

# Condition-based Maintenance Simulation Engine (Flanders Make *Costleap* project)

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## 1 Overview

The developed simulation engine provides a flexible framework for generating stochastic degradation indicators of engineering systems and modeling maintenance actions under both preventive and corrective strategies. It explicitly incorporates the effects of covariates on the degradation through log-linear parameter modulation. Specifically, the engine allows the user to simulate feature-dependent sample paths of a degradation process over a finite service horizon, and to obtain relevant measures such as the evolution of the degradation indicator, first hitting times to critical thresholds, and maintenance event logs. Multiple degradation mechanisms and maintenance policies are included to enable realistic modeling of condition-based maintenance systems.

### 1.1 Inputs and Outputs.

All parameters supplied to the simulator are user-defined inputs. These include degradation-model parameters, covariate specifications, maintenance-policy settings, repair models, cost parameters, and simulation controls. Also, machine-specific variation is supported in multi-machine simulations.

#### 1.1.1 Inputs

Tables 1–4 summarize the full set of input parameters for configuring a simulation run. The parameters are organized by category for clarity.

Table 1: Primary Simulation Parameters

Parameter	Default	Description
<code>degradation_type</code>	"gamma"	Type of degradation process: {"gamma", "inverse_gaussian", "wiener", "compound_poisson", "combined"}
<code>degradation_params</code>	None	Dictionary of process-specific parameters (see Table 2)
<code>covariate_specs</code>	None	List of covariates $Z(t)$ for degradation
<code>covariate_effects</code>	None	Coefficient vectors for covariates that affects process parameters
<code>dt</code>	0.01	Time step size for discretization
<code>PM_level</code>	2.0	Preventive maintenance threshold
<code>PM_interval</code>	None	Scheduled PM interval (if level-only policy: None)
<code>L</code>	5.0	Failure threshold
<code>x0</code>	0.0	Initial degradation level
<code>repair_func</code>	None	Repair function (typically <code>sample_post_repair_mixed</code> )
<code>repair_params</code>	None	Dictionary of repair strategy parameters (see Table 3).
<code>noise</code>	None	Observation noise model: {"none", "additive_normal", "brownian_increment"}, with $\sigma$ controlling noise intensity (e.g., {"type": "additive_normal", "sigma": 0.15})
<code>obs_time</code>	100.0	Observation horizon $T_{\text{obs}}$ .
<code>random_seed</code>	None	Random seed for reproducibility.

Table 2: Degradation Process Parameters by Type

Process Type	Parameter	Description
gamma	alpha	Shape parameter per unit time
	beta	Scale parameter
inverse_gaussian	mu	Drift rate per unit time
	lambda	Dispersion parameter
wiener	mu	Drift coefficient
	sigma	Diffusion coefficient
compound_poisson	lambda_shock	Shock arrival rate (can be PHM-adjusted by $Z(t)$ )
	shock_dist	Shock magnitude distribution: {"exponential", "gamma", "lognormal"}
	shock_scale	Scale parameter for exponential/gamma distributions
	shock_shape	Shape parameter (gamma only)
	shock_mu	Log-mean for lognormal shocks
	shock_sigma	Log-standard deviation for lognormal shocks
combined	base_process	Underlying process: {"gamma", "inverse_gaussian", "wiener"}
	alpha, beta, mu, sigma	Parameters inherited from the base process
	lambda_shock, shock_dist, ...	Parameters for the additional shock component

Table 3: Mixed Repair Strategy Parameters

Parameter	Default	Description
p_major	0.2	Probability of major repair (float $\in [0,1]$ )
dist_minor	"uniform"	Minor repair distribution: {"uniform", "beta", "proportional"}
dist_major	"uniform"	Major repair distribution: {"uniform", "beta", "proportional"}
a_minor, b_minor	2.0, 2.0	Beta distribution parameters for minor repairs
a_major, b_major	2.0, 5.0	Beta distribution parameters for major repairs
rho_minor	0.5	Proportional reduction factor for minor repair (float $\in [0,1]$ )
rho_major	0.7	Proportional reduction factor for major repair (float $\in [0,1]$ )

Table 4: Cost Model Parameters

Parameter	Default	Description
pm_shape, pm_scale	2.0, 50.0	Shape and scale parameters for perfect PM cost (Gamma distribution)
cm_shape, cm_scale	2.0, 200.0	Shape and scale parameters for CM cost (Gamma distribution)
cost_covariate_effects	None	Coefficients of covariates $Z(t)$ affecting the Gamma location parameter in Perfect PM and CM cost
c_0	100.0	Maximum cost in imperfect maintenance
cost_covariate_specs	None	List of covariates $W(t)$ affecting imperfect maintenance fixed cost
gamma_coeffs	None	Coefficients for the fixed-cost component $c_{\text{fix}}(t) = \exp(\gamma^\top W(t))$
epsilon_std	5.0	Standard deviation of random noise in imperfect maintenance cost

### 1.1.2 Outputs

The simulation produces the following outputs:

- **Degradation trajectories:** latent  $X(t)$  and observed  $X_{\text{obs}}(t)$  for  $t \in [0, T_{\text{obs}}]$ .

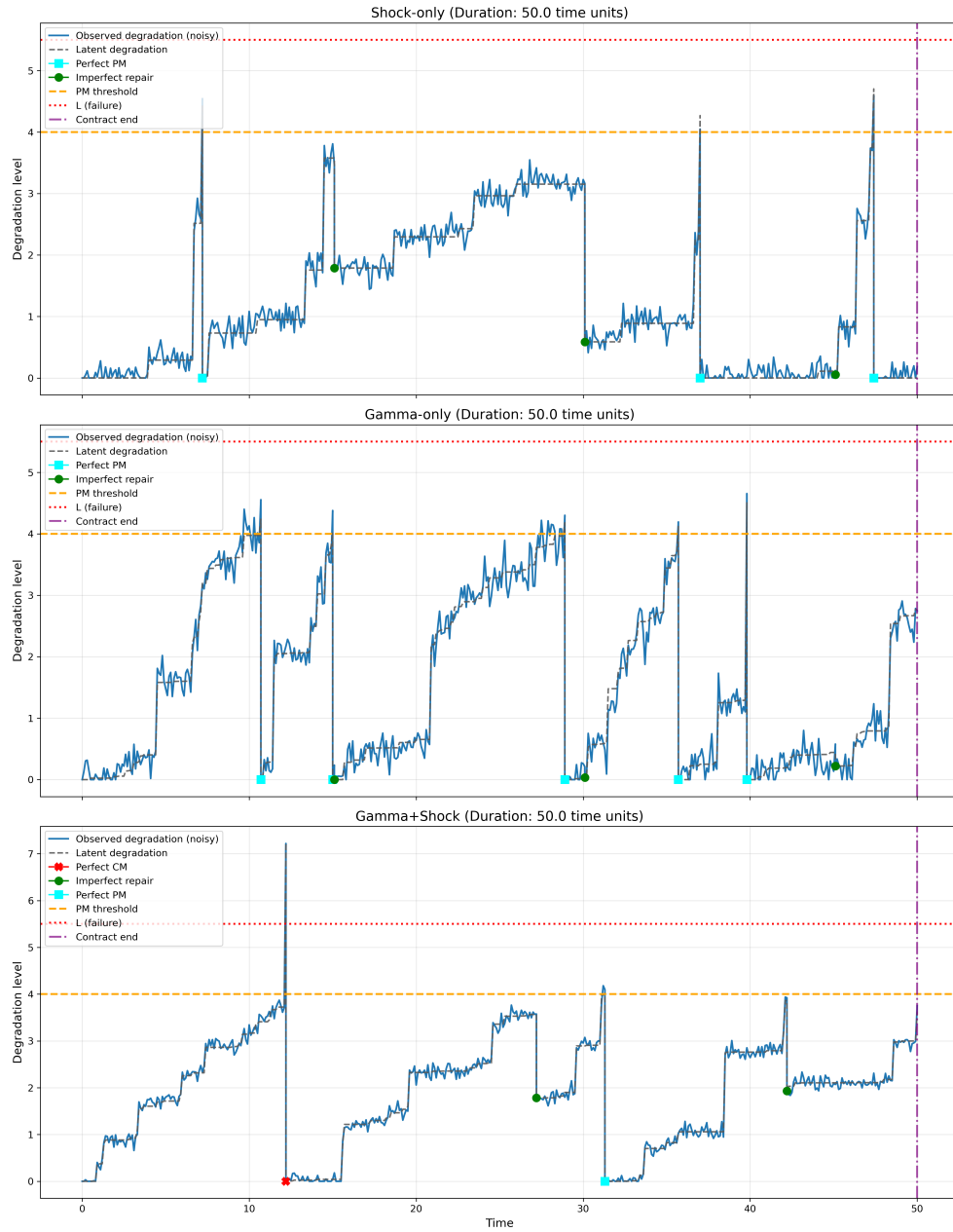


Figure 1: Examples of degradation path of a machine.

