Université de Sousse
Polytechnique Sousse: Cycle Préparatoire

**A.U: 2019-2020**
Section: 1 PC

# Développements limités usuels en 0

**Formule de Taylor-Young :** Si f est une fonction de classe $C^n$ au voisinage de 0, alors elle admet un développement limité à l'ordre $n$, donné par

$$f(x) = f(0) + xf'(0) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + o(x^n) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k + o(x^n).$$

$$\begin{aligned}
(1+x)^a &= 1 + ax + \frac{a(a-1)}{2!}x^2 + \cdots + \frac{a(a-1)(a-2)...(a-n+1)}{n!}x^n + o(x^n) \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n) \\
\frac{1}{1+x} &= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n) \\
\ln(1-x) &= -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} + o(x^n) \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n) \\
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + o(x^n) \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots + \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!}x^{2n-1} + o(x^{2n}) \\
\operatorname{ch} x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
\operatorname{sh} x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
\operatorname{th} x &= x - \frac{x^3}{3} + \frac{2x^5}{15} + o(x^5) \\
\arcsin x &= x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3\cdot x^5}{2\cdot 4\cdot 5} + \cdots + \frac{1\cdot 3\cdot 5\cdots(2n-1)x^{2n+1}}{2\cdot 4\cdot 6\cdots(2n)\cdot(2n+1)} + o(x^{2n+2}) \\
\arccos x &= \frac{\pi}{2} - x - \frac{x^3}{2\cdot 3} - \frac{1\cdot 3\cdot x^5}{2\cdot 4\cdot 5} - \cdots - \frac{1\cdot 3\cdot 5\cdots(2n-1)x^{2n+1}}{2\cdot 4\cdot 6\cdots(2n)\cdot(2n+1)} + o(x^{2n+2}) \\
\operatorname{Argsh} x &= x - \frac{x^3}{2\cdot 3} - \cdots + (-1)^n \frac{1\cdot 3\cdot 5\cdots(2n-1)x^{2n+1}}{2\cdot 4\cdot 6\cdots(2n)\cdot(2n+1)} + o(x^{2n+2}) \\
\operatorname{Arctan} x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2}) \\
\operatorname{Argth} x &= x + \frac{x^3}{3} + \cdots + \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})
\end{aligned}$$

# Fonctions usuelles

$$\begin{aligned}
\sin^2(x) &+ \cos^2(x) = 1\\
\cos(2x) &= \cos^2(x) - \sin^2(x)\\
\sin(2x) &= 2\sin(x)\cos(x)\\
\cos^2(x) &= \frac{1+\cos(2x)}{2}\\
\sin^2(x) &= \frac{1-\cos(2x)}{2}\\
\tan(2x) &= \frac{2\tan(x)}{1-\tan^2(x)}
\end{aligned}$$

$$\begin{aligned}
\cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b)\\
\cos(a-b) &= \cos(a)\cos(b) + \sin(a)\sin(b)\\
\sin(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b)\\
\sin(a-b) &= \sin(a)\cos(b) - \cos(a)\sin(b)\\
\tan(a+b) &= \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}\\
\tan(a-b) &= \frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}
\end{aligned}$$

$$\boxed{\begin{aligned}
\cos(\arccos(x)) &= x \quad \forall x\in[-1,1]\\
\arccos(\cos(x)) &= x \quad \forall x\in[0,\pi]\\
\arccos'(x) &= \tfrac{-1}{\sqrt{1-x^2}} \quad \forall x\in\,]-1,1[
\end{aligned}}$$

$$\boxed{\begin{aligned}
\sin(\arcsin(x)) &= x \quad \forall x\in[-1,1]\\
\arcsin(\sin(x)) &= x \quad \forall x\in\left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right]\\
\arcsin'(x) &= \tfrac{1}{\sqrt{1-x^2}} \quad \forall x\in\,]-1,1[
\end{aligned}}$$

