самостоятельное решение (случайно 1 номер оттуда решил)

1

a)

$$\lim_{n\to\infty} \sqrt[2^n]{\prod_{k=1}^{2^n} (1+\frac{k}{2^n})} =$$

$$\lim_{n\to\infty} \prod_{k=1}^{2^n} \sqrt[2^n]{(1+\frac{k}{2^n})} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{\prod_{k=1}^{2^n} (1+\frac{k}{2^n})^{2^n}} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{\prod_{k=1}^{2^n} e^k} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{e^{\sum k}} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{e^{\frac{2^n(2^n+1)}{2}}} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{e^{2^{n-1}(2^n+1)}} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{e^{2^{2n-1}+2^{n-1}}} =$$

$$\lim_{n\to\infty} \sqrt[4^n]{e^{\frac{4^n}{2}}e^{\frac{2^n}{2}}} =$$

$$\lim_{n\to\infty} e^{\frac{3}{2}} \sqrt[4^n]{e^{\frac{1}{4}}} =$$

$$\lim_{n\to\infty} e^{\frac{3}{2}} \sqrt[4^n+1]{e} = e^{\frac{3}{2}}$$

DZ 13

1

a)

$$\int_1^2 \frac{dx}{x^2} =$$

$$\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{1}{(1+\frac{k}{n})^2}\frac{1}{n}=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{n}{(n+k)^2}$$

$$<\sum_{k=0}^{n-1}\frac{n}{(n+k)(n+k+1)}<$$

$$\sum_{k=0}^{n-1}\frac{n}{(n+k)^2}<$$

$$<\sum_{k=0}^{n-1}\frac{n}{(n+k)(n+k-1)}$$

$$\sum_{k=0}^{n-1}\frac{n}{(n+k)(n+k+1)}=$$

$$n\sum_{k=0}^{n-1}\frac{1}{n+k}-\frac{1}{n+k+1}=n(\frac{1}{n}-\frac{1}{2n})=\frac{1}{2}$$

$$\sum_{k=0}^{n-1}\frac{n}{(n+k)(n+k-1)}=$$

$$n\sum_{k=0}^{n-1}\frac{1}{n+k-1}-\frac{1}{n+k-1}=$$

$n(\frac{1}{n-1}-\frac{1}{2n-1})\to\frac{1}{2}$

по т. о двух миллиционерах наш интеграл равен $\frac{1}{2}$

b)

$$\int_1^e lnxdx=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}ln(1+\frac{e-1}{n}k)\frac{e-1}{n}=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}ln((1+\frac{e-1}{n}k)^{\frac{e-1}{n}})=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}ln(((1+\frac{e-1}{n}k)^{\frac{n}{e-1}})^{\frac{e-1}{n}^2})=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}ln((e^k)^{\frac{e-1}{n}^2})=$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}k\frac{(e-1)^2}{n^2}=$$

$$\lim_{n\to\infty}\frac{(e-1)^2}{n^2}\sum_{k=0}^{n}k=$$

$$\lim_{n\to\infty}\frac{(e-1)^2}{n^2}\frac{n(n+1)}{2}=$$

$$\lim_{n\to\infty}\frac{(e-1)^2(1+\frac{1}{n})}{2}=\frac{(e-1)^2}{2}$$

2

a)

$$\lim_{n\to\infty}\frac{1}{n^2}\sum_{k=1}^{n}\sqrt{k(n-k)}=$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}(1-\frac{k}{n})}=$$

