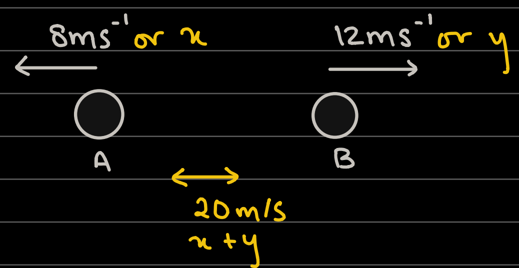


S.O.A $= x - y$

$= 12 - 7$

$= 5ms^{-1}$

Examples:
(Separation)

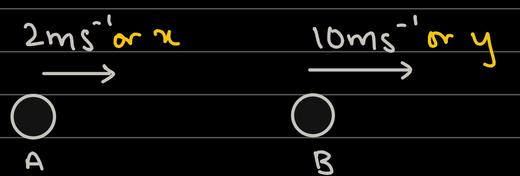


⇒ Speed of separation

S.O.S = 12 + 8

$= 20ms^{-1}$

$= x + y$

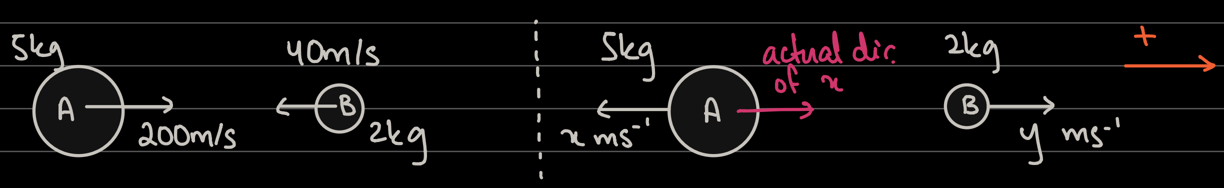


S.O.S = 10 − 2

$= 8ms^{-1}$

$= y - x$

Example Application of properties of elastic collisions to solve questions



Before Collision

After collision

The collision is perfectly elastic

Q1. Form an equation based on the speed of approach being equal to the speed of separation.

$$\text{SOA (before collision)} = \text{SOS (after collision)}$$
$$200 + 40 = x + y$$
$$① \; 240 = x + y \rightarrow \text{equation formed}$$

Q2. Form an equation based on the principle of conservation of momentum

(taking right as the positive direction)

$$(5)(200) + (2)(-40) = (5)(-x) + (2)(y)$$
$$1000 - 80 = 2y - 5x$$
$$② \; 920 = 2y - 5x \rightarrow \text{equation formed}$$

Q3. Solve the two equations simultaneously to find the values of x and y.

① $240 = x + y$ $\qquad 240 - y = x$

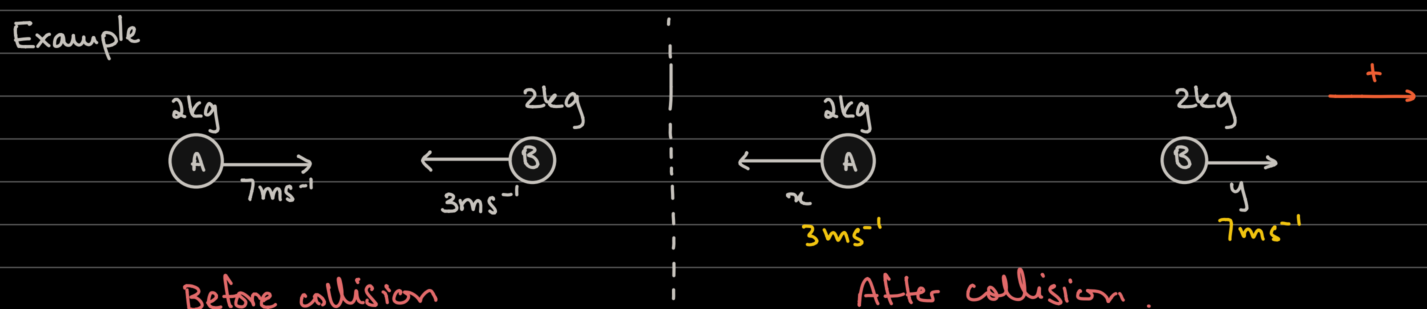
② $920 = 2y - 5x$

$$920 = 2y - 5(240 - y)$$
$$920 = 2y - 1200 + 5y$$
$$920 + 1200 = 7y$$
$$2120 = 7y$$
$$[302.9 = y]$$

$$240 = x + y$$
$$240 = x + 302.9$$
$$240 - 302.9 = x$$
$$[-62.9 = x]$$

* Since the values for x and y, the way we solved it, give us the speed of A and B in opposite directions to each other and to their initial direction, a negative value for x implies that A did not, in fact, change direction, but instead continued to move in the same direction but at a reduced speed, as a result of the collision.

