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In [1]: import sympy as sp
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Homework #3

Problem 1

Theorem 5.2 in TB states that the range of A is spanned by the left singular vectors.

Expand on the proof given in the text book by showing that $\mathbf{x} \in \text{Col}(A)$ if and only if we can write \mathbf{x} as a linear combination of left singular vectors.

Solution

Problem 2 (TB 5.3)

Consider the matrix

$$A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$$

- (a) Compute the real SVD (either by hand, or using Sympy).
- (b) List the singular values, left singular vectors, and right singular vectors of A . Draw a picture of unit disk in \mathbb{R}^2 and its image under A , together with their singular vectors. Mark the coordinates of the vertices of the singular vectors.
- (c) What are the 1-norm, 2-norm, ∞ -norm and Frobenius norms of A ? Verify Theorem 5.3 in TB.
- (d) Find A^{-1} using the SVD.
- (e) Find the eigenvalues λ_1 and λ_2 of A .
- (f) Verify that $\det A = \lambda_1 \lambda_2$ and $|\det A| = \sigma_1 \sigma_2$ (Theorem 5.6 (TB)).
- (g) What is area of the ellipsoid onto which A maps the unit disk in \mathbb{R}^2 ?
- (h) Express A as the sum of two outer product matrices. See Theorem 5.7 (TB).

Solution

In []: