# Finding the surface area and volume of different shape types of chocolate and how the surface area affects the melting time of the chocolates

Number of pages: 20

shows good structure and organization A+

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## 1. Introduction

### 1.0 Interest and aim for the project

A+

Since childhood, my sister and I, whenever our mom made the Sachertorte[1], we would be in the kitchen with her to help out, later, we learned how to make it on our own. One of the most important ingredients in the dessert is the chocolate glaze on top. In the process of making the glaze, we need to melt chocolate to prepare the Sachertore, the chocolate glaze is really the final touch and a very important ingredient in making the dessert. After many makings and experimenting, we found the perfect recipe, with the chocolate being the one variable left. We use either the normal chocolate bar (Example 1.0 a) or the fallen teardrop type of chocolate (Example 1.0 b), which makes the dessert just right. Nowadays I am making the dessert on my own, and I realised that one of the chocolate takes longer to melt. Learning some calculus enabled me to get the perspective of math, where the shape a and I got the urge to see how I can calculate the volume and surface of the chocolates, specifically of the flat raindrop, and see how these affect the melting time. With this research, I can see which chocolate shape takes less time to melt and by how much. That is why I decided to do my mathematics Internal Assessment and discover which chocolate I should use when making our dessert so the process would take less time.

Example 1.0 a

Example 1.0 b

Aim stated

[1] The classic Sacher Torte is made with chocolate cake layers, apricot preserves and a shiny chocolate-glaze finish. (Gray)

well-researched - C+

According to research, the surface area and the energy transfer of a body are directly proportional[2], hence with the process of heat transfer, the same happens. The object, i.e. the chocolate, with a larger surface area should melt first. The shape determines the object's surface area, in my case, the shape of the chocolate. Thus, the control variable will be the surface, while the volume, hence mass, will remain the same with both chocolates. In order to prove this I need to find the different surface areas for the same amount of chocolate.

Since I do not have a scale to measure the masses of the chocolate and the mass and volume are directly proportional, the amount of chocolate will be measured by finding the volume of the chocolates. The real-life application of integral calculus has enabled me to see how the volume of the peculiarly shaped chocolate can be accurately measured. To achieve this, I will be using calculus, precisely integral calculus and deriving the formulas for the volume and the area. Since the flat raindrop-like shape is a very particular shape and manipulating its volume would be more complicated, I decided first to calculate the volume of the flat raindrop chocolate and then do calculations and find the measurement of the exact same volume of the chocolate bar.

## 2. Background Information

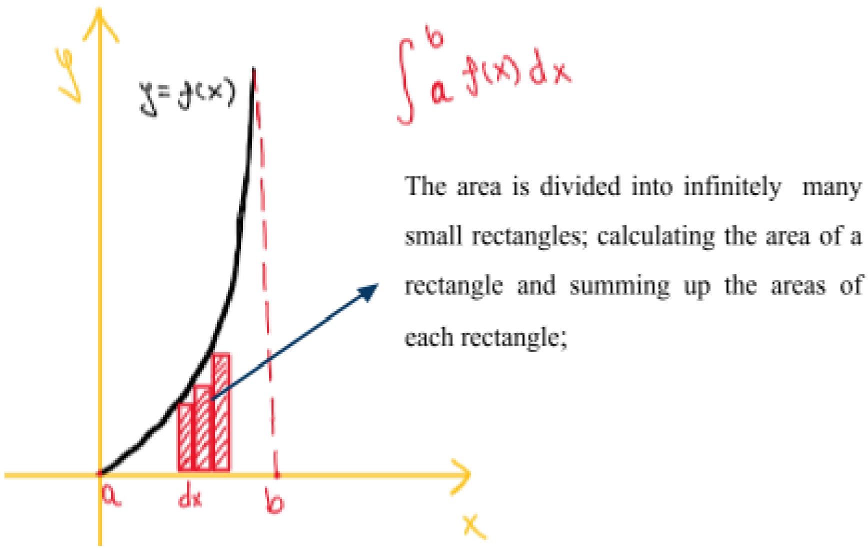
### 2.0 Definition and explanation of a definite integral

The definite integral is used to find the area under a curve of a function, with fixed limits assigned, hence its name - definite integral. The according limits represent the starting and ending points under which the area will be calculated. The process is done by representing the area under the curve as a sum of the area of the infinitely many rectangles at a specific given point. The small

[2] ("The Physics Classroom Tutorial")

rectangles are obtained as a derivative of the function, which takes an indefinitely small distance of a given point.

Using the drawn graph given below (Example 1.0), the explanation is visually presented. The area of a single rectangle is the product of the width (which represents the instantaneous rate of change at a particular point - a derivative of the function at the given point) - $dx$ and the height - $f(x)$, which is the corresponding output for the particular $dx$ (shown in example 2.0).



