Introduction to Computer Graphics

Assoc. Prof. Dr. Natasha Dejdumrong

COURSE DESCRIPTION

- Basic concepts in computer graphics including the techniques for creation, storage, transformation, translation, rotation, and animation of computer models and images.

- Line and polygon drawing algorithms, the basic transformation of translation, scaling and rotation in two and three dimensional space, windowing and clipping, and parallel and perspective projection.

- The 3D aspects of computer graphics will be emphasized, covering issues on what makes an image appear 3D and the various techniques for visual realism.

LEARNING OUTCOMES

- Students who have completed this course should be able to:

  1. Explain the basic principles of computer graphics.

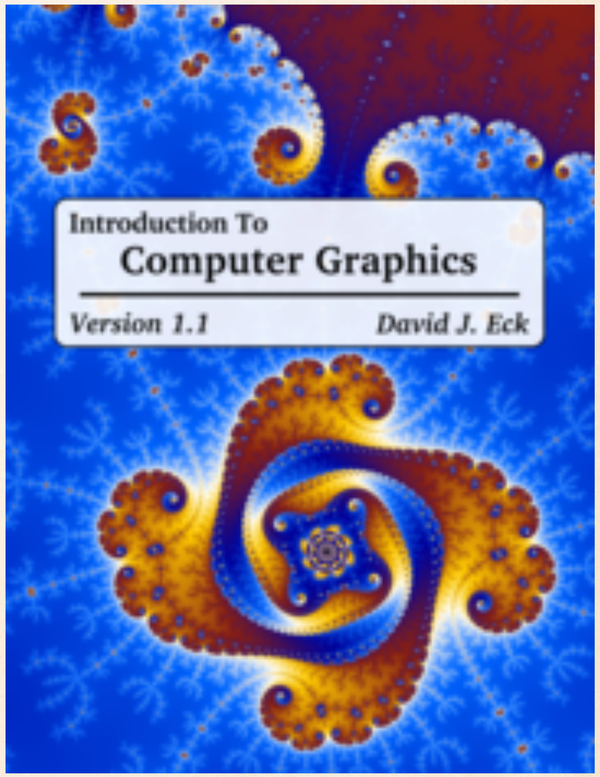
  2. Evaluate, select and implement appropriate algorithms to efficiently solve computer graphics problems.

  3. Apply computer graphics algorithms to create the corresponding programs.

MATERIALS

- Lecture Notes, Computer Graphics Fundamentals

- Introduction to Computer Graphics (2018)
  by David J. Eck



Handouts, slides, and assignments are posted at http://leb2.kmutt.ac.th. Regularly check for updates

GRADING SYSTEM

- Midterm exam 30 %
- Final exam 30 %
- Assignments & Quizzes 20 %
- Active Learning/Projects 20 %

Module 1: Two Dimensional Transformations

Assoc. Prof. Dr. Natasha Dejdumrong

CPE 361 Computer Graphics

Topics

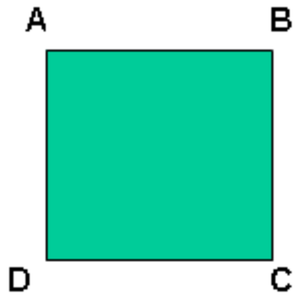
- Geometric Transformation
- Homogeneous Coordinates
- Coordinate Transformation
- Inverse Transformations
- Composite Transformations

Transformation

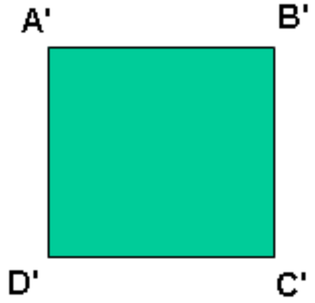
- To transform something is to change it.
- In geometry, there are specific ways to describe how a figure is changed.
- The transformations you will learn about include:
  - Translation
  - Rotation
  - Mirror Reflection
  - Scaling

Renaming Transformation

- It is common practice to name shape using capital letters:



It is common practice to name transformed shapes using the same letters with a “prime” symbol:



Geometric Transformation

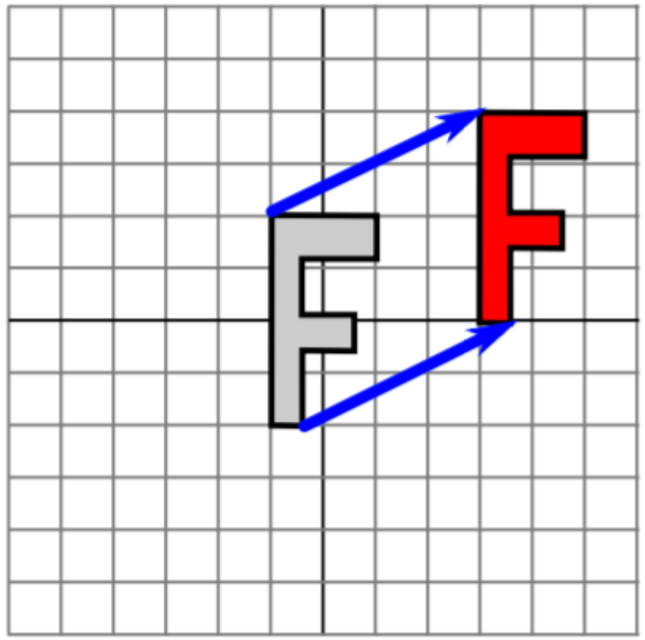
- Translation
- Rotation around the origin
- Scaling about the origin
- Mirror Reflection
- Inverse Transformations

Geometric Transformation

Translation

A translation transform simply moves every point by a certain amount horizontally and a certain amount vertically. If (x,y) is the original point and $(x1,y1)$ is the transformed point, then the formula for a translation is

```
x1 = x + e
y1 = y + f
```



Geometric Transformation

Translation

- What are the coordinates for A, B, C?

  A (-4,5) B (-1,1) C (-4,-1)

- What are the coordinates for A', B'. C'?

