LMECA2550 - Propulsion : First Homework

Victor COLOGNESI (82151100)

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1 Induction factors

Each blade element produces a thrust force and a torque. The force and the torque can both be calculated two different ways : on the first hand, we can use aerodynamics and the definitions of the lift and drag coefficients, and on the other hand, we can use momentum conservation. By comparing the two expressions derived for both the thrust and the moment, we can produce two equations that let us find the induction factors.

1.1 Thrust force equations

The thrust force provided by a blade element is given by :

$$dT = \frac{1}{2}\rho\lambda_1||v||^2 cdr = \frac{1}{B}2\pi r\rho\left(V + u_{x,0}\right)2u_{x,0}dr$$

where the first expression comes from aerodynamics and the second one from momentum.

Simplifying the terms present on both sides and using the definition of the blade solidity ($s = \frac{Bc}{2\pi r}$) gives :

$$\frac{u_{x,0}\left(V + u_{x,0}\right)}{||v||^2} = \frac{1}{4}s\lambda_1$$

By using the speed triangle at $x = 0$, we get

$$||v||^2 = \left(\frac{V + u_{x,0}}{\sin\phi}\right)^2$$

By replacing this in the previous equation and using the definition of the induction factor $a = \frac{u_{x,0}}{V}$, we get the following equation :

$$\frac{u_{x,0}\sin^2\phi}{V + u_{x,0}} = \frac{a\sin^2\phi}{1+a} = \frac{1}{4}s\lambda_1$$

Finally, accounted that $\sin^2\phi = \frac{1-\cos(2\phi)}{2}$, we get the expected equation :

$$\frac{a}{1+a} = \frac{s\lambda_1}{2(1-\cos(2\phi))}$$

1.2 Torque equations

The torque provided by a blade element is given by :

$$dQ = \frac{1}{2}\rho\lambda_2||v||^2 crdr = \frac{1}{B}2\pi r\rho\left(V + u_{x,0}\right)2\omega_0 r^2 dr$$

where the first expression comes from aerodynamics and the second one from momentum.

Simplifying the terms present on both sides and using the definition of the blade solidity ($s = \frac{Bc}{2\pi r}$) gives :

$$\frac{\omega_0 r\,(V + u_{x,0})}{||v||^2} = \frac{1}{4}s\lambda_2$$

By using the speed triangle at $x = 0$, we get :

$$||v||^2 = \left(\frac{V + u_{x,0}}{\sin\phi}\right)\left(\frac{\Omega r - u_{\theta,0}}{\cos\phi}\right)$$

By replacing this in the previous equation, considering the definition $u_\theta = \omega r$ and using the definition of the induction factors $a' = \frac{\omega_0}{\Omega}$, we get the following equation :

$$\frac{\omega_0 r\sin\phi\cos\phi}{\Omega r - \omega r} = \frac{a'\sin\phi\cos\phi}{1 - a'} = \frac{1}{4}s\lambda_2$$

Finally, accounted that $\sin\phi\cos\phi = \frac{\sin(2\phi)}{2}$, we get the expected equation :

$$\frac{a'}{1 - a'} = \frac{s\lambda_2}{2\sin(2\phi)}$$

2 Blade Elements Momentum : application

2.1 Method for the coding

To solve the equations for the induction factors, we use the given method and hence the equations :

$$\begin{cases} a_{new} &= (1 + a)\dfrac{s\lambda_1}{2(1 - \cos(2\phi))} \\ a'_{new} &= (1 - a')\dfrac{s\lambda_2}{2\sin(2\phi)} \end{cases}$$

and the use of under-relaxation :

$$a^k = (1 - \omega)a^{k-1} + \omega a_{new}$$

with $\omega = 0.5$.

So we first wrote a routine that solves the equations for the induction factors. We use fixed point iterations with the function presented here above. We stop when the iterations converge (we know that if the iterations converge then we have a solutions of the equation) and that the relative difference between two steps is smaller than a fixable tolerance or when reaching a maximum number of iterations, producing a warning in the latter case.

