Projekt IEL

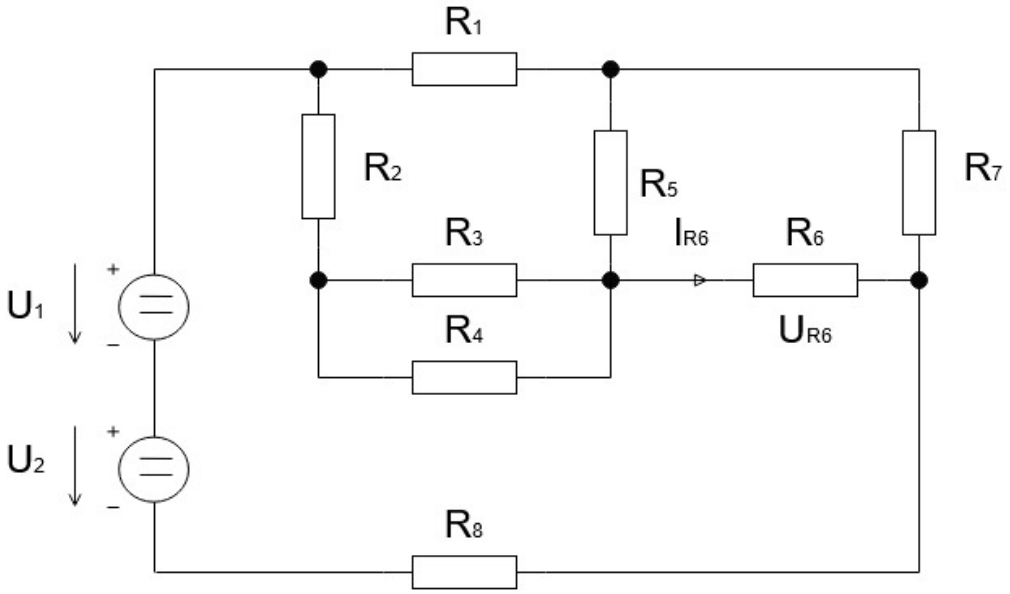
VOJTĚCH HÁJEK

xhajek51

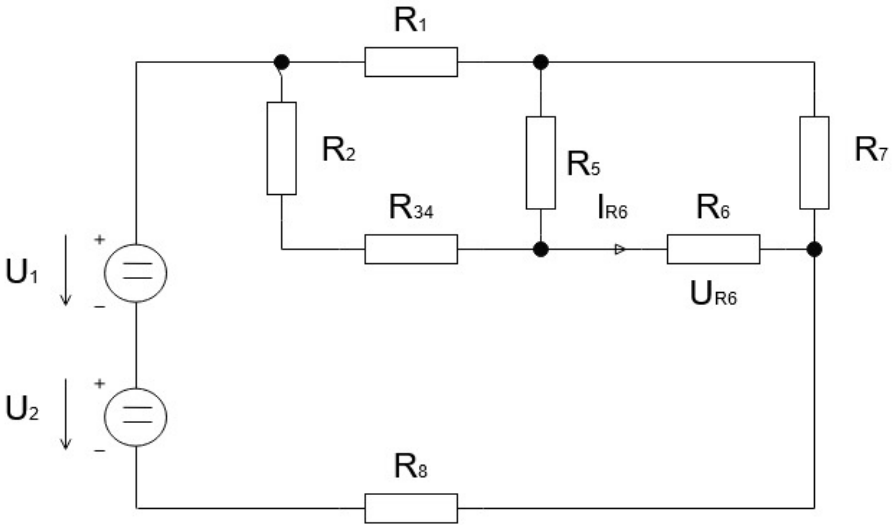
Příklad 1

Stanovte napětí U_{R6} a proud I_{R6}. Použijte metodu postupného zjednodušování obvodu.

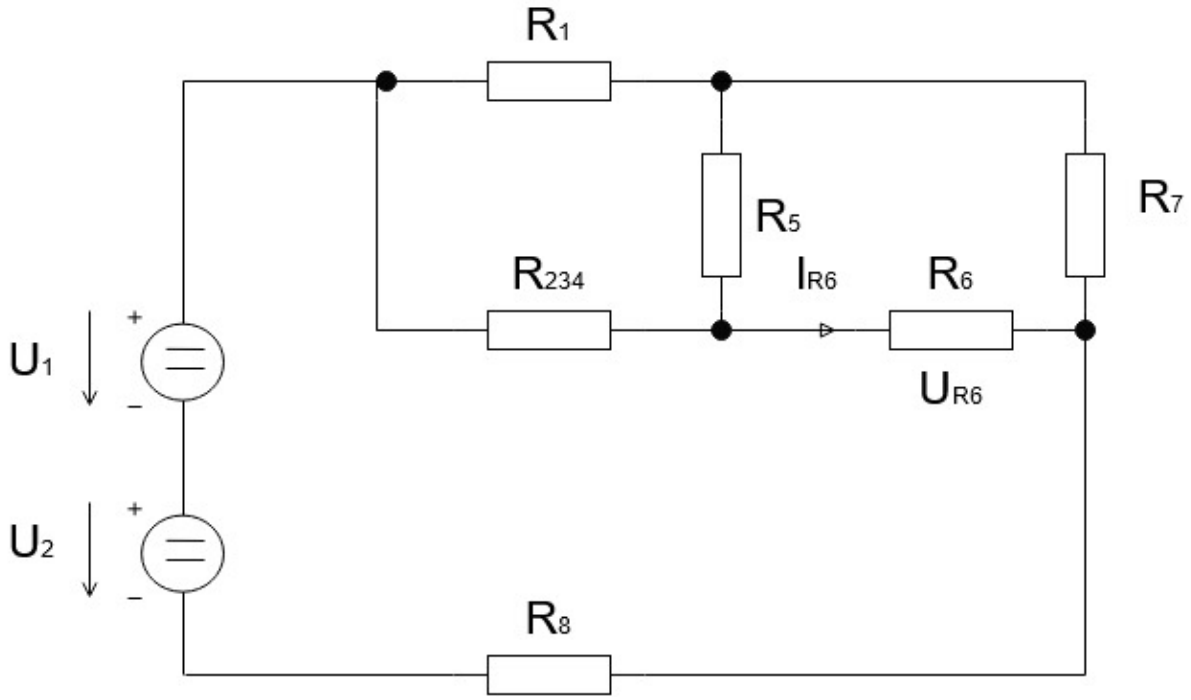
sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
G	130	60	380	420	330	440	450	650	410	275

