MICHEAL BEAR, HW3, DATE

☐ **Problem 1** (Real Functions)**.** Define $D \subseteq \mathbb{R}$ by $D = \mathbb{R} \setminus \{-1\} = \{x \in \mathbb{R} \ : \ x \neq -1\}$. Consider the real function $f : D \to \mathbb{R}$ defined by the formula

$$f(x) = \frac{x-2}{x+1}.$$

1.1 [10 points] Prove that f is injective

1.2 [10 points] Prove that $1 \notin \text{range}\,(f)$.

1.3 [10 points] Prove that $\text{range}\,(f) = \mathbb{R} \setminus \{1\}$.

1.4 [10 points] Is $\forall x \in D\ \exists u \in \mathbb{R}\ \ f(x) \leq u$ true or false? Give a proof.

1.5 [10 points] Is $\exists u \in \mathbb{R}\ \ \forall x \in D\ f(x) \leq u$ true or false? Give a proof.

solution

1.1)

1.2)

1.3)

1.4)

1.5)

☐ **Problem 2.** [Compositions, Implications, and Counterexamples]
Let X, Y, and Z be three sets (possibly infinite)
and let $f\ :\ X \to Y$ and $g\ : Y \to Z$ be two functions
since codomain(F) $= Y =$ domain(g), $g \circ f\ : X \to Z$ is a well defined function
prove that each given implication below is false by providing an explicit counterexample

2.1 [10 points] if f is constant and g is bijective then $g \circ f$ is surjective

2.2 [10 points] if $|X| \leq |Z|$ then $g \circ f$ is injective

2.3 [10 points] if $g \circ f$ is bijective then $|X| = |Y|$

solution

2.1)

2.2)

2.3)

☐ **Problem 3.** [Compositions, Surjectivity, and Injectivity]
Let X,Y, and Z be three sets (possibly infinite)
and let $f : X \to Y$ and $g : Y \to Z$ be two functions.

3.1 [10 points] Prove that if f is surjective and g is surgective then $g \circ f$ is surjective.

3.2 [10 points] prove that if f is injective and g is injective then $g \circ f$ is injective

solution

3.1

3.2