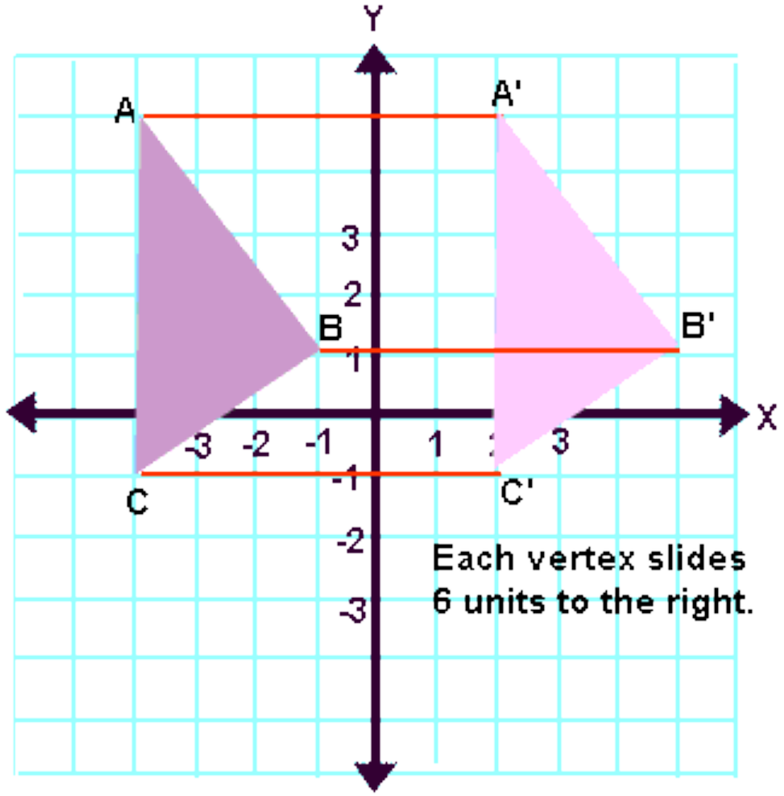
  A' (2,5) B' (5,1) C' (2,-1)

- How are they **alike**?

  They are similar triangles

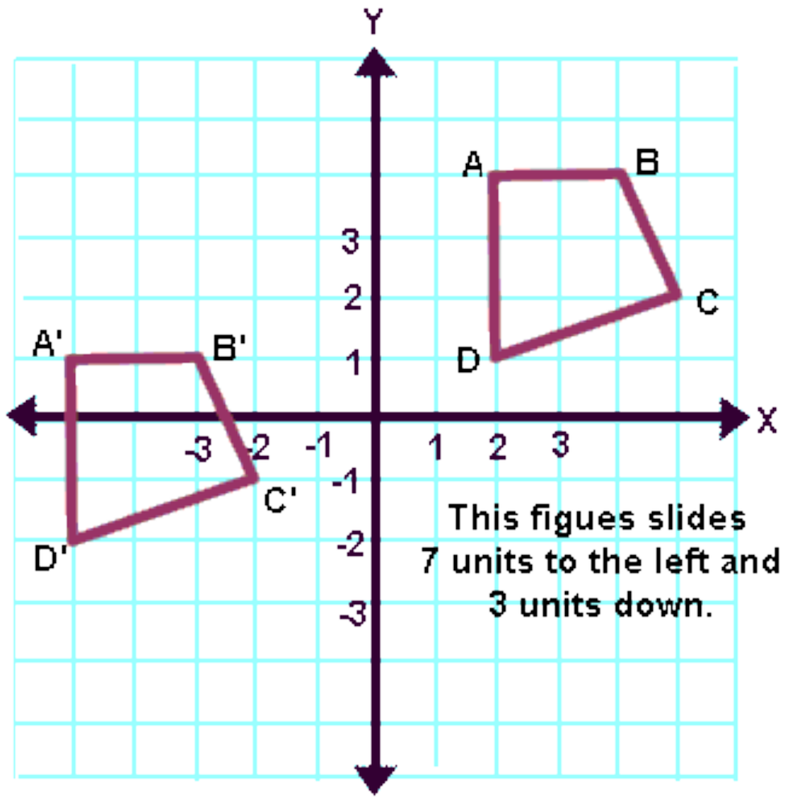
- How are they **different**?

  Each vertex slides 6 units to the right



Geometric Transformation

- Translation



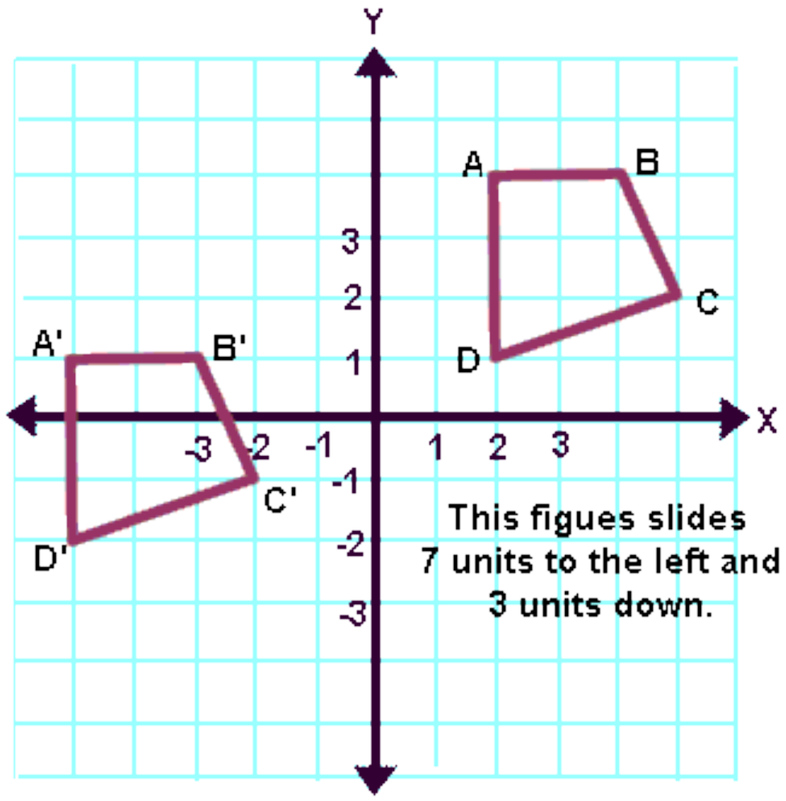
In this example, the "slide" moves the figure

7 units to the left and 3 units down

(or 3 units down and 7 units to the left.)