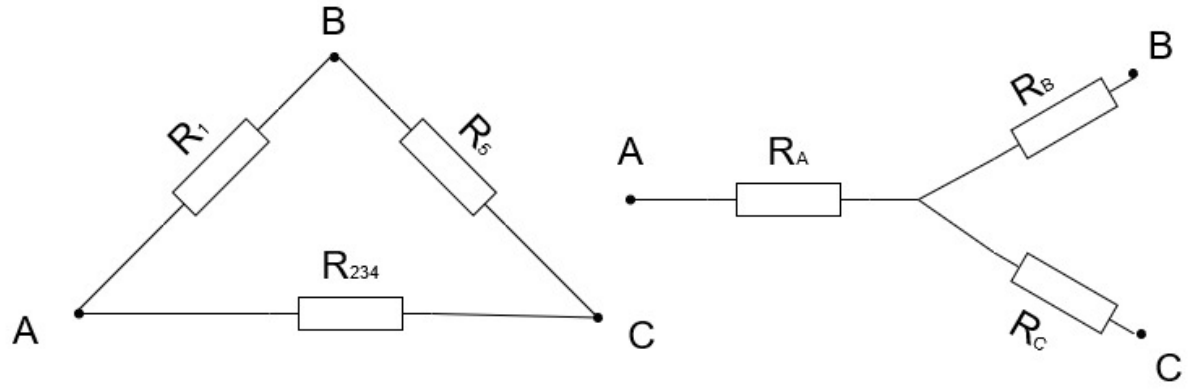


$R_{34} = \frac{R3 * R4}{R3 + R4}$ = 330 * $\frac{440}{330}$ + 440 = 188.5714 Ω



$R_{234} = R_2 + R_{34} = 420 + 188.5714 = 608.5714\ \Omega$

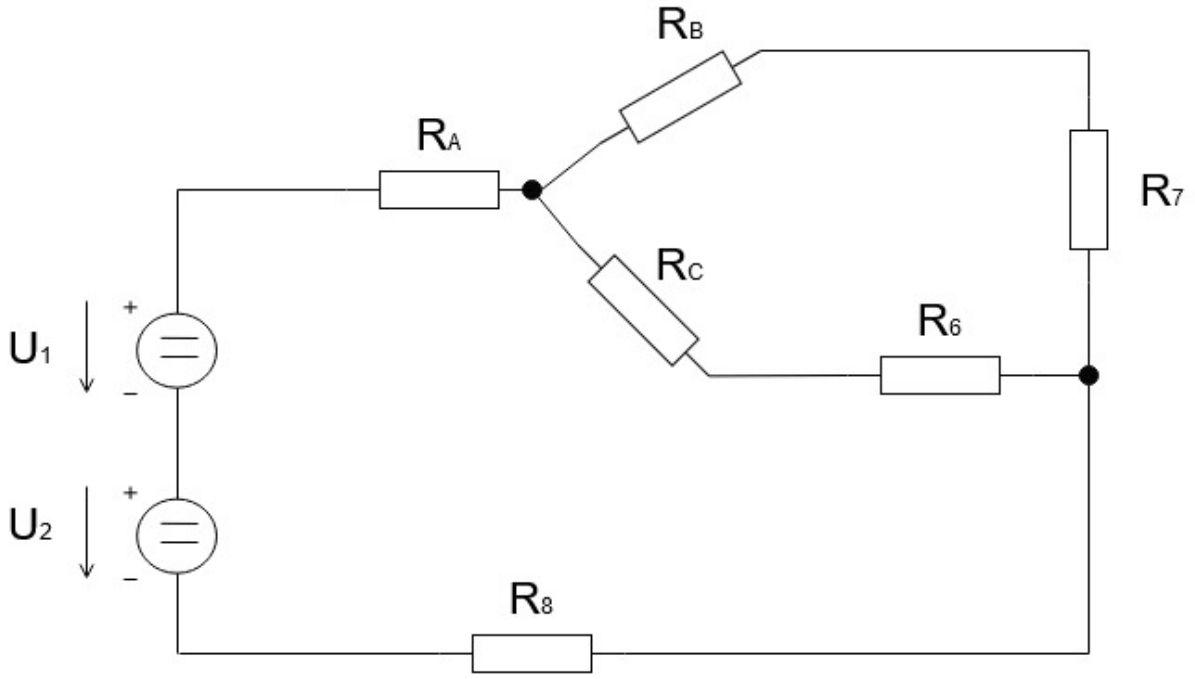




$R_A = \frac{R1 * R234}{R1+R5+R234}$ = 380 * 608.5714 / 380 + 450 + 608.5714 = 160.7547 Ω

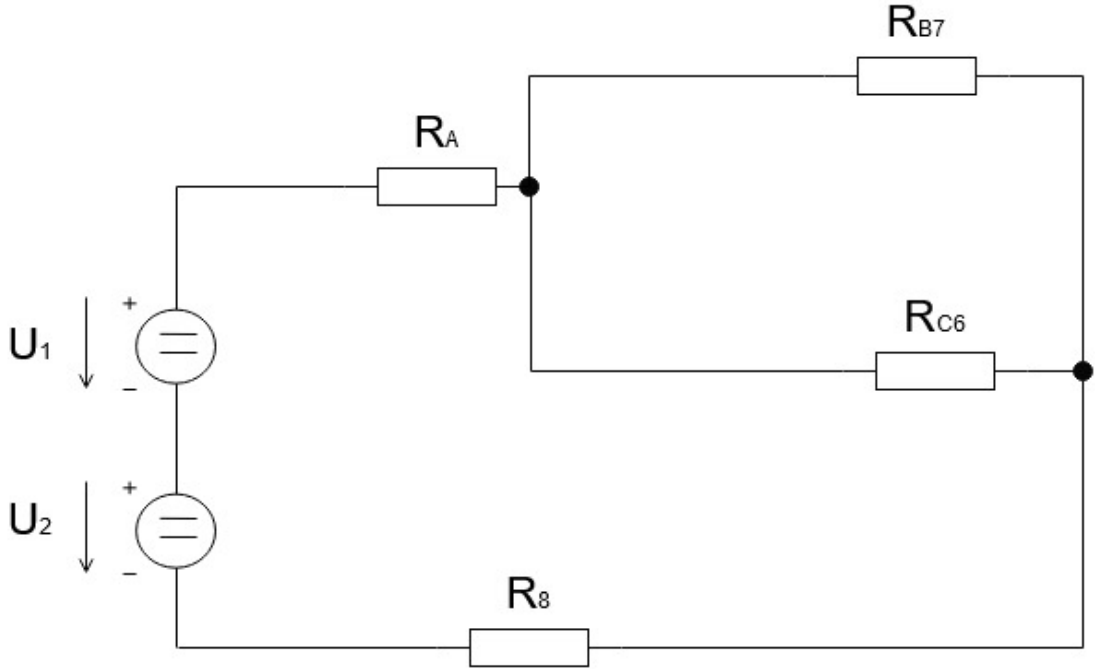
$R_B = \frac{R1 * R5}{R1+R5+R234}$ = 380 * 450 / 380 + 450 + 608.5714 = 118.8679 Ω

$R_C = \frac{R5 * R234}{R1 + R5+R234}$ = 450 * 608.5714 / 380 + 450 + 608.5714 = 190.3674 Ω

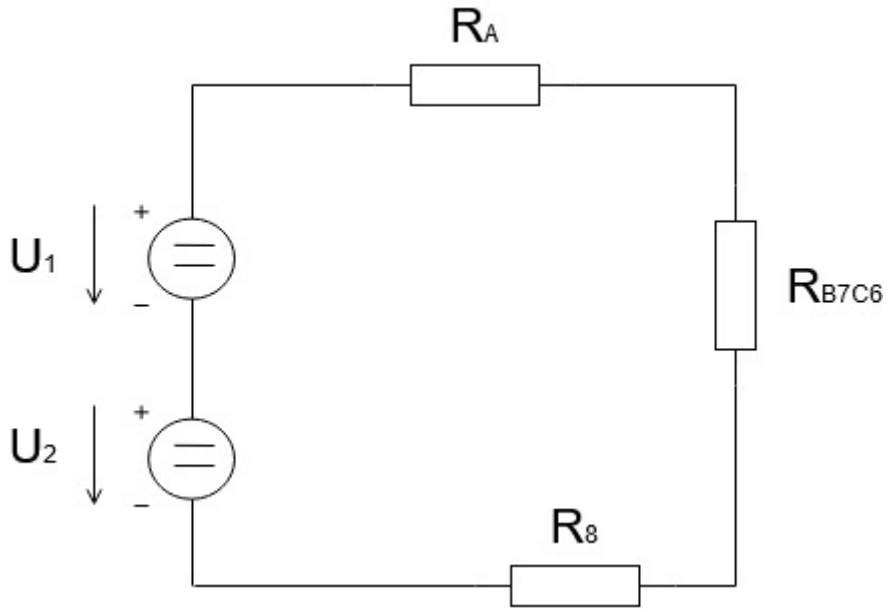


$R_{B7} = R_B + R_7 = 118.8679 + 410 = 528.8679\ \Omega$

$R_{C6} = R_C + R_6 = 190.3674 + 650 = 840.3674\ \Omega$



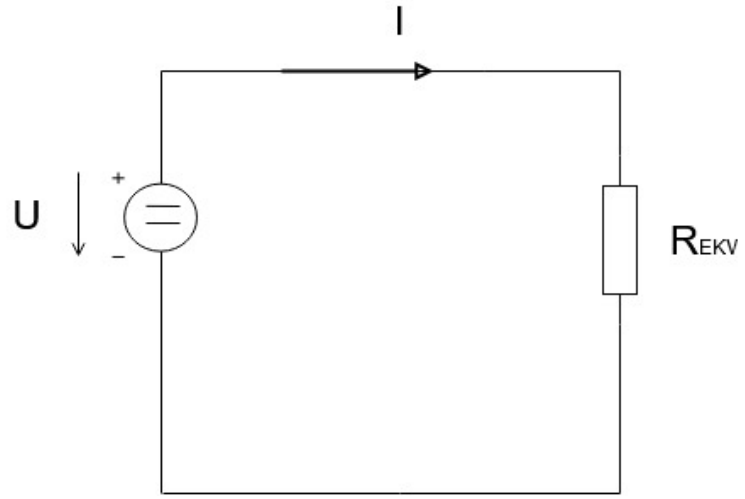
$R_{B7C6} = \frac{RB7 * RBC6}{RB7 + RC6}$ = 528.8679 * 840.3674 / 528.8679 + 840.3674 = 324.5923 Ω



$R_{EKV} = R_A + R_{BC76} + R_8 = 160.7547 + 324.5923 + 275 = 760.3471\ \Omega$

$U = U_1 + U_2 = 130 + 60 = 190\ V$

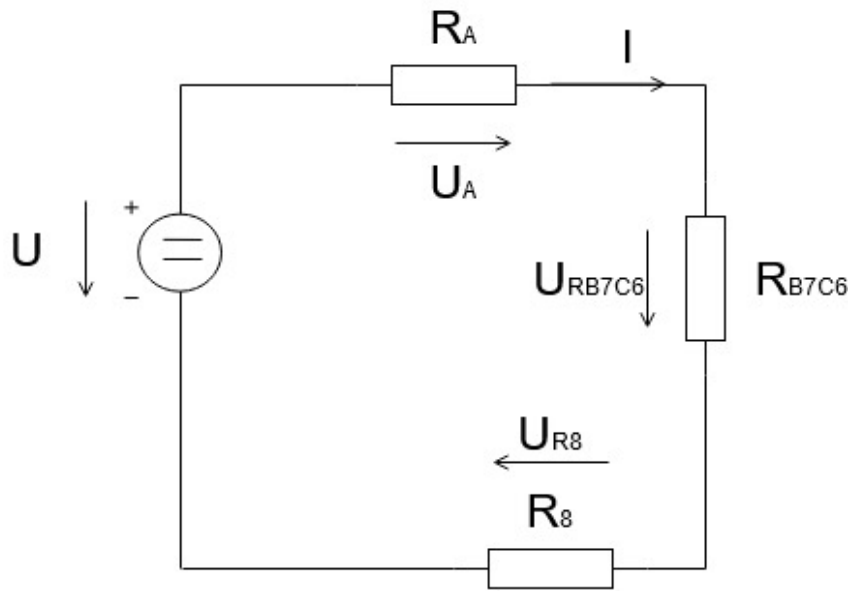
$I = \frac{U}{REKV} = 190 / 760.3471 = 0.2498\ A$



$U_{RA} = R_A * I = 160.7547 * 0.2498 = 40.1703\ V$

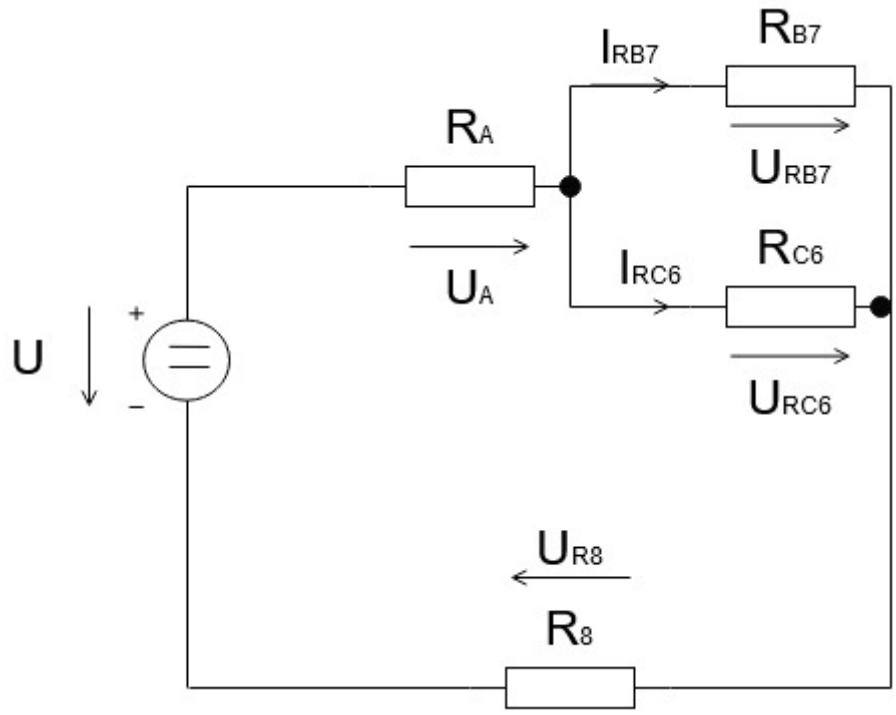
$U_{R8} = R_8 * I = 275 * 0.2498 = 68.7186\ V$