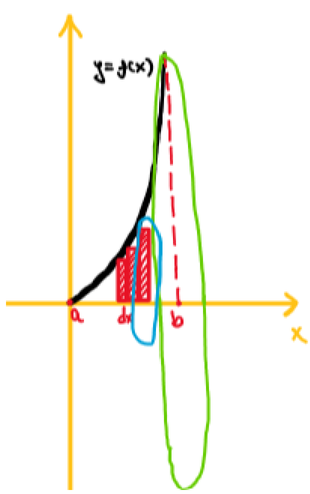

**Example 2.0**

## 2.1 Introduction to disk integration

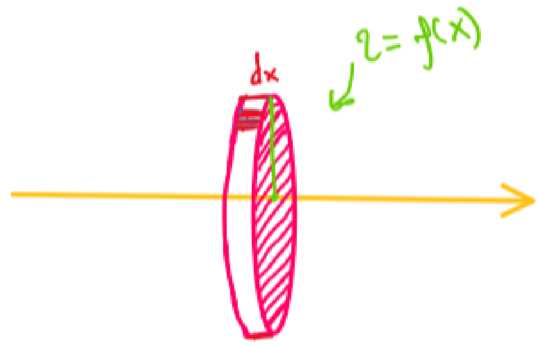
In order to fulfill the purpose of my project, I need to find a way in which I can calculate the volume of the object using the concepts of calculus explained above. For this reason, another concept of calculus needs to be introduced, and this is the disk integration method. For this concept to be explained, the definite integral needs to be analyzed again, where the explanation is given for how the area under the graph can be obtained.

Using the definite integral, the disk method is derived. The disk method is a slicing technique that creates cross-sections of a solid of revolution by slicing perpendicular to the axis of rotation and calculating the volume of the solid by adding the volumes of the infinitely many thin cross-sections (*Disk Method in Calculus: Formula & Examples - Video & Lesson Transcript*, 2021). To achieve this, imagination and visualisation are required, thus when rotation is applied to a general point on a function which follows a curve trajectory, the derivative $dx$ around the $x$- axis (Example 2.1a), the cross-section is obtained, which has a coin-like form(Example 2.1b). The width of the coin, i.e. cross-section, is given by the distance $dx$ and the radius of the cross section of the disk would just be $f(x)$ at the specific point and value for x.

mathematics from beyond the syllabus - C+
student explains ideas in their own words - E+




**Example 2.1a**




**Example 2.1b**

With this the area of the circle which has the form $A = \pi \cdot r^2$ , becomes:

Substitute r for the particular case: $r = f(x)$

$$A = \pi \cdot (f(x))^2$$

And the volume is the area multiplied by the width of the object, in this case, the width of the object is $dx$:

$$V_{rec.} = A \cdot dx \quad \Leftrightarrow$$

The volume is the area multiplied by the width of the disk. Now, the value for the area will be substituted:

$$V_{rec.} = \pi \cdot (f(x))^2 \cdot dx \text{ - volume for 1 rectangle}$$

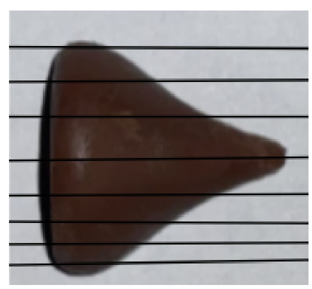
The above formula is the formula for the volume for one rectangle, in order to get the whole volume for all infinite rectangles, Integration must be used, hence if $a$ and $b$ are the limits of the function it follows:

$$V = \int_a^b \pi \cdot (f(x))^2 \cdot dx$$

**2.2 Implementation of disk integration with my problem**

linking statements to the aim of the exploration - A+

The reason for the introduction of the concepts above is because they align with the aim of this project. In order to calculate the volume of the tear shape like chocolate, the calculus of the disk method will be used. The shape of the chocolate allowed me to see that it can be divided into infinitely many "coins" or disks. This is using the disk method in the inverse direction, so first, we imagine the chocolate getting split into infinitely many disks (**Example 2.2**), then the idea is that the three-dimensional object is presented as a two-dimensional object, so the disk method can be applied. Finding the function which describes the curve is essential for the process of finding the volume of the function. I found 2 ways in which I can obtain the function,



**Example 2.2**

one way is to precisely draw the curve on sketch paper and then with the help of technology find the best-fit line.

### 2.3 Regression analysis

Initially the function that the curve of the chocolate is describing needs to be found. To obtain this I decided to manually sketch a quarter of the chocolate using pencil and sketch paper (Example 2.3a) since it is convenient and precise. I marked the chocolate in the middle and then, using a sharp knife, I split it into two equal parts. Taking into account that the disk method makes the disk rotate around an axis I choose to sketch the quarter of the chocolate with respect to the $x - axis$ (Example 2.3b). As if the disks would be rotating around $x - axis$. The curve is part of a function, in order to get the function described by the curve I had to take measurements of the points in the drawing. As shown below in Examples 2.3b and 2.3c I obtained those values. Very precisely I found them and noted them down, later I used GeoGebra's calculator where I input the numbers for both the x and y-axis.

