Radix Sort and Sorting Algorithms for Positive Integers

Exercise 1: Radix Sort Example

Sort the following arrays using the Radix Sort algorithm. Recall that Radix Sort processes digits from the least significant to the most significant.

1. **Array 1:** 34, 9134, 20134, 29134, 4, 134
2. **Array 2:** 4, 34, 134, 9134, 20134, 29134
3. **Array 3:** 29134, 20134, 9134, 134, 34, 4

For each example:

- Sort the numbers based on the least significant digit first.
- After sorting by each digit, move to the next most significant digit.
- Continue until all digits have been sorted.

Exercise 2: O(n) Sorting Algorithm for Positive Integers

Present an $O(n)$ algorithm to sort n positive integer numbers $a_1, a_2, \ldots, a_n$ that are known to be bounded by $n^2 - 1$ (i.e., $0 \leq a_i \leq n^2 - 1$ for all $i = 1, \ldots, n$).

Key Idea:

- We want each number from 0 to $n^2 - 1$ to require exactly two digits in base k.
- The largest number, $n^2 - 1$, should be represented as a 2-digit number in base k.
- To achieve this, we must choose $k = n$. This ensures that $n^2 - 1$ is the largest number representable in two digits in base n.
- Therefore, in base n, each number from 0 to $n^2 - 1$ can be represented using exactly two digits, making the algorithm run in $O(n)$ time.

Step-by-Step Algorithm:

- Choose base $k = n$ based on n.
- Convert all numbers from base 10 to base k.
- Apply **Radix Sort** to sort the numbers.

Example Sequences:

(a) Sequence 1:

Input Sequence: 45, 98, 3, 82, 132, 71, 72, 143, 91, 28, 7, 45

- $n = 12$ (since the largest number is 143, and $12^2 - 1 = 143$).
- Choose base $k = 12$ to ensure 2-digit representation for all numbers.
- Step 1: Convert each number to base 12.
- Step 2: Sort by the least significant digit, then by the next digit.
- Step 3: Repeat sorting for all digits (if necessary).
- Final Sorted Sequence: ...

(b) Sequence 2:

Input Sequence: 45, 98, 3, 82, 132, 71, 72, 143, 91, 28, 7, 45, 151, 175, 145, 399, 21, 267, 346, 292

- $n = 20$ (since the largest number is 399, and $20^2 - 1 = 399$).
- Choose base $k = 20$ to ensure 2-digit representation for all numbers.
- Step 1: Convert each number to base 20.
- Step 2: Sort by the least significant digit, then by the next digit.
- Step 3: Repeat sorting for all digits (if necessary).
- Final Sorted Sequence: ...