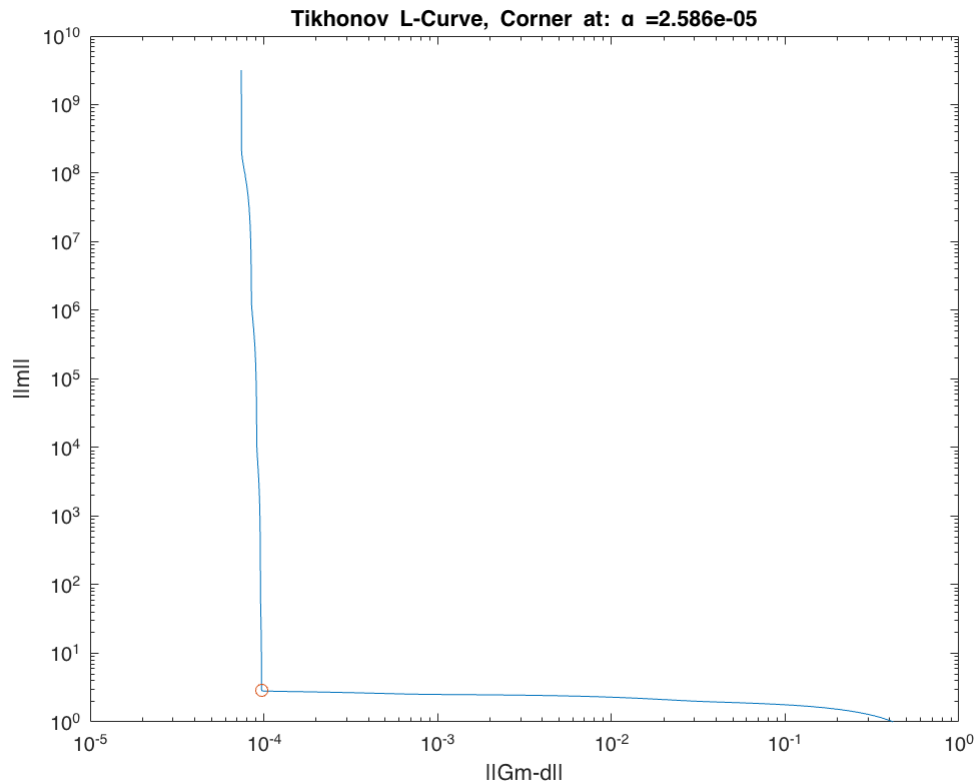


Ch4: Tikhonov regularization implementation, individual activity

1.



$$\alpha = 2.5860 \times 10^{-5}$$

2(a).

My choice is based on the residual norm corresponding to the chosen α . So based on this we used the heuristic approach ($\sigma \sqrt{m}$) to give us the value of $\delta = 4.47 \times 10^{-4}$.

2(b)

$\|Gm_\alpha - d\|_2^2 = 9.437 \times 10^{-9}$ and since $\|Gm_\alpha - d\|_2^2 < \delta$, then the choice of $\alpha = 2.5860 \times 10^{-5}$ satisfies the discrepancy chosen 2(a) above.

$$\text{ans} = 9.4370e-09$$

2(c)

This will require creating a nonlinear function $f(\alpha) = \|Gm_\alpha - d\|_2^2 - \delta$ and finding α so that $f(\alpha) = 0$ and it gives $\alpha = 2.5860 \times 10^{-5}$ the same as that obtained when using the L-curve.

Equation solved at initial point.

fsolve completed because the vector of function values at the initial point is near zero as measured by the value of the function tolerance, and the problem appears regular as measured by the gradient.

<stopping criteria details>

3. (a)

$\delta_{\text{reg}} = 20$ and this is based on the fact that $\|Gm_\alpha - d\|_2^2 / \sigma^2 + \alpha^2 \|m_\alpha\|_2^2$ is equivalent to the chi-square distribution with m degrees of freedom, hence δ_{reg} will be equal to the expected value of the chi-square distribution .

(b)

The choices of α satisfy the regularized discrepancy we chose in 3a, since the values of the regularized discrepancy obtained in 3b are less than the regularized discrepancy we chose in 3a.

