Optimal path planning for unmanned ground vehicles using potential field method and optimal control method

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Abstract: This paper presents an optimal path planning algorithm for unmanned ground vehicle (UGV) to control its direction during parking manoeuvres by employing artificial potential field method (APF) combined with optimal control theory. A linear two-degree-of-freedom vehicle model with lateral and yaw motion is derived and simulated in MATLAB. The optimal control theory is employed to generate an optimal collision-free path For UGV from starting to the desired locations. The obstacle avoidance technique is mathematically modelled using APF including both the attractive and repulsive potential fields. The inclusion of these two potential fields ends up with a new potential field which is implemented to control the steering angle of the UGV to reach to its target location. Several simulations are carried out to check the fidelity of the proposed technique. The results demonstrate the generated path for the UGV can satisfy vehicle dynamics constraints, avoid obstacles and reach the target location.

Keywords: optimal path planning; UGVs; unmanned ground vehicles; potential field methods; optimal control theory.

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1   Introduction

The past decade has witnessed an unprecedented development in the field of unmanned ground vehicles (UGVs) including wide applications for both civilian and military fields. Generally, UGV is self-driving vehicle fitted with sensors that enable it to observe and understand surrounding. These UGVs are used to replace humans in hazardous situations. The UGVs are used in different kind of applications such as surveillance, riot control, border patrol, etc. on the other hand several technologies, such as communications, navigation and embedded control systems, have been recently grown up rapidly. Path planning for UGVs has gained a substantial attention since several challenges are encountered such as road conditions, uncertainty in the vehicle states, and limited knowledge about the working environment (Hebert et al., 2012).

The research problem is to enable the UGV to move from position in the environment to reach the parking spot without hitting the parking border. In addition to make it more complicated, obstacles are added in the UGV. The UGV must be able to avoid these obstacles otherwise it will be collision with them. In order to solve this problem a motion planning algorithm is used which combined with optimal control theory in order to generate the optimal collision-free path that enable the UGV to park safely without collision with the obstacles or the parking spot borders. The introduced work includes two basic steps: motion planning for UGVs and optimal control. To be able to solve the path planning problem, artificial potential field (APF) method has been widely used for

mobile robots where, the obstacles are represented with repulsive potential field and the target by attractive potential field. The sum of those fields result in a force which will control the direction of the robot to find its path and avoid all the surrounding obstacles (Latombe, 2012). The popularity of potential field method arises from its simplicity and low computational time.

APF method is commonly used in unmanned vehicles path planning for it has many advantages, such as the simple algorithm and mathematical description, also it convenience for real-time control (Andrews and Hogan, 1983; Benamati et al., 2005; Khatib, 1986; Berry et al., 2010; Chen et al., 2016). For example, Tang et al. (2015) introduced a new potential field method for obstacle avoidance and path planning. This method is based on gravity chain which assumes that there is rubber band which connects the beginning and ending of the obstacle potential field space (Oland and Kristiansen, 2013) applying the APF method to solve the problem of collision with terrain for multiple UAVs also for avoiding collision with each other. All these applications explained that the APF method is a powerful method in the path planning and control of unmanned vehicles.

However, it also has its own problem (Koren and Borenstein, 1991; Park et al., 2001) point the significant problem using APF is the local minimum problem. Many research papers dealing with this problem. Besides this problem the APF cannot guarantee that the path planning is optimal path. Adeli et al. (2011) solve the path planning problem of mobile robots by introduce a new potential functions based on the distances from obstacles, goal point and start Point. The algorithm able to avoid obstacles but the generated path is not optimal according to use potential filed alone.

In order to solve the APF problems and improve the path planning, some scholars introduce away for this problem by combine other methods with APF method. Yang and Sukkarieh (2012) used the model predictive control (MPC) based on the improved APF method to solve the navigation problem of the UAV. Qu et al. (2014) obtaining an optimal path for UAV in 3D environments based on combination of flight space partitioning, Dijkstra algorithm and potential field method. Also, integrating the APF method with a bacterial evolutionary algorithm, Montiel et al. (2015) presented an algorithm to generate the optimal path for mobile robots using new concept of Parallel Evolutionary APF for path planning. Also obtain optimal solutions but its processing times might be excessive in complex real world situations. Later, Himawan et al. (2016) chose the APF method as a path planning and obstacle avoidance by integrating APF and control system under non-holonomic constraint to obtain the trajectory.