The we used this routine to compute the induction factors in the case of a given propeller. For a given blade configuration and advance ratio $J = \frac{V}{nD}$, we compute the induction factors on different points of the blade (we use roughly 50 points on the blade : the length of a blade element is a 50^{th} of the propeller radius and we only compute the factors on the blade itself). After computing the induction factors, we can compute the incidence angle, the factors λ_1 and λ_2 and the values $\frac{dT}{dr}$ and $\frac{dQ}{dr}$ on every point. Then, we can compute the contributions of each blade element to the thrust force and the torque with :

$$\begin{cases} dT_{r,r+dr} &= \dfrac{dr}{2}\left(\dfrac{dT}{dr}(r) + \dfrac{dT}{dr}(r + dr)\right) \\ dQ_{r,r/dr} &= \dfrac{dr}{2}\left(\dfrac{dQ}{dr}(r) + \dfrac{dQ}{dr}(r + dr)\right) \end{cases}$$

The sum of all these terms gives the total thrust force and torque. We can then compute the coefficients k_T, k_Q, k_P and the yield η_P.

When we compute the coefficients for multiple values of the advance ratio J, we each time use as starting values of a and a' for a given radius the values obtained for the previous value of J. Then when the value of k_T goes negative, we stop the iterations and change the value of J.

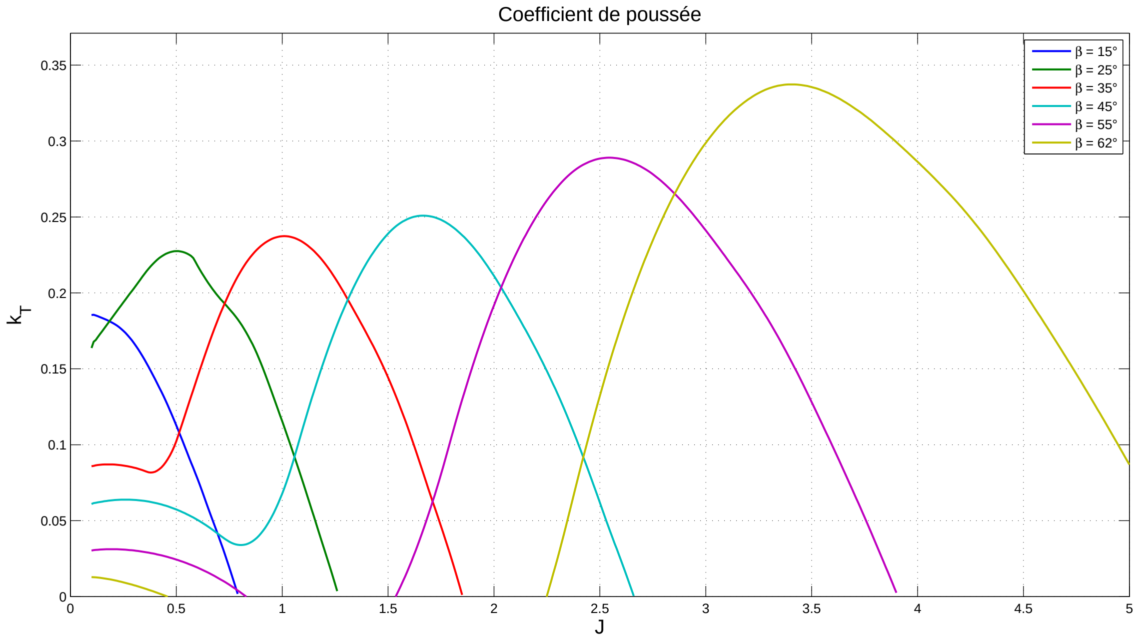

Figure 1: Thrust coefficient with respect to the advance ratio for multiple blade orientations

2.2 Results

Using this method allowed us to compute the curves for the thrust coefficient k_T, the torque coefficient k_Q, the power coefficient k_P and the propulsion yield η_P with respect to the advance ratio J for multiple orientations of the blades (the orientation of the blade is defined using the angle β between the chord of the blade and the airspeed at $r = 0.75R$). These curves are represented in the figures 1 to 4.