$$\int_0^1\sqrt{x(1-x)}dx$$

$$y=\sqrt{x(1-x)}$$

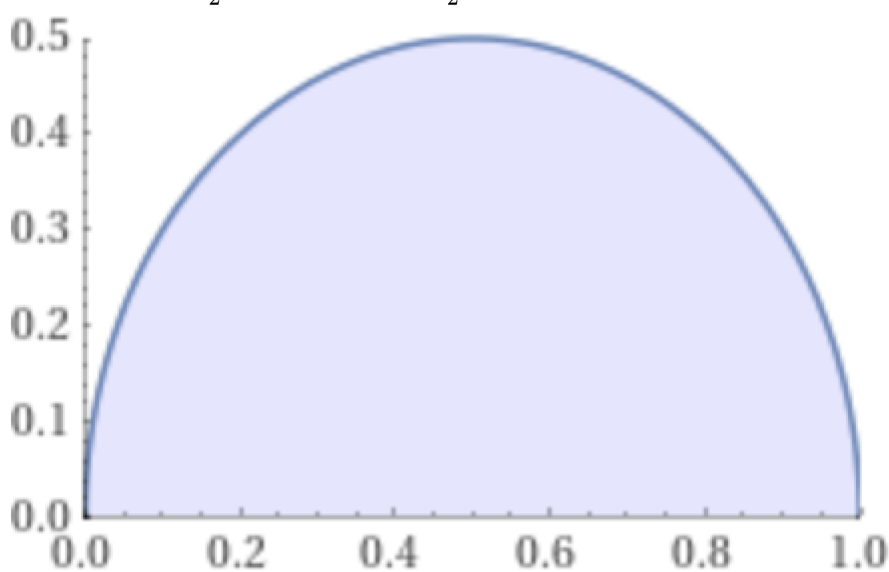
$$y^2=x(1-x)$$

$$x^2-x+y^2=0$$

$$x^2-x+\frac{1}{4}-\frac{1}{4}-y^2=0$$

$$(x-\frac{1}{2})^2+y^2=\frac{1}{4}$$

это окружность с центром в точке $(\frac{1}{2};0)$ и радиусом $\frac{1}{2}$ получается нужно посчитать площадь



верхнего полукруга

$S = \frac{\pi r^2}{2} = \frac{\pi}{4*2} = \frac{\pi}{8}$

b)

$$\lim_{n\to\infty}(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n})$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}(\frac{1}{n}) - lnn = \gamma$$

постоянная эйлера

тогда

$$\lim_{n\to\infty}(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n}) =$$

$$\gamma + ln3n - \gamma - lnn = ln3$$

3

a)

$$\int sin^4 x dx = \frac{sin4x}{32} - \frac{sin2x}{4} + \frac{3x}{8} + C$$

$$\int_0^{2\pi} sin^4 x dx = \frac{3\pi}{4}$$

b)

$$\int \frac{x^2}{1+x^6} dx =$$

$$\frac{1}{3}\int \frac{dx^3}{1+x^6} dx =$$

$\frac{1}{3} arctgx^3 + C$

$$\int_0^1 \frac{x^2}{1+x^6} dx = \frac{1}{3}(arctg1 - arctg0) = \frac{\pi}{12}$$

c)

$$\int \frac{x}{sin^2 x} dx =$$

$$-\int xdctgx =$$

$$-(xctgx - \int \frac{c}{t} gxdx) =$$

$$lnsinx - xctgx + C$$

$$\int_{\pi/4}^{\pi/3} \frac{x}{sin^2x}dx = ln\frac{\sqrt{3}}{2} - \frac{\pi\sqrt{3}}{4} - ln\frac{\sqrt{2}}{2} + \frac{\pi}{4} = ln\sqrt{\frac{3}{2}} + \frac{\pi}{4}(1-\sqrt{3})$$

d)

$$\int xatanxdx =$$

$$\frac{1}{2}\int atanxdx^2 =$$

$$\tfrac{1}{2}(x^2atanx - \int \frac{x^2}{x^2+1}dx) =$$

$$\tfrac{1}{2}(x^2atanx - \int 1 - \frac{1}{x^2+1}dx) = \frac{x^2atanx + atanx - x}{2} + C$$

$$\int_0^{\sqrt{3}} xatanxdx = \frac{4\frac{\pi}{6} - \sqrt{3}}{2}$$

e)

