MATH 568

Ch3: Generalized inverse uncertainty, individual activity

Consider problem 5. in your textbook.

1. Complete part a. but do not solve the system of equations. Instead, plot the singular values, and $1/j^{\alpha}$ for well chosen values of α on the same graph. Discuss if this problem is mildly or moderately ill-posed.

2. Plot the log of the singular values and identify a value for i above which the singular values can reasonably be considered to be nonzero, and below which the singular values can be considered to be zero.

3. Plot the log of the magnitude of the Picard ratios $\mathbf{U}_{:,i}^{T}\mathbf{d}/s_i$. These are the coefficients in the SVD series representation of the generalized inverse solution, and dictate the accuracy in the solution. Identify a value for i where these coefficients abruptly increase. Compare this to the value for i you identified in 2.

4. Use equation (3.80) to produce truncated SVD (TSVD) model parameter estimates, and use trial and error to choose the largest possible i so that you get a reasonable looking graph. Compare this value of i to those you identified in 3 and 2. Plot the best looking estimates.

5. The true parameters are given by $m(x) = e^{-10(x-0.2)^2} + 0.4e^{-10(x-0.9)^2}$. Plot the true parameters along with your best estimate from 4. On the same graph, also plot a TSVD estimate created by truncating at $i-1$ singular values, where i gave you the best TSVD estimate. Discuss how well your TSVD model parameters estimated the true parameters