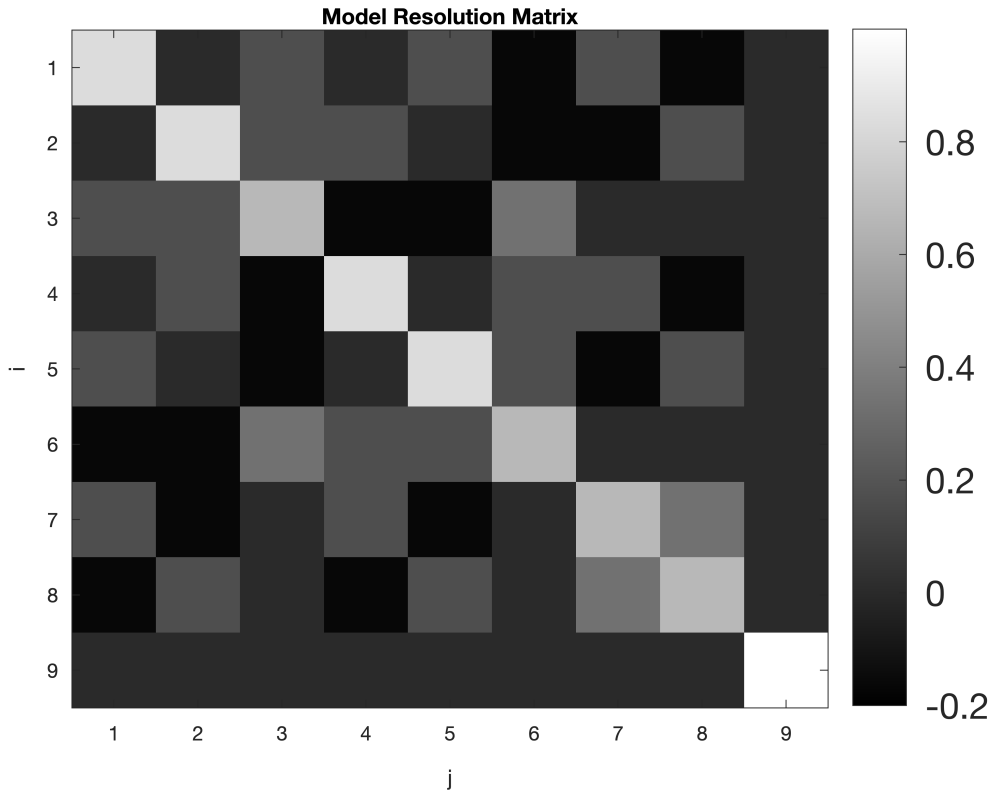
Generalized inverse uncertainty, individual activity

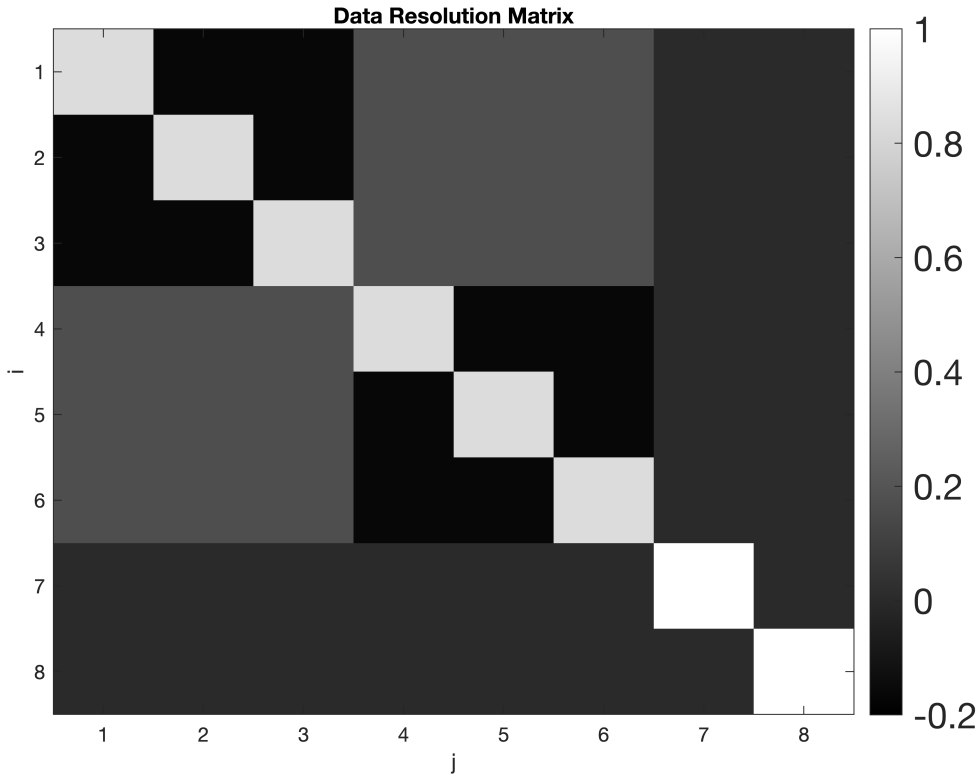
1.