On the other hand, using conventional optimal control method alone does not deal with obstacle avoidance directly. Consequently, we combined the two methods to generate the optimal path and avoid the obstacles. Optimal control theory is applied for UGVs path planning using an objective cost function which is minimised to satisfy the initial/final constrains (Wang et al., 2009; Raja and Pugazhenthi, 2012; Mashadi and Majidii, 2014; Xue and Sheu, 1988). The path planning problem can be formulated as an optimal control problem. Optimal control method is used as set of differential equations describing the paths of the control variables that minimise the cost function to satisfy the initial/final conditions and constrains for instance to generate the optimal collision-free path for UGV parking manoeuvre. The optimal control can be derived using Pontryagin's minimum principle (PMP) or simply Pontryagin's Principle (Ross, 2009) or by solving the Hamilton function.

The main contribution of this paper is that we combined artificial potential filed method with optimal control theory in order to avoid obstacles and achieve global optimisation in the same time. To the best of our knowledge, it is the first time these two methods are combined to solve the parking problems for the UGVs. Also to developing the optimal path for an UGV during the parking behaviour. Which the effective optimal path planning algorithm is proposed to generate the optimal collision-free path for an UGV to move from an initial point to the desired parking spot considering the parking border as obstacle or any other objects may be facing during parking scenario which defined by repulsive potential field function. This algorithm is applied for a simplified two-degree-of-freedom vehicle model with lateral and yaw dynamics.

2   Vehicle dynamics and modelling

A simplified single-track two-degree-of-freedom model (Wong, 2008) with linear tyre characteristics is derived to represent the vehicle motion considering the lateral and yaw dynamics as shown in Figure 1. The forward velocity (U) is assumed constant and the steering angle (δ_f) is applied to the front wheel directly and assumed small (Cos $\delta_f = 1$). The governing equations of motion for the vehicle body lateral and yaw dynamics are given in equations (1) and (2), respectively:

$$m\left(\dot{V} + U\,r\right) = F_{yf} + F_{yr} \tag{1}$$

$$I_{zz}\,\dot{r} = F_{yf}\,\cos\delta_f\,a - F_{yr}\,b. \tag{2}$$

A linear tyre model is employed to estimate the tyre cornering forces $(\boldsymbol{F}_{yf}, \boldsymbol{F}_{yr})$ based on the coefficients of tyre cornering stiffness $(\boldsymbol{C}_{\alpha f}, \boldsymbol{C}_{\alpha r})$ and the generated tyre slip angles $(\boldsymbol{\alpha}_f, \boldsymbol{\alpha}_r)$ for the front and rear tyres, respectively. The final states of the vehicle model can be derived and written according to the following matrix form:

$$\begin{bmatrix} \dot{V} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{C_{\alpha f} + C_{\alpha r}}{Um}\right) & -\left(\dfrac{a.C_{\alpha f} - b.C_{\alpha r}}{Um} - U\right) \\ -\left(\dfrac{a.C_{\alpha f} - b.C_{\alpha r}}{UI_z}\right) & -\left(\dfrac{a^2.C_{\alpha f} + b^2.C_{\alpha r}}{UI_z}\right) \end{bmatrix} \begin{bmatrix} V \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_{\alpha f}}{m} \\ \dfrac{a.C_{\alpha f}}{I_z} \end{bmatrix} \delta_f. \tag{3}$$

The coordinates of the vehicle body motion (x,y) in global coordinates can be derived using the heading angle (Ψ) and the vehicle velocity components (U,V) from the following equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} U & -(V + a.r) \\ (V + a.r) & U \end{bmatrix} \begin{bmatrix} \cos\Psi \\ \sin\Psi \end{bmatrix}. \tag{4}$$

MATLAB/Simulink is used to simultaneously solve the differential equations (3) and (4). Figure 2 shows the transient response of the vehicle model as a result of step-steer input manoeuvre at constant vehicle speed of 100 km/h.

