

WEEK-2

⇒ Notations :-

n = number of features
 x^i = input (features) of i^{th} training example
 x_j^i = value of feature j in i^{th} example
 m = no of rows in the training set
 y = output of training set.

⇒ Multivariate linear regression

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$(n+1) \times 1$ $(n+1) \times 1$

$$\Rightarrow h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$
$$= \boxed{\theta^T x}$$

$$\Rightarrow \text{Cost fn} \Rightarrow J(\theta_0, \theta_1, \theta_2, \dots, \theta_n) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^i) - y^i)^2$$

$$\downarrow$$
$$\frac{1}{2m} \sum_{i=1}^m (\theta^T x^i - y^i)^2$$

$$\frac{1}{2m} \sum_{i=1}^m \left(\sum_{j=1}^n \theta_j x_j^i - y^i \right)^2 \quad \text{OR}$$

⇒ Gradient Descent ($n \geq 1$) →

Repeat {

$$\theta_j := \theta_j - \alpha \left[\frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^i) - y^i) x_j^i \right]$$

↳ $\frac{\partial}{\partial \theta_j} (J(\theta))$

}

⇒ Feature Scaling - ^{sure} Make features are on a similar scale (have similar value ranges) then the Gradient Descent reduces quickly.

(divide the feature by its upper bound (range))

⇒ get in range of $-1 \leq x_i \leq 1$

$x_i = \frac{\text{size} - \text{min}}{(\text{max} - \text{min})}$ avg value ↑

⇒ Mean normalization :→ Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean.

Eg → $x_1 = \frac{\text{size} - 1000}{2000}$, $-0.5 \leq x_1 \leq 0.5$

(size of house) → (max-min)

⇒ The use of gradient descent is to find a value of θ that minimises the cost fn → $J(\theta)$

Finding the right α

- \Rightarrow If α is too small \rightarrow slow convergence
- \Rightarrow If α is too large $\rightarrow J(\theta)$ may not decrease on every iteration, may not converge

To help choose the suitable α ,
plot a graph of $J(\theta)$ vs no of iterations
(Y) (X-axis)

\Rightarrow POLYNOMIAL Regression

$$\Rightarrow h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

$$\hookrightarrow \Rightarrow \theta_0 + \theta_1 (\text{size}) + \theta_2 (\text{size})^2 + \theta_3 (\text{size})^3$$

OR

$$\Rightarrow \theta_0 + \theta_1 (\text{size}) + \theta_2 (\sqrt{\text{size}})$$

that means $\Rightarrow x_2 = (x_1)^2$ or $\sqrt{x_1}$ or $(x_1)^3$

\Rightarrow For linear regression, if we solve the ~~cost~~
 ~~$J(\theta)$~~ $h(\theta) = \theta_0 + \theta_1 x$ for the
value of θ , we get

$$\theta = (X^T \cdot X)^{-1} X^T y$$

X & y are matrices.

In this method,
no need for
feature scaling

$$\Rightarrow h_{\theta}(x) = \theta_0 + \theta_1 x \rightarrow \text{hypothesis } f^n$$

⇒ If n is large, use gradient descent.
(if $n > 10,000$) ⇒ Time $\Rightarrow O(kn^2)$

⇒ If n is ~~too~~ small, use Normal equation.
Time $\Rightarrow O(n^3)$

⇒ Sometime, $(X^T X)$ can be ~~non~~^{non-invertible}
↳ Reasons \Rightarrow

i) = two features have a relation
ie. $X_1 = k X_2$ (linearly dependent)

ii) = too many features ($m \leq n$)

Solutions \Rightarrow Delete some features or
regularization

⇒ Still the octave $f^n \rightarrow \text{pinv}(X^T X)$ will give
the right value of θ .
(even if $(X^T X)$ is non-invertible)

OCTAVE TUTORIAL

$a^T b = b^T a$ → if a & b are vectors

⇒ \sim means not equal to

⇒ ; semicolon suppresses the output.

⇒ `sprintf()` → used for formatting

⇒ Syntax is same as C language & MATLAB

⇒ `disp()` → used to display output on terminal

⇒ `format long` → changes the variable type to long.

