# DZ 11

## 1

$(\frac{1=i\sqrt{3}}{2})^n = (\frac{1}{2} - i\frac{\sqrt{3}}{2})^n = (cos(-\frac{\pi}{3}) + isin(-\frac{\pi}{3}))^n = cos(-\frac{n\pi}{3}) + isin(-\frac{n\pi}{3})$

## 2

$(cosx + isinx)^5 = cos5x + isin5x$

$(cosx + isinx)^5 = cos^5x + 5icos^4xsix + 10i^2cos^3xsin^2x + 10i^3cos^2xsin^3x + 5i^4cosxsin^4x + i^5sin^5x =$

$cos^5x + 5icos^4xsix - 10cos^3xsin^2x - 10icos^2xsin^3x + 5cosxsin^4x + isin^5x =$

$(cos^5x - 10cos^3xsin^2x + 5cosxsin^4x) + i(5cos^4xsins - 10cos^2xsin^3x + isin^5x)$

$cos5x + isin5x = (cos^5x - 10cos^3xsin^2x + 5cosxsin^4x) + i(5cos^4xsins - 10cos^2xsin^3x + isin^5x)$

$cos5x = cos^5x - 10cos^3xsin^2x + 5cosxsin^4x = cos^5 - 10cos^3(1 - cos^2x) + 5cosx(1 - cos^2x)^2 =$

$cos^5x - 10cos^3x + 10cos^5x + 5cosx - 10cos^3x + 5cos^5x = 16cos^5x - 20cos^3x + 5cosx$

## 3

$$\sqrt[4]{-4} = \sqrt[4]{4}(cos\frac{\pi+2\pi k}{4} + isin\frac{\pi+2\pi k}{4}) = \begin{bmatrix} \sqrt{2}(cos\frac{\pi}{4} + isin\frac{\pi}{4}) \\ \sqrt{2}(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}) \\ \sqrt{2}(cos\frac{5\pi}{4} + isin\frac{5\pi}{4}) \\ \sqrt{2}(cos\frac{7\pi}{4} + isin\frac{7\pi}{4}) \end{bmatrix} = \begin{bmatrix} 1+i \\ 1-i \\ -1+i \\ -1-i \end{bmatrix}$$

## 4

$$\sqrt[6]{64} = \sqrt[6]{64}(cos\frac{2\pi k}{6} + isin\frac{2\pi k}{6}) = \begin{bmatrix} 2(1+0i) \\ 2(\frac{1}{2} + i\frac{\sqrt{3}}{2}) \\ 2(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) \\ 2(-1+i0) \\ 2(-\frac{1}{2} + -i\frac{\sqrt{3}}{2}) \\ 2(\frac{1}{2} - i\frac{\sqrt{3}}{2}) \end{bmatrix} = \begin{bmatrix} 2 \\ 2+2i\sqrt{3} \\ -2+2i\sqrt{3} \\ -2 \\ -2-2i\sqrt{3} \\ 2-2i\sqrt{3} \end{bmatrix}$$

## 5

$ \sqrt[3]{2-2i} = (2(\frac{\sqrt2}{2} - i\frac{\sqrt2}{2}))^{\frac{1}{3}} = \sqrt[3]2(cos\frac{-\pi+8\pi k}
{43}+isin|frac{-|pi +8|pi k }{43}) = $

$$\begin{bmatrix} \sqrt[3]{2}(cos\frac{-\pi}{12} + isin\frac{-\pi}{12}) \\ \sqrt[3]{2}(cos\frac{7\pi}{12} + isin\frac{7\pi}{12}) \\ \sqrt[3]{2}(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) \end{bmatrix}$$

# 6

$(z+1)^n - (z-1)^n = 0$

Пусть $z = cos2a + isin2a$

Тогда $z + 1 = cos2a + isin2a + 1 = 2cos^2a - 1 + i2sinacosa + 1 = 2cos^2a + 2isinacosa$

а $z - 1 = cos2a + isin2a + 1 = 1 - 2sin^2a + i2sinacosa + 1 = -(2sin^2a - 2isinacosa)$

$\frac{z+1}{z-1} = -\frac{2cos^2a+2isinacosa}{2sin^2a-2isinacosa} = -\frac{cosa(cosa-isina)}{-isina(-isina+icosa)} = \frac{cosa}{isina} = -ictga$

$(\frac{z+1}{z-1})^n = 1$

$\frac{z+1}{z-1} = \sqrt[n]{1} = cos\frac{2\pi k}{n} + isin\frac{2\pi k}{n}$

$-ictga = cos\frac{2\pi k}{n} + isin\frac{2\pi k}{n}$

$a = arcctg(-\frac{cos\frac{2\pi k}{n}+isin\frac{2\pi k}{n}}{i})$

$z = cos2a + isin2a$

# 7

$256x^8 + 1 = 0$

$$x = \sqrt[8]{-\frac{1}{256}} = \sqrt[8]{\frac{1}{256}}(cos\frac{2\pi k}{8} + isin\frac{2\pi k}{8}) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2}(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) \\ \frac{1}{2}(0+i) \\ \frac{1}{2}(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) \\ -\frac{1}{2} \\ \vdots \end{bmatrix}$$

$256x^8 + 1 = (x - \frac{1}{2})(x - i\frac{1}{2})(x - \frac{\sqrt{2}+i\sqrt{2}}{4})(x - \frac{-\sqrt{2}+i\sqrt{2}}{4})(x + \frac{1}{2})(x + i\frac{1}{2})(x + \frac{\sqrt{2}+i\sqrt{2}}{4})(x + \frac{-\sqrt{2}+i\sqrt{2}}{4})$

$= (x - \frac{1}{2})(x + \frac{1}{2})(x^2 + \frac{1}{4})\frac{4x^2+2\sqrt{2}x+1}{4}\frac{4x^2-2\sqrt{2}x+1}{4}$