Example



Q. Given that it is an elastic collision, form two equations and solve them simultaneously to obtain the values for x and y.

S.O.A = S.O.S

$7+3 = x+y$

$10 = x+y$

total p = total p

$mv + mv = mv + mv$

$(2)(7)+(2)(-3) = 2y - 2x$

$14 - 6 = 2y - 2x$

$8 = 2y - 2x$

$8 = 2(10-x) - 2x$

$8 = 20 - 2x - 2x$

$4x = 12$

$x = 3$

$10 = x+y$

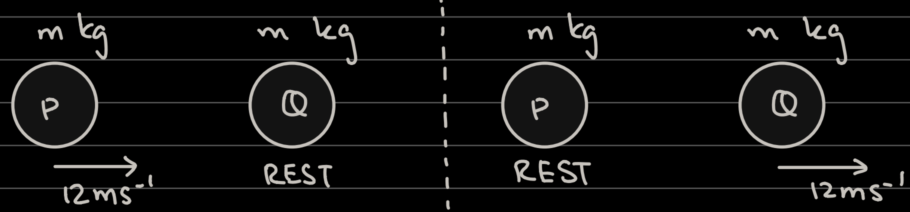
$10 - 3 = y$

$7 = y$

Conclusion: For identical masses performing a perfectly elastic collision, the speeds will be "interchanged"

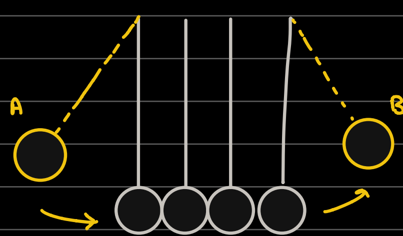
ie. initial speed of A becomes the final speed of B and the initial speed of B becomes the final speed of A.

Example:



Before collision After collision

Another example



Note: momentum of the first ball transfers into the last ball hence all balls in the middle remain at rest, only the first and last ball moves back and forth.

↳ This is called Newton's cradle

INELASTIC COLLISIONS

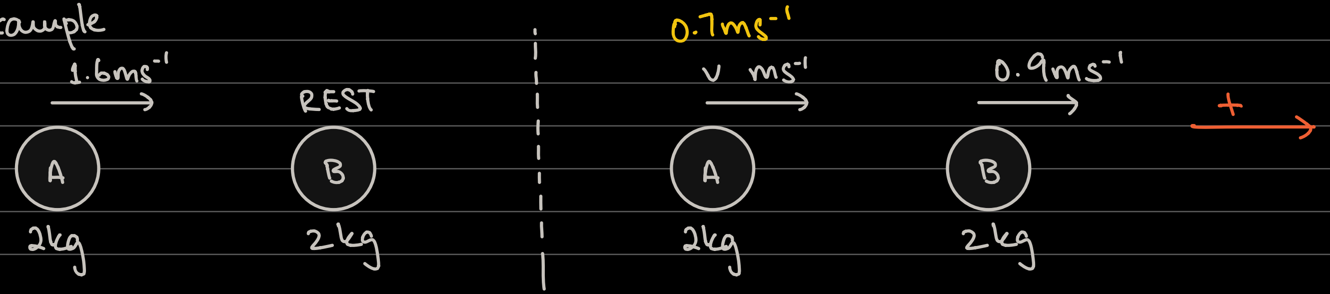
Properties of inelastic collisions:

- Momentum of the system remains conserved
- Total energy of the system remains conserved
- * Since there is a "loss" of kinetic energy, therefore the kinetic energy of the system is NOT conserved.

↳ The K.E after collision will be less than the K.E before the collision

- S.O.A ≠ S.O.S

Example



i) Calculate the value of v.

Principle of conservation of momentum

$$(2)(1.6) + (2)(0) = 2v + (2)(0.9)$$
$$3.2 = 2v + 1.8$$
$$3.2 - 1.8 = 2v$$
$$1.4 = 2v$$
$$0.7 = v$$

ii) Show that this is an inelastic collision

Method 1 (Using kinetic energy)

K.E before collision

$$\frac{1}{2}(2)(1.6)^2 + 0 = \underline{\underline{2.56\,J}}$$

K.E after collision

$$\frac{1}{2}(2)(0.7)^2 + \frac{1}{2}(2)(0.9)^2$$
$$= \underline{\underline{1.3\,J}}$$

Since the kinetic energy before and after the collision is not the same, it can be said that the collision is inelastic.

Method 2 : Using speeds of approach / separation

S.O.A $= \underline{\underline{1.6\,ms^{-1}}}$

S.O.S $= 0.9 - 0.7$
$= \underline{\underline{0.2\,ms^{-1}}}$

Since the S.O.A is not equal to the S.O.S, it can be said that the collision is inelastic.

Example

20 ms^{-1} | v ms^{-1}

14.3 ms^{-1}

Q. Given that the particles join up after collision, and they move with a common velocity v, calculate the value of v and determine whether the collision is elastic or inelastic?

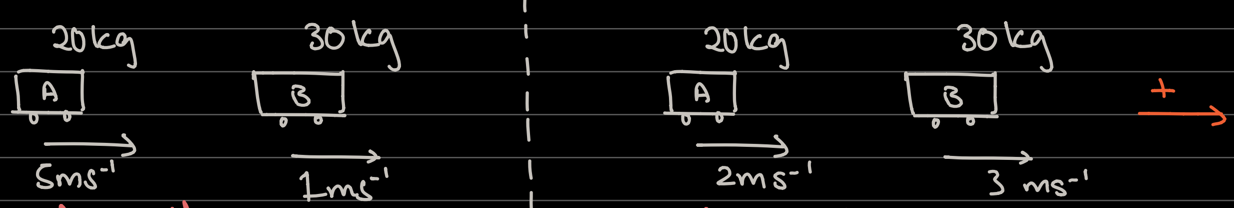
Principle of Conservation of Momentum

$$(5)(20) + (2)(0) = (7)v$$
$$100 = 7v$$
$$14.3 = v$$

SOA ≠ SOS because there is no speed of separation, as the two masses are stuck together, therefore the nature of the collision can be called inelastic.

Note: whenever two objects join up and move together with a common velocity, we can conclude without working that the nature of collision will be inelastic

Example



Before collision | After collision

i) Calculate change in momentum, but only for trolley A.

$$p_f - p_i$$
$$= mv - mu$$
$$= (20)(2) - (20)(5)$$
$$= 40 - 100$$
$$\Delta p = -60 \text{ Ns}$$

ii) Calculate Δp, but only for trolley B.

$$p_f - p_i$$
$$= mv - mu$$
$$= (30)(3) - (30)(1)$$
$$= 90 - 30$$
$$\Delta p = 60 \text{ Ns}$$

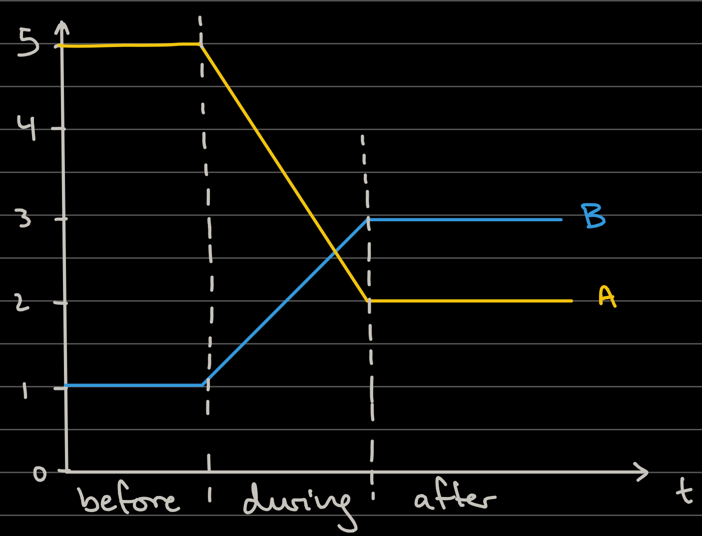
(iii) Using your answers to part (i) and (ii), show that the momentum of the system remains conserved

Ans. The loss in momentum of A is equal to the gain in momentum of B, thus the net change in momentum is 0. Since change in momentum is zero, it can be said that the momentum was conserved.