Figure 1 Two-degree-of-freedom bicycle vehicle model (see online version for colours)
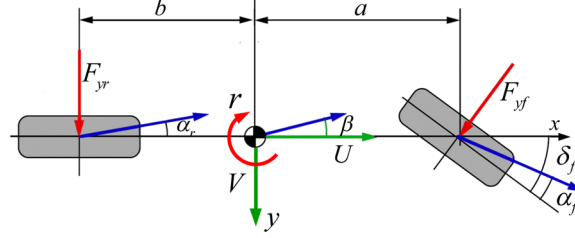
Figure 2 Step steer response of two-degree-of-freedom bicycle vehicle model where (a) lateral velocity; (b) yaw rate; (c) lateral acceleration and (d) side slip angle (see online version for colours)
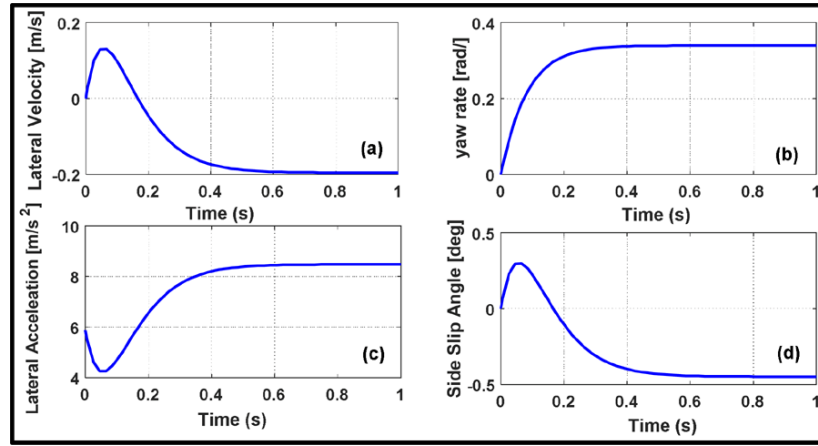
3 Potential field method

The potential field method is employed to define the surrounding obstacles (repulsive potential field) and the target location (attractive potential field). The potential field is based on two-dimensional Gaussian attractor ($f_A(x,y)$) and repulsor ($f_R(x,y)$) functions as follows (Ren et al., 2003):

$$f_A(x,y) = 1 - e^{\frac{-\left((x-a_x)^2+(y-a_y)^2\right)}{2\sigma^2}} \tag{5}$$

$$f_R(x,y) = e^{-0.5\left(\frac{(x-r_x)^2+(y-r_y)^2}{\sigma^2}\right)^c}, \tag{6}$$

where

(a_x,a_y): position centre of the attractor

(a_x,a_y): position centre of the repulsor

(σ): used to change the size of the obstacle

(c): determines the effect range of the given obstacle.

The implementation of the given optional field in the vehicle model requires the calculation of the coordinates of vehicle body, surrounding obstacles and target location. Figure 3 shows 3D Gaussian functions for the repulsor and attractor potential field. Furthermore, these surrounding obstacles are considered static with predefined coordinates. This information is useful for the future incorporation of UGV's sensors which permits the acquisition of these coordinates in real-time.

Figure 3   Potential field using Gaussian functions: (a) positive Gaussian repulsor function) and (b) negative Gaussian attractor function (see online version for colours)

(a)   (b)
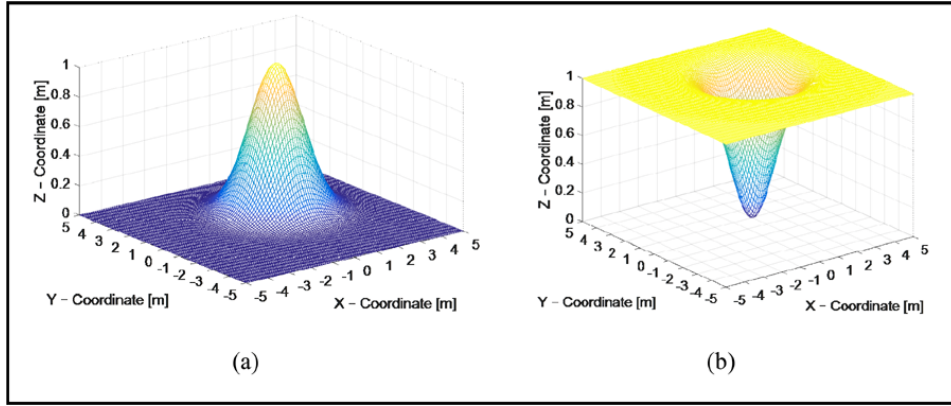
4   Optimal path planning algorithm