Regularized_discrepancy1 = 0.9437

(c)

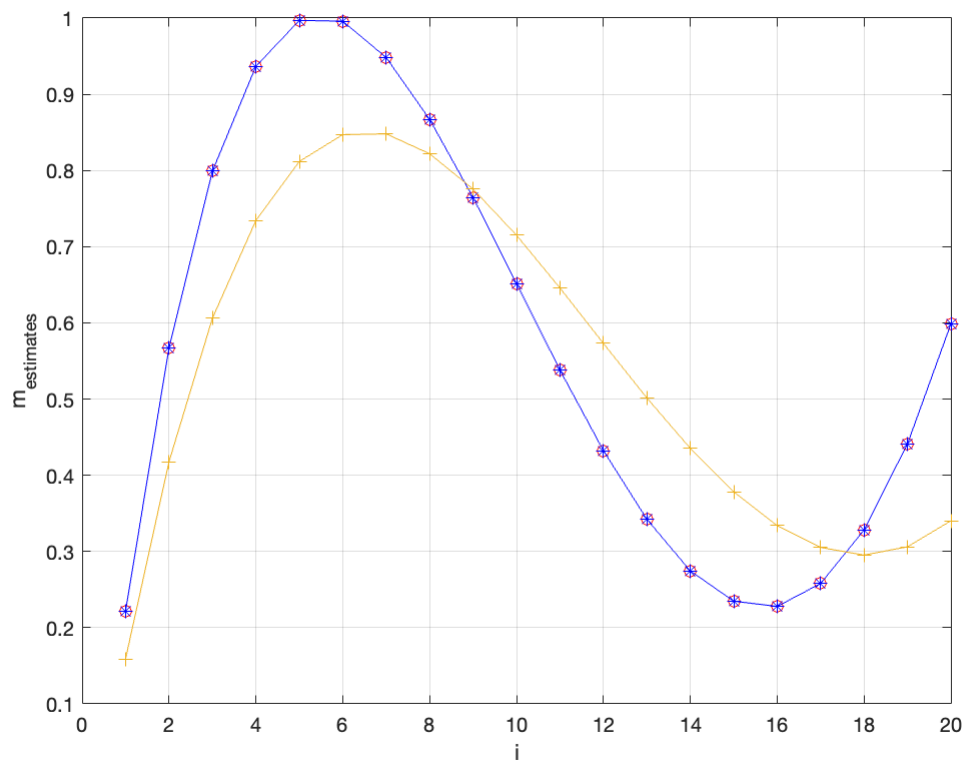
This will require creating a nonlinear function $f_{\text{reg}}(\alpha) = \|Gm_\alpha - d\|_2^2 / \sigma^2 + \alpha^2 \|m_\alpha\|_2^2 - \delta_{\text{reg}}$ and finding α so that $f_{\text{reg}}(\alpha) = 0$ which gives $\alpha = 6.7486 \times 10^{-4}$.

Equation solved.

fsolve completed because the vector of function values is near zero as measured by the value of the function tolerance, and the problem appears regular as measured by the gradient.