The first remark to make about the effect of the orientation of the blade is that it forces the use of higher advance ratio's and hence lower propeller rotation speed. For low values of β, the thrust coefficient k_T goes negative for high values of J. For higher values of β, k_T goes down for lower values of J and even negative for the highest values of the blade angle. Concerning the torque coefficient k_Q, we observe it reaches its maximum value for higher values of J. By observing the figure 4, we can realize that we could stay on the envelope of the propeller yield by continuously changing the blade angle β while we change the value of J. We could then always have a maximum yield because for higher blade angles, the maximum yield shifts towards higher values of J.

Globally, the appearance of the curves stays similar for the different values of β but they get wider and shift towards higher values of J as β increases. The maximum amplitudes of k_T and k_Q also go up as β increases but the maximum value of η_P stays roughly the same as the increases of k_T and k_Q have an opposed effect.

3 Maximum speed

The given data and the caracteristics of the propeller allow us to compute the following values for the flight regime :

P	n	k_P
1.12 MW	23.85 s^-1	0.2779

By using figure 3, we can find the value of J corresponding to each blade angle allowing us to reach the given value of k_P. The values of J are given in the following table, with the matching values of $V = J \cdot nD$ and $M = V/c$ (with c being here the speed of sound) :

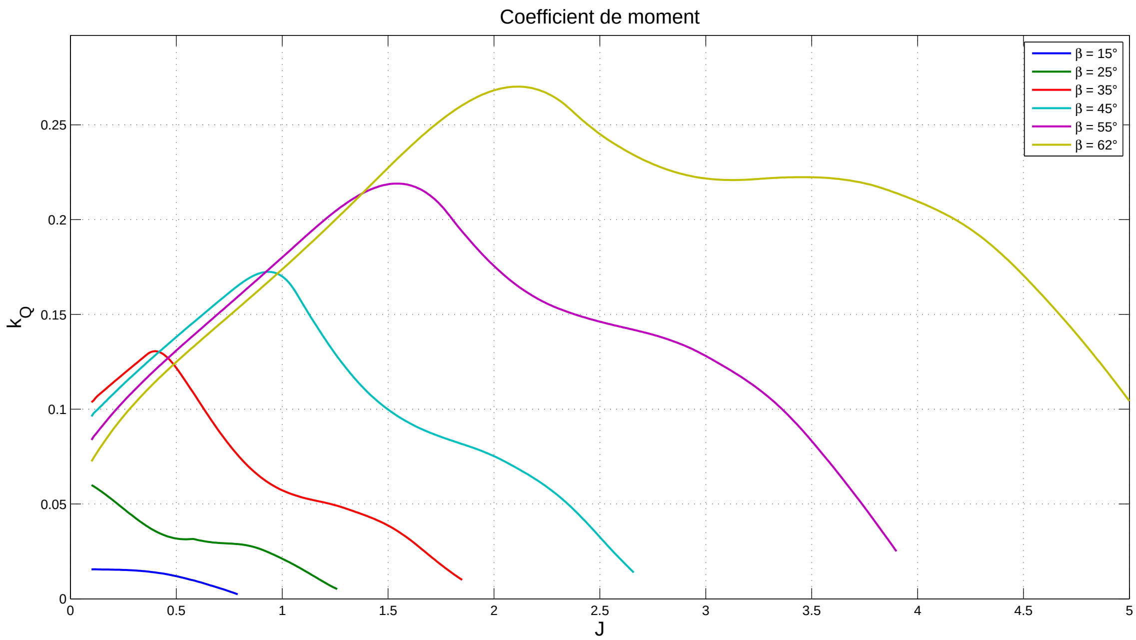

Figure 2: Torque coefficient with respect to the advance ratio for multiple blade orientations