The proposed path planning algorithm is based on the optimal control theory to generate the optimal path which will permit the vehicle to move between two points along with predefined constraints such as the coordinates of the obstacles, initial and final points and the heading angle. The control action is performed by applying the output control signal directly to the front wheel of the vehicle model ($u(t) = \dot{\delta}_f$). The proposed control system is aimed to minimise the cost (objective) function (J) from initial states vector $x(t_o)$ to final states vector $x(t_f)$ by minimising control signal $u(t)$ as follows:

$$\min u(t) J = \int_{t_o}^{t_f} L(X(t), u(t), t) dt. \tag{7}$$

The higher derivative of the state vector $\dot{X}(t)$ and the initial $X(t_o)$ and final $x(t_f)$ conditions are defined as follows:

$$\begin{cases} \dot{X}(t) = f(X(t), u(t), t) \\ \quad X(t_o) = X_o \\ \quad g(X(t_f), t_f) = 0 \end{cases} \tag{8}$$

The Lagrange function $L(x,u)$ is introduced to minimise the generated path according to the potential field's coordinates of both the obstacle (repulsor) and the target (attractor) as follows:

$$L(X,u) = L_1(X,u) + W_{po}\sum_i P_{oi}(X_c) + W_{PA}P_A(X_c), \tag{9}$$

where $X_c = (x_c, y_c)$, $P_{oi}(X_c) = f_A(x_c, y_c)$ is the repulsor potential field as defined in equation (5), $P_A(X_c) = f_R(x_c, y_c)$ is the attractive potential field in equation (6), W_{po} and W_{pA} are the weights.

$L_1(x, u)$ is introduced to define the states vector and control signal as follows:

$$L_1(X,u) = X^T Q X + u^T R u, \tag{10}$$

where (Q, R) are given positive weighting matrices. From equations (9) and (10), equation (7) can be rewritten as follows to define the cost function:

$$\min u(t) J = X^T Q X + u^T R u + W_{po}\sum_i P_{oi}(X_c) + W_{PA}P_A(X_c). \tag{11}$$

In order to solve equation (12), the PMP is employed as an optimal control solution (Rousseau et al., 2007; Serrao and Rizzoni, 2008; Serrao et al., 2009). For this purpose, the Hamiltonian function is considered as shown in equation (12).

$$H(X(t),u(t),\lambda(t),t) \triangleq L(X(t),u(t),t) + \lambda^T f(X(t),u(t),t), \tag{12}$$

where λ^T is the Lagrange multiplier vector for the vehicle dynamics constraint, the co-state equation ($\dot{P}$) can be presented in the following form:

$$\dot{P} = -\frac{\partial H}{\partial x}(X^*(t_f^*),u^*(t_f^*),\lambda(t_f^*),t_f^*. \tag{13}$$

The control input can be obtained from the first derivative of Hamiltonian function with respect to the control signal as follows:

$$\frac{\partial H}{\partial u} = 0 \quad \text{i.e.} \ \frac{\partial L}{\partial u} + \sum_i \lambda_i \frac{\partial f_i}{\partial u} = 0. \tag{14}$$

Both the initial and final conditions of the Hamiltonian function given in equation (14) are defined as follows:

$$\left.\begin{array}{l} X(t_f) = \begin{bmatrix} 0 & 0 & \Psi_0 & d_{xo} & d_{yo} & 0 \end{bmatrix} \\ X(t_o) = \begin{bmatrix} 0 & 0 & \Psi_f & d_{xf} & d_{yf} & 0 \end{bmatrix} \end{array}\right\}, \tag{15}$$

where (d_{xo}, d_{yo}), (d_{xf}, d_{yf}) are the initial vehicle positions in longitudinal and lateral directions, respectively. (Ψ_0, Ψ_f) are the initial and final heading angles of the vehicle. The Hamiltonian function is solved for the following states and co-states vectors which are given as follows:

$$\left.\begin{array}{l} \dot{X} = \begin{bmatrix} \dot{V} & \dot{r} & \dot{\Psi} & \dot{x} & \dot{y} & \dot{\delta} \end{bmatrix}^T \\ P = \begin{bmatrix} \dot{P}_1 & \dot{P}_2 & \dot{P}_3 & \dot{P}_4 & \dot{P}_5 & \dot{P}_6 \end{bmatrix}^T \end{array}\right\}. \tag{16}$$

MATLAB built-in boundary value problem function (bvp4c) is used to solve equations (9), (13)–(15) to find the state variables and the optimal path within time interval from (t_o) to (t_f) (Wang et al., 2009; Naidu, 2002).

