Radix Sort and Sorting Algorithms for Positive Integers

Exercise 1: Radix Sort Example

Sort the following arrays using the Radix Sort algorithm:

1. **Array 1:** 34, 9134, 20134, 29134, 4, 134
2. **Array 2:** 4, 34, 134, 9134, 20134, 29134
3. **Array 3:** 29134, 20134, 9134, 134, 34, 4

Exercise 2: O(n) Sorting Algorithm for Positive Integers

Present an $O(n)$ algorithm to sort n positive integer numbers $a_1, a_2, \ldots, a_n$ that are known to be bounded by $n^2 - 1$ (i.e., $0 \leq a_i \leq n^2 - 1$ for all $i = 1, \ldots, n$). This is based on the idea of **Radix Sort** (discussed in the textbook, Section 8.3).

Hint:

To obtain an $O(n)$ time complexity, you must use **Radix Sort** by selecting a **suitable base**. The runtime of **Radix Sort** is $O(d(n + k))$, where:

- d is the number of digits.
- k is the base.

The idea is to represent each number in **base k** such that each number in the range $\{0, 1, \ldots, n^2 - 1\}$ requires **only 2 digits**, so $d = 2$.

To achieve this, you need to choose a base k so that each number requires exactly 2 digits. You will need to:

- Explain which base k you select.
- Show how to convert numbers from base 10 to the chosen base.

Key Idea:

- The base 10 representation does not work because the largest number $n^2 - 1$ would require $\log_{10}(n^2 - 1)$ digits, which is not constant and would prevent achieving $O(n)$ time complexity.
- Since no constant base works, the base k must depend on n.
- Provide a formula for k as a function of n, and show that 2 digits are sufficient to represent all numbers from 0 to $n^2 - 1$.

Step-by-Step Algorithm:

- Choose base k based on n.
- Convert all numbers from base 10 to base k.
- Apply **Radix Sort** to sort the numbers.

Example Sequences:

(a) Sequence 1:

Input Sequence: 45, 98, 3, 82, 132, 71, 72, 143, 91, 28, 7, 45

- $n = 12$ (since the largest number is 143, and $12^2 - 1 = 143$).
- Sorting Steps:
  - Choose an appropriate base k that allows us to represent all numbers in 2 digits.
  - Sort the numbers by the digits at each position (starting from the least significant digit).
  - Continue sorting based on the next most significant digit.

(b) Sequence 2:

Input Sequence: 45, 98, 3, 82, 132, 71, 72, 143, 91, 28, 7, 45, 151, 175, 145, 399, 21, 267, 346, 292

- $n = 20$ (since the largest number is 399, and $20^2 - 1 = 399$).
- Sorting Steps:
  - Choose an appropriate base k for sorting the numbers using 2 digits.
  - Perform the Radix Sort step-by-step for each digit, sorting the numbers in increasing order.

Additional Notes for Conversion:

If you use a base b greater than 10, you **don't need to invent new symbols** for digits greater than 9. Instead, represent them using base 10 digits:

- For example, if you choose base 25, the digits will be: $0, 1, 2, \ldots, 9, 10, 11, \ldots, 24$.
- Numbers like "10", "11", etc., will be treated as single symbols in the radix sorting process.
- For instance, the number 9 23 written in base 25 has 2 digits: 9 and 23.