DTU - 31792 Advanced Optimization and Game Theory for Energy Systems Jan 21

Assignment 1

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Objective: A home energy management system which tries to performs a cost minimization approach to use the fullest potential of the household equipment. The household has Electric and heat demand with PV, Electric heater, Gas boiler and flexible load. The datasets are taken from various sources.

Status:

- The primal problem works smoothly while the dual throws a different result. KKT remains unsolved.

Primal problem:

$$\min_{P_i, P_e, P_{h,g}} Z = C_i^t P_i^t - C_e^t P_e^t + C_g^t P_{h,g}^t \quad \forall t \in T$$

s.t

λ_1: $P_i^t + P_s^t - P_e^t - P_{d,el}^t - P_{h,ele}^t - P_{d,flex}^t = 0 \quad \forall t \in T$

λ_2: $P_{h,ele}^t + P_{h,g}^t - P_{h,d}^t = 0 \quad \forall t \in T$

μ_1: $P_i^t \geq 0$ "

μ_2: $P_e^t \geq 0$ "

μ_3: $P_s^t \geq 0$ "

μ_4, μ_5: $0 \leq P_{h,g}^t \leq \overline{P_{h,g}}$ "

μ_6, μ_7: $0 \leq P_{h,ele}^t \leq \overline{P_{h,ele}}$

μ_8, μ_9: $0 \leq P_{d,flex}^t \leq \overline{P_{d,flex}} \quad \forall t \in T$

Primal problem; here:

C_i - Cost of electricity import

C_e - Cost of electricity export

C_g - Cost of gas import

P_i - Import (Net) from grid

P_e - Net electricity export to the grid

$P_{h,g}$ - ~~Boil~~ Boiler gas import for heat.

$P_{h,d}$ - Net heat demand

P_s - Net solar power

$P_{d,ele}$ - Electricity demand

$P_{h,ele}$ - Heat demand supplied by electric ~~boiler~~ heating

$P_{h,gas}$ - Heat demand supplied by gas

$P_{d,flex}$ - Shiftable electricity demand.

Lagrangian:

Lagrangian: $L(P_i, P_e, P_{h,g}, \lambda_1, \lambda_2, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_9) =$

$$C_i^t P_i^t - C_e^t P_e^t + C_g^t P_{h,g}^t + \lambda_1^t (P_i^t - P_e^t + P_s^t - P_{d,el}^t - P_{h,el}^t - P_{d,flex}^t)$$
$$+ \lambda_2^t (P_{h,ele}^t + P_{h,g}^t - P_{h,d}^t) - \mu_1^t P_i^t - \mu_2^t P_e^t - \mu_3^t P_s^t$$
$$- \mu_4^t P_{h,g}^t + \mu_5^t (P_{h,g}^t - \overline{P_{h,g}}) - \mu_6^t P_{h,ele}^t + \mu_7^t (P_{h,ele}^t - \overline{P_{h,ele}})$$
$$- \mu_8^t P_{d,flex}^t + \mu_9^t (P_{d,flex}^t - \overline{P_{d,flex}})$$

KKT condition:

PD

$$\frac{\partial L}{\partial P_i} = C_i^t + \lambda_1^t - \mu_1^t = 0 \quad \forall t \in T$$

$$\frac{\partial L}{\partial P_e} = -C_e^t - \lambda_1^t - \mu_2^t = 0 \quad \forall t \in T$$

$$\frac{\partial L}{\partial P_{h,g}} = C_g^t + \lambda_2^t + \mu_5^t - \mu_4 = 0 \quad \forall t \in T$$

Equality

$$P_i^t - P_e^t + P_s^t + P_{d,ele}^t + P_{h,ele}^t - P_{d,flex}^t = 0 \quad \forall t \in T$$

Lagrangian and KKT:

$$P_{h,ele}^t + P_{h,g}^t - P_{h,d}^t = 0$$

Complimentarity condition:

$\forall t \in T$ for all equations

$$0 \leq -P_i^t \perp \mu_1 \geq 0$$
$$0 \leq -P_e^t \perp \mu_2 \geq 0$$
$$0 \leq -P_s^t \perp \mu_3 \geq 0$$
$$0 \leq -P_{h,g}^t \perp \mu_4 \geq 0$$
$$0 \leq \overline{P_{h,g}} - P_{h,g}^t \perp \mu_5 \geq 0$$
$$0 \leq -P_{h,ele}^t \perp \mu_6 \geq 0$$
$$0 \leq \overline{P_{h,ele}} - P_{h,ele}^t \perp \mu_7 \geq 0$$
$$0 \leq -P_{d,flex}^t \perp \mu_8 \geq 0$$
$$0 \leq \overline{P_{d,flex}} - P_{d,flex}^t \perp \mu_9 \geq 0$$

Dual problem:

$\mu_1 \to \mu_9$: all μ_i $i = 1..9$

Dual problem

$$\max_{\lambda_1 \lambda_2, \mu_1 \to \mu_9} \ \min_{P_i, P_e, P_{h,g}} L(P_i, P_e, P_{h,g}, \mu_1 \to \mu_9, \lambda_1, \lambda_2)$$

$$\max_{\lambda_1 \lambda_2 \mu_1 \to \mu_9} \min_{P_i, P_e, P_{h,g}} \begin{cases} P_i^t(C_i^t + \lambda_1^t - \mu_1^t) + P_e^t(-C_e^t - \lambda_1^t - \mu_2^t) \\ + P_{h,g}^t(C_g^t - \lambda_2^t - \mu_4 + \mu_5) \\ + \lambda_1^t(P_s^t - P_{d,el}^t - P_{h,el}^t - P_{d,flex}^t) \\ + \lambda_2^t(P_{h,ele}^t - P_{h,d}^t) - \mu_3^t P_s^t \\ + \mu_5^t \overline{P_{h,g}} - \mu_6^t P_{h,ele}^t + \mu_7^t(P_{h,ele}^t - \overline{P_{h,ele}}) \\ - \mu_8^t P_{d,flex}^t + \mu_9^t(P_{d,flex}^t - \overline{P_{d,flex}^t}) \end{cases}$$

$$C_i^t + \lambda_1^t - \mu_1^t = 0 \qquad -C_e^t - \lambda_1^t - \mu_2^t = 0$$
$$C_i^t + \lambda_1^t \geq 0 \qquad C_e^t + \lambda_1^t \leq 0$$
$$\lambda_1^t \geq -C_i^t \qquad \lambda_1^t \leq -C_e^t$$

$$-C_i^t \leq \lambda_1^t \leq -C_e^t$$

Dual problem (final):

$$C_g^t - \lambda_2^t - \mu_4^t + \mu_5^t = 0 \qquad \forall t \in T$$

Therefore, dual problem is

$$\max_{\lambda_1, \lambda_2, \mu_1 \to \mu_9} \begin{cases} \lambda_1^t(P_s^t - P_{d,ele}^t - P_{h,ele}^t - P_{d,flex}^t) + \\ \lambda_2^t(P_{h,ele}^t - P_{h,d}^t) - \mu_3^t P_s^t \\ + \mu_5^t \overline{P_{h,g}} - \mu_6^t P_{h,ele}^t + \mu_7^t(P_{h,ele}^t - \overline{P_{h,ele}}) \\ + \mu_8^t P_{d,flex}^t + \mu_9^t(P_{d,flex}^t - \overline{P_{d,flex}}) \end{cases}$$

s.t

$$-C_i^t \leq \lambda_1^t \leq -C_e^t$$
$$C_g^t - \lambda_2^t - \mu_4^t + \mu_5^t = 0 \qquad \forall t \in T.$$

Machine parameters: Core i7 8550, Solver=Glpf, solution time: 30s, Total steps = 192 steps (15 min resolution) or 2 days. Total constraints = 1892.

Background: Due to time constraint and hectic work during I could not create a full-fledged document of equation in LATEX. I deeply regret and sorry for the same.