DZ 11

1

$(A + Bcosx)sinx = Asinx + Bsinxcosx = Asinx + \frac{B}{2}sin2x =$

$A \displaystyle\sum^{n=1}*{k=0}\frac{(-1)^kx^{2k + 1}}{(2л+1)!}+ \frac{B}{2} \displaystyle\sum^{n=1}*{k=0}\frac{(-1)^k + (2x)^{2k + 1}}{(2k + 1)!}+ o(x^4)$

$= Ax - A\frac{x^3}{6} + Bx - B\frac{2x^3}{3}$

$\begin{cases} \frac{A}{2} + 2B = 0 \\ A + B = 1 \end{cases}$

$A = \frac{4}{5}$

$B = \frac{1}{5}$

2

$x = \frac{2\pi}{5}$

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k + \frac{f^{n+1}(\epsilon)}{(n+1)!}x^{n+1}, \epsilon \in (0, x)$$

$\epsilon := \frac{\pi}{3}$

$$f(x) = \sum_{k=0}^{n} \frac{(-1)^k}{(2k!)}x^{2k} + \frac{cos^{2n+1}(\frac{\pi}{3})}{(2n+1)!}x^{2n+1}$$

$n = 1, f(x) \approx 1 - \frac{1}{2}\frac{4\pi^2}{25} - \frac{\sqrt{3}}{2*6}(\frac{2\pi}{5})^3 = 1 - \frac{2\pi^2}{25} - \frac{2\sqrt{3}\pi^3}{375}$

$\frac{2\sqrt{3}\pi^3}{375} \approx 0.28 > 10^{-3}$

$n = 2, f(x) \approx 1 - \frac{1}{2}\frac{4\pi^2}{25} + \frac{1}{24}\frac{16\pi^4}{5^4} - \frac{\sqrt{3}}{2*120}\frac{32\pi^5}{5^5}$

$\frac{\sqrt{3}}{2*120}\frac{32\pi^5}{5^5} = \frac{4\sqrt{3}\pi^5}{15*5^5} \approx 0.04 > 10^{-3}$

$n = 3, f(x) \approx 1 - \frac{1}{2}\frac{4\pi^2}{25} + \frac{1}{24}\frac{16\pi^4}{5^4} - \frac{1}{720}\frac{64\pi^6}{5^6} - \frac{1}{5040}\frac{128\pi^7}{5^7}$

$\frac{1}{5040}\frac{128\pi^7}{5^7} \approx 0.0009 < 10^{-3}$

$n = 3$ подходит, посчитаем :

$f(x) \approx 0.309$

3

$$\lim_{x\to 0}\frac{ln(e^{2x}+sinx)-3arcsinx+\frac{5x^2}{2}}{\sqrt[3]{8+x^3}-2}=$$

$$\lim_{x\to 0}\frac{ln(1+2x+2x^2+\frac{4x^3}{3}+o(x^3)+x-\frac{x^3}{6}+o(x^3))-3x-\frac{1}{2}\frac{3x^3}{3}+o(x^3)+\frac{5x^2}{2}}{2\sqrt[3]{1+(\frac{x}{2})^3}-2}$$

$$\lim_{x\to 0}\frac{ln(1+(3x+2x^2+\frac{7x^3}{6}))-3x-\frac{x^3}{2}+\frac{5x^2}{2}}{2+\frac{1}{3*8}x^3+o(x^3)-2}$$

$$\lim_{x\to 0}\frac{3x+\frac{7x^3}{3}-3x-\frac{x^3}{2}+o(x^3)}{\frac{x^3}{24}+o(x^3)}$$

$$\lim_{x\to 0}\frac{\frac{11x^3}{6}+o(x^3)}{\frac{x^3}{24}+o(x^3)}$$

$$\lim_{x\to 0}\frac{44x^3}{x^3}=44$$

4

$$_{x\to 0}\frac{\sqrt[3]{1+3x+x^2}+sinln(1-x)-e^{-7x^2/6}}{x-arctgx}=$$

$$\lim_{x\to 0}\frac{1+x-\frac{2x^2}{3}-\frac{2x^3}{3}+o(x^3)+sin(-x+\frac{x^2}{2}-\frac{x^3}{3}+o(x^3))-1+\frac{7x^2}{6}}{x-x+\frac{x^3}{3}+o(x^3)}$$

$$\lim_{x\to 0}\frac{x-\frac{x^2}{2}-\frac{2x^3}{3}+o(x^3)-x+\frac{x^2}{2}-\frac{x^3}{3}+o(x^3)}{\frac{x^3}{3}+o(x^3)}$$

