Дискра Бабушкин 16.md 2024-06-06

## **DZ 16**

1

рассмотрим последовательность  $[a_0,a_1,\dots]$ 

тогда 
$$\Delta Sa = \Delta[a_1,a_2,\dots] = [a_2-a_1,a_3-a_2\dots]$$

$$S\Delta a = S[a_1 - a_0, a_2 - a_1, \dots] = [a_2 - a_1, a_3 - a_2 \dots]$$

$$ightarrow S\Delta = \Delta S$$

2

 $\Delta^n$  всегда имеет вид

$$\Delta^n a_i = \sum_k^n C_k^n a_{i+k}$$

тогда 
$$\Delta^t a_m = C_0^t a_m + C_1^t a_{m+1} + \dots$$

однако 
$$orall k > m: a_k = 0$$

тогда старший коэффициент равен lpha

2.

m=0:

 $n, 2n, 3n, 4n, \ldots$ 

m = 1

 $n, n, n, \ldots$ 

 $m \geq 2$ 

 $0, 0, 0, \dots$ 

4

$$\Delta rac{a_n}{b_n} = rac{a_{n+1}}{b_{n+1}} - rac{a_n}{b_n} =$$

$$rac{a_{n+1}b_{n}-a_{n}b_{n+1}}{b_{n+1}b_{n}}=$$

$$\tfrac{a_{n+1}b_n-a_nb_{n+1}+b_{n+1}a_{n+1}-b_{n+1}a_{n+1}}{b_{n+1}b_n}=$$

$$rac{b_n(a_n-a_{n+1})-a_n(b_n-b_{n+1})}{b_{n+1}b_n} =$$

$$\frac{b_n\Delta a_n\!-\!a_n\Delta a_n}{b_{n+1}b_n}$$

5

$$\Delta \frac{n^2}{(-3)^n} = \frac{(n+1)^2}{(-3)^{n+1}} - \frac{n^2}{(-3)^n} =$$

$$\frac{n^2 + 2n + 1 + 3n^2}{(-3)^{n+1}} = \frac{4n^2 + 2n + 1}{(-3)^{n+1}}$$

6

$$egin{aligned} \Delta cos(lpha n + eta) &= \ cos(lpha n + lpha + eta) - \cos(lpha n + eta) \ -2sin(lpha n + eta + rac{lpha}{2})sin(rac{lpha}{2}) \end{aligned}$$

7

вначале посчитаем

$$x^5 - (x+1)^5 = 5x^4 - 10x^3 + 10x^2 - 5x + 1$$

сложим такие уравнения  $\forall x \in \mathit{n}$ 

$$n^5 = 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + 1$$
  $\sum n^4 = rac{n^5 + 10\sum n^3 - 10\sum n^2 + 5\sum n + 1}{5} = rac{n^5 + 5rac{n^2(n+1)^2}{2} - 5rac{n(n+1)(2n+1)}{3} + 5rac{n(n+1)}{2} + 1}{5} = rac{n(n+1)(2n+1)(3n^2 - 3n + 1)}{30}$ 

8

a)

$$\sum \frac{1}{(n-1)(n-3)} = \sum \left(\frac{1/2}{n+1} - \frac{1/2}{n+3}\right) =$$

$$\frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} + \frac{1}{6} - \dots = \frac{1}{2} + \frac{1}{4} - \frac{1}{2n+2} - \frac{1}{2n+4}$$

$$n \to \infty : \sum = \frac{3}{4}$$

b)

$$\sum (b_n\Delta c_n)=(b_nc_n-b_0c_0)-\sum c_{n+1}\Delta b_n$$
 $b_n=n^2$  $c_n=(-rac{1}{3})^n=-rac{3}{4}\Delta(rac{1}{3})$  (заметим что)

$$egin{align} \sum rac{n^2}{(-3)^n} &= -rac{3}{4} \sum n^2 rac{1}{(-3)^n} = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - \sum (-rac{1}{3}^{n+1} (2n+1))) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - \sum (-rac{1}{3}^{n+1} (2n+1))) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - \sum (-rac{1}{3}^{n+1} (2n+1))) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - rac{1}{3} (2n+1)) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - rac{1}{3} (2n+1)) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - rac{1}{3} (2n+1)) = \ldots = \ &= -rac{3}{4} (rac{n^2}{(-3)^n} - rac{1}{3} (2n+1)) = \ldots = \ &= -rac{3}{4} (2n+1) = -$$

$$= \frac{21}{32} + \frac{8n^2 - 4n - 5}{32(-3)^{n-1}}$$

при 
$$n o\infty:\sum orac{21}{32}$$

c)

$$b_n=sin(lpha n+\gamma)$$
;  $\gamma=eta-rac{lpha}{2}$ 

$$\Delta b_n = 2 sin rac{lpha}{2} cos (lpha n + \gamma + rac{lpha}{2})$$

$$\Delta b_n = 2 sin rac{lpha}{2} cos (lpha n + eta)$$

$$cos(lpha n + eta) = rac{\Delta sin(lpha n + \gamma)}{2sinrac{lpha}{2}}$$

$$\sum cos(lpha n + eta) = rac{1}{2sinrac{lpha}{2}} \sum \Delta sin(lpha n + \gamma) =$$

$$rac{1}{2sinrac{lpha}{2}}(sin(lpha n+b-rac{lpha}{2})-sin(eta-rac{lpha}{2}))=$$

$$\frac{sin\frac{\alpha n}{2}cos(\frac{\alpha n}{2}\!-\!\frac{\alpha}{2}\!+\!\beta)}{sin\frac{\alpha}{2}}$$