Data Structures and Algorithms

COSC 336 Assignment 4

Instructions.

1. Due date and time: As indicated on Blackboard.
2. This is a team assignment. Work in teams of 3-4 students. Submit on Blackboard one assignment per team, with the names of all students making the team.
3. The exercises will not be graded, but you still need to present your best attempt to solve them. If you do not know how to solve an exercise, say it. This will give me feedback about your understanding of the theoretical concepts.
4. Your programs must be written in Java.
5. Write your programs neatly - imagine yourself grading your program and see if it is easy to read and understand.

   Comment your programs reasonably: there is no need to comment lines like "i++" but do include brief comments describing the main purpose of a specific block of lines.
6. You will submit on **Blackboard** 32 files.

   The **1-st file** is a pdf file (produced ideally with latex and Overleaf) and it will contain the following:

   (a) The solution to the Exercises (see the remark above).

   (b) A short description of your algorithm for the Programming Task, where you explain the dynamic programing approach. Focus on how you have modified MERGE.

   (c) A table with the results your program gives for the data sets indicated for the programming task.

   (d) The java code (so that the grader can make observations) of the program.

   The **2-nd file** is the .java file containing the java source code for Programming Task.

Exercise 1. For each of the following functions, give a $\Theta(t(n))$ estimation with the simplest possible $t(n)$ (for example $3n^2 + 5n \log n = \Theta(n^2)$).

1. $13n^2 - 2n + 56$
2. $2.5 \log n + 2$
3. $n(12 + \log n)$
4. $1 + 2 + 3 + \ldots + 2n$
5. $1 + 2 + 3 + \ldots + n^2$
6. $\log(n^3) + 10$
7. $\log(n^3) + n \log n$
8. $n \log(n^3) + n \log n$
9. $2^{2 \log n} + 5n + 1$

Exercise 2.

1. Evaluate the following postfix arithmetic expression: 10 3 4 $-$ 5 $*$ /
2. Convert the following infix arithmetic expression to postfix notation: $(((2+3)*5)-15)$

Exercise 3.

Consider the following algorithms A and B for the problem of computing $2^n \pmod{317}$ (This is the modular exponentiation problem that we have discussed in class. Algorithm A is the one that we have seen in class, and algorithm B is a variant of it.

```
Algorithm A.

mod_exp_A(n) {
   if (n== 0) return 1;
   else {
          t = mod_exp_A(n/2);
          if (n is even) return t*t (mod 317);
          if (n is odd) return t*t*2 (mod 317);
}}

Algorithm B.

mod_exp_B(n) {
   if (n== 0) return 1;
   else {
          if (n is even) return   mod_exp_B(n/2) * mod_exp_B(n/2) (mod 317);
          if (n is odd) return  mod_exp_B(n/2) * mod_exp_B(n/2) *2 (mod 317);
}}
```

1. Write the recurrence for the runtime $T_A(n)$ of algorithm A, and solve the recurrence to find a $\Theta(\cdot)$ estimation of $T_A(n)$.
2. Write the recurrence for the runtime $T_B(n)$ of algorithm B, and solve the recurrence to find a $\Theta(\cdot)$ estimation of $T_B(n)$.
3. Which algorithm is faster? (Note: There is a huge difference between T_A and T_B.)

Exercise 4. Give a $\Theta(\cdot)$ evaluation for the runtime of the following code:

```
 i= 1; x=0;
 while(i <= n) {
       j=1;
       while (j <= i) { x=x+1; j= 2*j; }
       i= 2*i;
}
```

Hint: You can assume that n is a power two. Then i from the outer loop takes successivley the values: $1, 2, 2^2, 2^3, ..., 2^{\log n}$.)

Programming task 1.

Your program needs to solve the *Rod Cutting* problem, described in the textbook in Section 14.1, page 364. In this problem, a rod needs to be cut into segments that yield the maximum revenue, given a list or prices for each segment length. See the description of the problem for details. You need to implement the algorithm that in addition to computing the maximum revenue, also produces a list of piece sizes that produce the max revenue. This algorithm is presented in pseudocode in the textbook on page 372 (Extended-Bottom-Up-Cut-Rod (p,n) and Print-Cut-Rod-Solution (p,n). Thus your program needs to return the maximum revenue, and a list of cuts yielding this revenue (this is the list printed by Print-Cut-Rod-Solution (p,n).

Test your program on the following data sets:

1. Data set 1: 1,5,8,9,10,17,17,20,24,30. (This is the example on page 364).

2. Data set 2: file input-4-2 on Blackboard. The first line is the value of n (the length of the rod), and the second line is the list p of n prices for segments of lengths 1,2, $\ldots, n$.

3. Data set 3: file input-4-3 on Blackboard. As above, the first line is the value of n (the length of the rod), and the second line is the list p of n prices for segments of lengths 1,2, $\ldots, n$.