$\text{trace}\,(R_m) = 7$

From the plot of R_m above, it is observed that the diagonal is lighter (whiter) than any other part of the plot. Along the diagonal It is observed that the block for m_9 with a resolution of 1 (highly resolved) is whiter than any other model parameter which means that the model paremeter estimate for m_9 will always be equal to the true parameter value and it will always be well recovered unlike other parameters. Model parameters $m_1,\ m_2, m_4$ and m_5 have a resolution of 0.833 which is higher than m_3, m_6, m_7 and m_8 with the least resolution of about 0.667. Therefore we expect loss of resolution for all other parameters except m_9.

2.



$\text{trace}\,(R_d) = 7$

From the plot of R_d above, it is observed that the diagonal is lighter (white) than any other part of the plot. It is observed that the diagonal is lighter at predicted travel times t_7 and t_8 with a resolution of 1 (highly resolved) than any other travel time. Which means that the predicted travel times t_7 and t_8 will always be equal to their corresponding travel time observations and they will always be well recovered than the rest of the travel times. The rest of predicted travel times t_1 to t_6 have a resolution of 0.833 and we will always expect loss of resolution for all these travel times.

3.

Matrix $\text{Rm} - I$ is given as below;

```
ans = 9×9
   -0.1667    0.0000    0.1667    0.0000    0.1667   -0.1667    0.1667   -0.1667 ···
    0.0000   -0.1667    0.1667    0.1667    0.0000   -0.1667   -0.1667    0.1667
    0.1667    0.1667   -0.3333   -0.1667   -0.1667    0.3333    0.0000    0.0000
    0.0000    0.1667   -0.1667   -0.1667         0    0.1667    0.1667   -0.1667
    0.1667    0.0000   -0.1667         0   -0.1667    0.1667   -0.1667    0.1667
   -0.1667   -0.1667    0.3333    0.1667    0.1667   -0.3333    0.0000    0.0000
    0.1667   -0.1667    0.0000    0.1667   -0.1667    0.0000   -0.3333    0.3333
   -0.1667    0.1667    0.0000   -0.1667    0.1667    0.0000    0.3333   -0.3333
    0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000
```

Matrix $-V_0V_0^T$ is given as below;