Example 2.3a

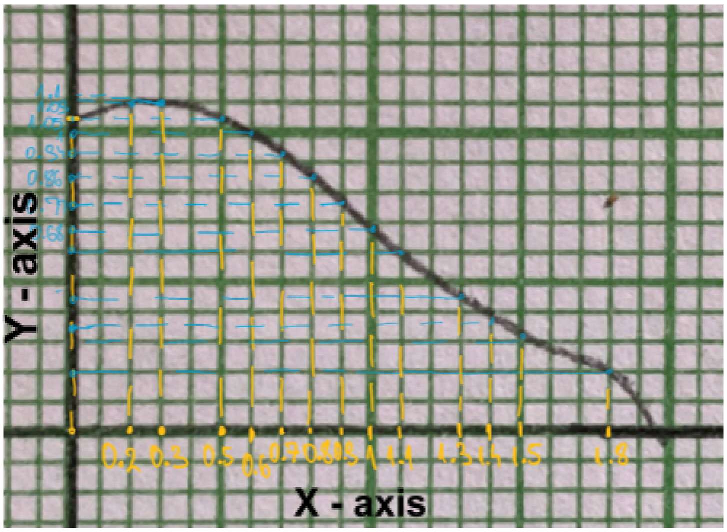
Using GeoGebra's calculator I used the regression analysis[3] tool to obtain the best-fit curve[4] and find the function which best describes the curve of the chocolate.

To represent the function of the curve, I used the model of the chocolate by dividing it in half

[3] Regression analysis is a set of statistical methods used for the estimation of relationships between a dependent variable and one or more independent variables. (*Regression Analysis - Formulas, Explanation, Examples and Definitions*, 2022)

[4] In regression analysis, curve fitting is the process of specifying the model that provides the best fit to the specific curves in your dataset. (Frost, n.d.)

and drawing the shape of its curve on graph paper and representing it on a cartesian plane (Example 2.3).



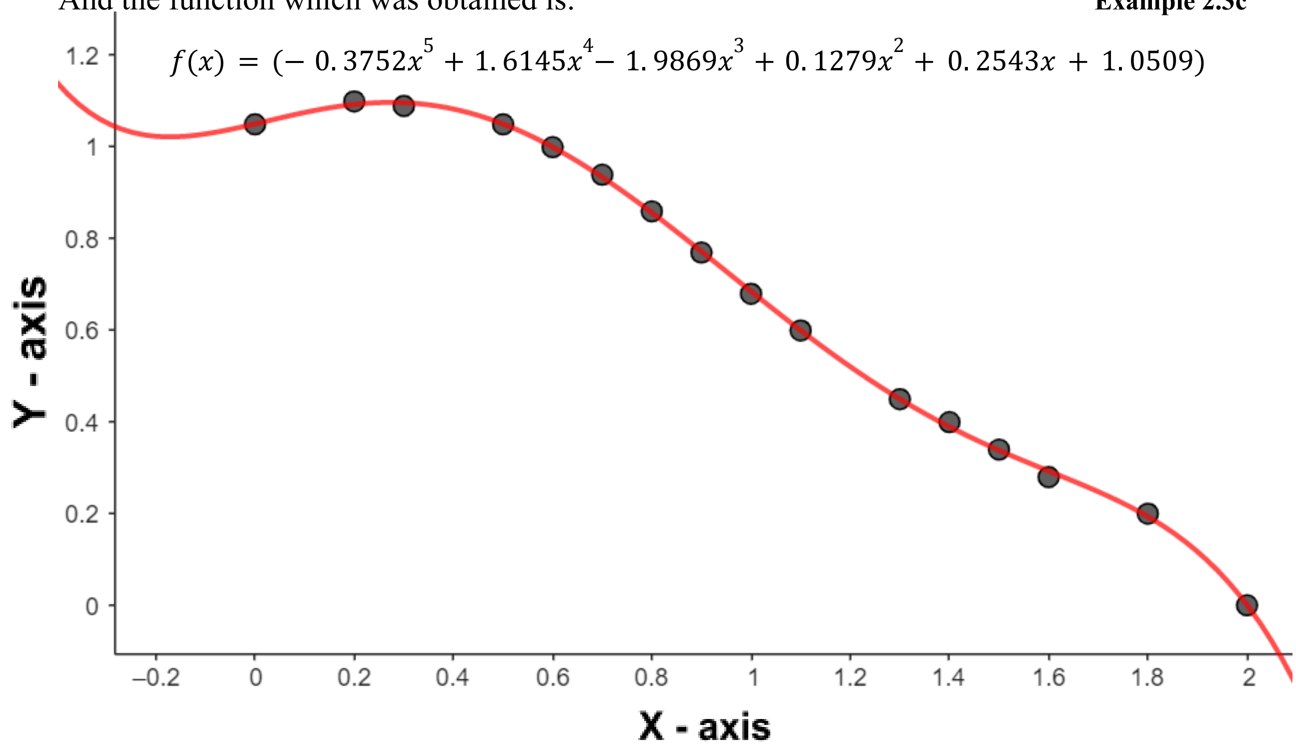

Example 2.3b

| | X | Y |
|---|---|---|
| 1 | 0 | 1.05 |
| 2 | 0.2 | 1.1 |
| 3 | 0.3 | 1.09 |
| 4 | 0.5 | 1.05 |
| 5 | 0.6 | 1 |
| 6 | 0.7 | 0.94 |
| 7 | 0.8 | 0.86 |
| 8 | 0.9 | 0.77 |
| 9 | 1 | 0.68 |
| 10 | 1.1 | 0.6 |
| 11 | 1.3 | 0.45 |
| 12 | 1.4 | 0.4 |
| 13 | 1.5 | 0.34 |
| 14 | 1.6 | 0.28 |
| 15 | 1.8 | 0.2 |
| 16 | 2 | 0 |

