Exercises to familiarize with Sparse Grids

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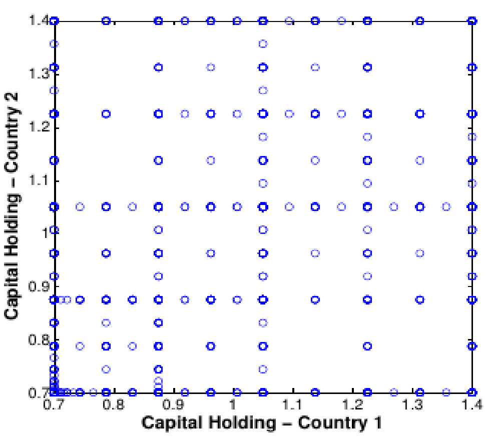

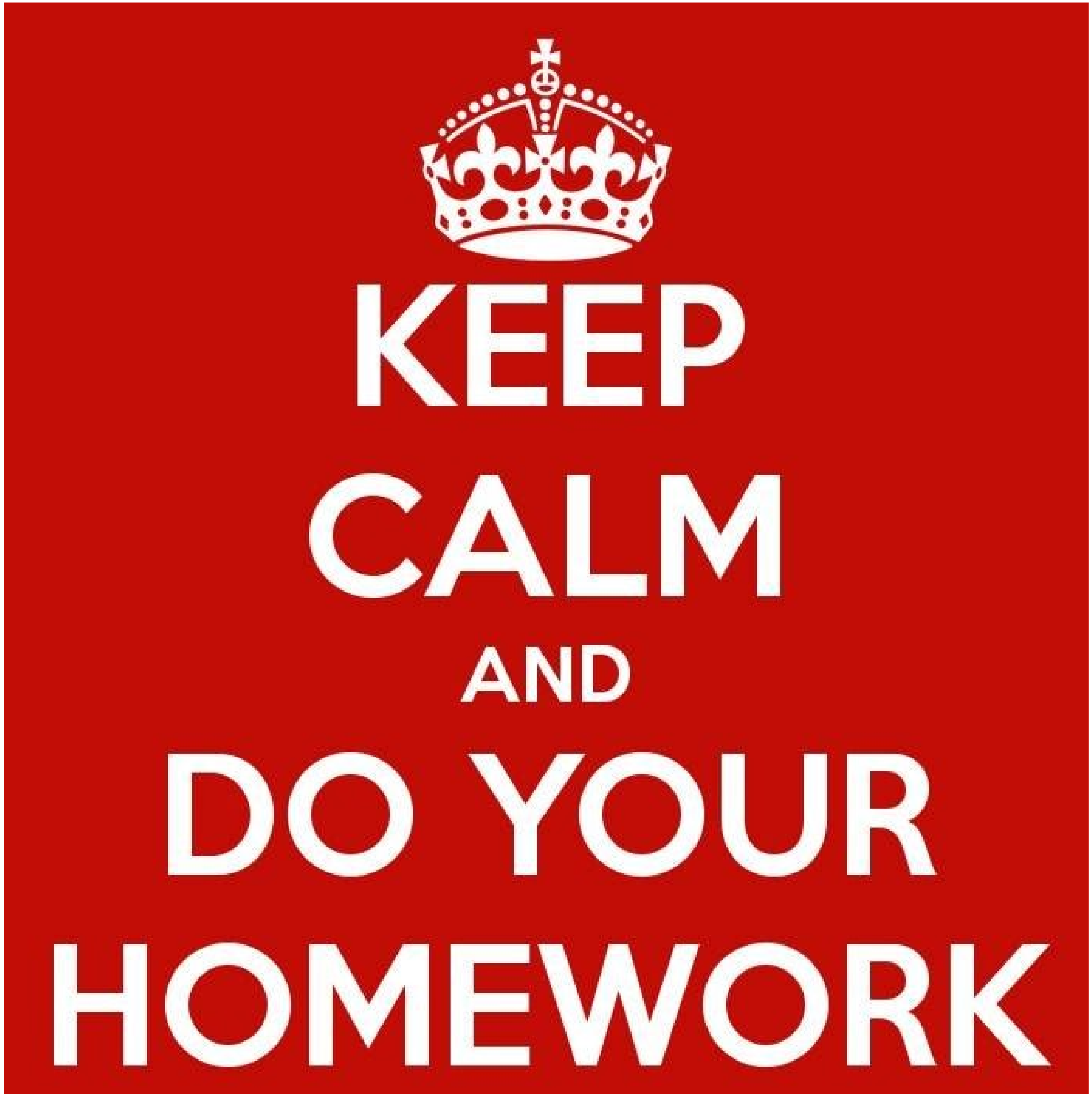
KEEP
CALM
AND
DO YOUR
HOMEWORK

1. Analytical examples

Create sparse grids based on different analytical test functions, e.g. Genz (1984).

→ different test functions can be obtained by varying $c = (c_1, \ldots, c_d)$ (c> 0) and $w = (w_1, \ldots, w_d)$

→ difficulty of functions is monotonically increasing with c.

→ randomly generate 1,000 test points and compute error(s): $e = \max_{i=1,\ldots,1000} |f(\vec{x}_i) - u(\vec{x}_i)|.$

→ play with adaptive/non-adaptive sparse grids/refinement level and criterion.

→ generate convergence plots (number of points versus error – as done above).

Genz (1984) test functions

1. OSCILLATORY: $f_1(x) = \cos\left(2\pi w_1 + \sum_{i=1}^{d} c_i x_i\right),$
2. PRODUCT PEAK: $f_2(x) = \prod_{i=1}^{d} \left(c_i^{-2} + (x_i - w_i)^2\right)^{-1},$
3. CORNER PEAK: $f_3(x) = \left(1 + \sum_{i=1}^{d} c_i x_i\right)^{-(d+1)},$
4. GAUSSIAN: $f_4(x) = \exp\left(-\sum_{i=1}^{d} c_i^2 t(x_i - w_i)^2\right),$
5. CONTINUOUS: $f_5(x) = \exp\left(-\sum_{i=1}^{d} c_i |x_i - w_i|\right),$
6. DISCONTINUOUS: $f_6(x) = \begin{cases} 0, & \text{if } x_1 > w_1 \text{ or } x_2 > w_2, \\ \exp\left(\sum_{i=1}^{d} c_i x_i\right), & \text{otherwise.} \end{cases}$

2. Growth model – Homework (I)

- I implemented you the model in Python (TASMANIAN)

→ OSM_Lab/SparseGrid/SparseGridCode/growth_model

II) Familiarize with the code (it will show up again :))

a) run the model with different settings

→vary the dimensionality of the problem

→vary the refinement level of the problem

→compute the average and maximum errors ("contraction mapping")

b) Add adaptivity to the code (cf. the analytical examples)

2. Growth model – Homework (II)

→ Add stochastic production to the model

$$f(k_i, l_i, \theta_i) \;=\; \theta_i A k_i^{\psi} l_i^{1-\psi}$$

→ Here we assume 5 possible values of
Θ_i = {0.9, 0.95, 1.00, 1.05, 1.10}

→ for simplicity, we assume **Π(*,*) = 1/5**

→ solve $$V_t(k,\theta) = \max_{c,l,I} u(c,l) + \beta \mathbb{E}\left\{ V_{t+1}(k^+, \theta^+) \mid \theta \right\}$$