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DZ 11

1

$$(A+Bcosx)sinx=Asinx+Bsinxcosx=Asinx+rac{B}{2}sin2x=$$

 $A \bigg| \frac{n-1}{k=0} \bigg| \frac{(-1)^k x^{2k+1}}{(2\pi+1)!} + \bigg| \frac{B}{2} \bigg| \frac{n-1}{k=0} \bigg| \frac{n-1}{k+0} \bigg| \frac{(-1)^k x^{2k+1}}{(2k+1)!} + \frac{n-1}{k+0} \bigg| \frac{n-1}{k+0} \bigg|$

$$=Ax-Arac{x^3}{6}+Bx-Brac{2x^3}{3}$$

$$\begin{cases} \frac{A}{2} + 2B = 0\\ A + B = 1 \end{cases}$$

$$A = \frac{4}{5}$$

$$B=\frac{1}{5}$$

2

$$x = \frac{2\pi}{5}$$

$$f(x) = \sum_{k=0}^n rac{f^{(k)}(0)}{k!} x^k + rac{f^{n+1}(\epsilon)}{(n+1)!} x^{n+1}, \epsilon \in (0,x)$$

$$\epsilon := \frac{\pi}{3}$$

$$f(x) = \sum_{k=0}^{n} rac{(-1)^k}{(2k!)} x^{2k} + rac{cos^{2n+1}(rac{\pi}{3})}{(2n+1)!} x^{2n+1}$$

$$n=1, f(x)pprox 1-rac{1}{2}rac{4\pi^2}{25}-rac{\sqrt{3}}{2*6}(rac{2\pi}{5})^3=1-rac{2\pi^2}{25}-rac{2\sqrt{3}\pi^3}{375}$$

$$rac{2\sqrt{3}\pi^3}{375}pprox 0.28>10^{-3}$$

$$n=2, f(x)pprox 1-rac{1}{2}rac{4\pi^2}{25}+rac{1}{24}rac{16\pi^4}{5^4}-rac{\sqrt{3}}{2*120}rac{32\pi^5}{5^5}$$

$$\tfrac{\sqrt{3}}{2*120} \tfrac{32\pi^5}{5^5} = \tfrac{4\sqrt{3}\pi^5}{15*5^5} \approx 0.04 > 10^{-3}$$

$$n=3, f(x)pprox 1-rac{1}{2}rac{4\pi^2}{25}+rac{1}{24}rac{16\pi^4}{5^4}-rac{1}{720}rac{64\pi^6}{5^6}-rac{1}{5040}rac{128\pi^7}{5^7}$$

$$rac{1}{5040} rac{128\pi^7}{5^7} pprox 0.0009 < 10^{-3}$$

n=3 подходит, посчитаем :

$$f(x) \approx 0.309$$

3

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$$\lim_{x o 0}rac{ln(e^{2x}+sinx)-3arcsinx+rac{5x^2}{2}}{\sqrt[3]{8+x^3}-2}=$$

$$\lim_{x o 0}rac{ln(1+2x+2x^2+rac{4x^3}{3}+o(x^3)+x-rac{x^3}{6}+o(x^3))-3x-rac{1}{2}rac{3x^3}{3}+o(x^3)+rac{5x^2}{2}}{2\sqrt[3]{1+(rac{x}{2})^3}-2}$$

$$\lim_{x o 0}rac{ln(1+(3x+2x^2+rac{7x^3}{6}))-3x-rac{x^3}{2}+rac{5x^2}{2}}{2+rac{1}{3*8}x^3+o(x^3)-2}$$

$$\lim_{x o 0}rac{3x+rac{7x^3}{3}-3x-rac{x^3}{2}+o(x^3)}{rac{x^3}{24}+o(x^3)}$$

$$\lim_{x o 0}rac{rac{11x^3}{6}+o(x^3)}{rac{x^3}{24}+o(x^3)}$$

$$\lim_{x
ightarrow 0}rac{44x^3}{x^3}=44$$

4

$$_{x o0}rac{\sqrt[3]{1+3x+x^2}+sinln(1-x)-e^{-7x^2/6}}{x-arctgx}=$$

$$\lim_{x o 0}rac{1+x-rac{2x^2}{3}-rac{2x^3}{3}+o(x^3)+sin(-x+rac{x^2}{2}-rac{x^3}{3}+o(x^3))-1+rac{7x^2}{6}}{x-x+rac{x^3}{3}+o(x^3)}$$