$$\lim_{x\to 0}\frac{-3x^3+o(x^3)}{x^3+o(x^3)}=-3$$

5

$$\lim_{x\to 0}(\frac{2e^{x-x^2}-2}{2x-x^2})^{\frac{sinx}{x^3}}=$$

$$\lim_{x\to 0}(2\frac{e^{x-x^2}-1}{2x-x^2})^{\frac{1}{x^2}}=$$

$$\lim_{x\to 0}(2\frac{1+x-x^2+\frac{(x-x^2)^2}{2}+o(x^2)-1}{2x-x^2})^{\frac{1}{x^2}}=$$

$$\lim_{x\to 0}(2\frac{x-x^2+\frac{x^2}{2}+o(x^2)}{2x-x^2})^{\frac{1}{x^2}}=$$

$$\lim_{x\to 0}(2\frac{x-\frac{x^2}{2}+o(x^2)}{2x-x^2})^{\frac{1}{x^2}} =$$

$$\lim_{x\to 0}(\frac{2x-x^2+o(x^2)}{2x-x^2})^{\frac{1}{x^2}} =$$

$$\lim_{x\to 0}(1)^{\frac{1}{x^2}} = 1$$

6

$$\lim_{x\to\infty}(\frac{\sqrt{x^2-x}}{x}+\frac{1}{4}sin\frac{2}{x})^{x^2+sin3x} =$$

$$\lim_{x\to\infty}(\sqrt{1-\frac{1}{x}}+\frac{1}{4}sin\frac{2}{x})^{x^2+sin3x}$$

$t = \frac{1}{x}$

$$\lim_{t\to 0}(\sqrt{1-t}+\frac{1}{4}sin2t)^{\frac{1}{t^2}+sin\frac{3}{t}}$$

$$\lim_{t\to 0}(1+\frac{-t}{2}-\frac{t^2}{8}+o(t^2)+\frac{t}{2}+o(t^2))^{\frac{1}{t^2}+\frac{3}{t}+o(t^2)} =$$

$$\lim_{t\to 0}(1-\frac{t^2}{8})^{\frac{1}{t^2}}(1-\frac{t^2}{8})^{\frac{3}{t}} =$$

$$e^{-\frac{1}{8}}((1-\frac{t^2}{8})^{\frac{1}{t^2}})^{3t} = e^{-\frac{1}{8}}(e^{-\frac{1}{8}})^{3t} = e^{-\frac{1}{8}}$$

7

$tgx = \frac{sinx}{cosx} = \frac{1-\frac{\pi^2}{8}+\frac{\pi t}{4}+\frac{t^2}{2}}{x} + o(x^2)$

$t = \frac{\pi}{4} - x$

$$\lim_{t\to 0}\frac{ln(ctg(\frac{\pi}{4}-t))-2t}{(1-tgx)^2} =$$

$$\lim_{t\to 0}\frac{ln(\frac{\frac{\pi}{4}-t}{1-\frac{\pi^2}{8}+\frac{\pi t}{4}+\frac{t^2}{2}}+o(t^2))-2t}{(1-\frac{1-\frac{\pi^2}{8}+\frac{\pi t}{4}+\frac{t^2}{2}}{\frac{\pi}{4}-t})^2} =$$

$$\lim_{t\to 0}\frac{\frac{\frac{\pi}{4}-t}{1-\frac{\pi^2}{8}+\frac{\pi t}{4}+\frac{t^2}{2}}+o(t^2)-2t}{(\frac{\frac{\pi}{4}-t-1+\frac{\pi^2}{8}-\frac{\pi t}{4}-\frac{t^2}{2}}{\frac{\pi}{4}-t})^2} =$$

$$\lim_{t\to 0}\frac{\frac{\frac{\pi}{4}-t-2t+\frac{\pi^2 t}{4}-\frac{\pi t^4}{2}+o(x^2)}{1-\frac{\pi^2}{8}+\frac{\pi t}{4}+\frac{t^2}{2}}}{(\frac{\frac{\pi}{4}-t-1+\frac{\pi^2}{8}-\frac{\pi t}{4}-\frac{t^2}{2}}{\frac{\pi}{4}-t})^2}=$$

$$\frac{\pi}{4}*\frac{8}{8-\pi^2}*\frac{16}{(\pi-4)^2}*\frac{\pi^2}{16}=\frac{2\pi^3}{(8-\pi^2)(\pi-4)^2}$$