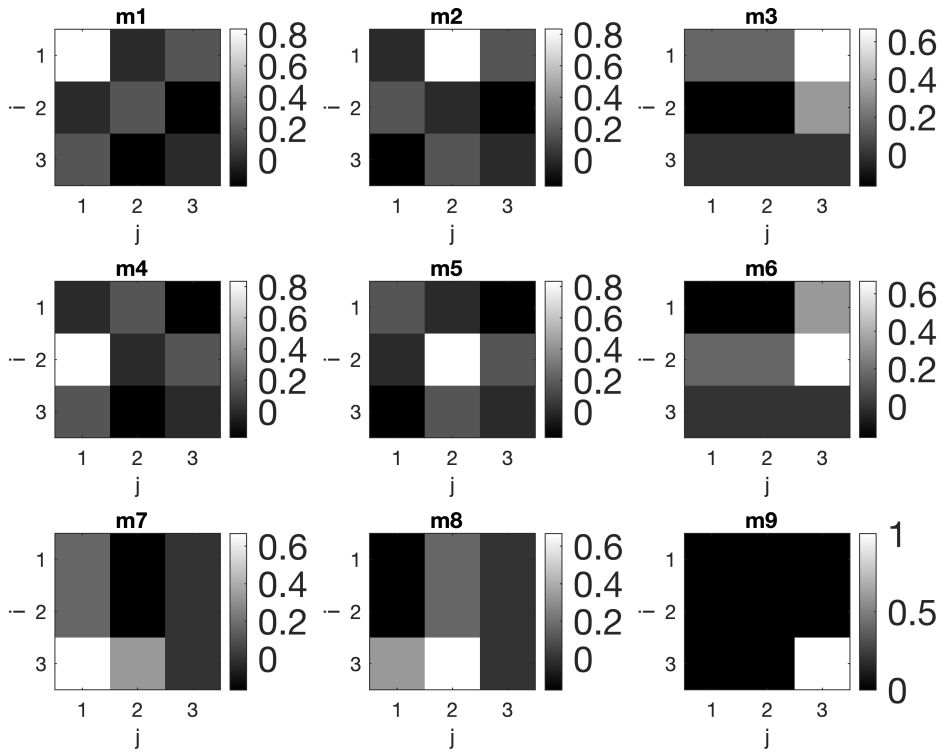
```
ans = 9×9
   -0.1667    0.0000    0.1667   -0.0000    0.1667   -0.1667    0.1667   -0.1667 ···
    0.0000   -0.1667    0.1667    0.1667   -0.0000   -0.1667   -0.1667    0.1667
    0.1667    0.1667   -0.3333   -0.1667   -0.1667    0.3333    0.0000    0.0000
   -0.0000    0.1667   -0.1667   -0.1667    0.0000    0.1667    0.1667   -0.1667
    0.1667   -0.0000   -0.1667    0.0000   -0.1667    0.1667   -0.1667    0.1667
   -0.1667   -0.1667    0.3333    0.1667    0.1667   -0.3333   -0.0000   -0.0000
    0.1667   -0.1667    0.0000    0.1667   -0.1667   -0.0000   -0.3333    0.3333
   -0.1667    0.1667    0.0000   -0.1667    0.1667   -0.0000    0.3333   -0.3333
    0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000
```

Since the matrices $\mathrm{Rm} - I$ and $-V_0V_0^T$ are the same, the it has been verified that $\mathrm{Rm} - I = -V_0V_0^T$.

$\|\mathrm{Rm} - I\| = 1$

If the matrix $\mathrm{Rm} - I$ was to be a zero matrix or $\|\mathrm{Rm} - I\| = 0$, then this would incate that there is no bias in the parameter estimates. But for this problem $\mathrm{Rm} - I$ is non zero with $\|\mathrm{Rm} - I\| = 1$meaning that the parameter estimates have an expected value hence an indication of the bias in parameter estimates.

4.



From the figure above, we observe that the recovered model from the spike test of parameter estimate m_9 in position $3\mathrm{x}3$ is the only one that doesnot smear information in other blocks because of its high resolution unlike

other parameter estimates m_i for $i = 1, 2, 3, 4, 5, 6, 7, 8$ that show that limited resolution causes information about their respective i^{th} block to smear into other blocks.

5.

The condition numbers of $G\dagger$ and G are $1.6815x\,10^{16}$ and $2.2298x\,10^{16}$ respectively. and since both condition numbers are large, it means that any small change in the data will change the parameter estimates significantly. Hence the model parameters are unstable.