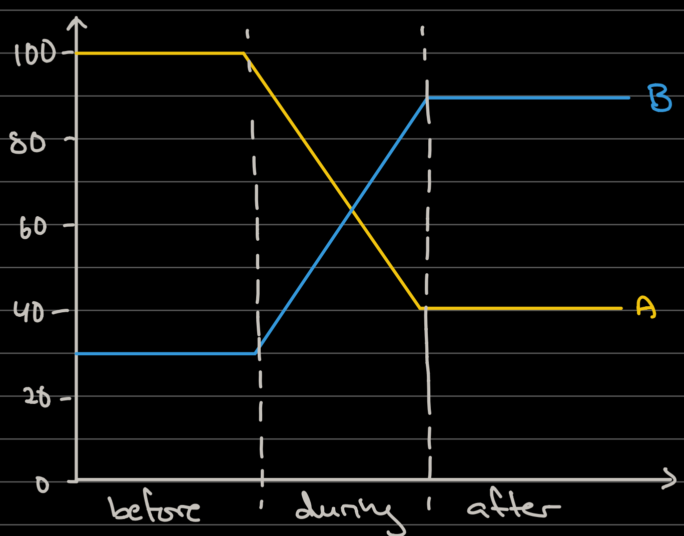
in mathematical terms

$\Delta P_A = -\Delta P_B$ [change in momentum of A is equal and opposite to the change in momentum of B]

iv) Sketch velocity/time graph before, during, and after the collision.



v) Sketch momentum vs. time graph before, during, and after collision



vi) Show that during collision, force which A applies on B is equal and opposite to the force which B applies on A.

Ans. From the previous working(s), it has been established that change in momentum of A is equal and opposite to the change in momentum of B.

$\Delta P_A = -\Delta P_B$

Since $\Delta p = Ft$ ↓ ↓

$F_A \times t_A = -F_B \times t_B$

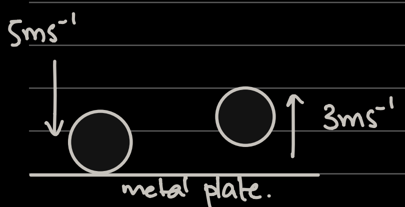
Since $t_A = t_B$, we can cancel them out

hence, $F_A = -F_B \rightarrow$ proved

[Newton's 3rd Law] →
- The two forces are equal in magnitude and opposite in direction
- The two forces act on different bodies ie. one force acts on "A" while the other force acts on "B"

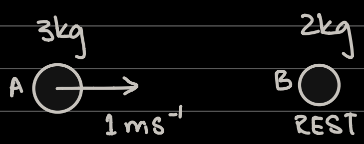
Example Q on →

Q. A ball falls vertically and strikes a metal plate. It rebounds from the plate as shown. Explain how the Principle of Conservation of momentum applies in this case. [3]

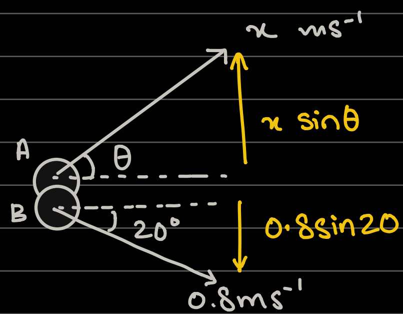


- The ball loses momentum upon impact, ie. momentum of the ball changes
- This loss in momentum of the ball is transferred/ gained by the metal plate
- ∴, according to the law of conservation of momentum, the total momentum of the system is conserved

Example (Two Dimensional Application).



Before collision



After collision

i) Apply the Law of Conservation of Momentum to find θ and x

In the horizontal plane :

$$(3)(1) + 2(0) = (3)(x\cos\theta) + 2(0.8\cos 20)$$
$$3 = 3x\cos\theta + 1.5$$
$$1.5 = (3)(x\cos\theta)$$
$$0.5 = x\cos\theta \rightarrow \text{equation 1}$$

In the vertical plane :

$$0 + 0 = (3)(x\sin\theta) + (2)(-0.8\sin 20)$$
$$0 = (3)(x\sin\theta) - 0.55$$
$$0.55 = (3)(x\sin\theta)$$
$$0.18 = x\sin\theta \rightarrow \text{equation 2.}$$