$U_{RB7C6} = R_{B7C6} * I = 324.5923 * 0.2498 = 81.1110\ V$



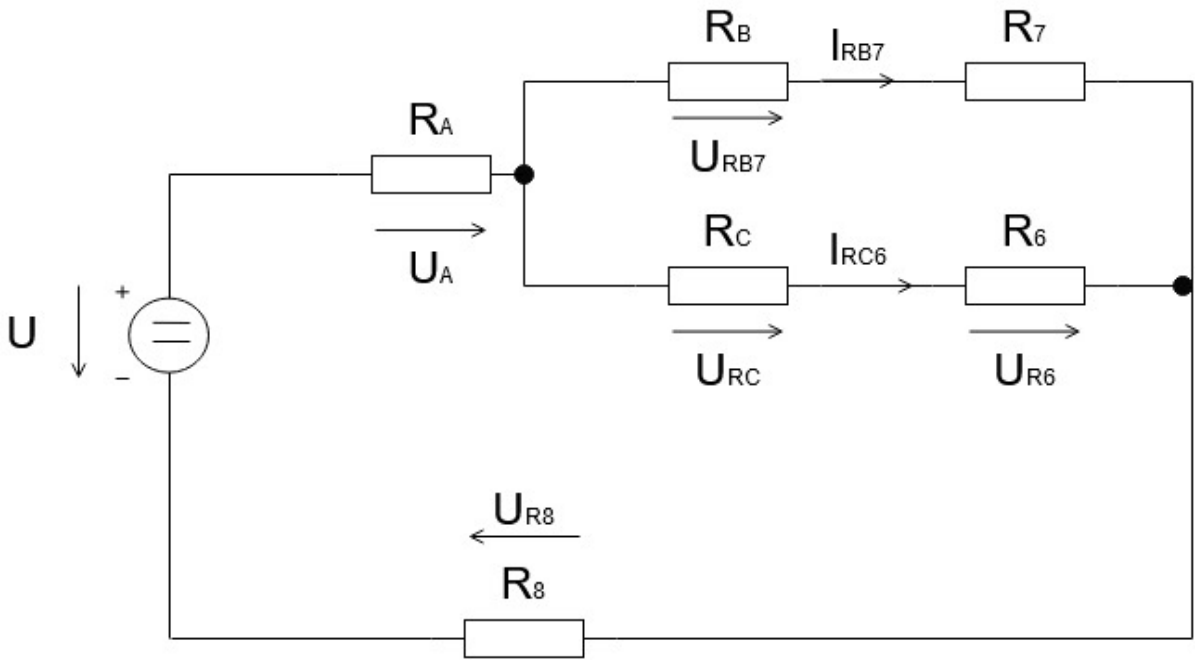
$I_{RB7} = \frac{URB7C6}{RB7}$ = 81.1110 / 528.8679 = 0.1533 A

$I_{RC6} = \frac{URB7C6}{RC6}$ = 81.1110 / 840.3674 = 0.0965 A



$U_{R6} = R_6 * I_{RC6} = 650 * 0.0965 =$ 62.7370 V

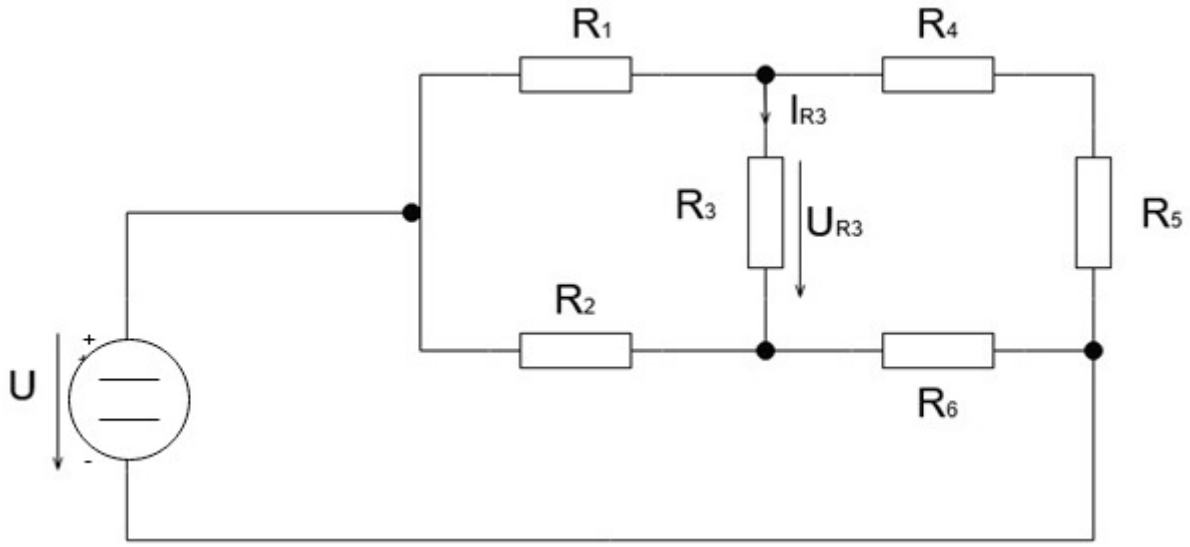
$I_{R6} = I_{RC6} =$ 0.0965 A


R_B
I_RB7
R_7
R_A
U_RB7
R_C
I_RC6
R_6
U_A
U
+
−
U_RC
U_R6
U_R8
R_8

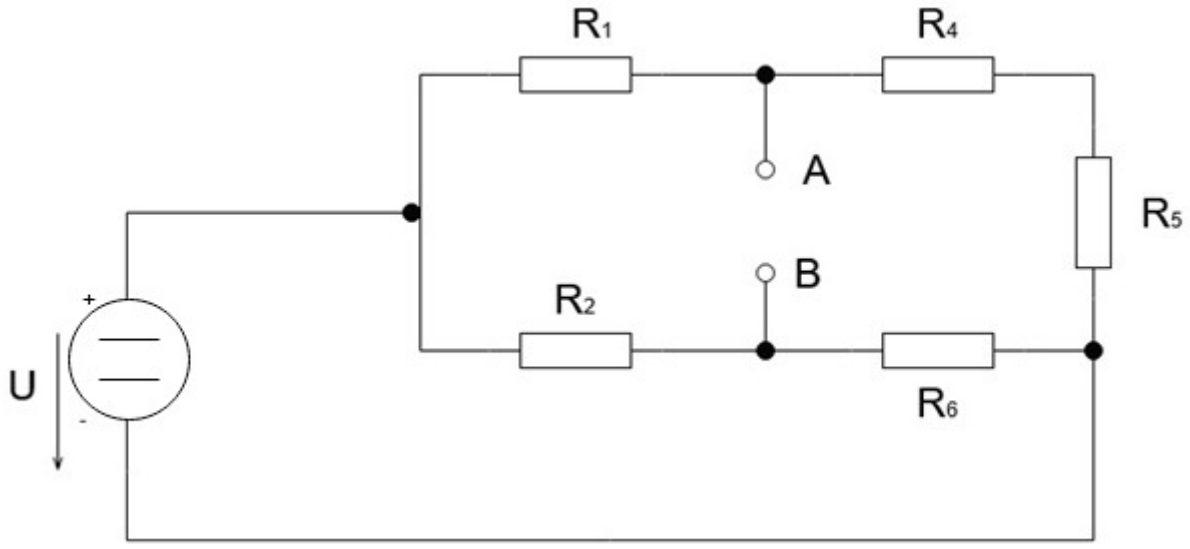
Příklad 2

Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu Théveninovy věty.

