SOLVING A SIMPLE TRANSPORT MODEL USING REPRESENTERS

by

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Project pre-proposal

submitted in partial fulfillment

of the requirements for the Inverse theory course

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0.1 Formulation of the inverse problem

0.1.1 The forward model

The transport model that will be used for this project is a linear advection partial differential equation with zero forcing as shown below;

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \quad x \in [0, L], \quad t \in [0, T] \tag{1}$$

with initial and boundary conditions as below;

$$\mathbf{u}(x, 0) = I(x) \quad x \in [0, L] \tag{2}$$

$$\mathbf{u}(0, t) = B(t) \quad t \in [0, T] \tag{3}$$

The circulation field u is a function of two variables (x, t), u can also represent the concentration of a given pollutant ($PM_{2.5}$ for this particular project) at position x and time t. c will be assumed to be a fixed constant and the boundary conditions will be assumed to be periodic.

0.1.2 Measurements

Simulated

A finite number of observations or data $\mathbf{d}_m$ collected at points (x_m, t_m) will be ~~assumed.~~ Following Furtado *et al.* (2010), these observations are imperfect point measurements collected at M points in space and these are related to the "true" circulation field $u(x, t)$ by;

$$\mathbf{d}_m = \mathbf{u}(x_m, t_m) + \epsilon \qquad 1 \le m \le M$$

Specify how you will generate $u(x_m, t_m)$ (i.e. Donna's code) and how you will simulate ϵ, e.g. $N(0, \sigma^2)$

data error ε is different than error in processes + Forcing

In addition there are

0.1.3 Accounting for model errors

The measurement error ϵ may arise from an imperfect measuring system, unresolved processes, and the forcing. This means that if u_F is the exact solution to the forward model 0.1.1, ~~in reality~~ $\mathbf{d}_m \neq \mathbf{u}_F(x_m, t_m)$. Therefore to cater for these uncertainties, errors will be added to the model, observations, the initial and boundary estimates as shown in equation 4 ;

in the transport model.

In addition to data error

$$\begin{cases} \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} & = f \quad x \in [0, L], \quad t \in [0, T] \\ \mathbf{u}(x, 0) & = I(x) + i(x) \quad x \in [0, L] \\ \mathbf{u}(0, t) & = B(t) + b(t) \quad t \in [0, T] \\ \mathbf{d}_m & = \mathbf{u}(x_m, t_m) + \epsilon \end{cases} \tag{4}$$

0.2 Project objectives

You haven't defined the inverse problem

The main objective for this project will be to solve the above inverse problem that is;

- To solve for the adjoint equation backward in time and for the optimal estimates forward in time using representers under the condition that the model is trusted but the initial condition is not and vice versa.

$\hat{u} = \arg\min J$ write this out

this comes later

~~0.3 Solving~~ the inverse problem

~~0.3.1 The objective function~~

Since we seek to find the optimal estimates $\hat{\mathbf{u}}$ for the circulation field, we shall have to minimize the errors in the inverse problem above by minimizing the cost function (the weighted sum of squared errors) in a least-squares sense to produce the maximum likelihood estimate. Write this out

0.3.2 Euler Lagrange equations

The minimum of the generalized least squares objective function will then be derived subject to the constraint that the state equations are satisfied. This constraint can be incorporated into the minimization procedure thus generating the Euler Lagrange equations which will then be solved using representers to give the optimal circulation estimates. $\hat{\mathbf{u}}$. Write this out

Bennett (2005) Benedetti *et al.* (2009)

Explain the inverse problem & how you will solve it in more detail for the proposal, in addition to other requirements for the proposal.

REFERENCES

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