Spatial Diffusion in SIR type Models: Simulation for Covid-19 Data

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Introduction

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2022

Bachelor Degree in Mathematics at UniTO

2023-2024

Tutoring in Numerical Analysis and Calculus

2024

Master Degree in Mathematics at UniTO

2024-2025

Fellowship in Biostatistics at the Clinical Department Science - UniTO

Skills

Modeling

Bio-mathematics and Bio-statistics, Dynamical Systems, Complex Systems, Neural Networks, MonteCarlo Simulations, Assurance and Financial Mathematic

Coding

R, Pyhton, MATLAB, Maple, C++

Programs and Languages

LaTeX, Git, manim (python graphic tool.
English language, level C1

The SIR model

Total Population: $N = S(t) + I(t) + R(t)$

- Infected (**I**)
- Susceptibles (**S**)
- Removed (**R**)

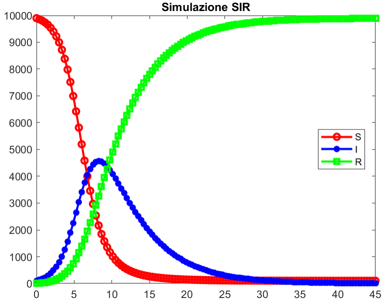


Figure: Numeric Simulation of the SIR model

Model Equations

$$\begin{cases} \dot{S} = -\frac{\beta}{N} S(t) I(t) \\ \dot{I} = -\frac{\beta}{N} S(t) I(t) - \gamma I(t) \\ \dot{R} = \gamma I(t) \end{cases}$$

- ✓ β is the infection rate I
- ✓ $\gamma \propto T^{-1}$ where T is the average time of recovery from the illness.

An Hidden Assumption

Perfect Mixing

S and I are uniformly distributed in space at all times

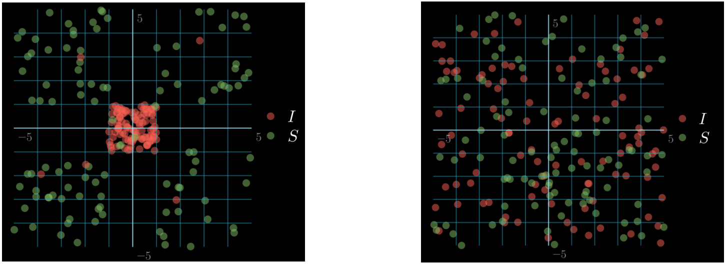
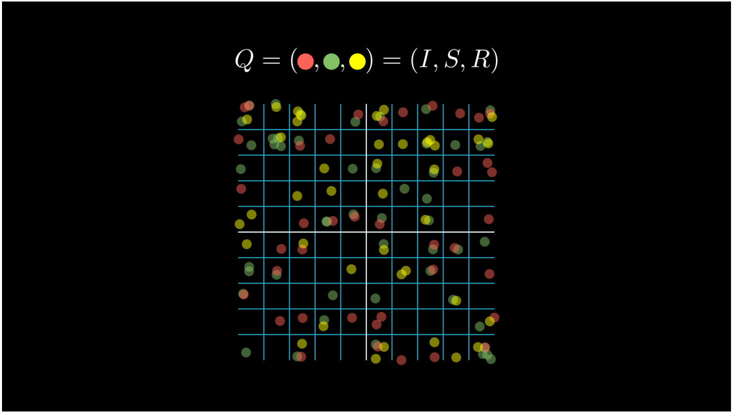


Figure: In the initial phases of an epidemic diffusion, the infected population I is not uniformly spread across space

Lattice Gas Cellular Automata (LGCA)

Particles and Cells

Particles moving on a lattice can interact only inside the same cell [Schneckenreither et al.]



Particles Laws of Motion

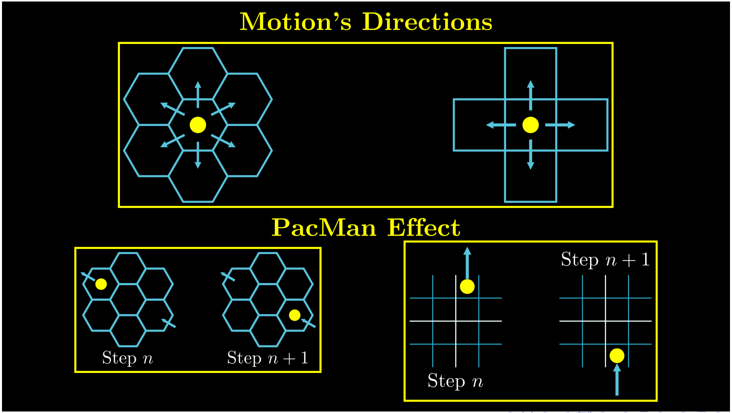
At each time step n particles move to a neighboor cell