$$\int_{\frac{1}{3}}^{3} \frac{atanx}{x^2-x+1}dx =$$

$$\int_{\frac{1}{3}}^{1} \frac{atanx}{x^2-x+1}dx$$

$$+\int_1^3 \frac{atanx}{x^2-x+1}dx =$$

$$\int_1^3 \frac{atan\frac{1}{t}}{\frac{1}{t^2}-\frac{1}{t}+1}d\frac{1}{t}$$

$$+\int_1^3 \frac{atanx}{x^2-x+1}dx =$$

$$\int_1^3 \frac{atan\frac{1}{t}}{\frac{t^2-t+1}{t^2}}\frac{1}{t^2}dt$$

$$+\int_1^3 \frac{atanx}{x^2-x+1}dx =$$

$$\int_1^3 \frac{atan\frac{1}{t}}{t^2-t+1}dt$$

$$+\int_1^3 \frac{atanx}{x^2-x+1}dx =$$

$$\int_1^3 \frac{\frac{\pi}{2} - atant}{t^2 - t + 1} dt$$

$$+ \int_1^3 \frac{atanx}{x^2 - x + 1} dx =$$

переменим t на x

$$\int_1^3 \frac{\frac{\pi}{2}}{x^2 - x + 1} dx$$

$$- \int_1^3 \frac{atanx}{x^2 - x + 1} dx$$

$$+ \int_1^3 \frac{atanx}{x^2 - x + 1} dx =$$

$$\int_1^3 \frac{\frac{\pi}{2}}{x^2 - x + 1} dx =$$

$$\frac{\pi}{2} \int_1^3 \frac{1}{x^2 - x + 1} dx =$$

$$\frac{\pi}{2} \int_1^3 \frac{d(x - \frac{1}{2})}{\frac{3}{4} + (x - \frac{1}{2})^2} =$$

$$\frac{\pi}{2} (\frac{2\sqrt{3}}{3} arctg \frac{2(x - \frac{1}{2})}{\sqrt{3}})|_1^3 =$$

$$\frac{\sqrt{3}\pi}{3} (atan \frac{5\sqrt{3}}{3} - \frac{\pi}{6})$$

f)

$$\int \frac{dx}{4 + cos^2 x} = \frac{\sqrt{5} arctg(\frac{2\sqrt{5} tgx}{5})}{10} + C$$

$$\int_0^{2\pi} \frac{dx}{4 + cos^2 x} = 0 - 0 = 0$$

4

a)

$$\int_0^{1/2} 6x^2 - 6x + 1 - cos\pi x dx =$$

$$2x^3 - 3x^2 + x - \frac{sin\pi x}{\pi}|_0^{1/2} =$$

$$\frac{1}{4} - \frac{3}{4} + \frac{1}{2} - \frac{\sqrt{3}}{2} =$$

$-\frac{\sqrt{3}}{2\pi}$

b)

$$\int \frac{x^2}{2}dx - \int \frac{1}{1+x^2}dx = \frac{x^3}{6} - arctgx$$

c)

$\begin{cases} x^2 + y^2 = 4 \\ 2y = x^2 \end{cases}$

$\sqrt{4-x^2} = \frac{x^2}{2}$

$4 - x^2 = \frac{x^4}{4}$

$16 - 4x^2 = x^4$

$x^4 + 4x^2 - 16 = 0$

$D = 16 + 64 = 80$

$x^2 = \frac{-4 \pm 4\sqrt{5}}{2} = -2 \pm 2\sqrt{5}$

$a = -\sqrt{2\sqrt{5} - 2}$

$b = \sqrt{2\sqrt{5} - 2}$

$$S = \int_a^b \sqrt{4-x^2}dx - \int_a^b \frac{x^2}{2}dx =$$

$\frac{x\sqrt{4-x}}{2} + 2arcsinx/2 - \frac{x^3}{6}|_a^b$

d)

$$S = \int_0^2 \frac{5x}{2}dx - \int_0^1 x^2 + x - 1dx - \int_1^2 x^2dx =$$

$\frac{45}{4} + \frac{1}{6} - \frac{7}{3}$

e)

$$\int sinx\sqrt{cosx}dx =$$

$$-\int \sqrt{cosx}dcosx =$$

$$-\int \sqrt{t}dt =$$

$-\frac{3t\sqrt{t}}{2} = -\frac{3cosx\sqrt{cosx}}{2}$

$$S = 4 * \int_0^{\pi/2} sinx\sqrt{cosx}dx =$$

$$-6cosx\sqrt{cosx}|_0^{\pi/2} = 6$$