Матан Бабушкин 3.md 2023-10-07

дз 3

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a)
$$\lim_{n \to \infty} rac{3n^2 + n - 1}{-4n^2 - n + 3} = \lim_{n \to \infty} rac{3 + rac{1}{n} - rac{1}{n^2}}{-4 - rac{1}{n} + rac{3}{n^2}} = -rac{3}{4}$$

$$\text{b)} \lim_{n \to \infty} n - \frac{3}{\frac{3}{n} - \frac{3}{n^2} + \frac{1}{n^3}} = \lim_{n \to \infty} n - \frac{3n^3}{3n^2 - 3n + 1} = \lim_{n \to \infty} \frac{-3n^3 + 3n^3 - 3n^2 + n}{3n^2 - 3n + 1} = \lim_{n \to \infty} \frac{-3 + \frac{1}{n}}{3 - \frac{3}{n} + \frac{1}{n^2}} = -1$$

c)
$$\lim_{n o \infty} rac{2n - \sqrt{4n^2 - 1}}{\sqrt{n^2 + 3} - n} = \lim_{n o \infty} 2 rac{1 - \sqrt{1 - rac{1}{4n^2}}}{\sqrt{1 + rac{3}{n^2}} - 1} = 2$$

$$\mathrm{d)}\lim_{n\to\infty}\sqrt[3]{n+1}-\sqrt[3]{n-1}=\lim_{n\to\infty}\frac{n+1-n+1}{(\sqrt[3]{n+1})^2+\sqrt[3]{(n+1)(n-1)}+(\sqrt[3]{n-1})^2}=0$$

$$\text{e)} \lim_{n \to \infty} \frac{10^n + n!}{2^n + (n+1)!} = \lim_{n \to \infty} \frac{\frac{10^n}{n!} + 1}{\frac{2^n}{n!} + n + 1} = \lim_{n \to \infty} \frac{1}{n+1} = 0$$

f)

$$\lim_{n\to\infty}\frac{\sqrt[n]{16}-1}{\sqrt[n]{8}-1}=\lim_{n\to\infty}\frac{2^{\frac{4}{n}}-1^4}{2^{\frac{3}{n}}-1^3}=\lim_{n\to\infty}\frac{(2^{\frac{1}{n}}-1)(2^{\frac{1}{n}}+1)(2^{\frac{2}{n}}+1)}{(2^{\frac{1}{n}}+1)(2^{\frac{2}{n}}+1)}=\lim_{n\to\infty}\frac{(2^{\frac{1}{n}}+1)(2^{\frac{2}{n}}+1)}{(2^{\frac{2}{n}}+2^{\frac{1}{n}}+1)}=\lim_{n\to\infty}\frac{2^{\frac{3}{n}}+2^{\frac{1}{n}}+2^{\frac{2}{n}}+1}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{3}{n}}+2^{\frac{1}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{3}{n}}+2^{\frac{1}{n}}+2^{\frac{1}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{3}{n}}+2^{\frac{1}{n}}+2^{\frac{3}{n}}+1}=\lim_{n\to\infty}1+\frac{2^{\frac{3}{n}}}{2^{\frac{3}{n}+2^{\frac{3}{n}}+$$

$$1 + \frac{1}{3} = \frac{4}{3}$$

простите первую строчку переписал неправильно, ответ $\frac{3}{4})))$

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$$\lim_{n o \infty} x_n o -\infty \Leftrightarrow orall C', \exists N(C'), orall n > N(C'), x_n < C'$$

$$x_n + y_n < C' + C$$

$$\forall C'+C, \exists N(C'+C), \forall n>N(C'+C), x_n+y_n < C'+C \Leftrightarrow \lim_{n\to\infty} (x_n+y_n) \to -\infty$$

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$$\lim_{n \to \infty} x_n \to \pm \infty \Leftrightarrow \forall C', \exists N(C'), \forall n > N(C'), |x_n| > |C'|$$

$$|x_n|y_n>|C'|C$$

$$|x_ny_n|>|C'C|$$

$$\forall C'C, \exists N(C'C), \forall n > N(C'C), |x_ny_n| > |C'C| \Leftrightarrow \lim_{n \to \infty} (x_ny_n) \to \pm \infty$$