Tikhonov regularization, individual activity

1.

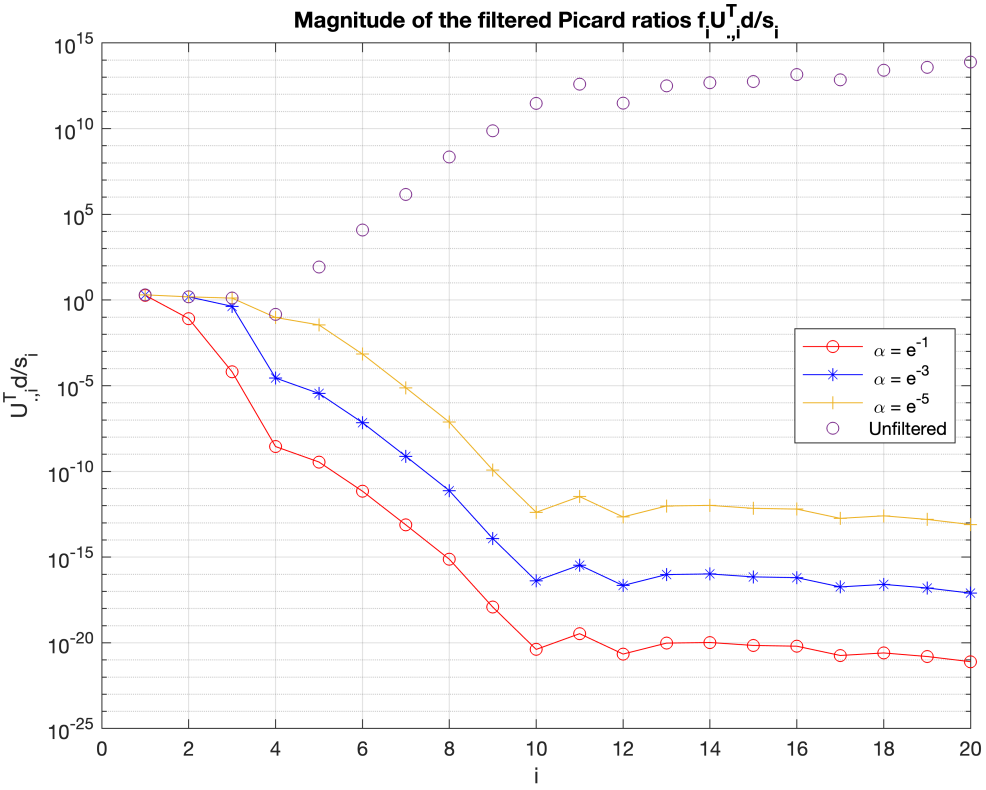
The χ^2_{obs} for the TSVD model parameter estimates with repect to i=4, is 0.9433, for the TSVD model parameter estimates with repect to i-1=3, is 0.9437 and for the true model parameter values, is 22.4482.

The expected value of the χ^2 statistics with degrees of freedom (m-n = 0) is given by; $E\left[\chi^2_{m-n}\right] = E\left[\chi^2_0\right] \approx 0$

The χ^2_{obs} for the true model parameter values is far greater than that of the two TSVD model parameter estimates. The χ^2_{obs} for the TSVD model parameter estimates with repect to i=4 is similar to that for the TSVD model parameter estimates with repect to i-1=3 with an absolute difference of 4.0812×10^{-4}.

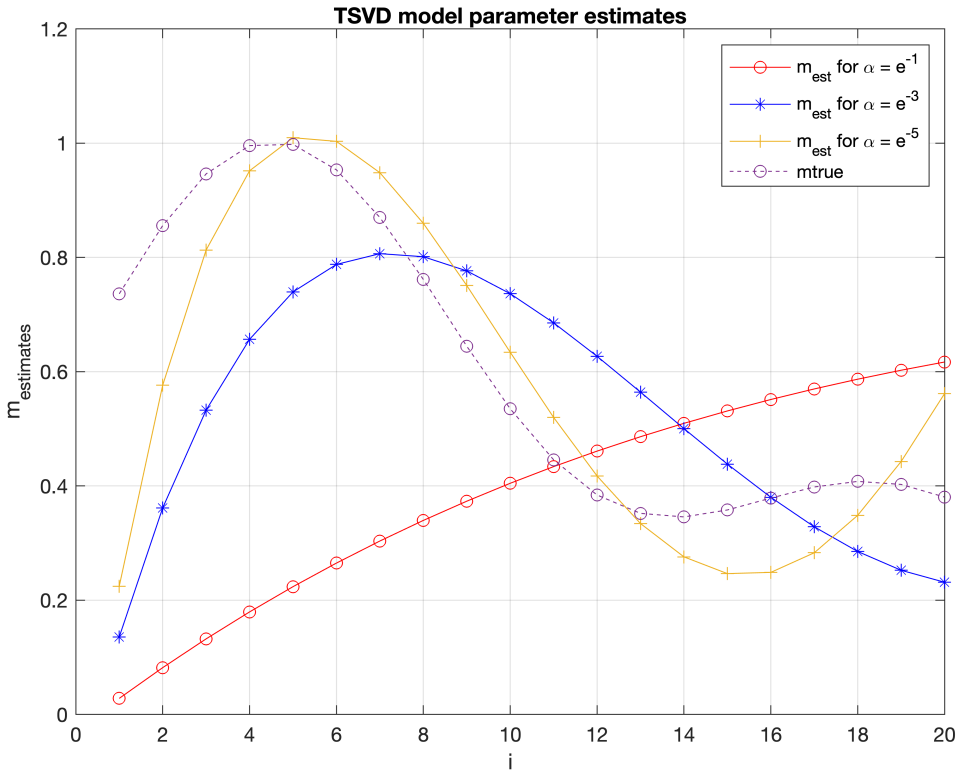
All the χ^2_{obs} are higher than the expected value although that of the two TSVD model parameter estimates is near the expected value. This means that the true model parameter values and the TSVD model parameter estimates ado not follow the χ^2 distribution.

2.



Both the filltered and unfiltered Picard ratios are the same for i=1. There after the unfiltered Picard ratios slightly decrease for i=4 and then abruptly increase while as the filltered Picard ratios decrease in a manner similar to the increase of the unfiltered Picard ratios and the bigger the alpha the more the decrease in the ratios.

3(a).



From above, we see that the plot for the estimates with respect to $\alpha = 10^{-5}$ has a similar shape to that of the true values although the estimates are out of phase with the true parameters except at i=5 and 13 where the values are almost the same. For bigger alpha that is $\alpha = 10^{-3},\ 10^{-1}$, the behaviour of the plots changes and deviate from that of the true parameters and the bigger the alpha the more the deviation. This behaviour is potrayed by the Picard ratios where the ratios decrease as alpha increases. All this means that the smaller alpha is the more accurate the parameter estimates are.

3(b).

The maginitudes of the residuals with respect to $\alpha = 10^{-1}10^{-3},\ 10^{-5}$ are $0.00324, 3.8418 \times 10^{-7}, 9.4333 \times 10^{-9}$ meaning that the residuals decrease as alpha decreases and we notice

that the maginitude of the residuals with respect to $\alpha = 10^{-5}$ is similar to that of the TSVD model parameter estimates with repect to i=4.