Ritangle

Final question

	Round 1		Round 2		Round 3	
(board 1 pairings)	D.01	C.01	E.01	A.01	B.01	E.01
	A.01	B.01	C.01	F.01	D.01	F.01
	F.01	E.01	B.01	D.01	C.01	A.01
(board 2 pairings)	C.02	B.02	A.02	E.02	A.02	C.02
	F.02	A.02	B.02	D.02	E.02	B.02
	E.02	D.02	F.02	C.02	D.02	F.02
(board 3 pairings)	E.03	A.03	F.03	D.03	F.03	B.03
	C.03	F.03	B.03	E.03	D.03	A.03
	B.03	D.03	A.03	C.03	C.03	E.03
(board 4 pairings)	D.04	A.04	D.04	C.04	A.04	F.04
	F.04	B.04	E.04	F.04	E.04	D.04
	C.04	E.04	A.04	B.04	B.04	C.04

Congratulations on reaching Stage 3 of Ritangle 2023. Welcome to the world of Jamboree Pairings.

In a head-to-head chess match, the players within each team are arranged in order of strength, with board 1 being the strongest. The two board 1 players play one another, as do the two board 2 players, and so on.

In a 'jamboree' chess tournament, N teams, of B players each, play R rounds, of B boards each. If $R < N - 1$ then there are not enough rounds for each team to play head-to-head matches against each of the others. Instead, the 'jamboree' system specifies a set of pairings. The first-named player in each pairing has the white pieces and the second plays black. The example in the table above shows pairings for six 4-player teams A to F over three rounds. In round 1 the board 1 player for team D (D.01) plays white against C.01, while A.01 plays B.01 and F.01 plays E.01. In general, the board 2 players have opponents from different teams. In the second round D.01 plays black against B.01, and in round 3, white against F.01. With an odd number of teams, one of the board 1 players would receive a 'down-float' to play against one of the board 2 players, who would get a corresponding 'up-float'.

There are several mandatory constraints on a set of pairings. Assuming that $R < N - 1$:
1. Nobody may play the same opponent twice, nor twice against opponents from the same team.
2. Nobody may play against a team-mate.
3. The number of times that each team plays each of the others shall be either $\mathbf{floor}\ \left(\frac{RB}{N-1}\right)$ or $\mathbf{ceiling}\ \left(\frac{RB}{N-1}\right)$, where '$\mathbf{floor}(x)$' is the largest integer less than or equal to x and '$\mathbf{ceiling}\ (x)$' is the smallest integer greater than or equal to x.
4. Among the players on any given board, there shall be no more than one up-float or down-float per round.
5. No player shall have more than one up-float or more than one down-float.
6. Over all the rounds taken together, no team shall have more than $2 + \ \mathbf{floor}\ \left(\frac{RB}{2N}\right)$ up-floats nor more than $2 + \ \mathbf{floor}\ \left(\frac{RB}{2N}\right)$ down-floats.
7. All up-floats shall have the white pieces.
8. The number of times a player is black shall differ by no more than 1 from the number of times a player

is white.

9. Over all the rounds taken together, the number of blacks for each team shall equal the number of whites.

You should check that all these conditions are satisfied for the jamboree above.

There are also a number of desirable characteristics to make the pairings as fair as possible for each team:

X. In each round, the number of blacks for each team should be similar to the number of whites.

Y. Over all the rounds taken together, the number of up-floats for each team should be as close as possible to the number of down-floats.

Z. For each team, the black games and white games should be evenly distributed down the team (i.e. it is undesirable for the higher boards to have a preponderance of whites and the lower boards to have a preponderance of blacks).

There are thus three desirable goals to achieve, X, Y and Z. Define a 'detriment' to be a measure of the amount by which one of these goals is missed.

To measure the total detriment Q_X against goal X, calculate

$$Q_X = \sum_{i=1}^{N} \sum_{k=1}^{R} |w_{ik} - b_{ik}|$$

where w_{ik} and b_{ik} are the number of whites and blacks for team i in round k, and $|x|$ means 'take the absolute value of x'. (Teams are numbered here for convenience; interpret team 1 as A, team 2 as B, etc.)

To measure the detriment Q_Y against goal Y, calculate

$$Q_Y = \sum_{i=1}^{N} \left| \sum_{k=1}^{R} u_{ik} - \sum_{k=1}^{R} d_{ik} \right|$$

where u_{ik} and d_{ik} are the number of up-floats and down-floats for team i in round k.
To measure the detriment Q_Z against goal Z, calculate

$$Q_Z = \sum_{i=1}^{N} \left| 1 - \frac{4}{RB(B+1)} \sum_{l=1}^{B} l w_{il} \right|$$

where w_{il} is the number of times that the player on board l of team i has white. (The final summation calculates the 'board count' of the whites for team i. The target number of whites is $\frac{R}{2}$ on each board and the average board number is $\frac{B+1}{2}$, so for each team, the target board count for the whites is the product of these two expressions multiplied by the number of boards. When calculating Q_Z, any downfloated board 1 player remains a board 1 player; any upfloated board 2 player remains board 2, etc.)

Your first task is to analyse the set of pairings given above, for six teams of four players with three rounds. Satisfy yourself that the mandatory constraints are respected, then calculate the total detriment $Q = Q_X + Q_Y + Q_Z$ to two decimal places. This will be the first line of your submission as given below. The school or college of any team that gives the correct value will be included in the list of successful teams on the Ritangle webpage after the competition closes.

Your second task is to find the best sets of pairings for 11 and 12 ten-player teams playing three rounds, i.e. the sets that minimise the total detriment $Q = Q_X + Q_Y + Q_Z$ for each case. You may find it easier to start with the case for 12 teams, as this does not have the added complication of up-floats and down-floats.

Label the teams A, B, C, ..., J, K and submit your results as follows.

The detriment Q, to two decimal places, for your first task.

The total detriment Q for both sets of pairings, to two decimal places, followed by the detriment Q and a set of 165 pairings for the 11-team jamboree (round 1, then round 2, then round 3), followed by the detriment Q and 180 pairings for the 12-team jamboree (now including team L; again, round 1, then round 2, then round 3). Each pairing must be in the format r,S.b,T.b, where r is the round number, S and T are the team letters and b gives the board number of the relevant player (for up- and down-floats the two b's will be different, see below). Each pairing must be on a new line. For example, if the pairings given above were submitted instead of the 11-team pairings, your submission would look like:

55.55 [you'll have to work it out!]
199.98 [you'll have to work this out, too, for your pairings]
99.99 [and this]
1,D.01,C.01
1,A.01,B.01
1,F.01,E.01
1,C.02,B.02
1,F.02,A.02
1,E.02,D.02
...
3,E.04,D.04
3,B.04,C.04

99.99 [and this; give just the numbers, omit the comments in square brackets]
1,...
[second set of pairings]

A first-round down-float of the board 9 player from team J (as black) to play the board 10 player from team K (as white) would be submitted as
1, K.10,J.09

Answer: *

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