

Tikhonov regularization, individual activity

1.

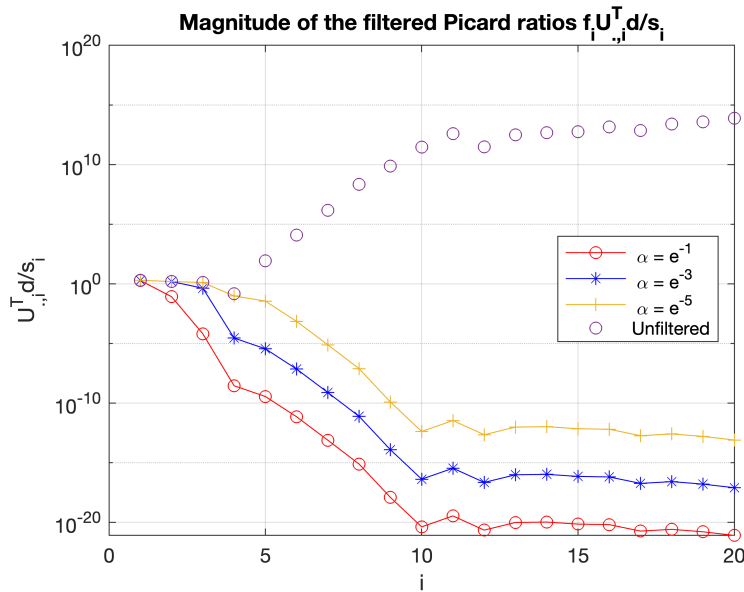
The χ^2_{obs} for the TSVD model parameter estimates with respect to $i=4$, is 0.9433, for the TSVD model parameter estimates with respect to $i-1=3$, is 0.9437 and for the true model parameter values, is 22.4482.

The expected value of the χ^2 statistics with degrees of freedom ($m-n = 0$) is given by; $E[\chi^2_{m-n}] = E[\chi^2_0] \approx 0$

The χ^2_{obs} for the true model parameter values is far greater than that of the two TSVD model parameter estimates. The χ^2_{obs} for the TSVD model parameter estimates with respect to $i=4$ is similar to that for the TSVD model parameter estimates with respect to $i-1=3$ with an absolute difference of 4.0812×10^{-4} .

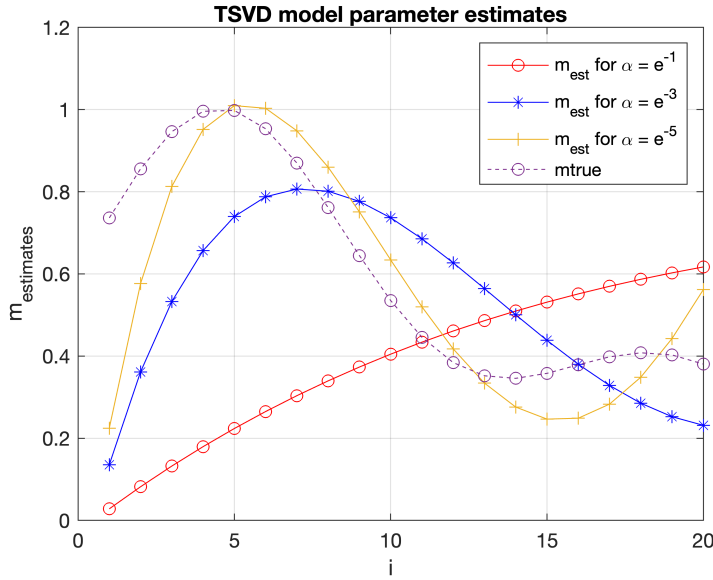
All the χ^2_{obs} are higher than the expected value although that of the two TSVD model parameter estimates are near the expected value. This means that the true model parameter values do not follow a χ^2 distribution.

2.



Both the filtered and unfiltered Picard ratios are the same for $i=1$. There after the unfiltered Picard ratios slightly decrease for $i=4$ and then abruptly increase while as the filtered Picard ratios decrease in a manner similar to the increase of the unfiltered Picard ratios and the bigger the alpha the more the decrease in the ratios.

3(a).



From above, we see that the plot for the estimates with respect to $\alpha = 10^{-5}$ has a similar shape to that of the true values although the estimates are out of phase with the true parameters except at $i=5$ and 13 where the values are almost the same. For bigger alpha that is $\alpha = 10^{-3}, 10^{-1}$, the behaviour of the plots changes and deviate from that of the true parameters and the bigger the alpha the more the deviation. This behaviour is portrayed by the Picard ratios where the ratios decrease as alpha increases. All this means that the smaller alpha is the more accurate the parameter estimates are.

3(b).

The χ^2_{obs} for the model parameter estimates with respect to $\alpha = 10^{-1}, 10^{-3}$ and 10^{-5} are 3.2498×10^5 , 38.418 and 0.9433 and we notice that χ^2_{obs} values decrease as alpha decreases.

The χ^2_{obs} for the model parameter estimates with respect to $\alpha = 10^{-1}$ and 10^{-3} are greater than all χ^2_{obs} values in question 1 above while as χ^2_{obs} with respect to $\alpha = 10^{-5}$ is the same as that of the TSVD model parameter estimates with respect to $i=4$.