5   Simulation and results

The numerical simulation is carried out in MATLAB environment in order to evaluate the robustness of the proposed path planning algorithm without collision including different scenarios of obstacles and initial conditions. Based on the APF method, the generated optimal path is experienced through the following three scenarios. The first scenario shows the vehicle optimal path between two points without obstacles. The second scenario shows the vehicle optimal path between two points with different obstacles imposed in the path of the vehicle. Last scenario examines the generated optimal path for different starting points.

5.1   Optimal path between two points without obstacles

The objective of this scenario is to check the ability of the proposed methodology to generate an optimal path for the vehicle from a given starting point and parking safely in a parking spot without hitting the parking walls. For this purpose, the parking walls are simulated using repulsor potential field which are represented by five circles as shown in Figure 4. The generated optimal path is drawn in blue line while the parking spot is drawn in red line. The results show the trace of the vehicle in x-y grid (m). Additionally, the time-history of the vehicle states in terms of longitudinal, lateral, heading and steering angles during parking manoeuvre are shown in Figure 5.

In order to explain the benefit that achieved from the combination between the optimal control and the APF the optimal path compared with the path generated using potential field alone is shown in Figure 6. The potential field is a local method and only reacts locally, when the obstacles/attractors are near. Our method using the optimal control and potential field is a global method. Judging from Figure 6, the resulting path is different. Our method can generate a much smoother path which is close to what real driver would do.

5.2   Optimal path between two points with obstacles

This scenario is aimed to check the generated optimal path when different obstacles are placed at different locations on the way of the vehicle during parking as shown in Figure 7(a). Furthermore, for each result, two paths are plotted and compared; one without obstacles (the blue line) and the other with obstacles (the red line). Also the steering angle for the two scenarios are shown in Figure 7(b). The results show that, the proposed algorithm is able to avoid the imposed obstacles with different locations and enable the vehicle to safely parking in the targeted area. It is clear that, the proposed algorithm not only generates the path according to the position of the initial and final points but also to avoid the imposed obstacles. It should be noted that, in this paper, the obstacles are assumed to be static with fixed coordinates in the *x-y* grid.

Figure 4 (a)–(d) show the optimal path between two points without obstacles (see online version for colours)
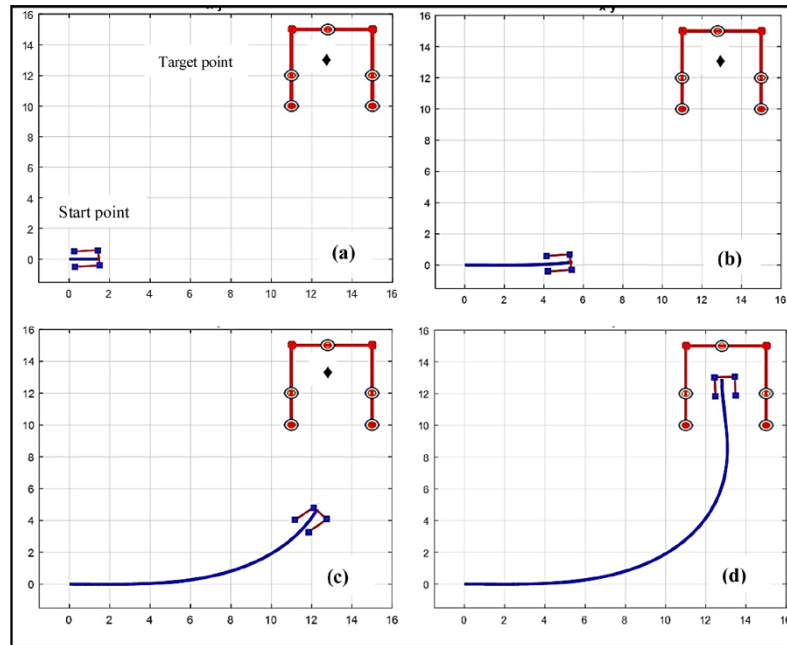

