Homework #9

Problem #1

Suppose $\bar{f}(x)$ is a *backward stable* algorithm for computing $f(x) = e^x$. Show that for $x \geq 0$, we will have

$$\bar{f}(x) \leq e^x. \tag{1}$$

Use Theorem 15.1, which for this problem can be stated as

$$\frac{|\bar{f}(x) - f(x)|}{|f(x)|} \leq \kappa(x)\frac{|\bar{x} - x|}{|x|} \tag{2}$$

Solution

If $\bar{f}(x)$ is a *backward stable* algorithm for computing $f(x) = e^x$, then $\bar{f}(x) = f(\bar{x}) = e^{\bar{x}}$.

Also,

$$\kappa(x) = \frac{|f'(x)||x|}{|f(x)|} = |x| \tag{3}$$

Substituting in equation (2) above, we get;

$$\begin{aligned}\frac{|f(\bar{x}) - f(x)|}{|f(x)|} &\leq |x|\frac{|\bar{x} - x|}{|x|} \\ \Rightarrow \frac{|e^{\bar{x}} - e^x|}{|e^x|} &\leq |\bar{x} - x|\end{aligned} \tag{4}$$

Assuming that $\bar{x} = x + \delta x$, equation (4) becomes;

$$\begin{aligned}\frac{|e^{x+\delta x} - e^x|}{|e^x|} &\leq |x + \delta x - x| \\ \frac{|e^x||e^{\delta x} - 1|}{|e^x|} &\leq |\delta x|\end{aligned} \tag{5}$$

$$\Rightarrow |e^{\delta x} - 1| \leq |\delta x| \tag{6}$$

But the inequality above holds if and only if $\delta x \leq 0$. Therefore if $|e^{\delta x} - 1| \leq |\delta x|$, then $\delta x \leq 0$ and we have that;

$$f(\overline{x}) = e^{\overline{x}} = e^{x+\delta x} \leq e^{x} \tag{7}$$

$$\therefore \overline{f}(x) \leq e^{x} \qquad for \;\; x \geq 0$$

In [ ]:

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