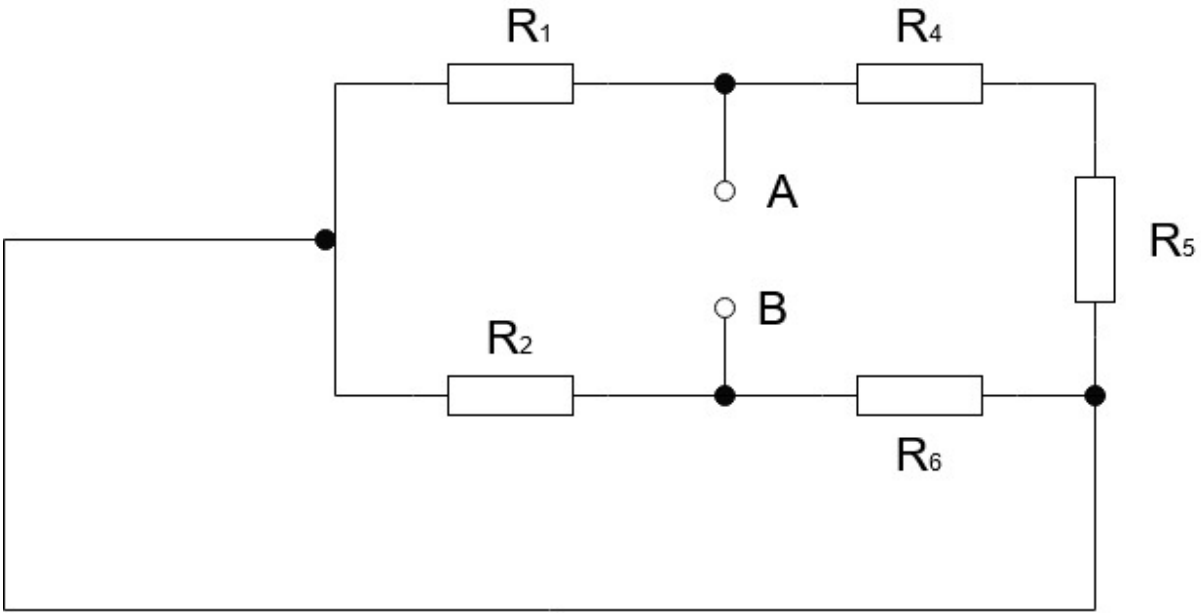
sk.	U [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]
H	220	190	360	580	205	560	180

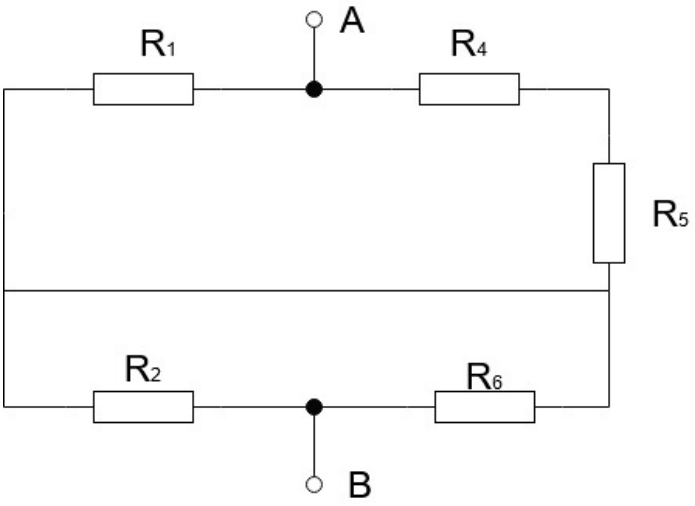


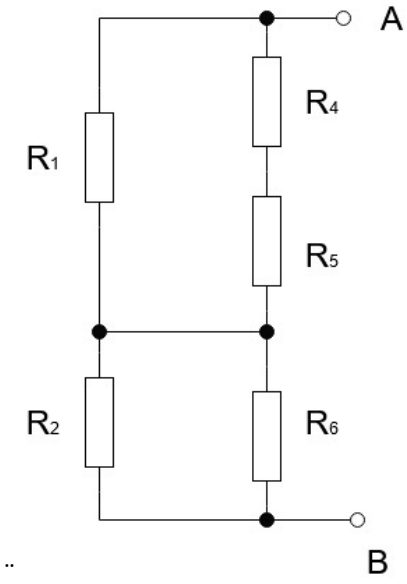
Obvod bez R3



Zkratujeme napěťové zdroje

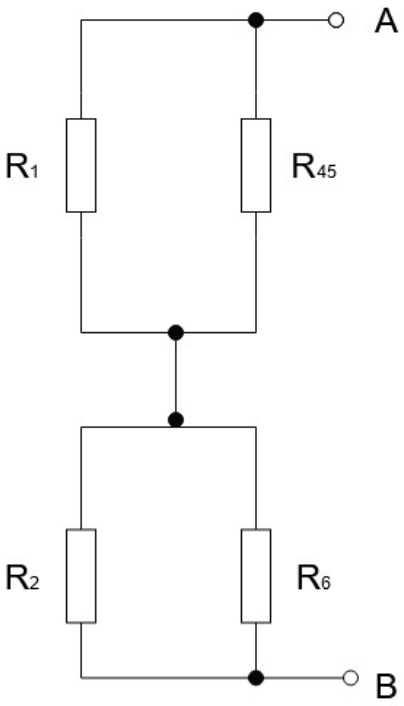






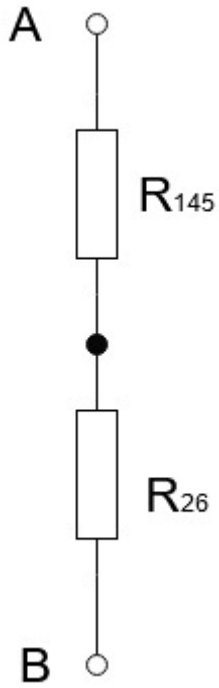
..