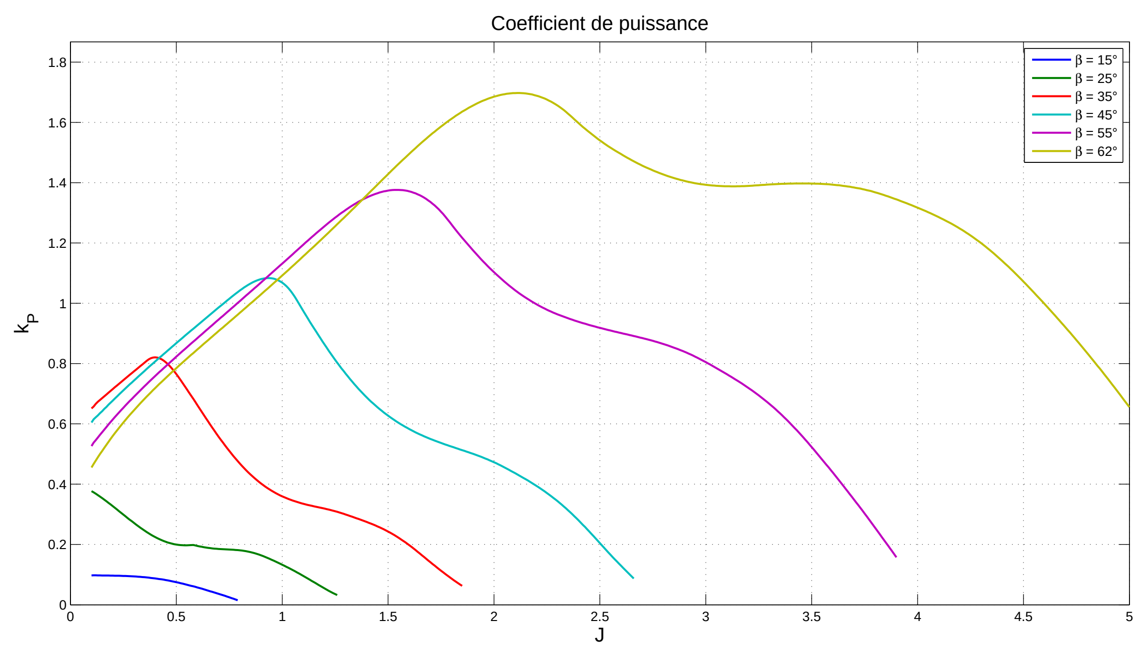

Figure 3: Power coefficient with respect to the advance ratio for multiple blade orientations