$$\lim_{x o 0}rac{x-rac{x^2}{2}-rac{2x^3}{3}+o(x^3)-x+rac{x^2}{2}-rac{x^3}{3}+o(x^3)}{rac{x^3}{3}+o(x^3)}$$

$$\lim_{x o 0}rac{-3x^3+o(x^3)}{x^3+o(x^3)}=-3$$

5

$$\lim_{x o 0}(rac{2e^{x-x^2}-2}{2x-x^2})^{rac{sinx}{x^3}}=$$

$$\lim_{x o 0}(2rac{e^{x-x^2}-1}{2x-x^2})^{rac{1}{x^2}}=$$

$$\lim_{x o 0}(2rac{1+x-x^2+rac{(x-x^2)^2}{2}+o(x^2)-1}{2x-x^2})^{rac{1}{x^2}}=$$

$$\lim_{x o 0}(2rac{x-x^2+rac{x^2}{2}+o(x^2)}{2x-x^2})^{rac{1}{x^2}}=$$

$$\lim_{x o 0}(2rac{x-rac{x^2}{2}+o(x^2)}{2x-x^2})^{rac{1}{x^2}}=\ \lim_{x o 0}(rac{2x-x^2+o(x^2)}{2x-x^2})^{rac{1}{x^2}}=$$

$$\lim_{x
ightarrow 0}(1)^{rac{1}{x^2}}=1$$

6

$$\lim_{x o\infty}(rac{\sqrt{x^2-x}}{x}+rac{1}{4}sinrac{2}{x})^{x^2+sin3x}=\ \lim_{x o\infty}(\sqrt{1-rac{1}{x}}+rac{1}{4}sinrac{2}{x})^{x^2+sin3x}$$

$$t = \frac{1}{x}$$

$$\lim_{t o 0}(\sqrt{1-t}+rac{1}{4}sin2t)^{rac{1}{t^2}+sinrac{3}{t}}$$

$$\lim_{t o 0}(1+rac{-t}{2}-rac{t^2}{8}+o(t^2)+rac{t}{2}+o(t^2))^{rac{1}{t^2}+rac{3}{t}+o(t^2)}=$$

$$\lim_{t o 0}(1-rac{t^2}{8})^{rac{1}{t^2}}(1-rac{t^2}{8})^{rac{3}{t}}=$$

$$e^{-rac{1}{8}}((1-rac{t^2}{8})^{rac{1}{t^2}})^{3t}=e^{-rac{1}{8}}(e^{-rac{1}{8}})^{3t}=e^{-rac{1}{8}}$$

7

$$tgx = rac{sinx}{cosx} = rac{1 - rac{\pi^2}{8} + rac{\pi t}{4} + rac{t^2}{2}}{x} + o(x^2)$$

$$t = \frac{\pi}{4} - x$$

$$\lim_{t\to 0}\frac{ln(ctg(\frac{\pi}{4}-t))-2t}{(1-tgx)^2}=$$

$$\lim_{t o 0}rac{ln(rac{rac{\pi}{4}-t}{1-rac{\pi^2}{8}+rac{\pi t}{4}+rac{t^2}{2}}+o(t^2))-2t}{(1-rac{1-rac{\pi^2}{8}+rac{\pi t}{4}+rac{t^2}{2}}{rac{\pi}{4}-t})^2}=$$

$$\lim_{t o 0}rac{rac{rac{\pi}{4}-t}{1-rac{\pi^2}{8}+rac{\pi t}{4}+rac{t^2}{2}}+o(t^2)-2t}{(rac{\pi}{4}-t-1+rac{\pi^2}{8}-rac{\pi t}{4}-rac{t^2}{2}}{rac{\pi}{4}-t})^2}=$$

$$\lim_{t o 0}rac{rac{\pi}{4}-t-2t+rac{\pi^2t}{2}-rac{\pi t^4}{2}+o(x^2)}{(rac{\pi}{4}-t-1+rac{\pi^2}{8}+rac{\pi t}{4}+rac{t^2}{2}}{(rac{\pi}{4}-t-1+rac{\pi^2}{8}-rac{\pi t}{4}-rac{t^2}{2}})^2}$$

$$\frac{\pi}{4} * \frac{8}{8-\pi^2} * \frac{16}{(\pi-4)^2} * \frac{\pi^2}{16} = \frac{2\pi^3}{(8-\pi^2)(\pi-4)^2}$$