Example 2.3c

And the function which was obtained is:

$$f(x) = (-0.3752x^5 + 1.6145x^4 - 1.9869x^3 + 0.1279x^2 + 0.2543x + 1.0509)$$




Example 2.4

The calculator has the precision of up to ninth degree polynomials. I used a five degree polynomial to describe the curve of the graph because first of all: it provided a good fit for all the data points in the

engaging with the process; thinking independently; C+
showing good mathematical understanding; E+

analysis; secondly, comparing the two models of five degree polynomial and its the goodness of fit measure is similar with the one of the ninth degree polynomial they proved to be fairly similar, thus choosing the five degree polynomial makes the process simpler. Third aspect as to why the five degree polynomial was chosen is because it reduces the risk of overfitting which occurs when there is an overly complex model. While the ninth degree polynomial would give a more precise answer, I do not have the equipment required to measure with such precision for the volume of the other chocolate. Lastly I used a five degree polynomial instead of a polynomial of the lesser degree because it was the polynomial with the least degree which included all the data set points in the curve.

## 3. Volume and surface area of flat raindrop-shaped chocolate

### 3.0 Volume of the flat raindrop-shaped chocolate

Now, looking at section 2.1, the obtained formula for calculating the volume was the following:

$$V = \int_{a}^{b} \pi \cdot (f(x))^2 \cdot dx$$

Here the $f(x)$ will be substituted with the function obtained above, the limits $a$ $and$ $b$ were also part of the measuring, hence the starting limit of the function is 0 and the upper limit is 2 cm.

Rewriting the formula so it matches the new data:

$$(f(x))^2 := g(x)$$

$$V = \int_{0}^{2} \pi \cdot g(x) \cdot dx$$

Since the function is complex, the first step would be to expand the function and get the constant - coefficient π in front of the integral.

For expanding the function the use of Graphic Display Calculator (abbreviation: GDC) is efficient, hence the obtained value is:

$$g(x) = (0.14077504x^{10} - 1.2115208x^{9} - 4.09758001x^{8} + 6.51167626x^{7} + 4.16993399x^{6} - 0.47503432x^{5} + 2.39627107x^{4} - 4.10744006x^{3} + 0.33325849x^{2} + 0.53403x + 1.1025)$$

The integral then gets the form:

$$V_{r.drop} = \pi\int_{0}^{2} (0.14077504x^{10} - 1.2115208x^{9} - 4.09758001x^{8} + 6.51167626x^{7} + 4.16993399x^{6} - - 0.47503432x^{5} + 2.39627107x^{4} - 4.10744006x^{3} + 0.33325849x^{2} + 0.53403x + 1.1025)\, dx$$

After computing it on my GDC I got the following value for the volume:

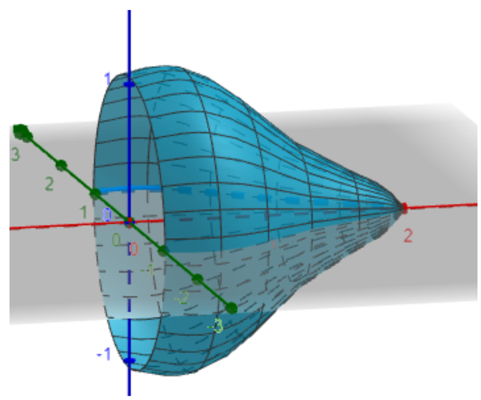
$$V_{r.drop} \approx 3.5158\, units^{3} \Leftrightarrow V_{r.drop} \approx 3.5158cm^{3}$$

student uses appropriate mathematical notation
B+

### 3.1 Surface area of the raindrop shaped chocolate

The next step is to find the surface area for the chocolate. Finding the area of the chocolate consists of multiple parts, firstly as it was with the case of the volume, to generate the surface of the chocolate there needs to be a revolution of the curve described by the chocolate around an axis. In the case of the raindrop shaped chocolate the curve is taken to be rotated around the x - axis. To show this I used the tool of Solid Revolution in GeoGebra which drafted the sketch (Example 3.1) after substituting the function of the curve of how the surface area looks.