Geometric Transformation

- Translation



- What are the coordinates for A, B, C, D?

  A (2, 4) B (4, 4) C (5, 2) D (2, 1)

- What are the coordinates for A', B', C', D'?

  A' (-5, 1) B' (-3, 1) C' (-2, -1) D' (-5, -2)

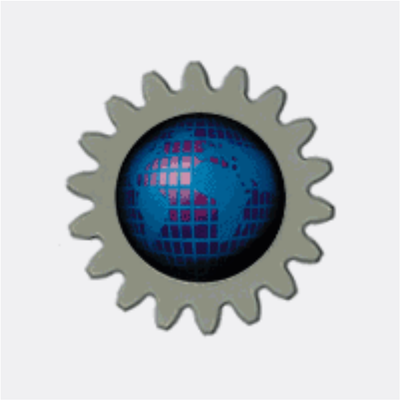
- How did the transformation change the points?

  The figure slides 7 units to the left and 3 units down

Geometric Transformation

- Rotation

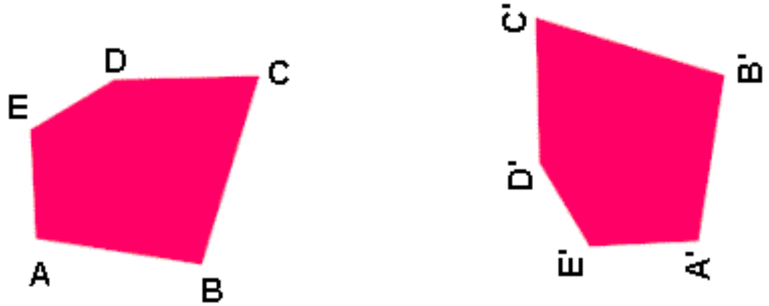
A Rotation is a transformation that turns a figure about a fixed point called the center of rotation. An object and its rotation are the same shape and size, but the figures may be turned in different directions

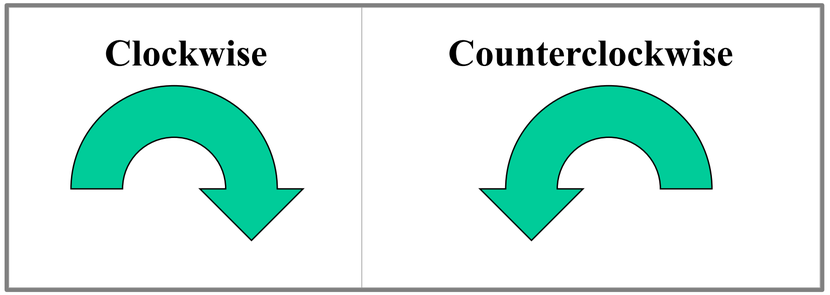


Geometric Transformation

Rotation

This rotation
is 90 degrees counterclockwise.



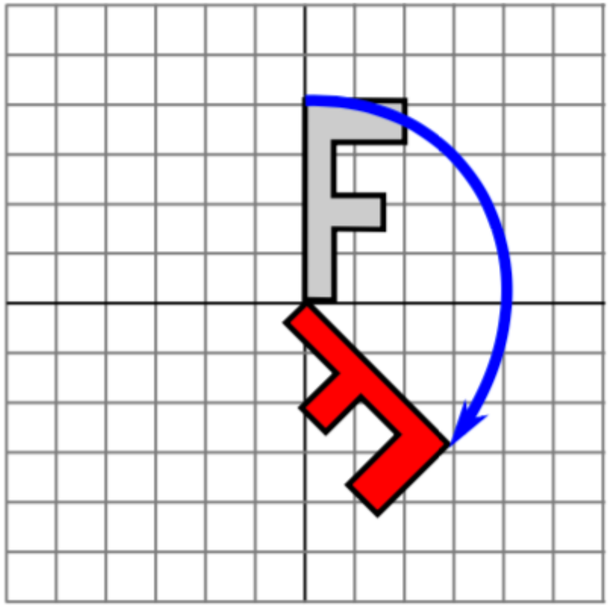


Geometric Transformation

Rotation about the origin

A rotation transform, for our purposes here, rotates each point about the origin, (0,0). Every point is rotated through the same angle, called the angle of rotation. For this purpose, angles can be measured either in degrees or in radians.

```
x1 = cos(r) * x - sin(r) * y
y1 = sin(r) * x + cos(r) * y
```

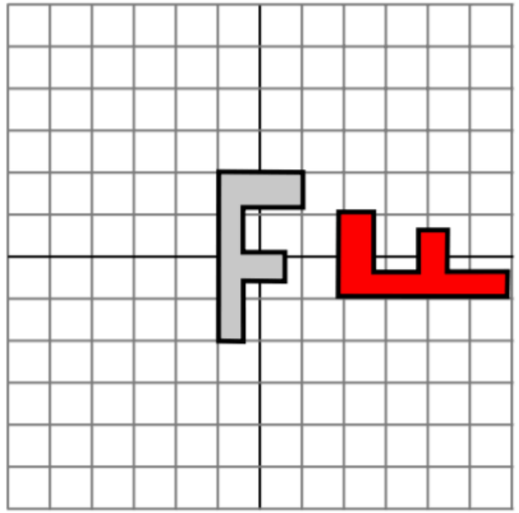


Geometric Transformation

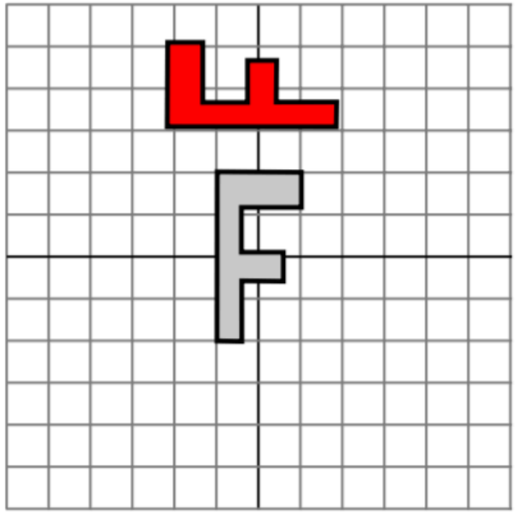
Combining translation and rotation

We are now in a position to see what can happen when you combine two transformations. Suppose that before drawing some object, you say

```
translate(4,0)
rotate(90)
```



