Supplement to "On finite mixture modeling and model-based clustering of directed weighted multilayer networks"

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S-1. PARAMETERS USED FOR SIMULATION STUDIES IN EXPERIMENTAL EVALUATION SECTION

1.1 Experiments in Section 3.1

For this experiment, a three-component mixture with mixing proportion $0.5, 0.3$, and 0.2 has been generated. Then, networks with sample sizes $50, 100$, and 200 are generated from this mixture. Four settings consisting of different levels of inter- and intra-cluster variability have been considered. High variability has been simulated by multiplying the covariance of the baseline variability case by two. For each setting, 100 data sets have been simulated. The model parameters are presented in Table S-1 and Table S-2.

Table S-1: Mean vector parameters for the simulation study in Section 3.1.

	Gray	Red	Khaki
Gray	3	5	6
Red	3	12	9
Khaki	10	13	15

1.2 Experiments in Section 3.2

For this experiment, a four-layer network consisting of 50 nodes has been generated from a two-component mixture with mixing proportions 0.6 and 0.4. Three settings with varying separation levels are considered. For each setting, 100 data sets have been simulated. The model parameters are presented in Table S-3 , Table S-4, and Table S-5.

Table S-2: Covariance matrix parameters for the simulation study in Section 3.1.

	Grey		Red		Khaki	
Grey	1.0	-0.5	1.0	-0.5	1.0	0.5
		1.0		1.0		1.0
Red			1.0	0.5	1.0	-0.5
				1.0		1.0
Khaki					1.0	0.5
						1

Table S-3: Mean matrix parameters for simulation study in Section 3.2

		Layer 1		Layer 2		Layer 3		Layer 4	
		Red	Gray	Red	Gray	Red	Gray	Red	Gray
Low Sep	Red	5	5	2	3	3	2	6	6
	Gray	5	5	4	2	3	4	6	4
Med Sep	Red	5	5	2	3	3	2	7	4
	Gray	5	5	4	2	3	4	5	2
High Sep	Red	5	5	2	3	3	2	7	4
	Gray	5	5	4	6	3	4	5	2

1.3 Experiments in Appendix B

For this experiment, a network consisting of 50 nodes has been generated from a three component mixture with mixing proportions $0.5, 0.3$, and 0.2. Four settings with $0\%, 10\%, 25\%$, and 50% of missing nodes are considered. For each setting, 100 data sets have been simulated. The model parameters are presented in Table S-6 and Table S-7.

S-2. CLUSTERING SOLUTIONS OBTAINED IN THE APPLICATION SECTION

2.1 Unilayer network considering trade volume

Table S-8 and Table S-9 provide the clustering solution associated to the unilayer network representing the total trade volume of 39 European countries.

Table S-4: Row covariance matrix parameters for simulation study in Section 3.2

	Red		Grey	
Red	1.00	-0.25	1.00	-0.25
		1.00		1.00
Grey			1.00	0.25
				1.00

Table S-5: Column covariance matrix parameters for simulation study in Section 3.2

	Red				Grey			
	L1	L2	L3	L4	L1	L2	L3	L4
Red	1.0000	-0.3651	0.3651	-0.6000	1.0000	-0.2041	0.2041	-0.5000
		1.0000	-0.3333	0.3651		1.0000	0.0000	0.2041
			1.0000	0.3651			1.0000	-0.2041
				1.0000				1.0000
Grey					1.0000	-0.3651	0.3651	-0.6000
						1.0000	-0.3333	0.3651
							1.0000	0.3651
								1.0000

Table S-9: Partition obtained by three-cluster solution (unilayer approach on trade volume)

Cluster 1 (Khaki)	Austria	Belarus	Bulgaria
	Croatia	Czech Republic	Denmark
	Estonia	Finland	Greece
	Hungary	Ireland	Latvia
	Lithuania	Norway	Poland
	Portugal	Romania	Serbia, FR(Serbia/Montenegro)
	Slovak Republic	Slovenia	Spain
	Sweden	Switzerland	Ukraine
Cluster 2 (Grey)	Albania	Andorra	Bosnia and Herzegovina
	Iceland	Luxembourg	Macedonia, FYR
	Malta	Moldova	Montenegro
Cluster 3 (Red)	Belgium	France	Germany
	Italy	Netherlands	United Kingdom

Table S-6: Mean vector parameters for simulation study in Appendix C

	Component 1	Component 2	Component 3
Component 1	13	15	16
Component 2	13	22	19
Component 3	20	23	25

Table S-7: Covariance matrix parameters for simulation study in Appendix C

	Component 1		Component 2		Component 3	
Component 1	1.0	-0.5	1.0	-0.5	1.0	-0.5
		1		1		1
Component 2			1.0	-0.5	1.0	0.5
				1		1
Component 3					1.0	-0.5
						1

Table S-8: Mean trade volume for all products in the three-cluster solution (unilayer approach)

	Gray	Khaki	Red
Gray	\$14,355	\$47,622	\$377,714
Khaki	\$99,735	\$1,145,940	\$6,859,876
Red	\$640,585	\$7,144,463	\$52,769,862

2.2 Unilayer network considering relative trade

Table S-10 and Table S-11 provide the clustering solution associated to the unilayer network consisting of relative trade volume of 39 European countries.