good use of technology - A+



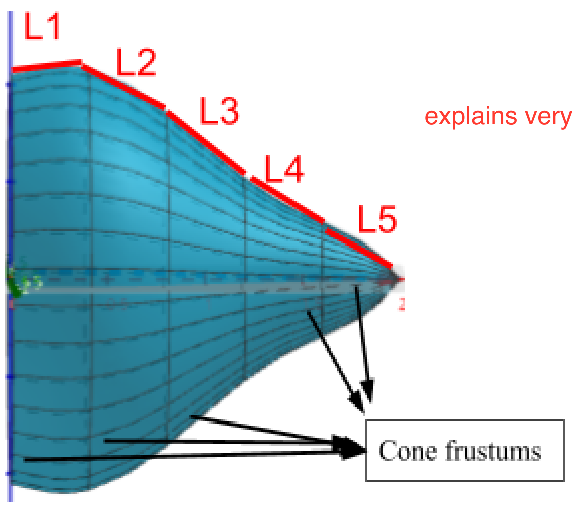
**Example 3.1**

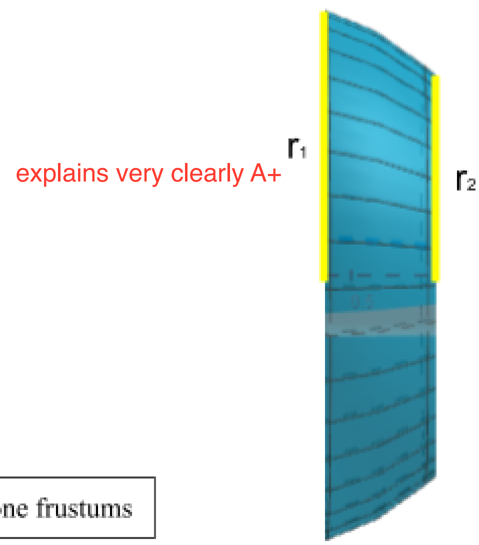
Then the surface of the solid obtained is divided in many parts which have the shape of a frustum of a cone[5], as seen in example 3.1b. Now to obtain the lateral surface area of a frustum the used formula is:

$$S = 2\pi r L$$

[5] The part of the cone when it is cut by a plane into two parts. The upper part of cone remains same in shape but the bottom part makes a frustum ("Frustum of A Cone (Volume, Surface Area & Problem) - Maths")

Where $r = r_1 + r_2$ and it represents the average radius of the average function values, that is why later it gets substituted with $f(x)$ (Example 3.1b), it also presents the height of the frustum; while $L$ presents the length of the frustum;



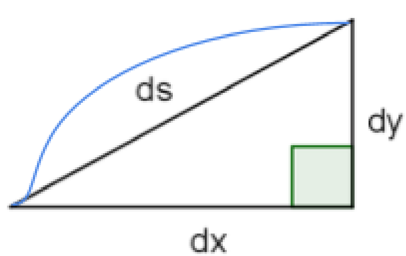




**Example 3.1b** **Example 3.1c**

To get the sum of the surface area of the chocolate, there needs to be a sum of all the different frustum surface areas. Hence, if the sum of all is taken, it is obtained:

$\sum_{i=1}^{n} \Delta S_i = \sum_{i=1}^{n} 2\pi r_i \Delta L_i = \sum_{i=1}^{n} 2\pi f(x)_i \Delta L_i$ where $\Delta S = 2\pi r \Delta L$ is the value of one single frustum;

The length of the frustum is the line $L$, and in this case presents the length of an arc line; in order to find the length of the arc line, a line connecting both of the ends of the frustum is drawn (Example 3.1d);




**Example 3.1d** ("Derivative of Arc Length")

then, taking into account that these frustums are representing infinitesimally small distances we take the values to be equal to the very small change in $x$ and very small change in $y$, which in the field of calculus is expressed as $dx$ and $dy$; now using the Pythagoras theorem the line can be written in the form as:

$$L_i^2 \approx dx^2 + dy^2 \quad \Leftrightarrow L_i \approx \sqrt{dx^2 + dy^2} \approx \sqrt{dx^2(1 + \frac{dy^2}{dx^2})}$$

now this $\frac{dy^2}{dx^2}$ can be rewritten as $\frac{dy^2}{dx^2} = f'(x)$ which is showing the derivative of the function.

The part of the derivative of the function used here is the tangent line at the specific point of where the average radius $r$ is on the curve line; then the sum of all of the arc length can be presented as:

mathematics is commensurate and relevant E+

$$L = \int_a^b \sqrt{1 + f'(x)}dx$$

Now if we substitute the value for the arc lengths and use the integral symbol instead of the sigma symbol in the upper surface area sum equation, the following formula for the Surface Area[6] is obtained:

$$A_{r.drop} = \int_0^2 2\pi[f(x)\sqrt{1 + (f'(x))^2}]dx = 2\pi\int_0^2 [f(x)\sqrt{1 + (f'(x))^2}]dx$$

$$f'(x) = -1.876x^4 + 6.458x^3 - 5.9607x^2 + 0.2558x + 0.2543$$

Substituting the values for $f(x)$ and $f'(x)$ the form changes to:

$$A_{r.drop} = 2\pi(\int_0^2 [(-0.3752x^5 + 1.6145x^4 - 1.9869x^3 + 0.1279x^2 + 0.2543x + 1.0509)$$

$$\sqrt{1 + (-1.876x^4 + 6.458x^3 - 5.9607x^2 + 0.2558x + 0.2543)}\,]dx$$

Now with the use of GDC the value obtained is:

[6](“6.4: Areas of Surfaces of Revolution - Mathematics LibreTexts”) (“6.4 Arc Length of a Curve and Surface Area | Calculus Volume 1”)

$$A_{r.drop} \approx 9.58043\ units^2 \approx 9.58 units^2$$

Since all the measurements were done in centimetres the units here are centimetres, hence:

$$A_{r.drop} \approx 9.58043\ cm^2 \approx 9.58 cm^2$$

## 4. Volume and surface area of a chocolate bar

### 4.0 Volume of chocolate bar

In order to fulfil the needs of the research the volume of the chocolate bar must be the same as the flat rain-shaped chocolate, hence the volume of the chocolate must be $3.5158 cm^3$ . Since I do not

considering the limitations carefully D+

have the required equipment to measure with such precision I will take the volume of the chocolate bar to be $3.5 cm^3$ rounding it to two significant figures. I am doing this because the ruler I am using gives precision up to one decimal place, hence it is not possible to measure more than this. The volume of the chocolate bar can be obtained with the formula for the volume of:

$$V_{c.bar} = w \cdot l \cdot h \quad where\ w\ is\ width,\ l\ is\ length\ and\ h\ is\ height$$

I can manipulate the length of the chocolate bar in this equation, while the height and the width of the chocolate remain unchanged. The chocolate bar I use for cooking is 0.6 centimetres in height (Example 4.0a) and 2.0 centimetres in width(Example 4.0b), this leaves the length to be:

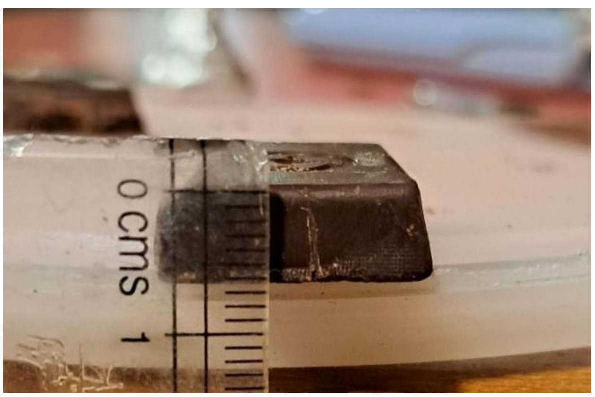
$V_{c.bar} = V_{r.drop} = 3.5\ cm^3$ where $V_{c.bar} = w \cdot l \cdot h$ (where the height and width are known)

$h = 0.5cm \Rightarrow 3.5cm^3 = 2.0 \cdot l \cdot 0.6cm \Leftrightarrow l = \frac{3.5cm^3}{1.2cm^2} = 2.91\underline{6}cm \approx 2.9cm$ (rounding to 2 sig. figs)

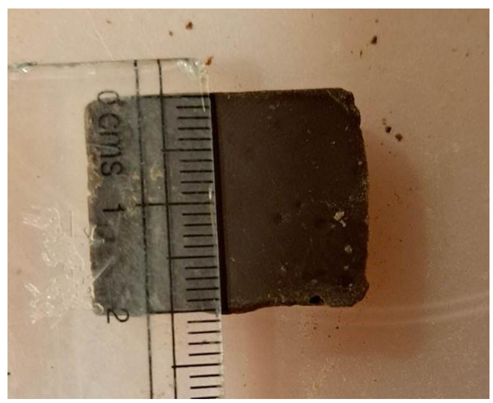
The chocolate bar is naturally divided in the unconventionally so-called 'cubes' which are, right rectangular prisms also known by the name cuboid; where the length of one is 2.6 centimetres (Example 4.0c). Since the length needs to be 2.9 centimetres I will need to add

$l_{extra} = (2.9 - 2.6)cm = 0.3cm$ extra in lenght from another chocolate cuboid;

**Example 4.0 a**

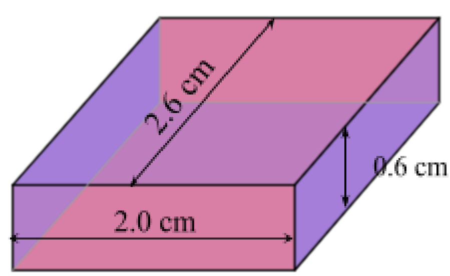



**Example 4.0b**




**Example 4.0c**


The following drawing imitates the shape of the chocolate 'cube' presenting the measurements of the chocolate cuboid. The model of the chocolate cuboid is presented in Example 4.0d;




demonstrates good understanding and knowledge of mathematics E+


**Example 4.0d**

Now we finally get the volume of the chocolate bar to be $3.5cm^3$ , the same as the volume of the fallen raindrop.

