дз 3

1

a) $\lim\limits_{n\to\infty}\dfrac{3n^2+n-1}{-4n^2-n+3}=\lim\limits_{n\to\infty}\dfrac{3+\frac{1}{n}-\frac{1}{n^2}}{-4-\frac{1}{n}+\frac{3}{n^2}}=-\dfrac{3}{4}$

b) $\lim\limits_{n\to\infty} n-\dfrac{3}{\frac{3}{n}-\frac{3}{n^2}+\frac{1}{n^3}}=\lim\limits_{n\to\infty} n-\dfrac{3n^3}{3n^2-3n+1}=\lim\limits_{n\to\infty}\dfrac{-3n^3+3n^3-3n^2+n}{3n^2-3n+1}=\lim\limits_{n\to\infty}\dfrac{-3+\frac{1}{n}}{3-\frac{3}{n}+\frac{1}{n^2}}=-1$

c) $\lim\limits_{n\to\infty}\dfrac{2n-\sqrt{4n^2-1}}{\sqrt{n^2+3}-n}=\lim\limits_{n\to\infty}2\dfrac{1-\sqrt{1-\frac{1}{4n^2}}}{\sqrt{1+\frac{3}{n^2}}-1}=2$

d) $\lim\limits_{n\to\infty}\sqrt[3]{n+1}-\sqrt[3]{n-1}=\lim\limits_{n\to\infty}\dfrac{n+1-n+1}{(\sqrt[3]{n+1})^2+\sqrt[3]{(n+1)(n-1)}+(\sqrt[3]{n-1})^2}=0$

e) $\lim\limits_{n\to\infty}\dfrac{10^n+n!}{2^n+(n+1)!}=\lim\limits_{n\to\infty}\dfrac{\frac{10^n}{n!}+1}{\frac{2^n}{n!}+n+1}=\lim\limits_{n\to\infty}\dfrac{1}{n+1}=0$

f)

$\lim\limits_{n\to\infty}\dfrac{\sqrt[n]{16}-1}{\sqrt[n]{8}-1}=\lim\limits_{n\to\infty}\dfrac{2^{\frac{4}{n}}-1^4}{2^{\frac{3}{n}}-1^3}=\lim\limits_{n\to\infty}\dfrac{(2^{\frac{1}{n}}-1)(2^{\frac{1}{n}}+1)(2^{\frac{2}{n}}+1)}{(2^{\frac{1}{n}}-1)(2^{\frac{2}{n}}+2^{\frac{1}{n}}+1)}=\lim\limits_{n\to\infty}\dfrac{(2^{\frac{1}{n}}+1)(2^{\frac{2}{n}}+1)}{(2^{\frac{2}{n}}+2^{\frac{1}{n}}+1)}=\lim\limits_{n\to\infty}\dfrac{2^{\frac{3}{n}}+2^{\frac{1}{n}}+2^{\frac{2}{n}}+1}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=\lim\limits_{n\to\infty}1+\dfrac{2^{\frac{3}{n}}}{2^{\frac{2}{n}}+2^{\frac{1}{n}}+1}=$

$1+\frac{1}{3}=\frac{4}{3}$

простите первую строчку переписал неправильно, ответ $\frac{3}{4}$)))

2

$\lim\limits_{n\to\infty}x_n\to-\infty\Leftrightarrow\forall C',\exists N(C'),\forall n>N(C'),x_n<C'$

$x_n+y_n<C'+C$

$\forall C'+C,\exists N(C'+C),\forall n>N(C'+C),x_n+y_n<C'+C\Leftrightarrow\lim\limits_{n\to\infty}(x_n+y_n)\to-\infty$

3

$\lim\limits_{n\to\infty}x_n\to\pm\infty\Leftrightarrow\forall C',\exists N(C'),\forall n>N(C'),|x_n|>|C'|$

$|x_n|y_n>|C'|C$

$|x_ny_n|>|C'C|$

$\forall C'C,\exists N(C'C),\forall n>N(C'C),|x_ny_n|>|C'C|\Leftrightarrow\lim\limits_{n\to\infty}(x_ny_n)\to\pm\infty$