- **Covariate histories:** time series of degradation covariates  $Z(t)$  and, if applicable, cost covariates  $W(t)$ . Many covariates are *time-varying* or *path-dependent* and they update during the simulation.

	machine_id	time	stress	environment_class	labor_cost_index	material_availability
	0	0.0	1.500000	1.0	0.992314	0.9
	1	0.1	1.502506	1.0	1.010014	0.9
	2	0.2	1.505025	1.0	0.999704	0.9
	3	0.3	1.507557	1.0	1.011336	0.9
	4	0.4	1.510101	1.0	1.027530	0.9
	...	...	...	...	...	...
	4995	49.5	6.940854	1.0	1.487014	0.9
	4996	49.6	6.970632	1.0	1.512563	0.9
	4997	49.7	7.000560	1.0	1.497458	0.9
	4998	49.8	7.030638	1.0	1.538980	0.9
	4999	49.9	7.060867	1.0	1.471267	0.9

Figure 2: Example of covariate outputs.

- **Event logs:** sequence of all PM/CM events across the fleet, including timestamps, event types, triggering mechanism, pre-/post-condition states, repair effectiveness, and event costs.

machine_id	PM_level	PM_interval	time	type	trigger_reason	level_before_latent	level_before_observed	level_after_latent	level_at
0	0	4.0	15.0	perfect_preventive_maintenance	level_threshold_observed	4.751474	4.796827	0.000000	
1	0	4.0	15.0	imperfect_repair	scheduled_time	1.176669	1.301039	0.995994	
2	0	4.0	15.0	imperfect_repair	scheduled_time	2.613625	2.405190	1.804810	
3	0	4.0	15.0	perfect_preventive_maintenance	level_threshold_observed	4.364901	4.432757	0.000000	
4	0	4.0	15.0	imperfect_repair	scheduled_time	0.134978	0.163415	0.067489	
5	1	3.5	12.0	perfect_preventive_maintenance	level_threshold_observed	3.541137	3.563063	0.000000	
6	1	3.5	12.0	imperfect_repair	scheduled_time	0.000000	0.000000	0.000000	
7	1	3.5	12.0	imperfect_repair	scheduled_time	1.779304	1.641513	0.889652	
8	1	3.5	12.0	catastrophic_failure_replacement	N/A	5.509543	5.465794	0.000000	
9	1	3.5	12.0	imperfect_repair	scheduled_time	0.339320	0.437897	0.169660	
10	1	3.5	12.0	perfect_preventive_maintenance	level_threshold_observed	4.177658	4.252024	0.000000	
11	2	4.0	15.0	perfect_preventive_maintenance	level_threshold_observed	4.155105	4.253645	0.000000	
12	2	4.0	15.0	perfect_preventive_maintenance	level_threshold_observed	4.835767	4.988472	0.000000	

Figure 3: Example of event logs for the fleet.

- **Machine logs:** per-machine summaries reporting event counts and costs by type, total maintenance cost, final degradation levels, and the maintenance-policy parameters applied to each machine.

	machine_id	PM_level	PM_interval	strategy	total_cost	final_level_latent	final_level_observed	n_perfect_pm	n_imperfect_pm	n_cm	total_events	cost_perfect
0	0	4.0	15.0	time_and_level	1781.130685	0.442856	0.579879	5	2	1	8	970.655
1	1	4.0	15.0	time_and_level	593.973457	2.110078	2.344733	3	3	0	6	463.219
2	2	4.0	15.0	time_and_level	580.416263	2.376325	2.535941	3	3	0	6	444.655
3	3	4.0	15.0	time_and_level	777.485749	0.805230	0.972556	5	3	0	8	713.342
4	4	4.0	15.0	time_and_level	733.621553	1.839913	1.562459	3	3	0	6	581.010
5	5	4.0	15.0	time_and_level	736.424842	0.845578	1.081965	4	3	0	7	603.555
6	6	4.0	15.0	time_and_level	1024.674267	2.937488	2.908638	4	3	0	7	900.456
7	7	4.0	15.0	time_and_level	770.438887	3.148987	3.098680	4	3	0	7	607.880
8	8	4.0	15.0	time_and_level	855.493222	1.295296	1.471597	3	3	0	6	728.340
9	9	4.0	15.0	time_and_level	920.273523	1.523254	1.354616	4	3	0	7	782.758

Figure 4: Example of machine logs for the fleet.