### 4.1 Area of the chocolate bar

The formula which can be used to calculate the area is the formula for the area of a cuboid which is:

$$A = 2(hl + wl + hw)$$

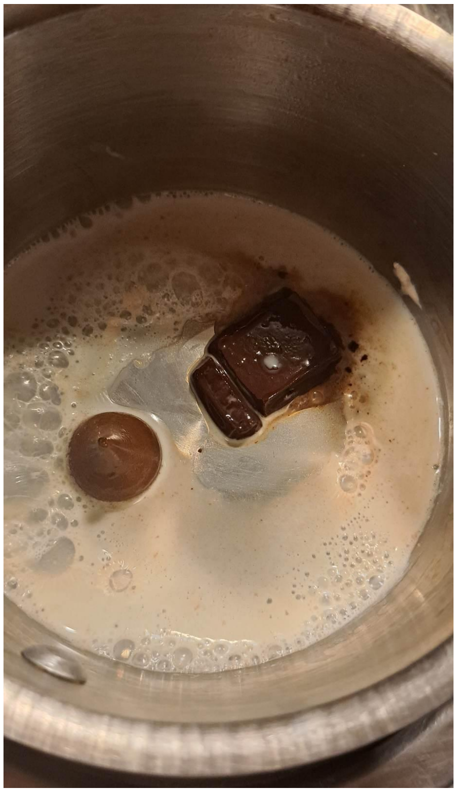
The value for the height is the sum of the height of the 1 cubicle and the 0.2 part of another cubicle is:

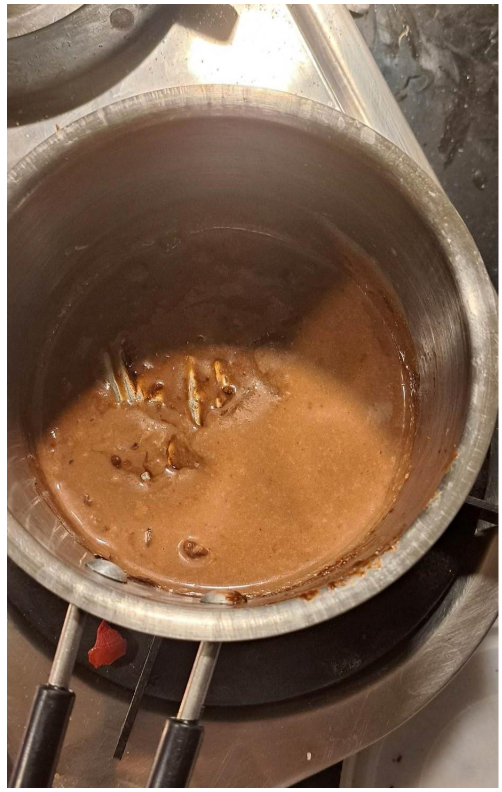
$$A_{c.bar} = 2 \cdot (0.6 \cdot 2.9 + 2.0 \cdot 2.9 + 0.6 \cdot 2.0)cm^2 \Leftrightarrow A = 17.48cm^2 \approx 17.5cm^2$$

## 5. Collecting data and making conclusions

extending beyond just finding volume and surface area shows significant personal engagement; also, shows creative thinking by using the volume of the teardrop chocolate to find the length of the chocolate bar C+

Now both the chocolates need to be melted, I put the two equal in volume and mass, chocolates in the same pan at the same temperature (Example 5.0a), then I waited to see which one would melt faster (Example 5.0b). The measures I got are presented below (Example 5.0c)





**Example 5.0a**

**Example 5.0b**

| **Type of chocolate** | **Surface area of chocolate - A** | **Timing - t** |
|---|---|---|
| Bar of chocolate | $17.48cm^2$ | 1 min; 22 seconds |
| Hershey's fallen raindrop chocolate | $9.580cm^2$ | 3min; 6 seconds *(Example 5.0b)* |

**Example 5.0c**

The ratio obtained between the time and surface area of the chocolate is the following:

$$\frac{A_{c.bar}}{t_{c.bar}} = \frac{17.48cm^2}{1.36min} \approx \frac{17.48cm^2}{1.37min} \approx 12.7591\frac{cm^2}{min} \approx 12.8\frac{cm^2}{min}$$

