DZ 16

1

рассмотрим последовательность $[a_0, a_1, \dots]$

тогда $\Delta Sa = \Delta[a_1, a_2, \dots] = [a_2 - a_1, a_3 - a_2 \dots]$

$S\Delta a = S[a_1 - a_0, a_2 - a_1, \dots] = [a_2 - a_1, a_3 - a_2 \dots]$

$\rightarrow S\Delta = \Delta S$

2

Δ^n всегда имеет вид

$\Delta^n a_i = \sum_k^n C_k^n a_{i+k}$

тогда $\Delta^t a_m = C_0^t a_m + C_1^t a_{m+1} + \dots$

однако $\forall k > m : a_k = 0$

тогда старший коэффициент равен α

2.

$m = 0:$

$n, 2n, 3n, 4n, \dots$

$m = 1$

$n, n, n, \dots$

$m \geq 2$

$0, 0, 0, \dots$

4

$\Delta \frac{a_n}{b_n} = \frac{a_{n+1}}{b_{n+1}} - \frac{a_n}{b_n} =$

$\frac{a_{n+1}b_n - a_n b_{n+1}}{b_{n+1}b_n} =$

$\frac{a_{n+1}b_n - a_n b_{n+1} + b_{n+1}a_{n+1} - b_{n+1}a_{n+1}}{b_{n+1}b_n} =$

$\frac{b_n(a_n - a_{n+1}) - a_n(b_n - b_{n+1})}{b_{n+1}b_n} =$

$\frac{b_n \Delta a_n - a_n \Delta a_n}{b_{n+1}b_n}$

5

$\Delta \frac{n^2}{(-3)^n} = \frac{(n+1)^2}{(-3)^{n+1}} - \frac{n^2}{(-3)^n} =$

$\frac{n^2+2n+1+3n^2}{(-3)^{n+1}} = \frac{4n^2+2n+1}{(-3)^{n+1}}$

6

$\Delta cos(\alpha n + \beta) =$

$cos(\alpha n + \alpha + \beta) - \cos(\alpha n + \beta)$

$-2sin(\alpha n + \beta + \frac{\alpha}{2})sin(\frac{\alpha}{2})$

7

вначале посчитаем

$x^5 - (x+1)^5 = 5x^4 - 10x^3 + 10x^2 - 5x + 1$

сложим такие уравнения $\forall x \in \underline{n}$

$n^5 = 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + 1$

$\sum n^4 = \frac{n^5+10\sum n^3 - 10\sum n^2 + 5\sum n + 1}{5} =$

$\frac{n^5 + 5\frac{n^2(n+1)^2}{2} - 5\frac{n(n+1)(2n+1)}{3} + 5\frac{n(n+1)}{2} + 1}{5} =$

$\frac{n(n+1)(2n+1)(3n^2-3n+1)}{30}$

8

a)

$\sum \frac{1}{(n-1)(n-3)} = \sum(\frac{1/2}{n+1} - \frac{1/2}{n+3}) =$

$\frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} + \frac{1}{6} - \dots = \frac{1}{2} + \frac{1}{4} - \frac{1}{2n+2} - \frac{1}{2n+4}$

$n \to \infty : \sum = \frac{3}{4}$

b)

$\sum(b_n \Delta c_n) = (b_n c_n - b_0 c_0) - \sum c_{n+1} \Delta b_n$

$b_n = n^2$

$c_n = (-\frac{1}{3})^n = -\frac{3}{4}\Delta(\frac{1}{3})$ (заметим что)

$\sum \frac{n^2}{(-3)^n} = -\frac{3}{4}\sum n^2 \frac{1}{(-3)^n} =$

$= -\frac{3}{4}(\frac{n^2}{(-3)^n} - \sum(-\frac{1}{3}^{n+1}(2n+1))) = \dots =$

$= \frac{21}{32} + \frac{8n^2-4n-5}{32(-3)^{n-1}}$

при $n \to \infty : \sum \to \frac{21}{32}$

c)

$b_n = sin(\alpha n + \gamma); \gamma = \beta - \frac{\alpha}{2}$

$\Delta b_n = 2sin\frac{\alpha}{2}cos(\alpha n + \gamma + \frac{\alpha}{2})$

$\Delta b_n = 2sin\frac{\alpha}{2}cos(\alpha n + \beta)$

$cos(\alpha n + \beta) = \frac{\Delta sin(\alpha n+\gamma)}{2sin\frac{\alpha}{2}}$

$\sum cos(\alpha n + \beta) = \frac{1}{2sin\frac{\alpha}{2}} \sum \Delta sin(\alpha n + \gamma) =$

$\frac{1}{2sin\frac{\alpha}{2}}(sin(\alpha n + b - \frac{\alpha}{2}) - sin(\beta - \frac{\alpha}{2})) =$

$\frac{sin\frac{\alpha n}{2}cos(\frac{\alpha n}{2}-\frac{\alpha}{2}+\beta)}{sin\frac{\alpha}{2}}$