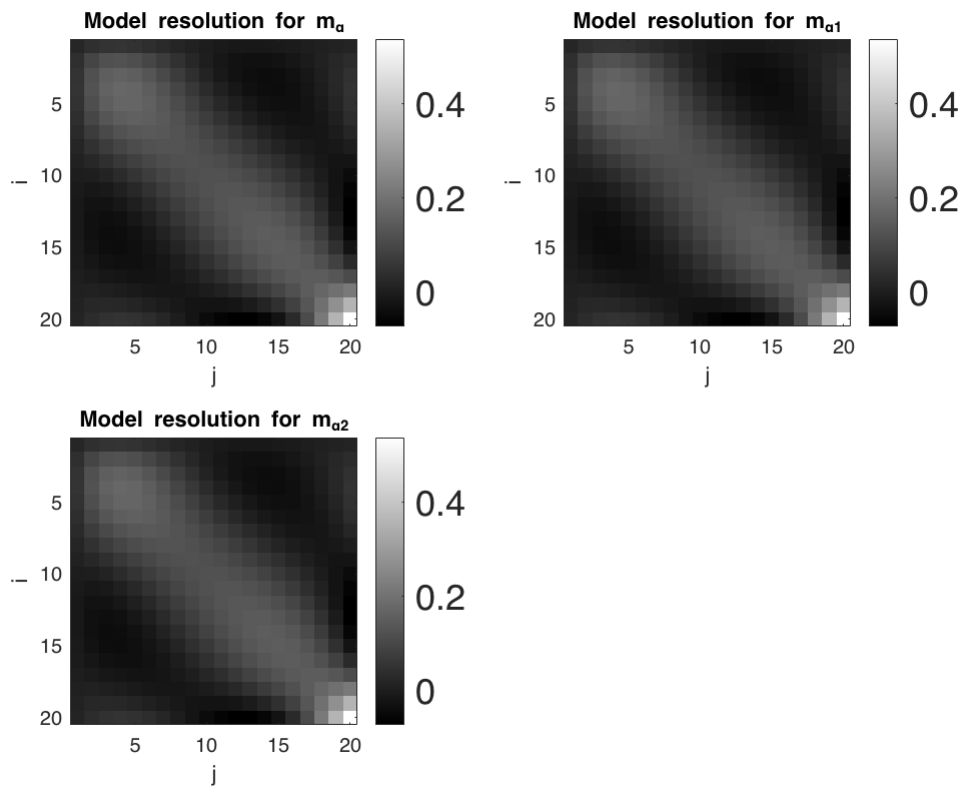
<stopping criteria details>

4.



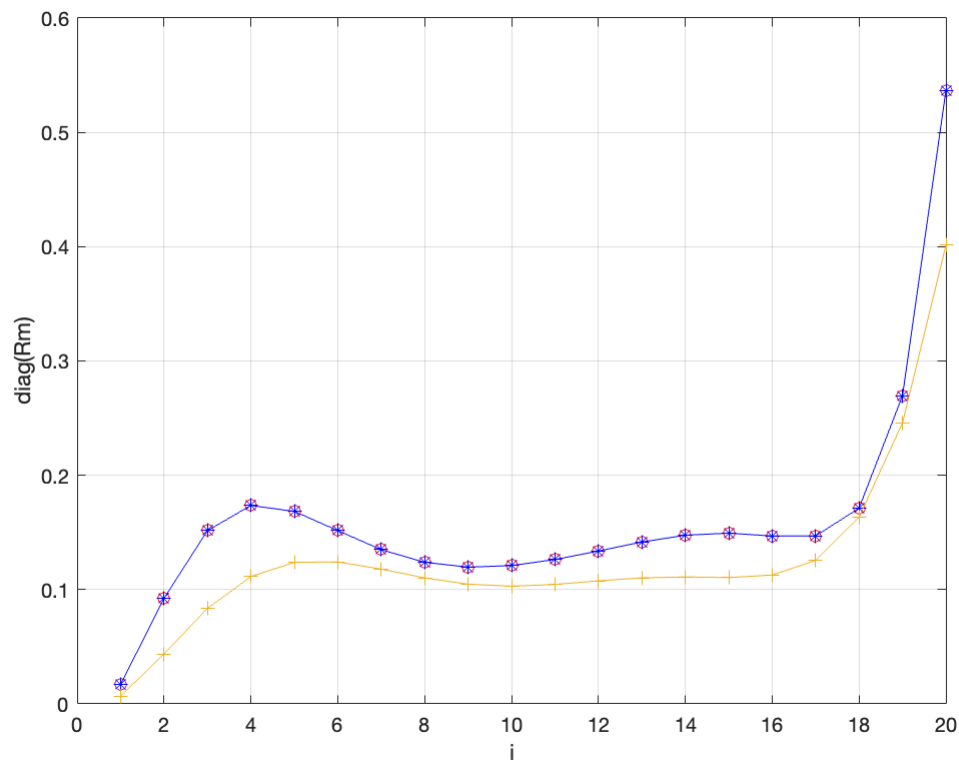
Based on the graph, smaller values of alpha yielded better curve shapes compared to the larger values .

5 (a).



The resolution is best at the 20th model parameter estimate.

5 (b).



The resolution is best at diagonal entries of the parameter estimates, and is worse at off diagonal entries. The model parameters can be well resolved, if we use better methods of finding alpha, since the estimates depend on the values of alpha. Therefore better values of alpha will yeild good parameter estimates, thereby achieving better resolution.