## 2 System Description

We consider a stochastic process  $\{X(t), t \geq 0\}$  to characterize the underlying deterioration of the system over a finite observation horizon  $[0, T_{\text{obs}}]$ . A finite horizon is appropriate because, in practice, maintenance service contracts and warranty periods are typically defined over limited time spans.

The degradation level is initialized at  $X(0) = 0$ , representing the *as-good-as-new* condition. System failure occurs when the degradation level reaches or exceeds a catastrophic failure threshold  $L$ , at which point the system is assumed to fail and must be replaced.

In practice, the degradation level of a system cannot be measured perfectly. To capture the practical limitations of real-world condition monitoring, the simulation introduces an *observed degradation indicator* that incorporates measurement noise. To distinguish it from the latent degradation level  $X(t)$ , the observed indicator is defined as

$$X_{\text{obs}}(t) = X(t) + \epsilon(t),$$

where  $\epsilon(t)$  represents random measurement noise added to the latent degradation level. The observed degradation indicator also starts from  $X_{\text{obs}}(0) = 0$  and terminates when the latent degradation level satisfies  $X(t) \geq L$ .

To decrease the likelihood of such failures, preventive maintenance (PM) strategies are introduced. The system can be monitored continuously and PM can be triggered either by:

1. observed degradation indicator exceeding a threshold, i.e.,  $X_{\text{obs}}(t) > PM_{\text{level}}$ ,
2. scheduled periodic intervals  $\tau_{\text{PM}}$ ,
3. or both (combined rule).

The simulator supports *different PM policies per machine* in a fleet: each machine can be assigned its own threshold and/or interval.

Although degradation evolves continuously, the system is observed at discrete time points  $t_k = k\Delta t$ ,  $k = 0, 1, \dots, \lfloor T/\Delta t \rfloor$ .

### 3 Degradation Process Models

The engine supports several types of degradation processes to describe the latent degradation path. Machine-specific features can also be incorporated to account for machine heterogeneity.

#### 3.1 Degradation Processes

**Gamma Process** Over a small interval  $\Delta t$ , the degradation increment  $\Delta X(t) = X(t + \Delta t) - X(t)$  is modeled as

$$\Delta X(t) \sim \text{Gamma}(\alpha\Delta t, \beta),$$

where  $\alpha > 0$  is the shape rate per unit time and  $\beta > 0$  is the scale parameter. The Gamma process is stochastically non-decreasing almost surely, meaning that the degradation path is monotone increasing. This makes it suitable for cumulative damage such as fatigue, corrosion, or wear.

**Inverse Gaussian Process** For the inverse Gaussian (IG) process, the increment is distributed as

$$\Delta X(t) \sim \text{IG}(\mu\Delta t, \lambda\Delta t^2),$$

where  $\mu > 0$  is the drift rate (expected degradation per unit time) and  $\lambda > 0$  is the dispersion parameter controlling the variability of the degradation path. The IG process is another type of monotonically increasing stochastic process.

**Wiener Process** For the Wiener process, the increment follows

$$\Delta X(t) = \mu\Delta t + \sigma\sqrt{\Delta t} Z, \quad Z \sim \mathcal{N}(0, 1),$$

where  $\mu \in \mathbb{R}$  is the drift rate and  $\sigma > 0$  is the diffusion coefficient. In contrast to the Gamma and IG processes, the Wiener process is not strictly monotone: the degradation path fluctuates up and down due to Gaussian noise, though the long-term trend is governed by  $\mu$ . This model is appropriate for systems where degradation exhibits random fluctuations around a linear trend, for example due to environmental disturbances or measurement noise.



**Compound Poisson Process** When degradation is resulted from random shocks, compound Poisson process is appropriated to be used. Over  $\Delta t$ ,

$$\Delta X(t) = \sum_{i=1}^{N(\Delta t)} S_i, \quad N(\Delta t) \sim \text{Poisson}(\lambda_{\text{shock}} \Delta t),$$

where  $N(\Delta t)$  is the number of shocks in the interval, and the shock sizes  $S_i$  are i.i.d. from a specified distribution, e.g.