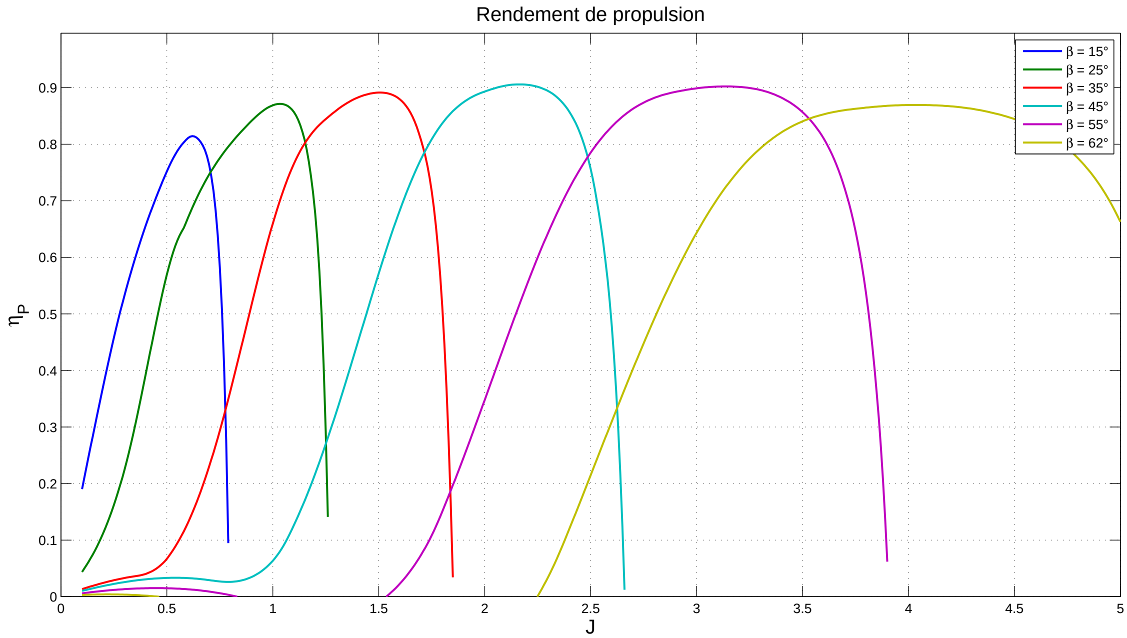

Figure 4: Propulsion yield with respect to the advance ratio for multiple blade orientations

β [degrees]	15	25	35	45	55	62
J [/]	/	0.287	1.385	2.42	3.375	>5
V [m/s]	/	23.27	112.3	196.2	273.7	>405.5
M [/]	/	0.0736	0.3553	0.6209	0.8659	>1.2828

These configurations are theoretically all acceptable except for the last one since the Mach number is greater than 1 and it's not possible to make thrust with an open flow propeller in supersonic conditions.

We choose the angle of 35 degrees which gives us a Mach number of 0.3553. At this mach number, we should not have transonic flows at the edges of the blades. Moreover, this is the configuration wich will give us the maximum value for η_P.

In this situation, we computed several parameters. Their values are presented in the following table :

k_T	k_Q	k_P	η_P	Poussée T [kN]	Couple Q [kNm]	Puissance consommée P [MW]
0.1762	0.0441	0.2774	0.8798	8.7463	7.4502	1.12

The pale anle distribution $\beta(r)$ is showed figure 5. The distribution would be the same in any case as, if we modify the angle of the blade, it moves as one and we only add a constant angle to the whole distribution.

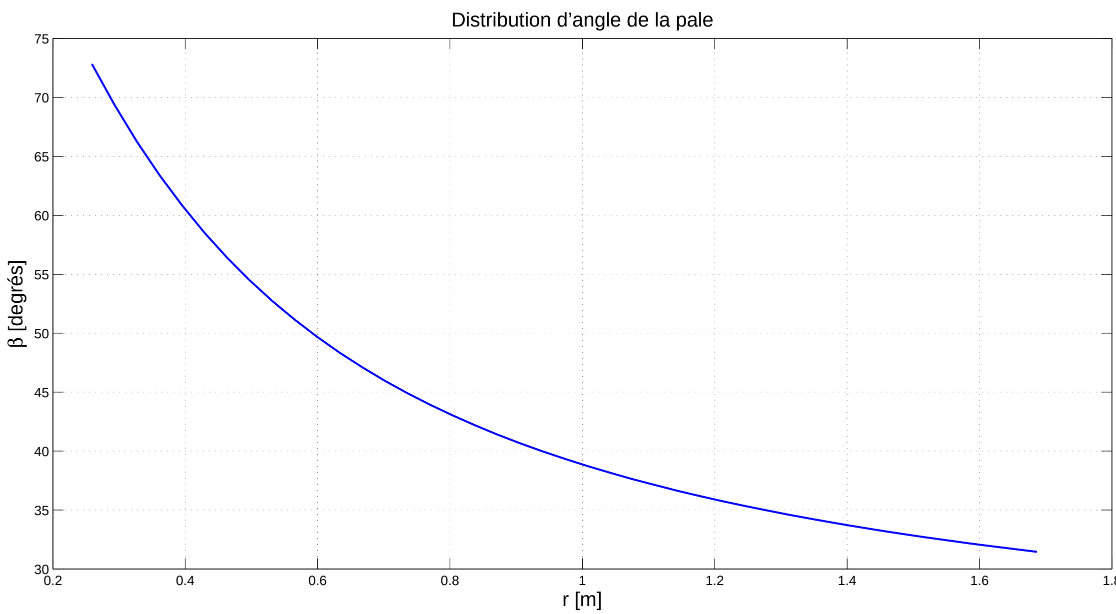

Figure 5: Blade angle distribution with respect to the radius.