Rotate then translate



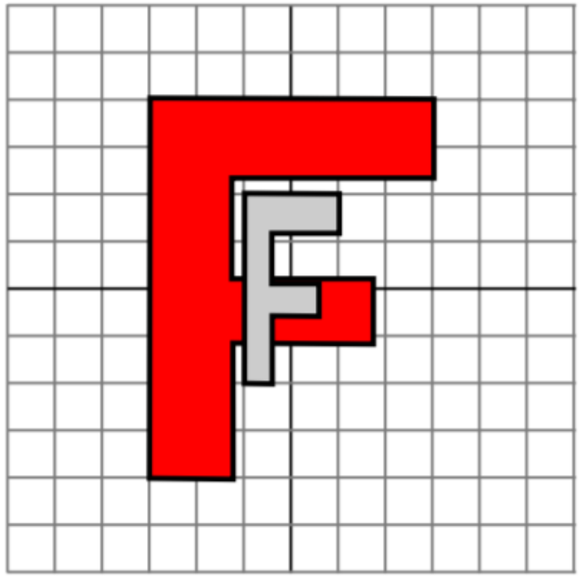
Translate then rotate

Geometric Transformation

Scaling about the origin

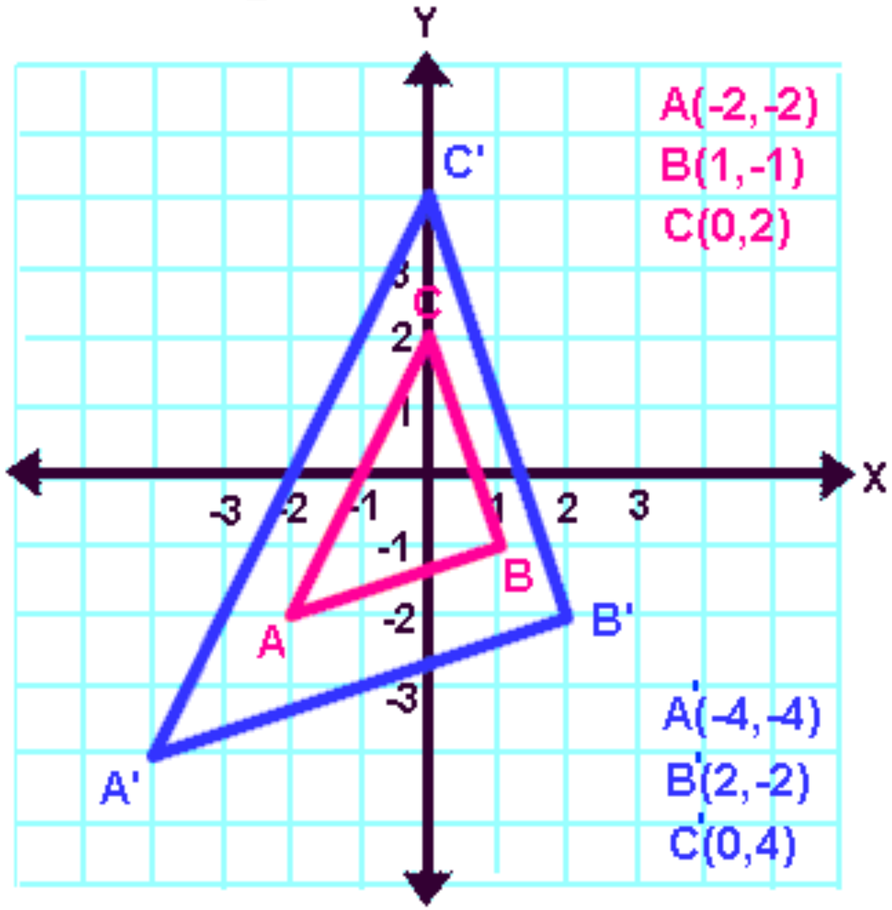
A scaling transform can be used to make objects bigger or smaller. Mathematically, a scaling transform simply multiplies each x-coordinate by a given amount and each y-coordinate by a given amount. That is, if a point (x,y) is scaled by a factor of a in the x direction and by a factor of d in the y direction, then the resulting point $(x1,y1)$ is given by

```
x1 = a * x
y1 = d * y
```



