MECA2550 Aircraft propulsion systems

HW1 : Blade Element Momentum theory

Version 1.
Hand-out : Oct 18, 2016
Hand-in : Nov 8, 2016

Guidelines

Submission : Submit a written or typed report with your answers and graphics.

Code submission : Submit your code on the icampus website.

1 Induction factors

Derive the set of equations for the induction factors a and a'

$$\frac{a}{1+a} = \frac{\frac{1}{2}\, s\, \lambda_1}{1-\cos 2\phi} \tag{1}$$

$$\frac{a'}{1-a'} = \frac{\frac{1}{2}\, s\, \lambda_2}{\sin 2\phi} \tag{2}$$

2 Blade Element Momentum theory

Implement your own BEM code in matlab. Use the algorithm presented in class and the following hints

– Write your own non-linear solver for a and a'. The equations **??** and **??** above can be easily re-written into an iterative procedure

$$a_{new} = (1+a)\frac{\frac{1}{2}\, s\, \lambda_1}{1-\cos 2\phi} \tag{3}$$

$$a'_{new} = (1-a')\frac{\frac{1}{2}\, s\, \lambda_2}{\sin 2\phi} \tag{4}$$

– Make this non-linear solver **robust** by under-relaxing every step :

$$a^k = (1-\omega)a^{k-1} + \omega a_{new}$$

where ω is the relaxation factor, chosen here < 1, e.g. 0.3.

– Be efficient. The results ($a(r)$, $a'(r)$,...) for a given advance ratio J will be good guesses for the next point in your curves.

3 Application : P-51D propeller curves

We consider the North-American P-51D Mustang. The data of its Hamilton-Standard propeller is summarized in the table below.

TABLE 1 – Data for the Hamilton-Standard 24D50 - 6813

Parameter	Value	Remarks
Diameter	3.4m	
Hub diameter	0.45m	
Number of blades	4	
Chord	0.25m	constant over r
Pitch angle at $r = 0.75R$, $\beta_{\text{ref},0}$	15^o	Reference pitch angle
Airfoil	NACA 16-509	matlab routine on icampus

This propeller has a constant geometric pitch p when the airfoil angle at a radius $r = 0.75R$ is equal to a reference value $\beta_{\text{ref},0}$. The pitch is then given by

$$p_{\text{ref},0} = 2\,\pi\,0.75\,R\;\tan(\beta_{\text{ref},0})$$

and we can write for the rest of

$$\beta(r) = \arctan\left(\frac{p_{\text{ref},0}}{2\,\pi\,r}\right).$$

- Apply your code to this propeller at a flight Mach number $M = 0.5$ and an altitude of $z = 20000$ft. (the standard atmosphere is also provided in a matlab routine on the icampus website)
- Produce the characteristic curves k_T,k_P, k_Q, and η_P vs J, over the range $[0, 5]$,
- For a series of pitch settings : $\beta_{0.75R} = [15^o(\beta_{\text{ref},0})\ 25^o\ 35^o\ 45^o\ 55^o\ 62^o]$.
  Remember, we take the blade defined earlier and **rotate** it about the blade axis. (we are not **deforming** it !)
- What is the effect of this pitch setting ? Comment your results.

4 Maximum speed

As a test of our approach, let us now estimate the performance of the propeller when the P-51D is flying at its maximum speed at that altitude. We know that the Rolls-Royce Merlin engine can develop a power of 1500 bhp at 3000 rpm at the altitude of 20000 ft (Remember that the engine rpm are fixed) and that the propeller rotation speed is reduced to 0.477 that of the engine through a gearbox.

- From these numbers, and the k_T curves, find potential flight regimes at given pitch settings.
- Are they all achievable ? What would the Mach be for certain pitch settings ?
- Pick the most realistic one, and analyse it. Compute the thrust, distributions of angle of attack, etc.