Lecture 4

Multivariable linear regression

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http://hunkim.github.io/ml/

Video (Korean): https://youtu.be/kPxpJY6fRkY

Recap

- Hypothesis
- Cost function
- Gradient descent algorithm

Recap

- Hypothesis

$$H(x) = Wx + b$$

- Cost function

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} (H(x^{(i)}) - y^{(i)})^2$$

- Gradient descent algorithm

Predicting exam score: regression using one input (x)

one-variable
one-feature

x (hours)	y (score)
10	90
9	80
3	50
2	60
11	40

Predicting exam score:

regression using three inputs (x1, x2, x3)

multi-variable/feature

x_1 (quiz 1)	x_2 (quiz 2)	x_3 (midterm 1)	Y (final)
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

Hypothesis

$$H(x) = Wx + b$$

Hypothesis

$$H(x) = Wx + b$$

$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

Cost function

$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$cost(W, b) = \frac{1}{m}\sum_{I=1}^{m}(H(x_1^{(i)}, x_2^{(i)}, x_3^{(i)}) - y^{(i)})^2$$

Multi-variable

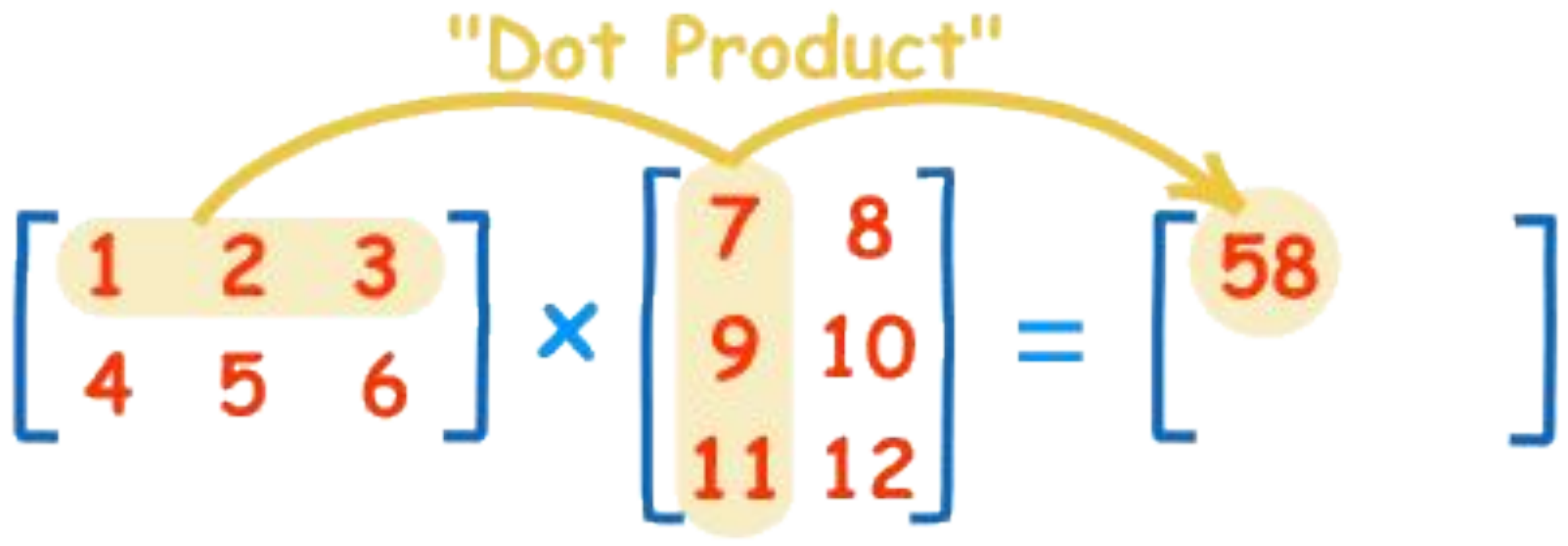
$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$H(x_1, x_2, x_3, ..., x_n) = w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n + b$$

Matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n$$

Matrix multiplication



https://www.mathsisfun.com/algebra/matrix-multiplying.html

Hypothesis using matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}$$

$$H(X) = XW$$

Hypothesis using matrix

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3$$

x_1	x_2	x_3	Y
73	80	75	152
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Test Scores for General Psychology

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3$$

x_1	x_2	x_3	Y
73	80	75	152
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Test Scores for General Psychology

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}$$

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Many x instances

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1 w_1 + x_2 w_2 + x_3 w_3 \end{pmatrix}$$

http://college.cengage.com/mathematics/brase/understandable_statistics/7e/students/datasets/mlr/frames/frame.html

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
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Hypothesis using matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n$$

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[5, 3] **[3, 1]** [5, 1]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$\begin{pmatrix} \boxed{\mathbf{X}} \end{pmatrix} \times \begin{pmatrix} \boxed{\mathbf{W}} \end{pmatrix} = \begin{pmatrix} \boxed{H(X)} \end{pmatrix}$$

[5, 3] [?, ?] [5, 1]

$$H(X) = XW$$

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[n, 3] **[3, 1]** [n, 1]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \; ? \; = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

[n, 3] [?, ?] [n, 2]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

[n, 3] **[3, 2]** [n, 2]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

WX vs **XW**

- Lecture (theory):

$$H(x) = Wx + b$$

- Implementation (TensorFlow)

$$H(X) = XW$$

Next

Logistic Regression

(Classification)