⇒ `V = 1:0.1:2` having values
V generates a vector starting from 1 to 2 with intervals of 0.1

⇒ `C = ones(2,3)` → $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

⇒ `W = zeros(2,3)` → $W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

⇒ `rand(1,3)` → random values less than 1

⇒ `randn(1,3)` → gives gaussian random values

⇒ `hist(W)` → plots a histogram

⇒ `eye(2)` = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ → generates an identity matrix

⇒ `help` → brings out documentation

⇒ $\text{size}(A)$ → gives size of matrix

$\text{size}(A, 1)$ → gives rows

$\text{size}(A, 2)$ → gives columns

⇒ $\text{length}(A)$ = gives size of longer dimension

⇒ who → shows variables in current workspace

~~whos~~ → gives detailed ~~new~~ view

⇒ ~~clear~~ $\text{clear } A$ → clears variable A from workspace

clear → deletes all variables

⇒ $\text{save file.mat } A$ → saves variable A in file.mat

⇒ $\text{save hello.txt } A -\text{ascii}$ → saves as text

⇒ $A(3, 2)$ → fetches the 6th element of A

⇒ $A(2, :)$ → fetch everything from 2nd row

⇒ $A([1 \ 3], :)$ → fetch everything from 1st & 3rd row

⇒ $A = [A, [100; 110; 120]]$ → appends another row to A . vector

⇒ $A(:)$ → puts all elements of A in single

⇒ $C = [A \ B]$ → concatenates 2 matrices (horizontally)

⇒ $C = [A; B]$ → concatenates vertically

$\Rightarrow A .* B \rightarrow$ multiplies every element of A with their corresponding element in B .

$\Rightarrow A.^2 \rightarrow$ gives element wise squaring

$\Rightarrow 1 ./ V \Rightarrow$ gives reciprocal of elements of V

$\Rightarrow \log(V) \Rightarrow$ element wise log
 $\exp(V) =$ " " exponent
 $\text{abs}(V) =$ absolute value

$\Rightarrow [\text{val}, \text{ind}] = \text{max}(A) \rightarrow$ gives value & index of max element of A

$\Rightarrow \text{find}(A < 3) \rightarrow$ gives elements of A , less than 3

$\Rightarrow A = \text{magic}(3) \rightarrow$ A magic matrix, has all the sum of rows & columns equal (even the diagonals add to same sum)

$\Rightarrow \text{sum}(A) \& \text{prod}(A) \rightarrow$ gives sum of all elements as same for product.

$\Rightarrow \text{max}(\text{max}(A)) \rightarrow$ gives max element of A
 $\text{max}(A) \rightarrow$ gives max elements of every row

$\Rightarrow \text{sum}(A, 1) \rightarrow$ gives sum of every element in row
 $\text{sum}(A, 2) \rightarrow$ in column

$\text{sum}(\text{sum}(A .* \text{eye}(9))) \rightarrow$ gives sum of diagonal

$\text{sum}(\text{sum}(A .* \text{fliplr}(\text{eye}(9)))) \rightarrow$ other diag sum

⇒ `fliplr(A)` → flips a matrix (mirror image)

⇒ `pinv(A)` → inverse of A

⇒ `plot(x, y)` → plots a graph.

⇒ `hold on` → makes all changes on same plot.

⇒ `xlabel('...')` & `ylabel('...')` → to label axis.

⇒ `print -dpng 'file.png'` → to save a plot.

⇒ `figure(1); plot(t, y1);` → helps plots more than 1 plot at a time.

⇒ `axis([0.5 1 -1 1])` → changes axis.

⇒	<pre>for i = 1:10; v(i) = 2^i; end;</pre>		<pre>@while i = 1; while i <= 5; v(i) = 100; i = i + 1; end;</pre>
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⇒ `addpath('c...')` → add the path to octave search.

⇒ Octave f^n can return multiple values.

⇒ Vectorization →
$$h_0(x) = \sum_{j=0}^n \theta_j x_j$$
$$= \theta^T x$$