- Exponential:  $S_i \sim \text{Exp}(\theta)$ ,
- Gamma:  $S_i \sim \text{Gamma}(k, \theta)$ ,
- Geometric:  $S_i \sim \text{Geom}(p)$ ,
- Lognormal:  $S_i \sim \text{Lognormal}(\mu, \sigma)$ .

Here  $\lambda_{\text{shock}} > 0$  is the arrival rate of shocks. The resulting degradation path is piecewise constant with sudden upward jumps, making it suitable for modeling systems subject to random shock loads.

**Combined Process** In the combined model, degradation is driven by a continuous base process plus random shocks:

$$\Delta X(t) = \Delta X_{\text{base}}(t) + \Delta X_{\text{shock}}(t).$$

The base process can be chosen as a Gamma, inverse Gaussian, or Wiener process, while  $\Delta X_{\text{shock}}(t)$  is generated by a compound Poisson process. This hybrid construction captures both gradual degradation trends (e.g., wear, corrosion) and sudden damage events (e.g., shocks). It is particularly useful in practical applications where equipment experiences gradual wear and fatigue over time and suffers from sudden damages caused by external events.

### 3.2 Covariates and Their Effects

Let  $\mathbf{Z}(t) = [Z_1(t), Z_2(t), \dots, Z_p(t)]$  denote the vector of covariates that influence the degradation dynamics at time  $t$ . The inclusion of covariates allows the degradation process to adapt to varying operational or environmental conditions.

Each covariate can be specified as one of the following types:

- **Fixed covariate:** a constant value (time-invariant) throughout the simulation;

- **Time-dependent covariate:** a function of time,  $Z_i(t) = f_i(t)$ , representing factors that vary deterministically or stochastically over time;
- **Path-dependent covariate:** a function of the current degradation state,  $Z_i(t) = g_i(X(t))$ , allowing feedback between the degradation process and the covariate evolution

Covariates affect the parameters of the degradation model through exponential (log-link) relationships. For a degradation-related parameter  $\theta_j$  with baseline value  $\theta_{j,0}$  and associated coefficient vector  $\beta_j$ , the adjusted parameter value at time  $t$  is given by

$$\theta_j(t) = \theta_{j,0} \exp(\beta_j^\top \mathbf{Z}(t)),$$

which ensures the parameter remains positive and allows its instantaneous adjustment to changing covariate conditions.

## 4 Maintenance Policies

### 4.1 Corrective Maintenance (CM)

A catastrophic failure occurs when the degradation level first exceeds the threshold  $L$ :

$$\tau_f = \inf\{t \geq 0 : X(t) \geq L\}.$$

At  $\tau_f$ , perfect corrective maintenance (CM) is performed, i.e.,

$$X(\tau_f^+) = 0,$$

and a new degradation process starts from zero.

### 4.2 Preventive Maintenance (PM)

In practical applications, preventive maintenance (PM) is often conducted either based on the system's degradation level or at fixed periodic intervals. To capture this reality, we incorporate both types of PM triggering mechanisms into the model:

- **Level-only PM:** triggered when the degradation process first exceeds the preventive threshold  $PM_{\text{level}}$ , provided  $X(t) < L$ . That is,  $\tau_p = \inf\{t \geq 0 : PM \leq X(t) < L\}$ . At  $\tau_p$ , the system is restored to a lower degradation level  $Y(\tau_p^+)$ . If the this level remains above  $PM_{\text{level}}$ , a perfect preventive maintenance is enforced immediately.

- **Time-and-level PM:** periodically scheduled at fixed intervals  $\tau_{\text{PM}}$ . Also, once the degradation level exceeds  $PM_{\text{level}}$ , a perfect PM is activated immediately.

### 4.3 Imperfect Repair Model

In practice, preventive maintenance does not always restore the system to an *as-good-as-new* condition. To capture the variability in maintenance effectiveness, we distinguish between two categories:

- **Major repair**, which may reduce the degradation to a level even lower than that achieved by the previous maintenance action, possibly close to the initial state.
- **Minor repair**, which only provides a limited reduction, keeping the degradation level no better than the most recent post-maintenance state.

