Evergreen
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Q1. $f(x_1, \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2} \times \sqrt{2\pi}} e^{-\frac{(x_i - \theta_i)^2}{2\theta_2}}$

likelihood function

$L(\theta_1, \theta_2) = \prod_{i=1}^{n} (x_i, \theta_1, \theta_2) :. \theta_2^{-n/2} (2\pi)^{-n/2} e^{-\frac{1}{2\theta_2} \sum (x_i - \theta_1)^2}$

$\log L(\theta_1, \theta_2) = \frac{-n}{2} \log \theta_2 - \frac{n}{2} \log 2\pi - \sum \frac{(x_i - \theta_1)^2}{2\theta_2}$

$$\frac{\partial \log 2(\theta_1 \theta_2)}{\partial \theta_1} = \frac{-2 \sum (x_i - \theta_1)(-1)}{2\theta_2} = 0$$

$\sum x_i - n\theta_1 = 0$

$\hat{\theta_1} = \hat{u} = \frac{\sum x_i}{n} = \bar{x}$

$$\frac{\partial \log_2 (\theta_1, \theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{\sum (x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$-n\theta_2 + \sum (x - \theta_1)^2 = 0$

$\hat{\theta_2} = \frac{\sum (x_i - \bar{x})^2}{n}$

$\theta_1 = \frac{\sum x_i}{n}, \quad \theta_2 = \frac{\sum (x - \bar{x})^2}{n}$

Q2. $P(y|Q) \therefore Bin(y, m, \theta)$

$$^mC_y \times \theta^y (1-Q)^{n-y}$$

log likelyhood function = $\log \theta (y|\theta)$

$$u(\theta) = \log {}^{m}C_y + y\log\theta + (m-y)$$

$$= \log(1-\theta)$$

Differentiating wrt θ

$$\frac{d\,u(\theta)}{d\theta} = \frac{y}{\theta} - \frac{(m-y)}{1-\theta} = 0$$

$$\frac{y}{\theta} - \frac{m-y}{1-\hat{\theta}} = 0$$

$$(m-y)(\hat{\theta}) = y(1-\hat{\theta})$$

$$m\hat{\theta} - y\hat{\theta} = y - y\hat{\theta}$$

$$m\hat{\theta} = y$$

$$\boxed{\hat{\theta} = \frac{y}{m}}$$