MICHEAL BEAR, HOMEWORK 1 , SEP, 4, 2025

☐ **Problem 1** [Logic and $\mathbb{D}$]. In L0, we encountered the finite set $\mathbb{D} = \{0,1,2,3,4,5,6,7,8,9\}$ of digits in base 10. Using d as symbol to denote an element of $\mathbb{D}$, consider the predicates

$$\begin{aligned} P(d) &= \text{“}|d-5| \leq 2\text{”} \\ Q(d) &= \text{“}|d-2| \leq 5\text{”} \end{aligned}$$

where $|x|$ is absolute value. *Careful: if x is a real number, x is not a set, so the notation “$|x|$” refers to “the absolute value of x” and cannot possibly mean “the cardinality of x”.* Determine the truth value of each of the following propositions.

- 1.1 [10 points] $P(3) \land (P(2) \lor P(1))$
- 1.2 [10 points] $\neg\big(P(3) \Rightarrow Q(3)\big)$
- 1.3 [10 points] $\exists d \in \mathbb{D}\ P(d)$
- 1.4 [10 points] $\forall d \in \mathbb{D}\ P(d)$
- 1.5 [10 points] $\forall d \in \mathbb{D}\ \big(P(d) \Rightarrow Q(d)\big)$.

solution

1)

$$\begin{gathered} P(3) \land (P(2) \lor P(1)) \\ (|3-5| \leq 2) \land \Big((|2-2| \leq 2) \lor (|1-5| \leq 2)\Big) \\ (2 \leq 2) \land \Big((0 \leq 2) \lor (4 \leq 2)\Big) \\ (True) \land (True \lor False) \\ True \land (True) \\ True \end{gathered}$$

2)

3)

4)

5)

☐ **Problem 2** [Logic and $\mathbb{N}$] In L0, we encountered the infinite set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, \ldots\}$. For each $k \in \mathbb{N}$, consider the truth set

$$A_k = \{n \in \mathbb{N} \,:\, k \leq n\}$$

To prove $a \in A_k$, you have to verify $k \leq a$. Prove each of the following propositions.

- 2.1 [10 points] $\exists \ell \in \mathbb{N}\ \ell \in A_7$
- 2.2 [10 points] $\exists \ell \in \mathbb{N}\ 7 \in A_\ell$
- 2.3 [10 points] $\neg\big(|\mathbb{N} \setminus A_{10}| = 9\big)$. *Careful: if S is a set, $|S|$ is the cardinality of S*
- 2.4 [10 points] $\forall m \in \mathbb{N}\ 2m + 1 \in A_m$
- 2.5 [10 points] $\forall m \in \mathbb{N}\ \big(m \in A_{112} \Rightarrow m \in A_{111}\big)$

solution

1)
2)
3)
4)
5)

$\square$ **Bonus** [X points] Is $\forall n \in \mathbb{Z}_+ \left(\exists q \in \mathbb{Q} \left((0 < q) \wedge (q < \frac{1}{n}) \right) \right)$ true or false? Explain.