Let  $y_{\text{lower}}$  denote the degradation level immediately after the last repair and  $y_{\text{upper}}$  the degradation level just before the current repair. To unify the formulation, we set

$$y_{\text{lower}} = \begin{cases} 0, & \text{major repair,} \\ \text{post-repair level of last maintenance,} & \text{minor repair.} \end{cases}$$

Then, the post-repair degradation level  $Y$  is modeled as a random draw from one of the following families:

**Uniform model.**

$$Y \sim U(y_{\text{lower}}, y_{\text{upper}}),$$

where  $U(a, b)$  denotes a uniform distribution on  $[a, b]$ .

**Beta model.**

$$Y = y_{\text{lower}} + (y_{\text{upper}} - y_{\text{lower}}) B(a, b),$$

where  $B(a, b)$  is a Beta-distributed random variable with shape parameters  $(a, b)$ .

**Proportional reduction model.**

$$Y = y_{\text{upper}} - (1 - \rho)(y_{\text{upper}} - y_{\text{lower}}),$$

where  $\rho \in (0, 1)$  represents the proportion of degradation reduced by the maintenance action.

## 5 Cost Modeling

### 5.1 Perfect PM and CM Cost

Perfect preventive maintenance (PM) and corrective maintenance (CM) are modeled using a *three-parameter Gamma distribution*, which is non-negative and right-skewed, allowing for occasional large cost realizations. This distribution also includes a *location parameter* that enables the entire cost curve to shift according to contextual factors.

Because both operating conditions (e.g., load, temperature, environment) and cost-related logistical factors (e.g., travel distance, access difficulty) can influence the baseline effort required for maintenance, we allow the cost features  $W(t)$  to enter the perfect PM and CM cost model through the location parameter representing the baseline cost level:

$$\ell_r(t) = \beta_r^\top W(t),$$

where  $\beta_r$  is the coefficient vector for maintenance type  $r$  (either PM or CM). Consequently, the maintenance cost for type  $r$  at time  $t$  is defined as

$$C_r(t) \mid W(t) \sim \text{Gamma}(k_r, \theta_r, \ell_r(t)),$$

where  $k_r$  and  $\theta_r$  denote the shape and scale parameters, respectively, and  $\ell_r(t)$  is the condition-dependent location parameter.

Perfect PM and CM share the same functional form but differ in parameter values. In general, PM corresponds to standardized, planned maintenance activities with lower variability in cost, while CM represents reactive interventions following unexpected breakdowns, typically exhibiting larger costs and higher variance.

### 5.2 Imperfect PM Cost

The cost of imperfect maintenance is primarily driven by the maintenance effect: the larger the reduction in degradation achieved by the action, the higher the associated cost. Accordingly, we model the cost as a function of the maintenance effect, with an additional fixed component and a random noise term.

The maintenance effect denoted by  $u \in [0, 1]$  is defined as

$$u = \frac{X_{\text{before}} - X_{\text{after}}}{X_{\text{before}}},$$

where  $X_{\text{before}}$  and  $X_{\text{after}}$  are the latent degradation levels immediately before and after the maintenance action, respectively. A larger value of  $u$  indicates a more effective maintenance intervention.

Based on this definition, the cost of imperfect maintenance at time  $t$  is given by

$$C_{\text{IPM}}(t) = c_{\text{fix}}(t) + c_0 u^\eta + \varepsilon,$$

with truncation at zero in implementation, i.e.,  $C_{\text{IPM}}(t) = \max\{0, C_{\text{IPM}}(t)\}$ . The components are explained as follows:

- $c_{\text{fix}}(t) = \exp(\gamma^\top W(t))$  is a strictly positive, fixed cost component (e.g., travel, access, and setup) influenced by operational covariates  $W(t)$ .
- $c_0 > 0$  is the scaling coefficient for the variable cost component; higher maintenance effect  $u$  (deeper restoration) results in higher maintenance costs.
- $\eta > 0$  is a shape parameter controlling the curvature of the cost–maintenance effect relationship. When  $\eta = 1$ , the variable cost increases linearly with maintenance effect  $u$ ; when  $\eta > 1$ , the marginal cost rises sharply at higher effect levels; and when  $0 < \eta < 1$ , the marginal cost increases more slowly, indicating diminishing returns to maintenance effort.
- $\varepsilon$  is a random noise term capturing unobserved variations in maintenance cost (e.g., small fluctuations in labor time or material usage).

This specification allows the imperfect maintenance cost to flexibly capture both systematic variations driven by observable factors and stochastic deviations inherent in maintenance operations.