

# Initial Thoughts on Invariant Polynomials

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## Initial Formula

I wanted to do this in LaTeX instead of just typing it into the email, for easiness in reading (and also to brush up my Tex skills, which I haven't used for a few weeks now).

We will begin with the complex function,  $G(z)$ .

$$G(z) = z^n = (x + iy)^n$$

Expanding this further, we see:

$$G(z) = \binom{n}{0} x^n y^0 i^0 + \binom{n}{1} x^{n-1} y^1 i^1 + \binom{n}{2} x^{n-2} y^2 i^2 + \dots + \binom{n}{n} x^0 y^n i^n$$

Then, for the polynomial  $R(x, y)$ , we have that:

$$R(x, y) = \binom{n}{0} x^n - \binom{n}{2} x^{n-2} y^2 + \binom{n}{4} x^{n-4} y^4 - \binom{n}{6} x^{n-6} y^6 + \dots + \binom{n}{2j} x^{n-2j} y^{2j}$$

With  $j = \lfloor \frac{n}{2} \rfloor \in \mathbb{N}$

A note here, I'm not quite sure how to notate the end of the polynomial. My aim is to show that the polynomial isn't endless, and that the exponent on  $x$  is greater than or equal to 0, as per the definition of a polynomial. The issue is that whether the last term of the polynomial has  $x^1$  or  $x^0$  is dependent on whether  $n$  is even or odd. I tried to use the notation with the floor function defining a value for  $j$ , and hopefully that's obvious, but is there a better way for me to notate this that I'm missing?

The polynomial  $I(x, y)$  is similar, it's the leftover from the original complex function:

$$I(x, y) = \binom{n}{1} x^{n-1} y^1 - \binom{n}{3} x^{n-3} y^3 + \binom{n}{5} x^{n-5} y^5 - \binom{n}{7} x^{n-7} y^7 + \dots + \binom{n}{2k-1} x^{n-2k+1} y^{2k+1}$$

With  $k = \lceil \frac{n}{2} \rceil$

The same problem exists here, where I don't quite know how to notate the final term. I'm also fairly certain that, if not for both, then at least for  $I(x, y)$ , the exponents on  $x$  and  $y$  are incorrect by 1, again simply because I couldn't think of a good way to notate it (maybe because of the time of night).

Are these polynomials correct? Are there more invariants? Also, are these a finite number of these invariant polynomials for  $A$ ? Should I also be trying to prove that I have all of them or something similar to that, or is it too early?