Figure 5 Vehicle states during the optimal path generation where (a) longitudinal displacement; (b) lateral displacement; (c) steering angle and (d) heading angle (see online version for colours)
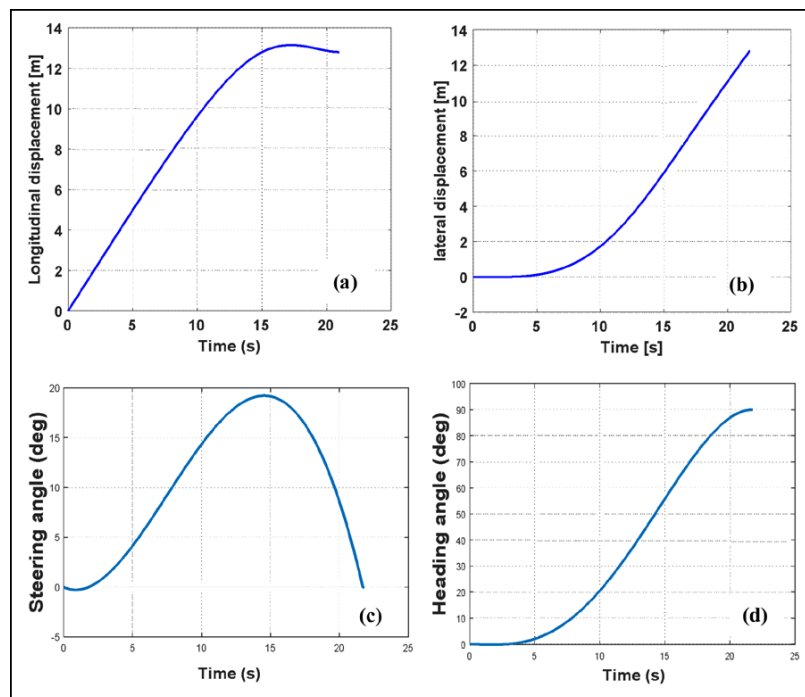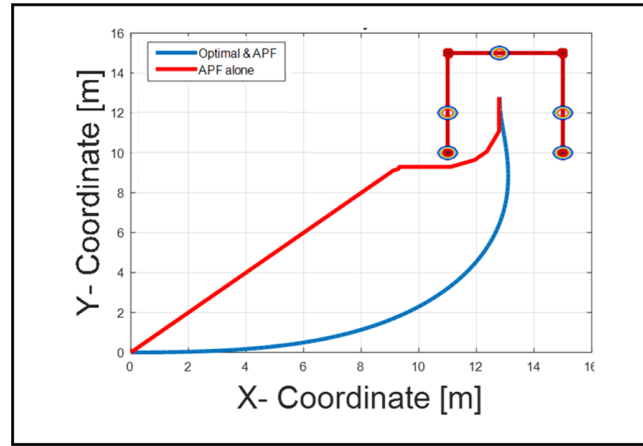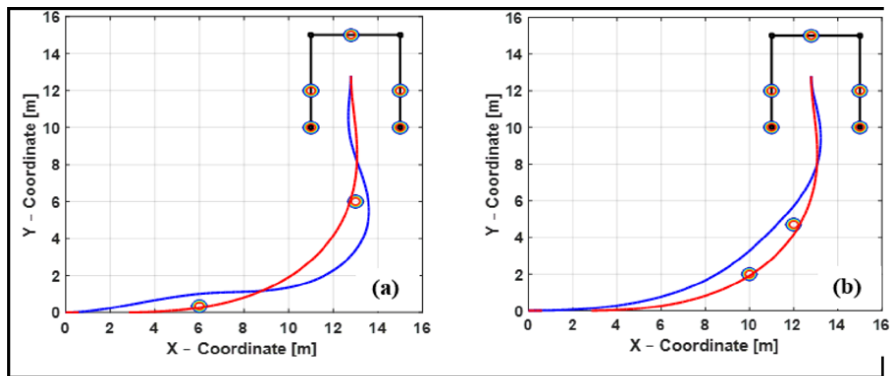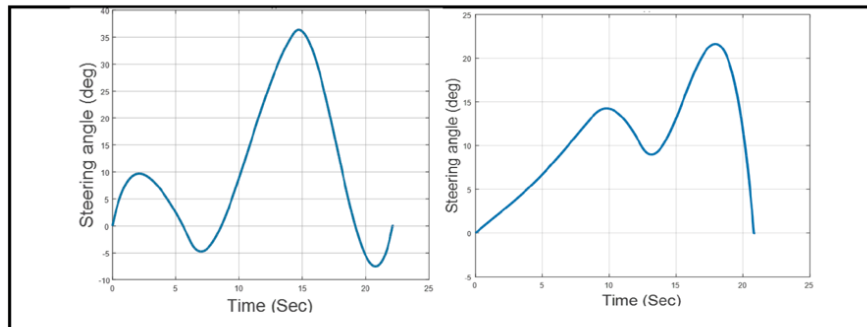
Figure 6   Vehicle path using optimal control and artificial potential field (see online version for colours)