Table S-10: Mean relative trade for all products in the two-cluster solution (unilayer approach)

	Khaki	Red
Khaki	0.2114	0.1388
Red	0.1249	0.0307

Table S-11: Partition obtained by two-cluster solution (unilayer approach on relative trade)

Cluster 1 (Khaki)	Belgium	Malta	Netherlands
	Slovenia		
Cluster 2 (Red)	Albania	Andorra	Austria
	Belarus	Bosnia and Herzegovina	Bulgaria
	Croatia	Czech Rep.	Denmark
	Estonia	Finland	France
	Germany	Greece	Hungary
	Iceland	Ireland	Italy
	Latvia	Lithuania	Luxembourg
	Macedonia	Moldova	Montenegro
	Norway	Poland	Portugal
	Romania	Serbia	Slovakia
	Spain	Sweden	Switzerland
	Ukraine	United Kingdom	

2.3 Multilayer network considering trade volume

Table S-12 and Table S-13 provide the clustering solution associated to the multilayer network consisting of trades in capital goods, consumer goods and intermediate goods category for 39 European countries.

Table S-12: Mean trade volume for the four-cluster solution (multilayer approach)

	Capital Goods				Consumer Goods				Intermediate Goods			
	Gray	Khaki	Magenta	Red	Gray	Khaki	Magenta	Red	Gray	Khaki	Magenta	Red
Gray	$0.00006	$562	$0.00015	$6,100	$11,994	$5,152	$522	$24,531	$0.00003	$1,000	$1,483	$6,102
Khaki	$3,485	$31,906	$26,983	$89,797	$26,739	$112,327	$111,792	$238,241	$6,087	$58,474	$48,615	$134,172
Magenta	$0.00151	$19,688	$36,572	$33,722	$13,315	$47,524	$256,449	$64,879	$3,272	$39,088	$131,409	$50,944
Red	$33,633	$169,586	$61,237	$2,493,202	$85,354	$285,254	$126,963	$3,689,111	$26,703	$161,728	$76,030	$2,242,728

Table S-13: Partition obtained in the four-cluster solution (multilayer approach on trade volume)

Cluster 1 (Grey)	Albania	Andorra	Malta
	Moldova		
Cluster 2 (Khaki)	Belarus	Bulgaria	Estonia
	Greece	Iceland	Latvia
	Lithuania	Luxembourg	Macedonia, FYR
	Serbia, FR(Serbia/Montenegro)	Slovenia	

Cluster 3 (Magenta)	Bosnia and Herzegovina	Croatia	Montenegro
Cluster 4 (Red)	Austria	Belgium	Czech Republic
	Denmark	Finland	France
	Germany	Hungary	Ireland
	Italy	Netherlands	Norway
	Poland	Portugal	Romania
	Slovak Republic	Spain	Sweden
	Switzerland	Ukraine	United Kingdom

2.4 Multilayer network considering relative trade

Table S-14 and Table S-15 provide the clustering solution associated to the multilayer network consisting of relative trades in capital goods, consumer goods and intermediate goods category for 39 European countries.

Table S-14: Mean relative trade for the three-cluster solution (multilayer approach)

	Capital Goods			Consumer Goods			Intermediate Goods		
	Gray	Khaki	Red	Gray	Khaki	Red	Gray	Khaki	Red
Gray	0.1752	0.0245	0.0052	0.1840	0.0248	0.0081	0.1813	0.0242	0.0061
Khaki	0.0328	0.0414	0.0843	0.0267	0.0410	0.0776	0.0269	0.0368	0.0876
Red	0.0089	0.0742	0.2271	0.0094	0.0787	0.2139	0.0108	0.0805	0.2255

Table S-15: Partition obtained by 3-cluster solution (multilayer approach on relative trade)

Cluster 1 (Grey)	Albania	Andorra	Bosnia and Herzegovina
	Croatia	Luxembourg	Macedonia
	Malta	Montenegro	Serbia, FR(Serbia/Montenegro)
	Slovenia		
Cluster 2 (Khaki)	Belgium	Czech Rep.	Estonia
	Latvia	Netherlands	Portugal
	Switzerland		
Cluster 3 (Red)	Austria	Belarus	Bulgaria
	Denmark	Finland	France
	Germany	Greece	Hungary
	Iceland	Ireland	Italy
	Lithuania	Moldova	Norway

Poland	Romania	Slovakia
Spain	Sweden	Ukraine
United Kingdom		