Rational Inattention and Perceptual Distance

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  - But what costly learning model?

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    - Can result in unrealistic predictions for behaviour

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  - Easy to reject experientially (Dean & Neligh, 2019)

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- New caveats for multinomial logit
- The natural way of generalizing Shannon Entropy to incorporate perceptual distance

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  - Representation of knowledge agent already has

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  - Supported by psych literature (Stewart et al., 2006; Noguchi & Stewart, 2014, 2018)

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- Cost of learning the realized event of a binary partition given the agents beliefs, $C(\mathcal{P}^b, \mu)$, is primitive of this paper

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  - True for any belief reached during learning process

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- Expected cost of a learning strategy:

$$C(S^b,\, \mu) \equiv C(\mathcal{P}_1^b,\, \mu) + \mathbb{E}\Big[C\Big(\mathcal{P}_2^b,\, \mu(\cdot|\mathcal{P}_1^b(\omega))\Big) + \cdots + C\Big(\mathcal{P}_n^b,\, \mu(\cdot|\cap_{i=1}^{n-1}\mathcal{P}_i^b(\omega))\Big)\Big]$$

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- Beliefs: $\mu_B \in \Delta(\Omega_B)$, $\mu_D \in \Delta(\Omega_D)$
  - $\mu \in \Delta(\Omega)$
- Assume policies are 'similar' enough to have same cost function: $C(\mathcal{P}^b_A, \cdot) = C(\mathcal{P}^b_B, \cdot) = C(\mathcal{P}^b_D, \cdot) = C(\mathcal{P}^b, \cdot)$

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Assume agent learns the state of world by successively learning about different policies

- If the realization of the policies is independent, then the cost of learning the state is:

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- What if policy realizations are not independent?

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- Suppose agent further knows only one policy is beneficial

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  - What should be directly learned and what indirectly learned?

Example:
If C is close to constant, then learning about policy A first then B if needed may minimize learning costs

- $$C(\mathcal{P}^b, \mu(A_1)) + (1 - \mu(A_1))C\left(\mathcal{P}^b, \frac{\mu(B_1)}{\mu(B_1) + \mu(D_1)}\right) < C(\mathcal{P}^b, \mu(B_1)) + (1 - \mu(B_1))C\left(\mathcal{P}^b, \frac{\mu(D_1)}{\mu(D_1) + \mu(A_1)}\right)$$

Example:
If C varies a lot depending on belief, then agent may want to instead avoid policy A and learn about B and then D if needed

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Axioms:

- **Axiom 1 (Measurement):** Given a binary partition $\mathcal{P}^b = \{A_1, A_2\}$, $C(\mathcal{P}^b, \mu)$ is determined by $\mu(A_1)$ and $\mu(A_2)$, and we can thus write $C(\mathcal{P}^b, \mu) = C(\mathcal{P}^b, \mu(A_1))$

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- A question is assumed to be 'similar' to itself

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- Very weak assumption
- Required of useful theory

Lemma: Given binary partition $\mathcal{P}^b = \{A_1, A_2\}$, if C satisfies our three axioms, then there exists a multiplier $\lambda(\mathcal{P}^b) \in \mathbb{R}_+$ such that for all probability measures μ:

$$C(\mathcal{P}^b, \mu) = -\lambda(\mathcal{P}^b)\Big(\mu(A_1)\log(\mu(A_1)) + \mu(A_2)\log(\mu(A_2))\Big)$$

The convention used here is to set $0\log(0) = 0$.

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- We can measure the cost of inattentive learning as the reduction it causes in the cost of learning the state of the world
- If inattentive learning results in posterior $\tilde{\mu}$, cost of learning is:

$$C(S^{b*}, \mu) - C(S^{b*}, \tilde{\mu})$$

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$$\max_{F \in \Delta(\mathbb{R} \times \Omega)} \sum_{\omega \in \Omega} \sum_{n \in \mathcal{N}} V(n|F) F(n|\omega) \mu(\omega) - \mathbf{C}(F(n, \omega), \mu),$$

$$\text{such that } \forall \omega \in \Omega : \sum_{n \in \mathcal{N}} F(n, \omega) = \mu(\omega)$$

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- For each options n, let the unconditional chance of selecting n be denoted:

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Theorem 2: If the agent's behaviour is optimal, then $\forall\, n \in \mathcal{N}$, and $\forall\, \omega \in \Omega$ such that $\mu(\omega) > 0$, the probability that option n is selected in state ω satisfies:

$$\Pr(n|\omega) = \frac{\Pr(n)^{\frac{\lambda_1}{\lambda_M}} \Pr(n|\mathcal{P}_{\lambda_1}(\omega))^{\frac{\lambda_2-\lambda_1}{\lambda_M}} \ldots \Pr(n|\cap_{i=1}^{M-1}\mathcal{P}_{\lambda_i}(\omega))^{\frac{\lambda_M-\lambda_{M-1}}{\lambda_M}} e^{\frac{\mathbf{v}_n(\omega)}{\lambda_M}}}{\sum_{\nu\in\mathcal{N}} \Pr(\nu)^{\frac{\lambda_1}{\lambda_M}} \Pr(\nu|\mathcal{P}_{\lambda_1}(\omega))^{\frac{\lambda_2-\lambda_1}{\lambda_M}} \ldots \Pr(\nu|\cap_{i=1}^{M-1}\mathcal{P}_{\lambda_i}(\omega))^{\frac{\lambda_M-\lambda_{M-1}}{\lambda_M}} e^{\frac{\mathbf{v}_\nu(\omega)}{\lambda_M}}}$$

- Generalization of Matějka and McKay's (2015) useful necessary condition for the optimal behaviour of the agent

Theorem 3: Optimal choice behaviour is identical to the behavior produced by a random utility model where $\forall \omega \in \Omega$ such that $\mu(\omega) > 0$, each option $n \in \mathcal{N}$ has perceived value:

$$u_n = \tilde{v}_n + \alpha_n + \epsilon_n,$$

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$$+ \cdots + \tfrac{\lambda_M - \lambda_{M-1}}{\lambda_M} \log(N\Pr(n| \cap_{i=1}^{M-1} \mathcal{P}_{\lambda_i}(\omega)))$$

- Fitted values can be biased even if $\Pr(n) = \frac{1}{N}$ $\forall n$

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- Fitted values can be biased even if $\text{Pr}(n) = \frac{1}{N} \ \forall n$
- Contradicts rational inattention model with Shannon Entropy (Matějka & McKay, 2015)

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  - Seems natural in many contexts
  - Cannot be identified with the unconditional choice probabilities

Thanks for your (in?)attention!

Questions or comments?

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