Solving the two equations simultaneously

① $x\cos\theta = 0.5$
② $x\sin\theta = 0.18$

$$\left(\frac{0.5}{\cos\theta}\right)\sin\theta = 0.18$$

$$0.5\,\frac{\sin\theta}{\cos\theta} = 0.18$$

$$0.5\tan\theta = 0.18$$

$$\theta = 20^\circ \rightarrow \underline{\underline{\text{Ans}}}$$

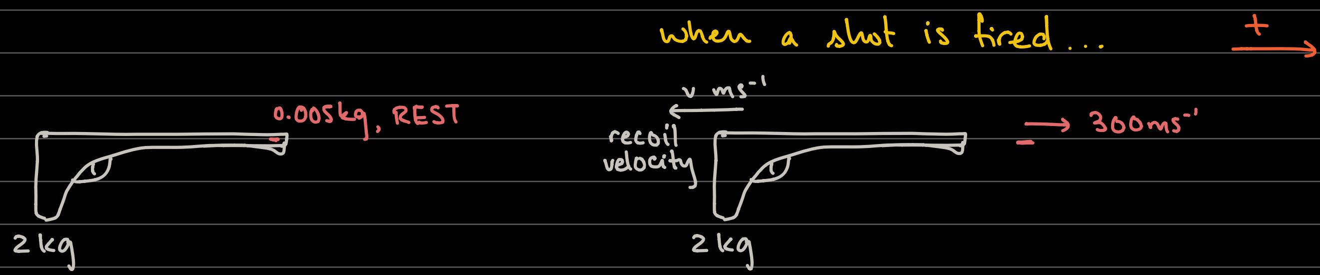
$$x\sin(20) = 0.18$$

$$x = \frac{0.18}{\sin(20)}$$

$$x = 0.53\,ms^{-1} \rightarrow \underline{\underline{\text{Ans}}}$$

How to apply Law of Conservation of Momentum in cases where the initial momentum of the system is zero

when a shot is fired ...



Reason: Since the initial momentum of the system is zero, therefore, for the law of conservation to be valid, the final momentum of the system must also remain 0.

This is only possible if the 2 bodies have equal momentum in the opposite direction so that they cancel out the effect of each other.
∴, when the bullet goes forward, the gun recoils backwards with equal momentum

Q. Calculate the speed v.

Principle of Conservation of Momentum

$$\Delta p_f = \Delta p_i$$

$$0 + 0 = (0.005)(300) + (2)(-v)$$

$$0 = 1.5 - 2v$$

$$-1.5 = -2v$$

$$0.75 = v \qquad \therefore,\ v = 0.75\,ms^{-1}$$

GENERAL FORMULA

$$mv = MV$$

where V = velocity of gun
M = mass of gun
v = velocity of bullet
m = mass of bullet

The above working can also be done using the ratio method

According to $p = mv$, mass and velocity are inversely proportional to each other

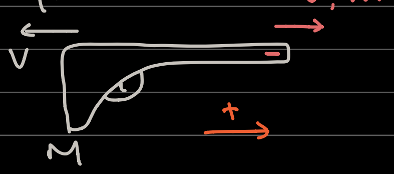
Hence :

$M = 2kg$ $\quad$ $m = 0.005kg$

Since mass of gun is "400 times" heavier than the mass of bullet, due to inverse relationship, the velocity of the gun might be 400 times lesser than the velocity of the bullet.

$$\frac{300}{400} = 0.75 \, m/s \rightarrow \text{Ans}$$

Example



(i) Show that $\frac{M}{m} = \frac{v}{V}$

Ans. Based on the law of conservation of momentum

$$0 + 0 = (m)(v) + (M)(-V)$$
$$MV = mv$$
$$\frac{M}{m} = \frac{v}{V} \rightarrow \text{shown}$$

(ii) Show that

$$\frac{\text{K.E of Gun}}{\text{K.E of bullet}} = \frac{V}{v}$$

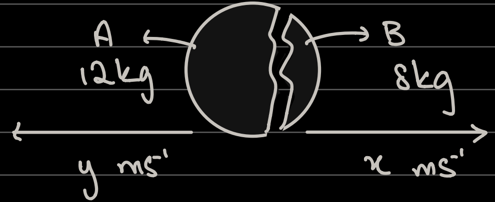
$$\frac{\text{K.E of Gun}}{\text{K.E of bullet}} = \frac{\frac{1}{2} M \cdot V \cdot V}{\frac{1}{2} m \cdot v \cdot v}$$ [from (i) we proved that $MV = mv$]

$$= \frac{V}{v}$$ → This answer suggests that the ratio of velocities determines the ratio of Kinetic Energy's.

Other examples where initial momentum is zero

Ex 1 : "A bomb before explosion"

After explosion, it breaks up into fragments



Ratio of masses: $\frac{M_A}{M_B} = \frac{12}{8}$ $\quad$ Hence, the ratio of velocities : $\frac{V_A}{V_B} = \frac{8}{12}$

Since kinetic energy is directly proportional to the velocity: $\frac{KE_A}{KE_B} = \frac{8}{12}$

Given that the total kinetic energy of the system is E, find, in terms of E, the fraction of kinetic energy....

i) possessed by A $= \frac{8}{20}E$

ii) possessed by B $= \frac{12}{20}E$