# MATH 568

Ch4: Tikhonov regularization, individual activity

Consider problem 5. in your textbook.

1. Calculate $\chi^2_{obs}$ for the two TSVD model parameter estimates you found in problem 5. of the Ch3: Discrete ill-posed problems individual activity, and for the true model parameter values. Identify the expected value of the $\chi^2$ statistic. Discuss the values you calculated in relationship to each other, and to the expected value.

2. Plot the log of the magnitude of the filtered Picard ratios $f_i \mathbf{U}_{:,i}^T \mathbf{d}/s_i$ where $f_i = \frac{s_i^2}{s_i^2+\alpha^2}$ for $\alpha = 10^{-1}, 10^{-3}, 10^{-5}$. Discuss how they compare to the unfiltered Picard ratios in the Ch3: Discrete ill-posed problems individual activity.

3. Use equation (4.7) to find model parameter estimates $\mathbf{m}_\alpha$ that optimize:

$$\min_{\mathbf{m}} \left\{ \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \alpha \|\mathbf{m}\|_2^2 \right\} \tag{1}$$

   for $\alpha = 10^{-1}, 10^{-3}, 10^{-5}$.

   (a) Plot the estimates for each value of $\alpha$ on the same graph. Discuss the accuracy of the model parameter estimates with different values of $\alpha$, and relate them to the Picard ratios in 2.

   (b) Report the values for the magnitude of the residual $\|\mathbf{d} - \mathbf{G}\mathbf{m}_\alpha\|_2^2$ for each value of $\alpha$. Interpret these $\chi^2_{obs}$ values, and their relationship to the values you found in 1.