$R_{45} = R_4 + R_5 = 205 + 560 = 765\ \Omega$

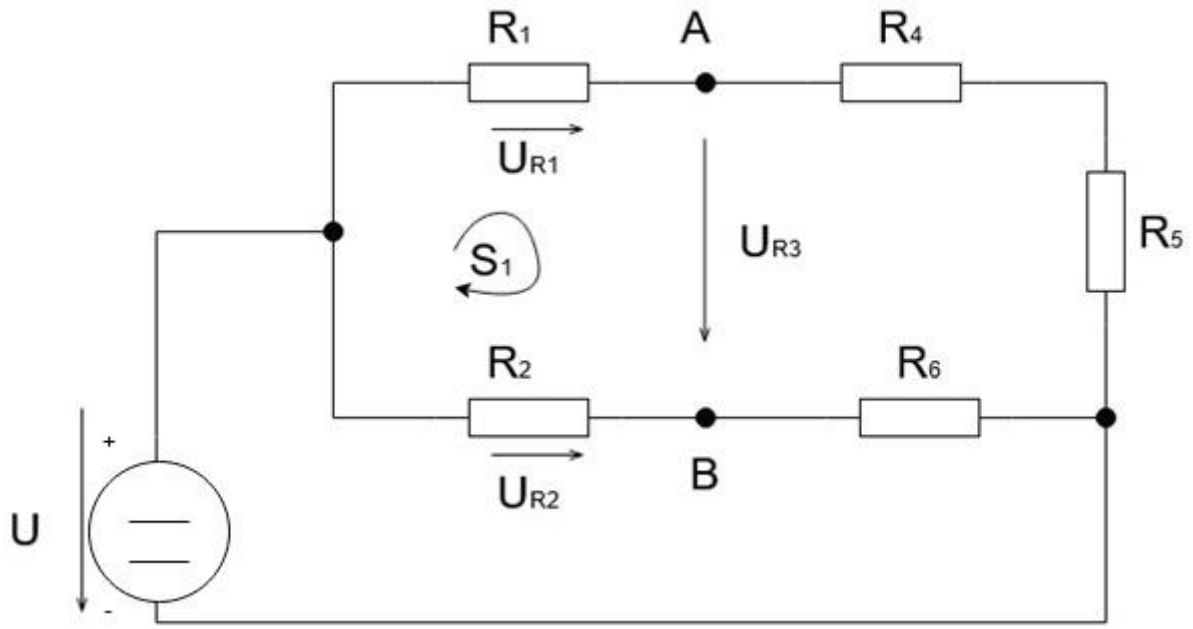


$R_{145} = \frac{R1 * R45}{R1 + R45} = 190 * 765 / 190 + 765 = 152.1989\ \Omega$

$R_{26} = \frac{R2 * R6}{R2 + R6} = 360 * 160 / 360 + 160 = 110.7692\ \Omega$



$R_i = R_{145} + R_{26} = 152.1989 + 110.7692 = 262.9681\ \Omega$



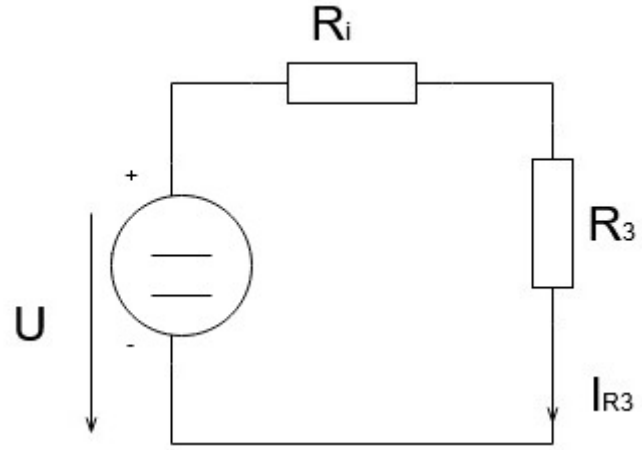
$U_{R1} = U * (\frac{R1}{R1 + R4 + R5}) = 220 * (190/ 190 + 205 + 560) = 43.7696$ V

$U_{R2} = U * (\frac{R2}{R2+R6}) = 220 * (360 / 360 + 180) = 146.6666$ V

S1:

$U_{R1} + U_{R3} – U_{R2} = 0$

U_{R3} = $U_{R2} – U_{R1}$ = 146.6666 - 43.7696 = **102.8970 V**

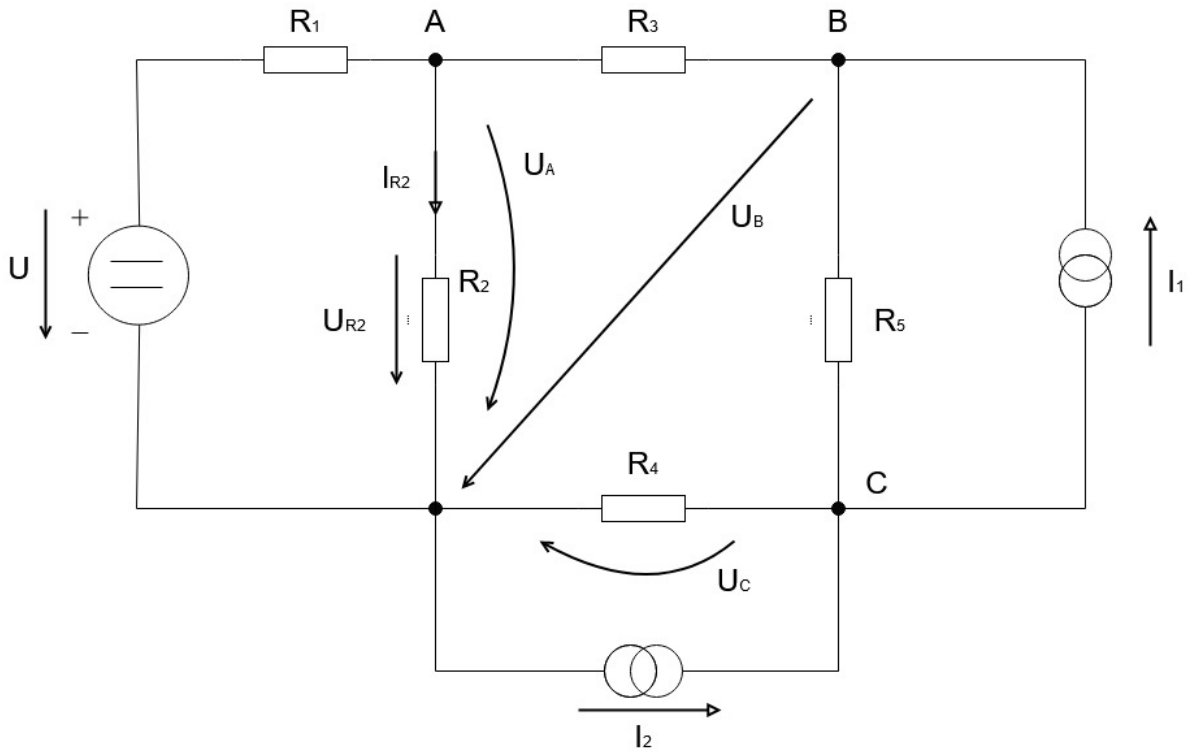


I_{R3} = $\frac{UR3}{Ri+R3}$ = 102.8970 / 262.9681 + 580 = **0.1220 A**

Příklad 3

Stanovte napětí U_{R2} a proud I_{R2}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [5]
C	110	0.85	0.75	44	31	56	20	30



Uzel

A: $I_{R1} - I_{R3} - I_{R2} = 0$

B: $I_{R3} + I_1 - I_{R5} = 0$

C: $I_2 - I_1 + I_{R5} - I_{R4} = 0$

Ohmův zákon

IR1 = $\frac{U - UA}{R1}$

IR2 = $\frac{UA}{R2}$

IR3 = $\frac{UA - UB}{R3}$

IR4 = $\frac{UC}{R4}$

IR5 = $\frac{UB - UC}{R5}$

I.KZ

$\frac{U-UA}{R1} - \frac{UA-UB}{R3} - \frac{UA}{R2} = 0$

$\frac{UA-UB}{R3} + 0.85 - \frac{UB-UC}{R5} = 0$

$(0.75 - 0.85) + \frac{UB-UC}{R5} - \frac{UC}{R4} = 0$

$$\begin{pmatrix} \frac{-1}{R_1} + \frac{-1}{R_3} + \frac{-1}{R_5} & \frac{1}{R_3} & 0 \\ \frac{1}{R_3} & \frac{-1}{R_3} + \frac{-1}{R_5} & \frac{1}{R_5} \\ 0 & \frac{1}{R_5} & \frac{-1}{R_3} + \frac{-1}{R_5} \end{pmatrix} \begin{pmatrix} U_A \\ U_B \\ U_C \end{pmatrix} = \begin{pmatrix} -2.5 \\ -0.85 \\ 0.1 \end{pmatrix}$$

