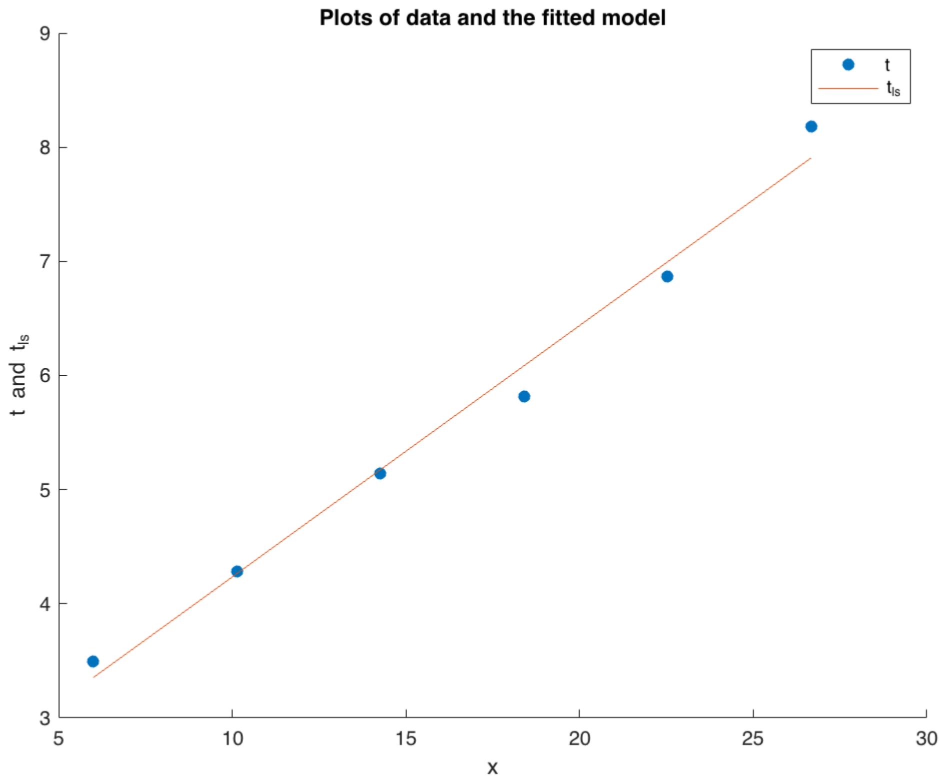
```
%rng(0)
t=[3.4935;4.2853;5.1374;5.8181;6.8632;8.1841];
x=[6;10.1333;14.2667;18.4000;22.5333;26.6667];
```

1.

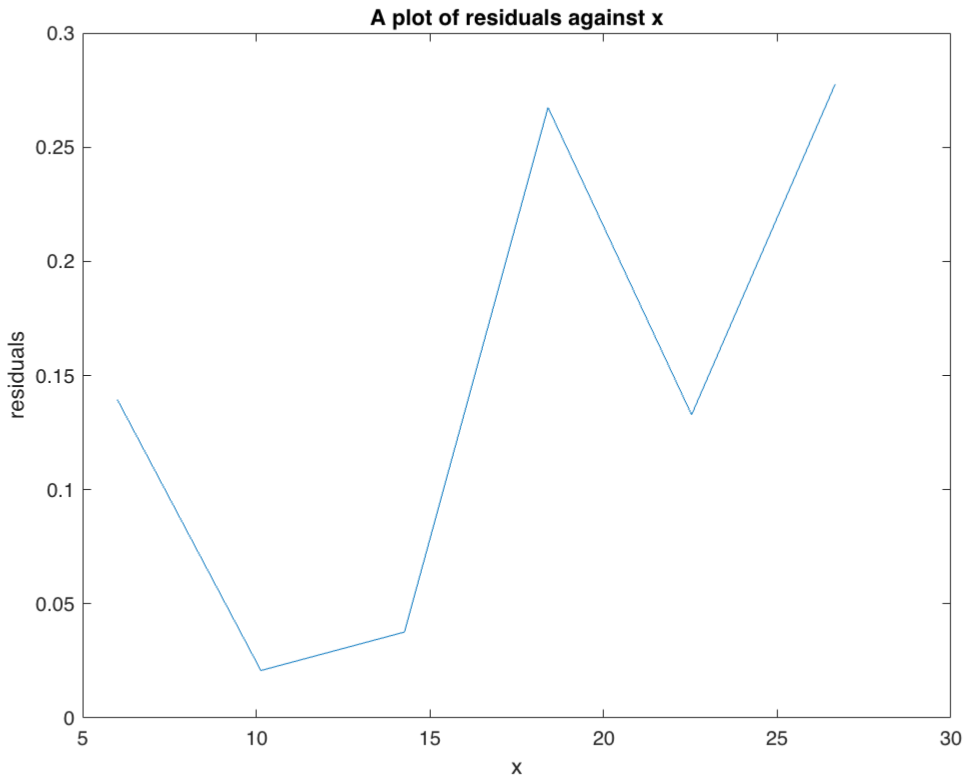
```
m = length(t);
n = 2;
G = [ ones(m,1) , x ];
%The least squares solution for the model parameters t_0 and s_2
m_ls = (G.'*G)\(G.'*t)
```

```
m_ls = 2×1
    2.0323
    0.2203
```

```
% Fitted model
t_ls = G*m_ls;


scatter(x,t,'filled')
title('')
xlabel('x')
ylabel('t and t_{ls}')

 hold on
plot(x,t_ls)
hold off
legend('t','t_{ls}')
title('Plots of data and the fitted model')
```




```
error = abs(t-t_ls);
plot(x,error)
title('A plot of residuals against x')
xlabel('x')
ylabel('residuals')
```