Geometric Transformation

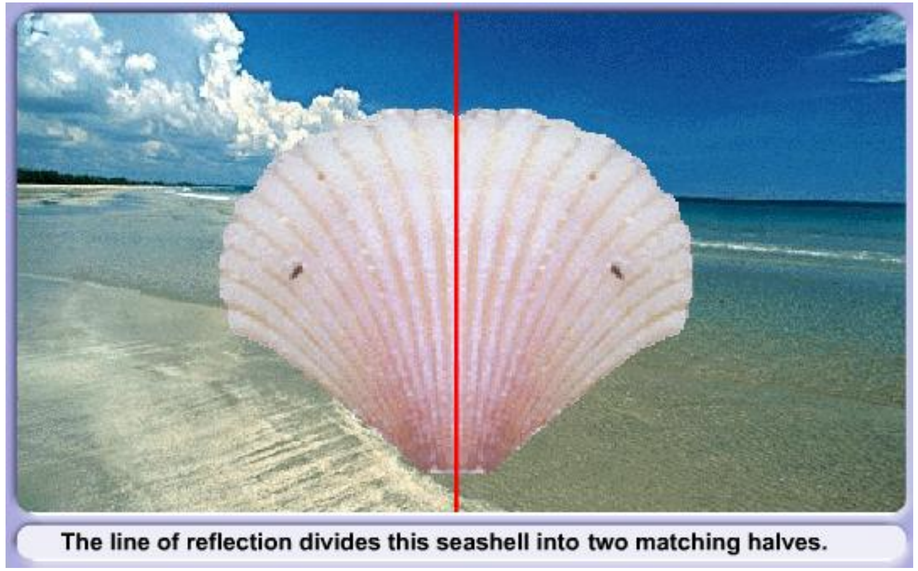
- Scaling about the origin



Geometric Transformation

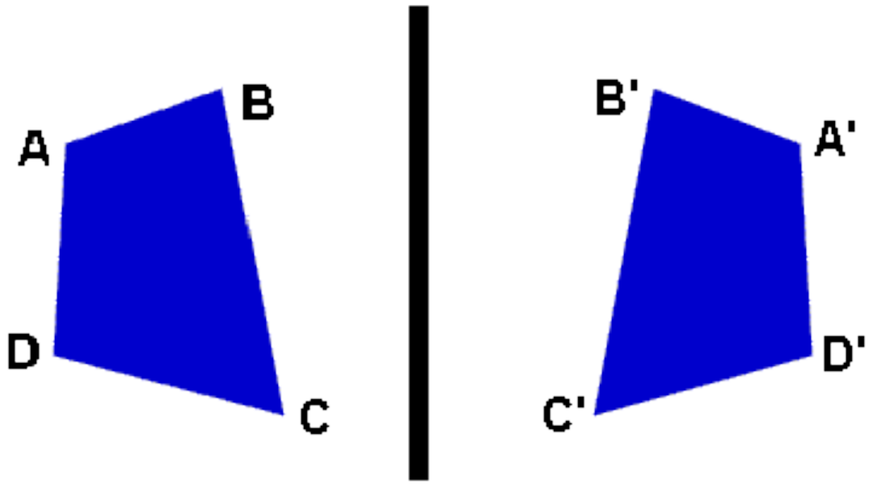
Mirror Reflection

A **reflection** can be seen in water, in a mirror, in glass, or in a shiny surface. An object and its reflection have **the same shape and size**, but the **figures face in opposite directions**. In a mirror, for example, right and left are switched.



The line (where a mirror may be placed) is called the **line of reflection**. The distance from a point to the line of reflection is the same as the distance from the point's image to the line of reflection.

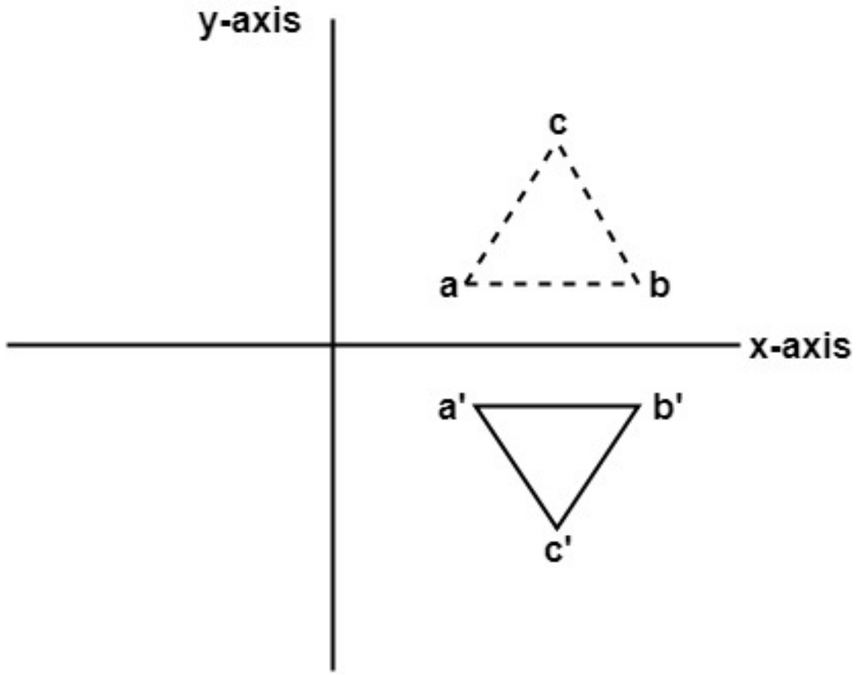
A reflection can be thought of as a "flipping" of an object over the line of reflection.



If you folded the two shapes together **line of reflection** **the two shapes would overlap exactly!**

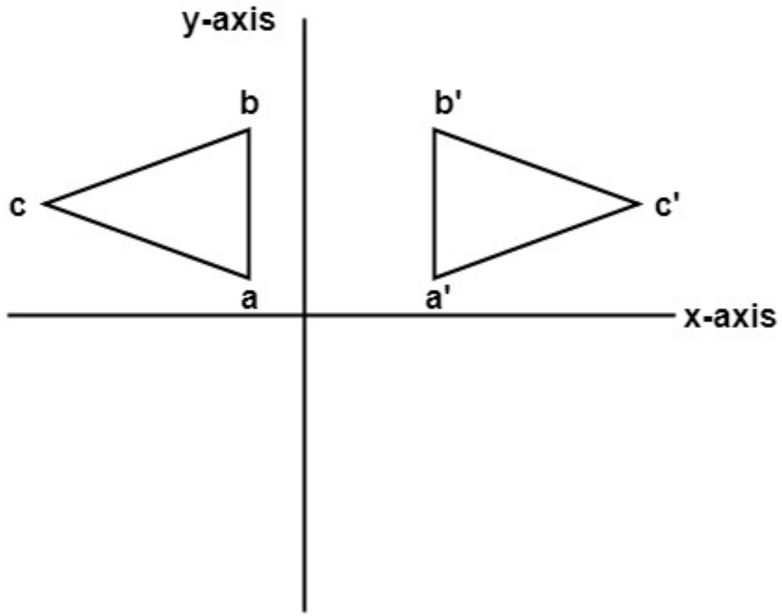
Geometric Transformation

- Mirror Reflection about x-axis



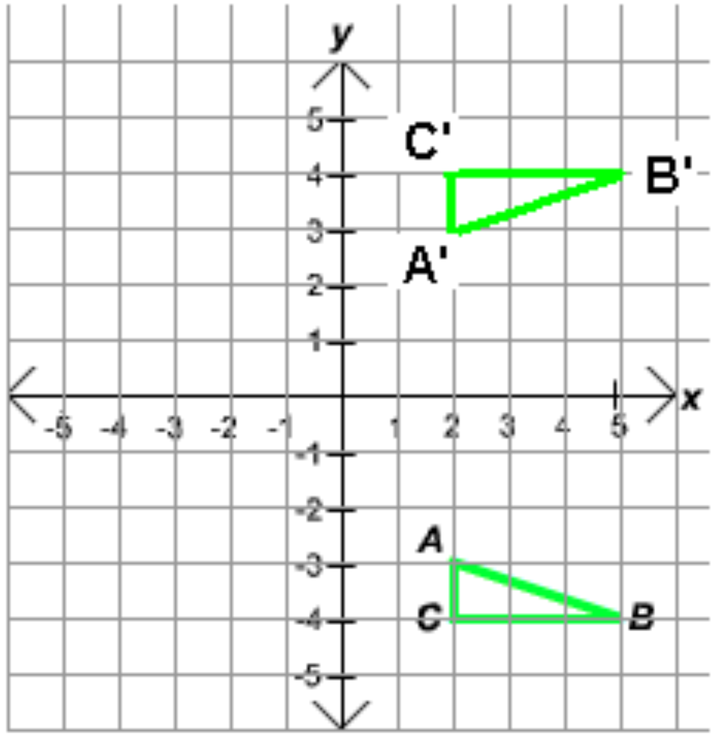
Geometric Transformation

- Mirror Reflection about y-axis



Geometric Transformation

- What happens to points in a Reflection about *x*-axis?



- Name the points of the original triangle.

  A (2,-3) B (5,-4) C (2,-4)

- Name the points of the reflected triangle.

  A' (2,3) B' (5,4) C' (2,4)

- What is the line of reflection?

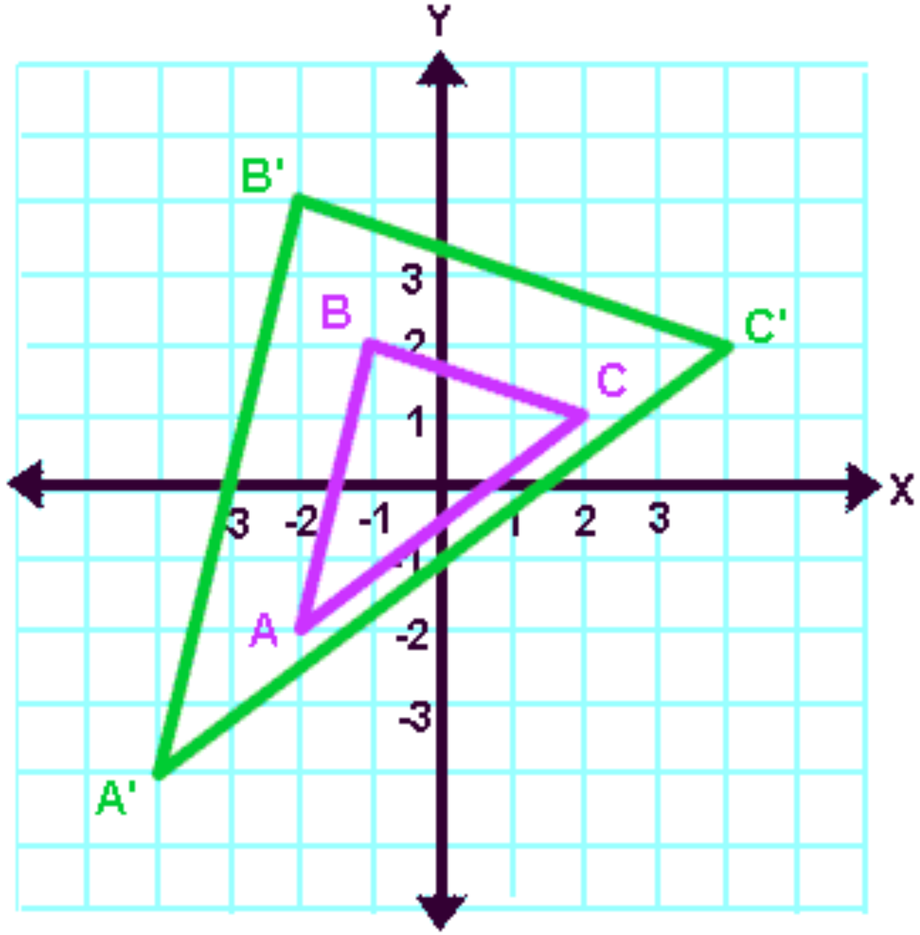
  x-axis

- How did the points change from the original to the reflection?