$U_A = 44.7393$ V

$U_B = 42.4997$ V

$U_C = 15.7999$ V

$I_{R1} = \frac{110 - 44.7393}{44} = 1.4832$ A

$I_{R2} = \frac{44.7393}{31} = 1.4432$ A

$\mathbf{I_{R3}} = \frac{44.7393-42.4997}{56} = \mathbf{0.0399\ A}$

$I_{R4} = \frac{15.7999}{20} = 0.7899$ A

$I_{R5} = \frac{42.4997 - 15.7999}{30} = 0.8899$ A

Zkouška:

A: 1.4832 - 0.0399 - 1.4432 = 0

B: 0.0399 + 0.85 - 0.8899 = 0

C: 0.75 – 0.85 + 0.8899 - 0.7899 = 0

$\mathbf{U_{R3}} = U_A = \mathbf{44.7393\ V}$

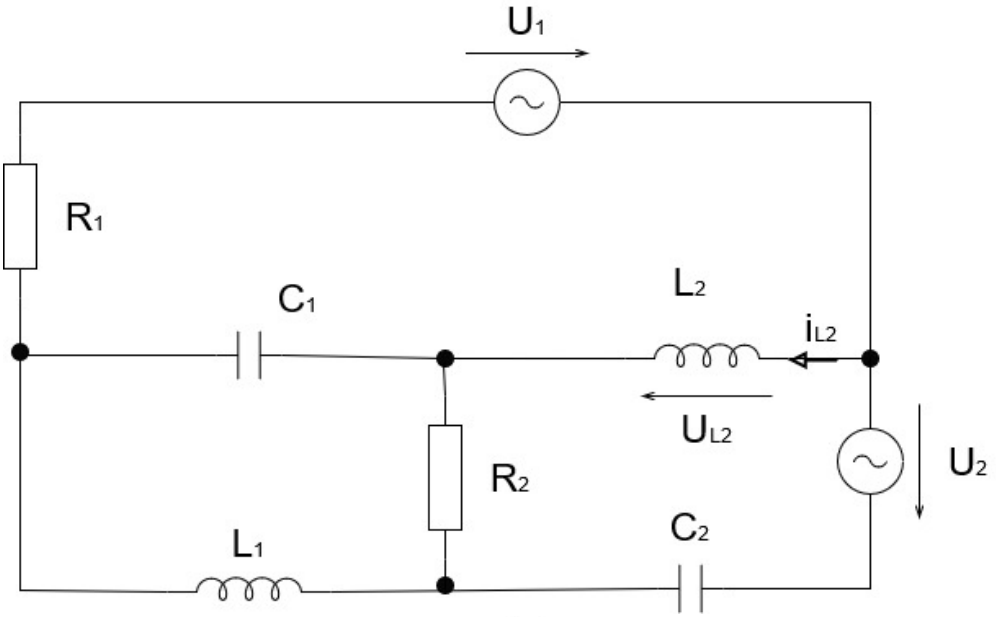
Příklad 4

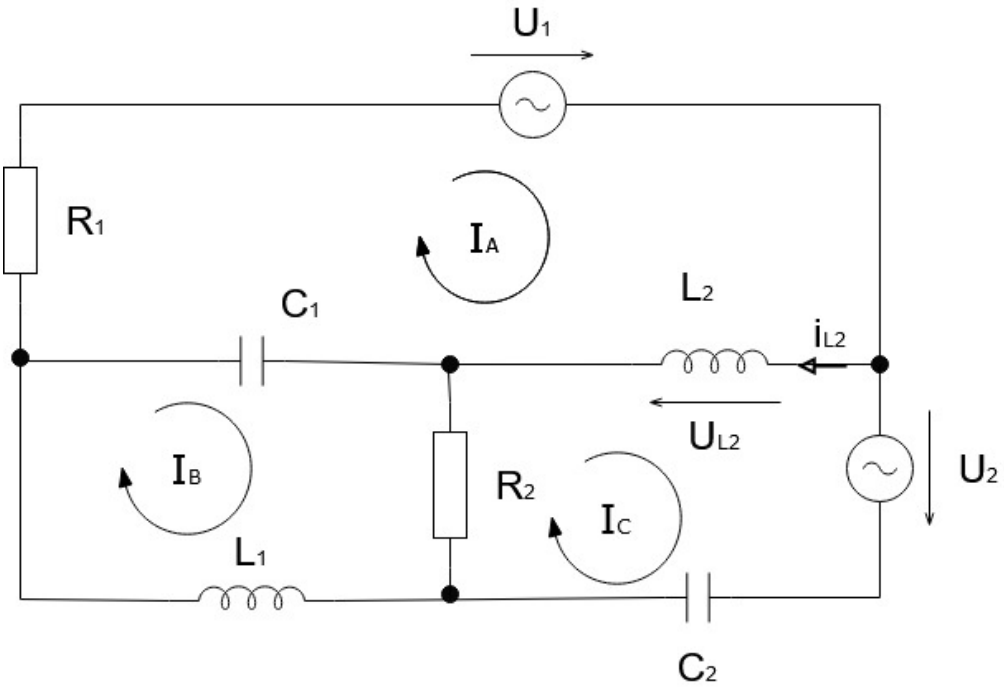
Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi f\, t)$, $u_2 = U_2 \cdot \sin(2\pi f\, t)$.

Ve vztahu pro napětí $u_{L2} = U_{L2} \cdot \sin(2\pi f\, t + \varphi_{L2})$ určete $|U_{L2}|$ a φ_{L2}. Použijte metodu smyčkových proudů.

Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \pi / 2\omega$).

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [µF]	C_2 [µF]	f [Hz]
G	55	50	13	12	140	60	160	80	60





$\omega = 2\pi f = 376.98$ [rad/s]

$ZL1 = j * \omega * L_1 = j * 52.7772$

$ZL2 = j * \omega * L_2 = j * 22.6188$

$ZC1 = -j * \frac{1}{\omega * C1} = -j * 1.6579$

$ZC2 = -j * \frac{1}{\omega * C2} = -j * 3.3158$

I_A: $R_1 * I_A + Z_{L2} * (I_A - I_C) + Z_{C1} * (I_A - I_B) + U_1 = 0$

I_B: $Z_{C1} * (I_B - I_A) + R_2 * (I_B - I_C) + Z_{L1} * I_B = 0$

I_C: $Z_{L2} * (I_C - I_A) + Z_{C2} * I_C + R_2 * (I_C - I_B) + U_2 = 0$

$$\begin{pmatrix} R_1 + Z_{L2} + Z_{C1} & -Z_{C1} & -Z_{L2} \\ -Z_{C1} & R_2 + Z_{L1} + Z_{C1} & -R_2 \\ -Z_{L2} & -R_2 & R_2 + Z_{L2} + Z_{C2} \end{pmatrix} \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -U_1 \\ 0 \\ -U_2 \end{pmatrix}$$

$$A = \begin{vmatrix} R_1 + Z_{L2} + Z_{C1} & -Z_{C1} & -Z_{L2} \\ -Z_{C1} & R_2 + Z_{L1} + Z_{C1} & -R_2 \\ -Z_{L2} & -R_2 & R_2 + Z_{L2} + Z_{C2} \end{vmatrix}$$

$$\begin{matrix} R_1 + Z_{L2} + Z_{C1} & -Z_{C1} & -Z_{L2} \\ -Z_{C1} & R_2 + Z_{L1} + Z_{C1} & -R_2 \end{matrix}$$

|A| = ((13+20,9609j)*(12+20,9609j) * (12+19,303j))+((1,6579j)*(-12)*(- 22,6188j))+ ((-22,6188j)* (1,6579j)*(-12))-((-22,6188j)*(12+20,9609j)*(-22,6188j))-((-12)*12* (13+20,9609j))-((12+19,303j)* (1,6579j)*(1,6579j))

|A| = -6371,2063+14613,8197j

$|I_A|$ = ((-55)*(12+20,9609j)*(12+19,303j))+(0*(-12)*(-22,6188j))+((-50)*(1,6579j)*(-12))-((-22,6188j)*(12+20,9609j)*(-50))-((-12)*(-12)*(-55))-((12+19,303j)*(1,6579j)*0)