Uniqueness

At most one particle can reach a fixed neighboor cell

Randomnes

The exit configuration is randomly selected



Epidemic Laws described

Evolution Rules

- Each particle of state I of a specific cell, infects a particle of state S of the same cell with probability β_{LG}
- With rate γ particles pass from state I to state R

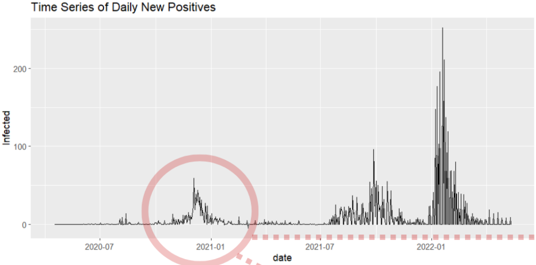
Epidemic Laws inside a cell

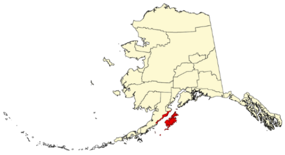
Let $I(c)$ be the number of infected in cell c and let $q_n(p)$ be the state of particle $p \in c$ at time step n, then

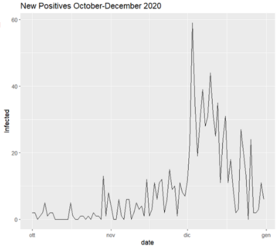
$$\begin{cases} \phi_c = \mathbb{P}\left(q_{n+1}(p) = S | q_n(p) = S\right) = (1-\beta_{LG})^{I(c)} \\ \psi_c = \mathbb{P}\left(q_{n+1}(p) = I | q_n(p) = S\right) = 1 - \phi_c \\ \mathbb{P}\left(q_{n+1}(p) = I | q_n(p) = I\right) = 1 - \beta_{LG} \\ \mathbb{P}\left(q_{n+1}(p) = R | q_n(p) = I\right) = \beta_{LG} \end{cases}$$

Covid-19 in Kodiak Island, Alaska

An overlook of data and estimated parameters



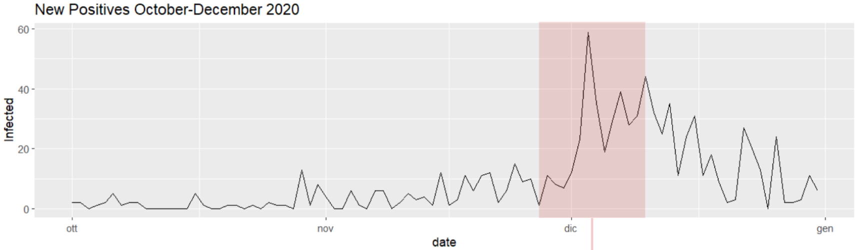




Parameter <chr>	Value <chr>
Starting Date	2020-10-01
Total Population	13100
Initial Population of Infected People	15
gamma	0.153846153846154
beta	0.345881323631 07

Covid-19 in Kodiak Island, Alaska

How to estimate β from new confirmed cases

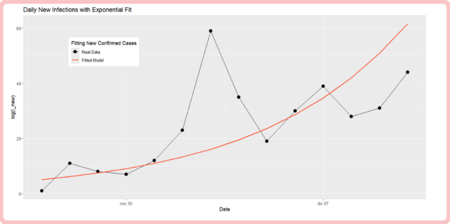


Assuming $S \simeq N$ in the beginning of epidemic peak

$$\frac{dI}{dt} = \frac{\beta}{N} SI - \gamma I \simeq (\beta - \gamma) I$$

Hence, if C is the number of new infected each day, we have

$$C = C_0 e^{(\beta - \gamma)t}$$



Covid-19 in Kodiak Island, Alaska

SIR and LGCA Simulation Results

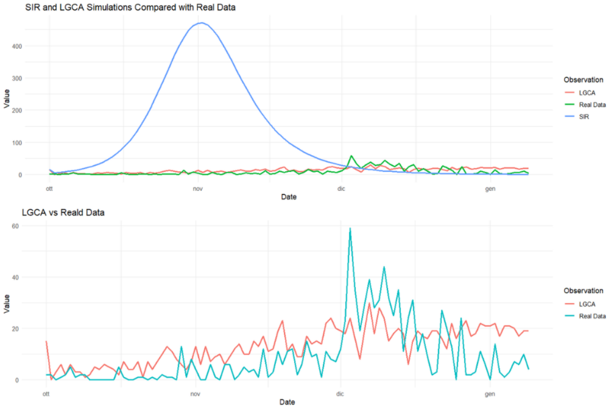
LGCA

Parameter <chr>	Value <chr>
Starting Date	2020-10-01
Population	12996
I_0	15
gamma	0.153846153846154
beta	0.34588132363107
cells	12996