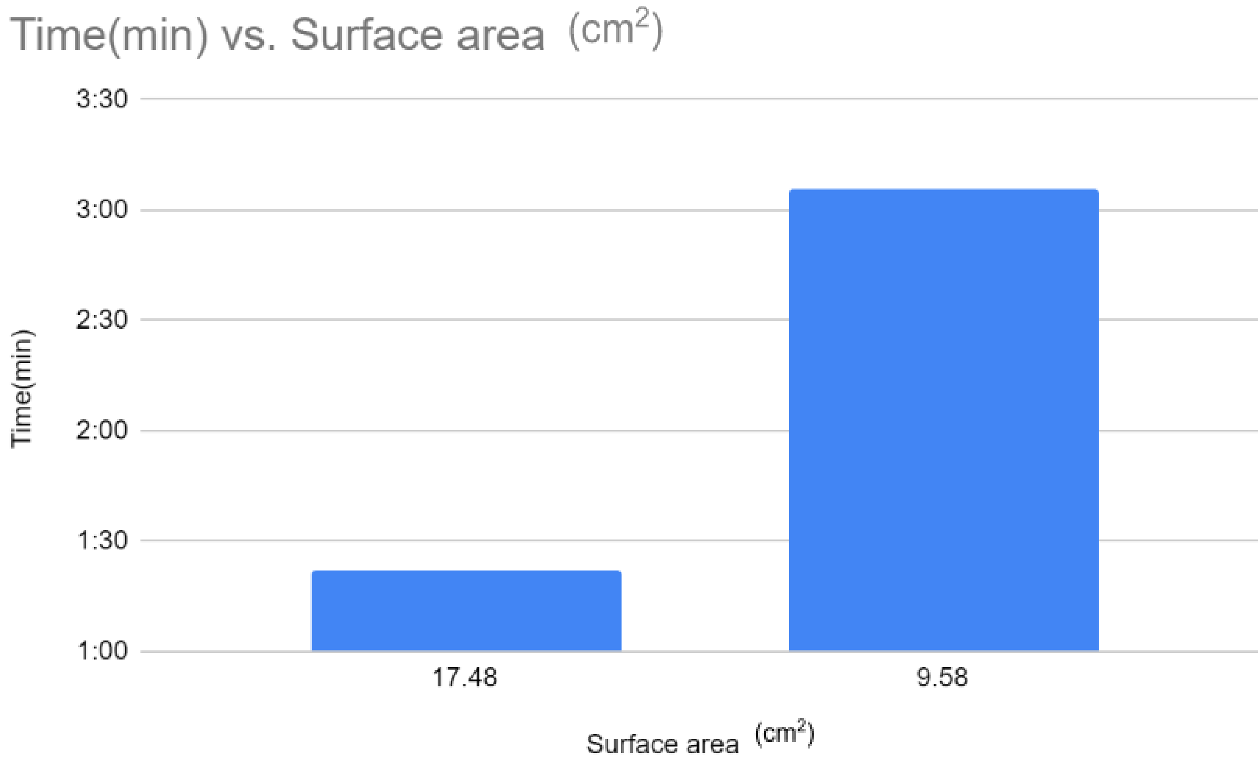
Displaying the speed of the melting process of the chocolate bar;

$$\frac{A_{r.drop}}{t_{r.drop}} = \frac{9.580cm^2}{3.1min} \approx 0.451613\frac{cm^2}{min} \approx 0.45\frac{cm^2}{min}$$

Displaying the speed of the melting process of the raindrop shaped chocolate;

Seeing the obtained result from the ratio, a significant difference can be noticed between the speed of the melting process between both the chocolates. This suggests why It was easily noticeable the difference in time when cooking the chocolates.  Using the data from the upper table about the surface area and the variable of time the following bar chart can be plotted:

using multiple forms pf representation - B+




**Example 5.0b**

This graph is just another way to present the data from the table, from the graph the following features can be noticed: the time and surface area are inversely proportional, less time more surface area, more surface area less time.

## 6. Conclusion

Doing this IA has achieved the purpose of measuring the surface areas and volume of both differently shaped chocolates. Later this information was used to see how the different surface area of both the different chocolates affects the melting time of the chocolates. In the case of the chocolates, the one with larger surface area melted faster. This lines up with the expectations that the surface area and the time required for the chocolate to be melted are inversely proportional. Overall, the estimation, in the beginning, was correct for the case of the chocolates, as it shows the desired relation between the surface area and the time. The things which I had taken into consideration are presented in the research above and in the next part I will do an evaluation of what could have been done differently and how that might have affected the results.

### 6.1 Evaluation

meaningful reflection - considers limitations and scope of the exploration D+

Firstly, the composition of the ingredients in the chocolate was not exactly the same, hence this is something that I did not take into account. Some ingredients are prone to make the melting faster[7] than others, this however is not in the scope of my knowledge, and the research is not focused on this part but more on the mathematical part.

Other things which might have influenced the outcome is whether the heat from the stove was equally distributed all over the pan, this can be made better if the stove was electrical instead of a gas stove, since I did not have the resources for the electrical stove, so I did it on a gas stove. While the melting time might have been different, it would not make a huge impact or change which might melt first, as the heat in my experiment was mostly equally, distributed and the difference was easily seen.

Another important point is how I determined the amount of chocolate. It would have been better if I used a scale and measured the mass of each of the chocolates instead of finding the amount of chocolate by finding the volume. Yet again, I could not find a scale and decided to continue the

[7] (Farrell)

task using mathematical skills, which would improve my skills and show my work in my Internal Assessment. While there are benefits to this method of obtaining the volume, the precision would have been better if a scale was used instead.
amount of chocolate by finding the volume. Yet again, I could not find a scale and decided to continue the task by using mathematical skills, which would benefit the improvement of my skills and show more content in my Internal Assessment. While there are benefits to this method of obtaining the volume, the precision would have been better if a scale was used instead.

As this investigation does not require excessive precision, the result is a good answer with some approximation in its responses. The conclusion is also accurate to a large extent; the only concern is the rate at which the answer can be considered exactly correct. This investigation can not answer the dependency of time rate on the surface area, and generalisations should not and can not be included in this scenario.

The conclusions brought in this investigation are strictly connected with the specific cases and examples included in the investigation.

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