  The sign of y switches

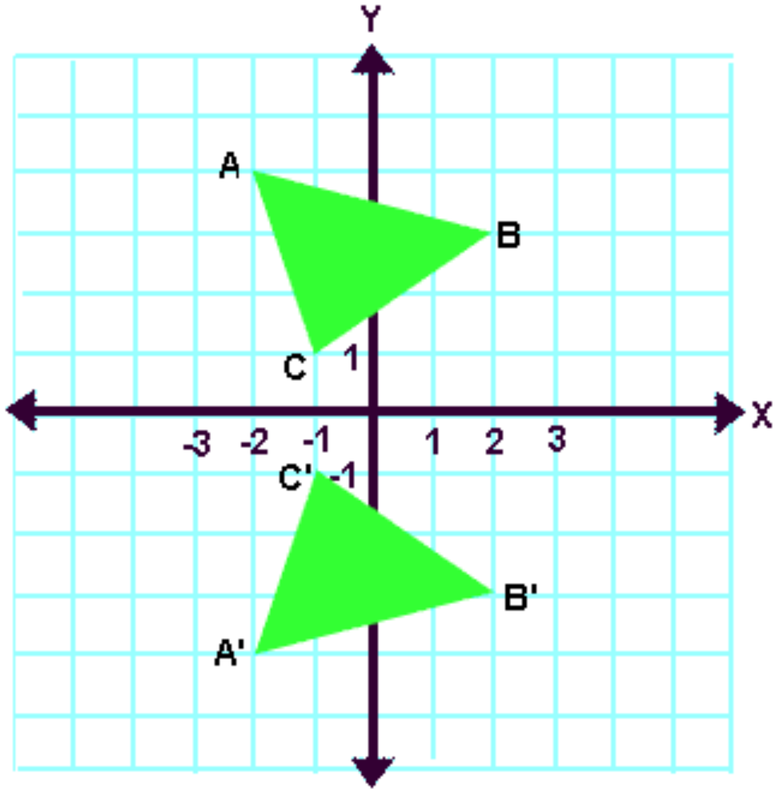
Review: Geometric Transformation

- Does this picture show a translation, rotation, scaling, or reflection?



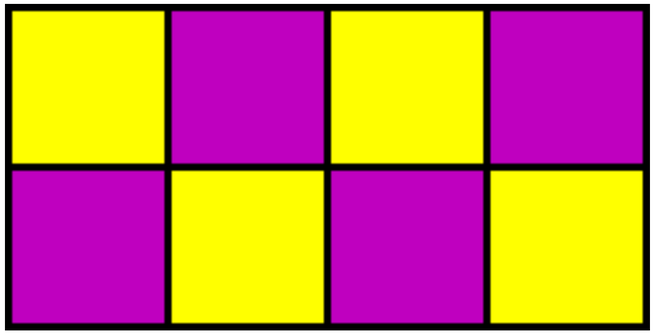
Review: Geometric Transformation

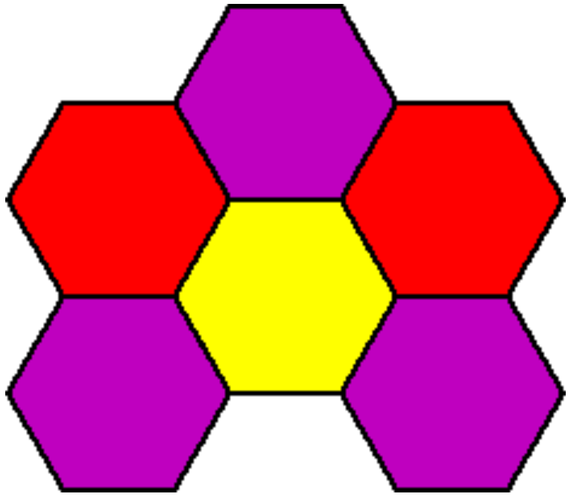
- Does this picture show a translation, rotation, scaling, or reflection?

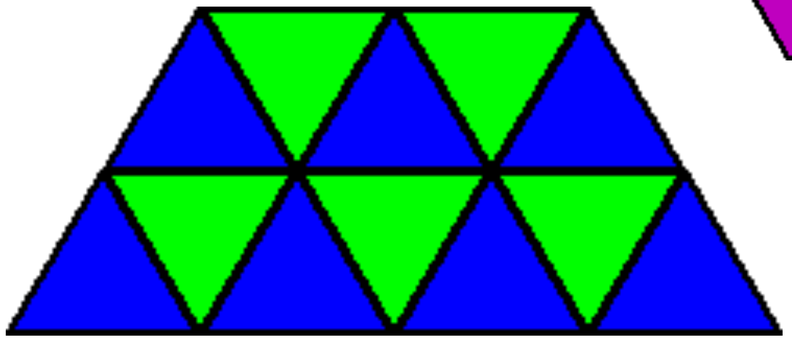


Applications

- Basically, a tessellation is a way to tile a floor (that goes on forever) with shapes so that there is no overlapping and no gaps.







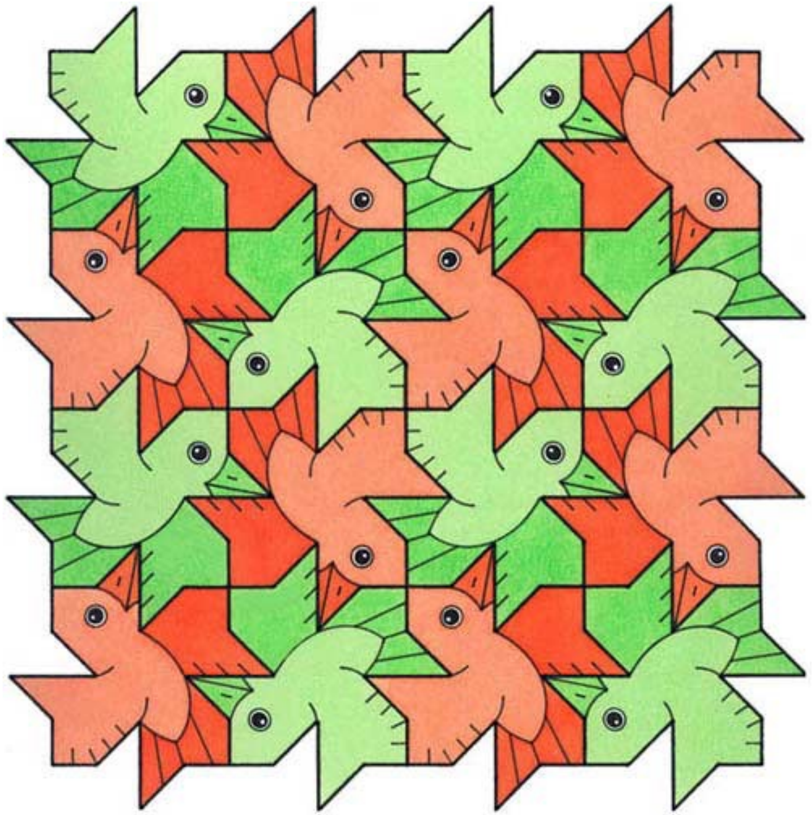
Applications

- Dutch graphic artist M. C. Escher (1898-1972) is known for his creative use of tessellations in his work.
- What transformations can you see in this picture?