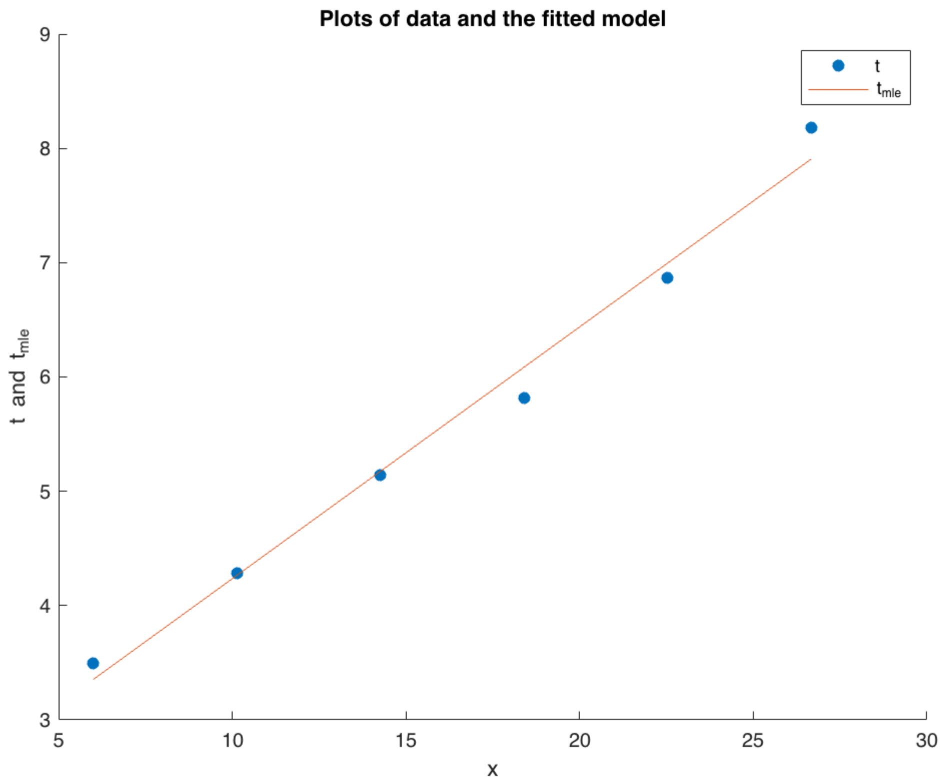
2.

```
sigma = 0.1;
I = eye(m);
W = sigma\I;
G_w = W*G;
t_w = W*t;

%The maximum likelihood estimates for the model parameters t_0 and s_2
%m_mle1 = (G_w.'*G_w)\(G_w.'*t_w)
m_mle = (G.'*W^2*G)\ G.'*W^2*t
```

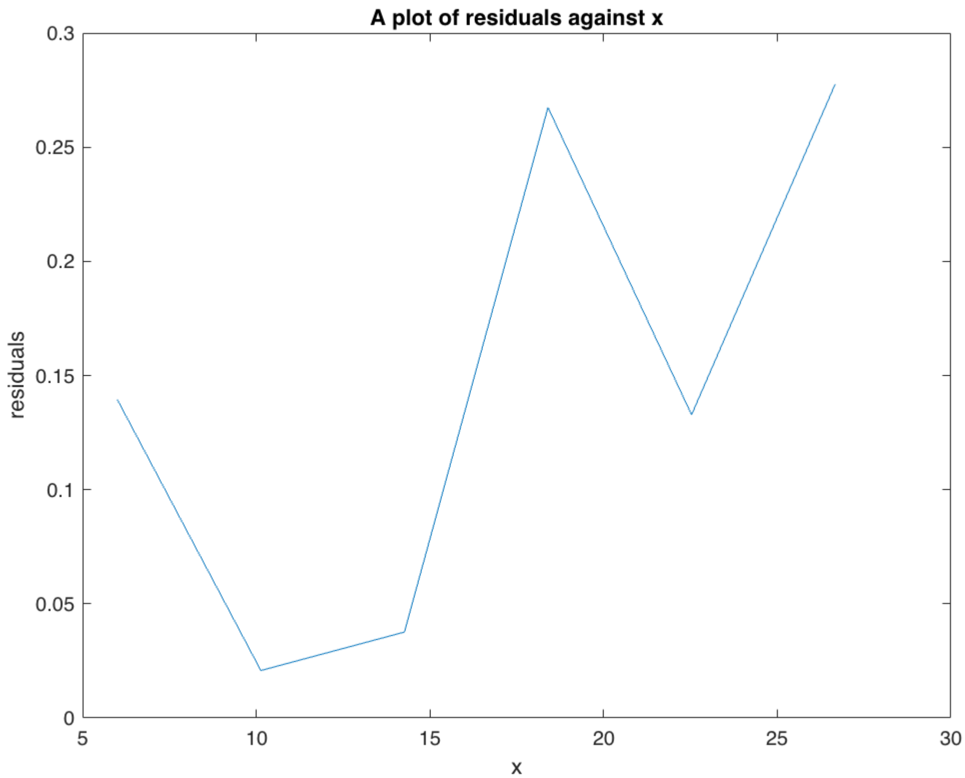
```
m_mle = 2×1
    2.0323
    0.2203
```

```
t_mle = G*m_mle;

scatter(x,t,'filled')
title('')
xlabel('x')
ylabel('t and t_{mle}')

 hold on
plot(x,t_mle)
hold off
legend('t','t_{mle}')
```

```
title('Plots of data and the fitted model')
```




```
plot(x,abs(t-t_mle))
title('A plot of residuals against x')
xlabel('x')
ylabel('residuals')
```



The graph in question (2) showing the travel time data (t) and travel time estimates corresponding to the weighted parameter estimates (t_{mle}) is exactly the same as that in question (1) showing the travel time data (t) and travel time estimates corresponding to the least squares parameter estimates (t_{ls}). This is so because the maximum likelihood estimates for the model parameters t_0 and s_2 in (2) above are exactly the same as the least squares estimates for the model parameters in (1) above. Getting the same estimates for the two cases came about because we are using a constant standard deviation that is ($\sigma = 0.1$) in calculating the maximum likelihood estimates and yet the idea of using a diagonal weight matrix whose inputs are the inverse standard deviations is the only difference between the two methods. This can be shown as below;

For the least squares method, the parameter estimates were obtained as in the equation;

$$m_{\text{ls}} = (G^T G)^{-1} G^T t \qquad (1)$$

While as for the maximum likelihood estimates were obtained obtained as in the equation;

$$m_{\text{mle}} = (G^T W^2 G)^{-1} G^T W^2 t \qquad (2)$$

But we know that $W = \sigma^{-1} I$ where I is an identity matrix. Substituting for W in equation (2) above we get;

$$m_{\text{mle}} = (G^T \sigma^{-2} I^2 G)^{-1} (G^T \sigma^{-2} I^2 t) \qquad (3)$$

Since $I^2 = I$, $G^T I G = G^T G$ and σ is a constant, equation (3) becomes;

$$m_{\text{mle}} = \sigma^2 (G^T\,I\,G)^{-1} * \sigma^{-2}\,(G^T\,I\,t)$$
$$= \ (G^T G)^{-1} G^T t \qquad (4)$$

Therefore since equations (1) and (4) are the same, then the maximum likelihood parameter estimates must be the same as the least squares parameter estimates. Hence giving the same graphs of the data and fitted models, and the residuals in question (1) and (2) above.

3.

```
% The Chisquare statistic with the weighted least squares parameter estimate found in
chisqure_mle = (t_w- G_w*m_mle).'*(t_w- G_w*m_mle)
```

```
chisqure_mle = 18.7502
```

```
% The Chisquare statistic with the least squares parameter estimate found in (1) above
chisquare_ls = norm((t - G*m_ls)./sigma)^2
```

```
chisquare_ls = 18.7502
```

Since the the maximum likelihood parameter estimates are the same as the the least squares parameter estimates, then the χ^2 statistics for the data is the same for both cases as shown above and it is given as $\chi^2_{\text{obs}} = 18.7502$.

The expected value of the random variable with appropriate degrees of freedom (m-n) is given by;

$E[\chi^2(m-n)] = E[\chi^2(4)] = 4$

And since $\chi^2_{\text{obs}} > \ E[\chi^2(4)]$, then χ^2_{obs} is not near $\chi^2(4)$.

4.

```
% Because there are 2 parameters to estimate, we have m-2 = 4 degrees of freedom.
dof = m-n;
% To find the p-value for this data set
p = 1-chi2cdf(chisqure_mle,dof)
```

```
p = 8.7992e-04
```

To verify if $\chi^2_{\text{obs}} \sim \ \chi^2(4)$, we do this using the $p-\text{value}$ and we consider the null hypothesis that:

Null hypothesis: $\chi^2_{\text{obs}} \sim \ \chi^2(4)$

Statistical test: If $p \approx 0$ or $p \approx 1$, then we reject the null hypothesis.

If $0 < p < 1$, we fail to reject the null hypothesis.

Basing on our $p-\text{value}$ above which is $p = 8.7992e-4$, and since $p \approx 0$, wetherefore reject that null hypothesis that $\chi^2_{\text{obs}} \sim \ \chi^2(4)$ and this conclusion is in line with that obtained in question (3) above.