Summary of Findings from Studying Group Generated by A and B

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Summary

The first step was proving that:

$$A^j = \begin{bmatrix} \cos\frac{2\pi}{n} & -\sin\frac{2\pi}{n} \\ \sin\frac{2\pi}{n} & \cos\frac{2\pi}{n} \end{bmatrix}^j = \begin{bmatrix} \cos\frac{2j\pi}{n} & -\sin\frac{2j\pi}{n} \\ \sin\frac{2j\pi}{n} & \cos\frac{2j\pi}{n} \end{bmatrix}$$

After this, it is simple enough to also prove that $A^j = A^{j+n}$. This means that A^j repeats for every n increase in the exponent.

It is fairly obvious that the group generated by B^k has 2 elements
We then prove that $A^jB = BA^{-j}$

Taking all these factors together, we can see that A^j has n unique values, as j = 1,2,3,4,...,n. Even though the group generated by A and B must consider both A^jB^k as well as B^kA^j, we know that the latter can be written instead as $A^{-j}B^k$, and then we can we can make the exponent of A positive by adding the appropriate multiple of n.