$$\boxed{\begin{aligned}
\tan(\arctan(x)) &= x \quad \forall x\in\mathbb{R}\\
\arctan(\tan(x)) &= x \quad \forall x\in\left]-\tfrac{\pi}{2},\tfrac{\pi}{2}\right[\\
\arctan'(x) &= \tfrac{1}{1+x^2} \quad \forall x\in\mathbb{R}
\end{aligned}}$$

$$\boxed{\begin{aligned}
\sin(\arccos(x)) &= \cos(\arcsin(x)) = \sqrt{1-x^2}\\
&\quad \forall x\in[-1,1]
\end{aligned}}$$

$$\mathrm{ch}(x) = \frac{e^x+e^{-x}}{2} \qquad \mathrm{sh}(x) = \frac{e^x-e^{-x}}{2}$$

$$\begin{aligned}
\mathrm{ch}^2(x) &- \mathrm{sh}^2(x) = 1\\
\mathrm{ch}(2x) &= \mathrm{ch}^2(x) + \mathrm{sh}^2(x)\\
\mathrm{sh}(2x) &= 2\,\mathrm{sh}(x)\,\mathrm{ch}(x)\\
\mathrm{ch}^2(x) &= \frac{\mathrm{ch}(2x)+1}{2}\\
\mathrm{sh}^2(x) &= \frac{\mathrm{ch}(2x)-1}{2}\\
\mathrm{th}(2x) &= \frac{2\,\mathrm{th}(x)}{1+\mathrm{th}^2(x)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{sh}'(x) &= \mathrm{ch}(x)\\
\mathrm{ch}'(x) &= \mathrm{sh}(x)\\
\mathrm{th}'(x) &= 1-\mathrm{th}^2(x) = \frac{1}{\mathrm{ch}^2(x)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{ch}(a+b) &= \mathrm{ch}(a)\,\mathrm{ch}(b) + \mathrm{sh}(a)\,\mathrm{sh}(b)\\
\mathrm{sh}(a+b) &= \mathrm{sh}(a)\,\mathrm{ch}(b) + \mathrm{ch}(a)\,\mathrm{sh}(b)\\
\mathrm{th}(a+b) &= \frac{\mathrm{th}(a)+\mathrm{th}(b)}{1+\mathrm{th}(a)\,\mathrm{th}(b)}
\end{aligned}$$

$$\boxed{\begin{aligned}
\mathrm{Argch}(\mathrm{ch}(x)) &= x \quad \forall x\in[0,+\infty[\\
\mathrm{ch}(\mathrm{Argch}(x)) &= x \quad \forall x\in[1,+\infty[\\
\mathrm{Argch}'(x) &= \tfrac{1}{\sqrt{x^2-1}} \quad \forall x\in\,]1,+\infty[\\
\mathrm{Argch}(x) &= \ln\left(x+\sqrt{x^2-1}\right)\\
&\quad \forall x\in[1,+\infty[
\end{aligned}}$$

$$\boxed{\begin{aligned}
\mathrm{Argsh}(\mathrm{sh}(x)) &= x \quad \forall x\in\mathbb{R}\\
\mathrm{sh}(\mathrm{Argsh}(x)) &= x \quad \forall x\in\mathbb{R}\\
\mathrm{Argsh}'(x) &= \tfrac{1}{\sqrt{1+x^2}} \quad \forall x\in\mathbb{R}\\
\mathrm{Argsh}(x) &= \ln\left(x+\sqrt{x^2+1}\right) \quad \forall x\in\mathbb{R}
\end{aligned}}$$

$$\boxed{\begin{aligned}
\mathrm{Argth}(\mathrm{th}(x)) &= x \quad \forall x\in\mathbb{R}\\
\mathrm{th}(\mathrm{Argth}(x)) &= x \quad \forall x\in\,]-1,1[\\
\mathrm{Argth}'(x) &= \tfrac{1}{1-x^2} \quad \forall x\in\,]-1,1[\\
\mathrm{Argth}(x) &= \tfrac{1}{2}\ln\left(\tfrac{x+1}{1-x}\right) \quad \forall x\in\,]-1,1[
\end{aligned}}$$