The birds and fish have been translated here.

Applications

- What transformations can you see in this Escher print?



Some birds have been translated and some have been rotated.

Homogeneous Coordinates

In homogeneous coordinates, we add a third coordinate to a point in the 2D space. Point P(x, y) is now represented by a triple (x, y, w). Moreover, two sets of homogeneous coordinates (x, y, w) and (x', y', w') represent the same point if and only if one is a multiple of the other.

Example: Points (2, 4, 7) and (4, 8,14) are the same point represented by different coordinate triples.

If the w coordinate is nonzero, we can divide through by it:

(x, y, w) represents the same point as (x/w, y/w, 1).

Geometric Transformation

- Translation can be represented in the matrix form as:

$$P' = \mathrm{T}P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathrm{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Geometric Transformation

- Rotation can be represented in the matrix form as:

$$P' = R_\theta P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Geometric Transformation

- Scaling can be represented in the matrix form as:

$$P' = SP$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Geometric Transformation

- Mirror Reflection about x-axis can be represented in the matrix form as:

$$P' = \mathrm{M}_{\mathrm{x}} P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathrm{M}_{\mathrm{x}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Geometric Transformation

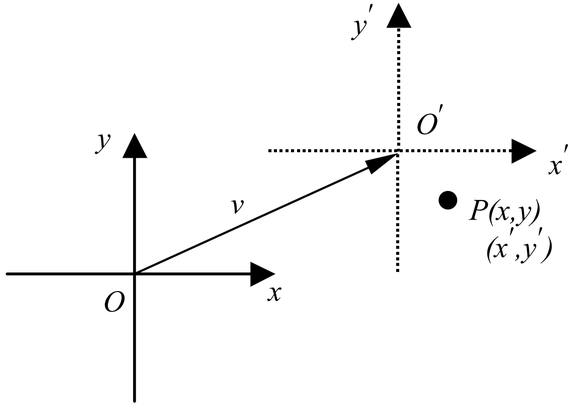
- Mirror Reflection about y-axis can be represented in the matrix form as:

$$P' = \mathrm{M}_{\mathrm{y}} P$$

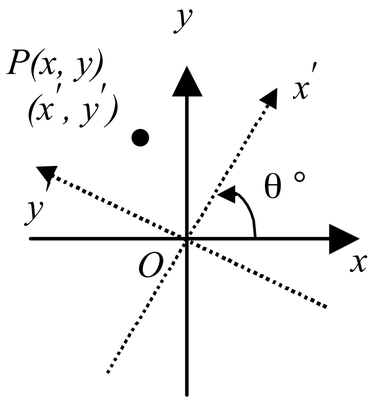
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathrm{M}_{\mathrm{y}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate Transformation



Coordinate Translation



Coordinate Rotation

Coordinate Transformation

- Translation can be represented in the matrix form as:

$$P' = \overline{\mathrm{T}} P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\overline{\mathrm{T}} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate Transformation

- Rotation can be represented in the matrix form as:

$$P' = \overline{R}_\theta P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \overline{R}_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate Transformation

- Scaling can be represented in the matrix form as:

$$P' = \overline{S} P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\overline{S} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate Transformation

- Mirror Reflection about x-axis can be represented in the matrix form as:

$$P' = \overline{\mathrm{M}}_{\mathrm{x}} P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\overline{\mathrm{M}}_{\mathrm{x}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate Transformation

- Mirror Reflection about *y*-axis can be represented in the matrix form as:

$$P' = \overline{\mathrm{M}}_{\mathrm{y}} P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\overline{\mathrm{M}}_{\mathrm{y}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Transformations

- Translation $\overline{T}_v^{-1} = \overline{T}_{-v}$
- Rotation $(\overline{R}_\theta)^{-1} = \overline{R}_{-\theta}$
- Scaling $\left(\overline{S}(s_x, s_y)\right)^{-1} = \overline{S}(1/s_x, 1/s_y)$
- Mirror Reflection about x-axis $(\overline{M}_x)^{-1} = \overline{M}_x$
- Mirror Reflection about y-axis $(\overline{M}_y)^{-1} = \overline{M}_y$

Composite Transformations

- COMPOSITION OF TWO TRANSLATIONS
- COMPOSITION OF TWO ROTATIONS
- COMPOSITION OF TWO SCALINGS
- ROTATION ABOUT ARBITRARY POINT
- SCALING ABOUT ARBITRARY POINT

Composite of Two Translations

- Let P be translated by T (tx, ty) to P' then let P' be translated by T'(t'x, t'y) to P". We now establish the transformation from P to P".

$$P' = TP \quad , \quad P'' = T'P'$$

so

$$P'' = T'(TP) = (T'T)P$$

Composite of Two Translations

In terms of homogeneous coordinates,

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t'_x \\ 0 & 1 & t'_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & t_x + t'_x \\ 0 & 1 & t_y + t'_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

T'T is a translation with vector **V** = **T**+**T**'.

Composite of Two Rotations

Let P be rotated by R_θ about the origin to P', and then P' be rotated by $R_{\theta'}$ to P". We can write:

$$P'' = R_{\theta'} P' = \left(R_{\theta'} R_\theta\right) P$$

Composite of Two Rotations

In terms of homogeneous coordinates,

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta' & -\sin\theta' & 0 \\ \sin\theta' & \cos\theta' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\ \cos\theta' - \sin\theta\ \sin\theta' & -(\sin\theta\ \cos\theta' + \sin\theta'\cos\theta & 0 \\ (\sin\theta\ \cos\theta' + \sin\theta'\cos\theta & \cos\theta\ \cos\theta' - \sin\theta\ \sin\theta' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + \theta) & -\sin(\theta + \theta') & 0 \\ \sin(\theta + \theta') & \cos(\theta + \theta') & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$R_{\theta'} R_{\theta} = R_{\theta' + \theta}$$

Composite of Two Scalings

In terms of homogeneous coordinates,

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} s'_x & 0 & 0 \\ 0 & s'_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_x s_{x'} & 0 & 0 \\ 0 & s_y s_{y'} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Active Learning

Rotation about an arbitrary point