Figure 7   (a) Optimal path between two points with different location of the obstacles and (b) vehicle steering angle during optimal path generation between two points with different location of the obstacles (see online version for colours)


(a)

(b)

5.3   *Optimal path with different starting points*

This scenario is aimed to check the robustness for the generated optimal path with respect to typical variable starting points and the same final parking spot. The numerical simulation is carried out as shown in Figure 8. It should be noted that, while the starting point is different, the controlled steering and heading angle ensure safe parking of the vehicle without hitting the side walls and this occurred with different turning radii.

Figure 8   Optimal path from different starting points (see online version for colours)
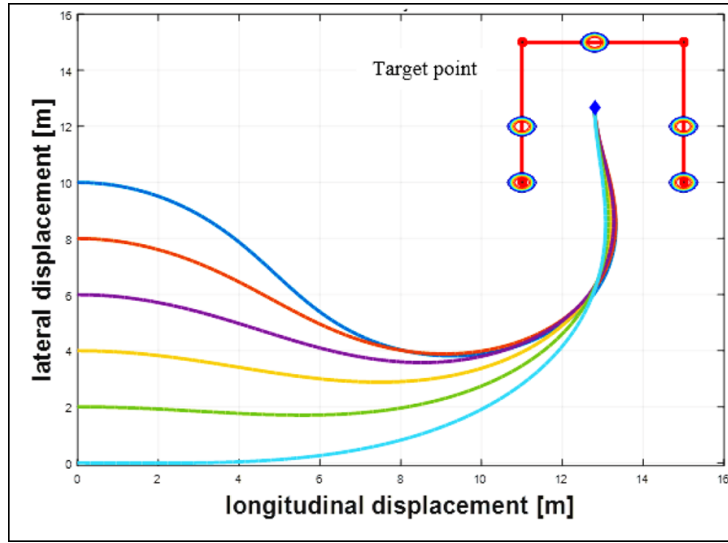
6   Conclusion

In this paper, the theory of optimal control has been successfully employed to generate an optimal path for a vehicle to move from a given starting point and safely parked in a parking spot. This algorithm is applied using a two-degree-of-freedom vehicle model. The proposed algorithm is mainly control the steering angle of the vehicle to follow the generated path. Additionally, the APF method is used to represent both the target location and different obstacles which are placed in different locations to check the robustness of the proposed optimal path planning algorithm. For this purpose, various numerical simulations are carried out such as different locations of the static obstacles and various starting points. For each simulation, the results demonstrated that the proposed control algorithm was able to optimally generate the required path without hitting the obstacles or the side border of the parking spot.

Acknowledgement

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Notations

a	Distance from the vehicle CG to the front axle
a_x, a_y	The centre position of the attractor
b	Distance from the vehicle CG to the rear axle
$C_{\alpha f}$	Front tyre cornering stiffness
$C_{\alpha r}$	Rear tyre cornering stiffness
d_{xo}, d_{yo}	Initial vehicle positions in longitudinal and lateral directions
d_{xf}, d_{yf}	Final vehicle positions in longitudinal and lateral directions
F_{yf}	Front tyre force
F_{yr}	Rear tyre force
I_{zz}	Moment of inertia
m	Vehicle mass
r	Yaw rate
U	Forward velocity
V	Lateral velocity
x	Longitudinal coordinates of the vehicle body motion
$\mathbf{X}(t)$	State vector

y	Lateral coordinates of the vehicle body motion
r_x, r_y	The centre position of the repulsor
δ	Steer angle
α_f	Front slip angle
α_r	Rear slip angle
θ	Angle between vehicle x-axis and direction of travel
Ψ	Heading angle
Ψ_o	Initial heading angle
Ψ_f	Final heading angle