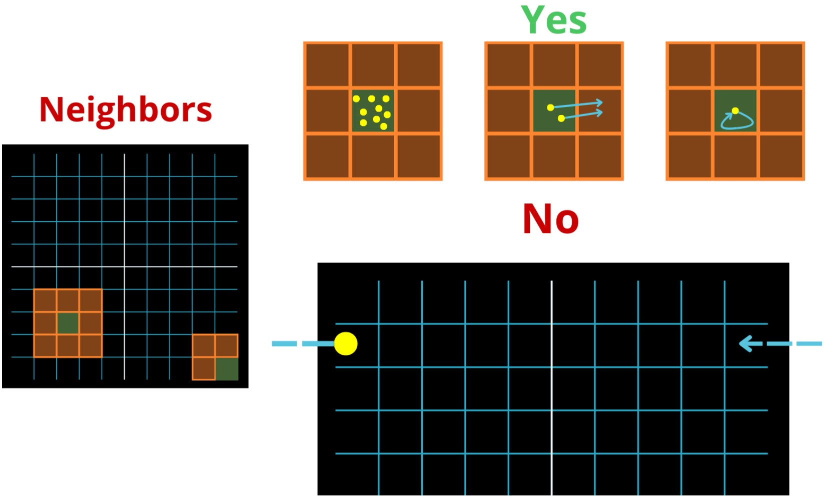
SIR

Parameter <chr>	Value <chr>
Starting Date	2020-10-01
Population	13100
I_0	15
gamma	0.153846153846154
beta	3.23807026513713e-05

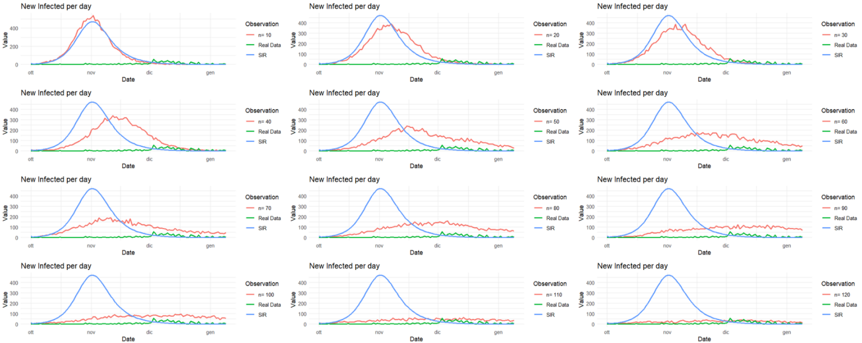
$$\beta_{LG} = \beta_{SIR} \cdot cells$$



Modified LGCA algorithm



Modified LGCA Simulations



cells <dbl>	beta_LG <dbl>
100	0.002640315
400	0.010561262
900	0.023762839
1600	0.042245047
2500	0.066007886
3600	0.095051356
4900	0.129375457
6400	0.168980189
8100	0.213865551
10000	0.264031545

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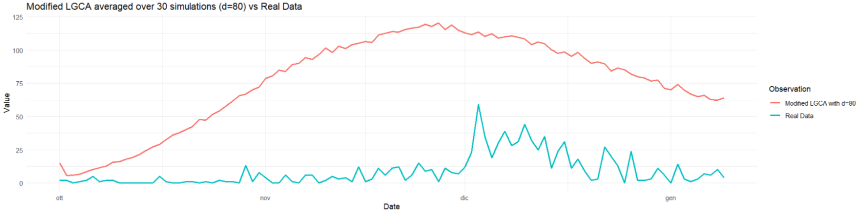

Parameter <chr>	Value <chr>
Starting Date	2020-10-01
Number of Simulations	30
Lattice Dimension	6400
beta_LG	0.168980188644187
gamma	0.153846153846154
I_0	15
Population	13100

Final Observations

Issues

Parameters and Initial Values Estimation,
Dataset Choice, Small Number of Simulations.

Open Questions

Link between parameters (e.g lattice dimension) and qualitative behaviour of Modified LGCA, Stability of the solutions and dependence on distributions and laws of motion

Conclusions

Modified LGCA seems to provide promising results into using lattice based structures to model spatial diffusion in epidemic peaks, specially for small populated areas

Bibliography

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Simulation Modelling Practice and Theory, 16(8):1014–1023, 2008.

Junling Ma.
Estimating epidemic exponential growth rate and basic reproduction number.
Infectious Disease Modelling, 5:129–141, 2020.