$|I_A|$= 45958,9741-39150,714j

$I_A = \frac{|IA|}{|A|}$ = -3,4032-1,6611j A

$|I_B|$ = ((13+20,9609j)*0*(12+19,303j)) + ((1,6579j)*(-50)*(-22,6188j))+((-22,6188j)*(-55)*(-12))-((-22,6188j)*0*(-22,6188j))-((-12)*(-50)*(13+20,9609j))-((12+19,303j)*(-55)*(1,6579j))

$|I_B|$= -11435,1198-26410,734j

$I_B = \frac{|IB|}{|A|}$ = -1,2319+1,3195j A

$|I_C|$ = ((13+20,9609j)*(12+20,9609j)*(-50))+((1,6579j)*(-12)*(-55))+((22,6188j)*(1,6579j)* 0)-((-55)*(12+20,9609j)*(-22,6188j))-(0*(-12)*(13+20,9609j))-((-50)*(1,6579j)*(1,6579j))

$|I_C|$ = 40106,6070-40035,319j

$I_C = \frac{|IC|}{|A|}$ = -3,3074-1,3024j A

I_{L2} = IA – IC = (-3,4032-1,6611j) - (-3,3074-1,3024j) = -0,0958-0,3587j A

$U_{L2} = I_{L2} * Z_{L2}$ = (-0,0958-0,3587j) * (22,6188j) = 8,1133-2,1668j V

$|U_{L2}|$, φ_{L2}

$|U_{L2}|$ = $\sqrt{((8,1133)^2-(2,1668j)^2)}$ = **8,3976**

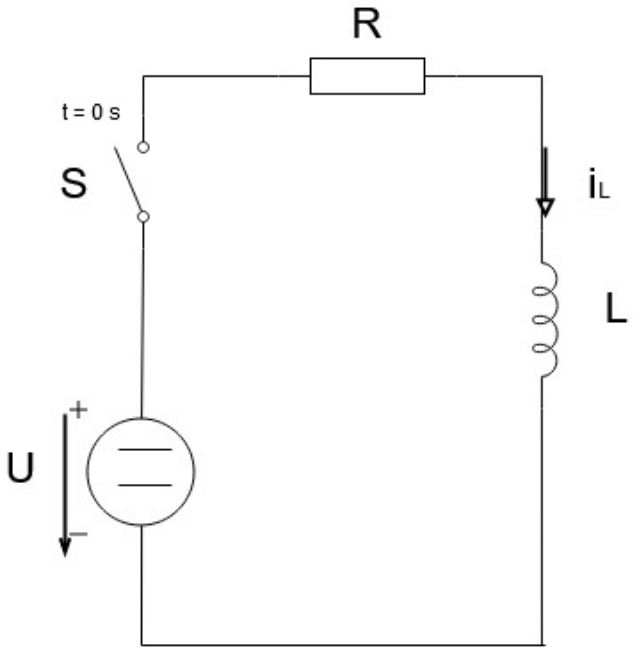
$\varphi = \tan^{-1}\frac{2,1668}{8,1133}$ = 14,95°

U_{L2} = 8,3976 ∠ 14,95°

Příklad 5

V obvodu na obrázku níže v čase t = 0 [s] sepne spínače S. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $i_L = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	U [V]	L [H]	R [Ω]	i_L (0) [A]
H	18	50	40	5



1) $i_L = \frac{UR}{R}$
2) $u_R + u_L - U = 0$
3) $i_L' = \frac{uL}{L}$, $i_L(0) = i_{LP}$

$L * i_L' + R * i_L = U$

Očekáváné řešení:

$i_L(t) = K(t) * e^{\lambda t}$

λ..?

$L\lambda + R = 0 \quad -> \quad \lambda = -\frac{R}{L}$

$i_L(t) = K(t) * e^{-\frac{R}{L}t}$

$i_L(t) = K'(t) * e^{-\frac{R}{L}t} + K(t) * (-\frac{R}{L}) * e^{-\frac{R}{L}t}$

dosazení rovnice

$L * (K'(t) * e^{-\frac{R}{L}t} + K(t) * (-\frac{R}{L}) * e^{-\frac{R}{L}t}) + R * K(t) * e^{-\frac{R}{L}t} = U$

$L * K'(t) * e^{-\frac{R}{L}t} - L * K(t) * (\frac{R}{L}) * e^{-\frac{R}{L}t} + R * K(t) * e^{-\frac{R}{L}t} = U$

$L * K'(t) * e^{-\frac{R}{L}t} - R * K(t) * e^{-\frac{R}{L}t} + R * K(t)\, e^{-\frac{R}{L}t} = U$

$L * K'(t) * e^{-\frac{R}{L}t} = U$

$K'(t) = \frac{U}{L} * e^{\frac{R}{L}t}$

$K(t) = \frac{\frac{U}{L}}{\frac{R}{L}} * e^{\frac{R}{L}t} + k$

$K(t) = \frac{U}{R} * e^{\frac{R}{L}t} + k$

Dosazení rovnice

$i_L = K(t) * e^{-\frac{R}{L}t}$

$iL = (\frac{U}{R} * e^{\frac{R}{L}t} + k) * e^{-\frac{R}{L}t}$

$iL = \frac{U}{R} + k * e^{-\frac{R}{L}t}$

určení integrační konstanty z počáteční podmínky

$i_L(0)=i_{LP}$ t=0s

$i_{LP} = \frac{U}{R} + k * e^0$

$k = i_{LP} - \frac{U}{R}$

Analytické řešení

$i_L = f(t)$

$$\boldsymbol{i_L = \frac{U}{R} + (i_{LP} - \frac{U}{R}) * e^{-\frac{R}{L}t}}$$

zkouška:

$i_L = \frac{U}{R} + (i_{LP} - \frac{U}{R}) * e^{-\frac{R}{L}t}$

$5 = \frac{18}{40} + (5 - \frac{18}{40}) * e^0$

5 = 5

$i_L(0) = i_{LP}$

TABULKA VÝSLEDKŮ

ÚLOHA	VARIANTA	VÝSLEDKY
1.	G	U_{R6} = 62.7370 V I_{R6} = 0.0965 A
2.	H	U_{R3} = 102.8970 V I_{R3} = 0.1220 A
3.	C	U_{R3} = 44.7393 V I_{R2} = 0.0399 A
4.	G	$\lvert U_{L2} \rvert$ = 8,3976 φ=14,95° U_{L2} = 8,3976 ∠ 14,95°
5.	H	$i_L = \frac{U}{R} + (i_{LP} - \frac{U}{R}) * e^{-\frac{R}{L} t}$