

**Course: Probability & Statistics****Faculty: Dr. Kota Venkata Ratnam****Module 5: Random Variables****Lesson 3: Continuous Random Variable****Reading Objectives:**

In this reading, you will be introduced to the probability density function of a continuous random variable and its mean, variance, and standard deviation with examples.

**Main Reading Section:**

**Definition 1:** We say that  $X$  is a continuous random variable if there exists a nonnegative function  $f(x)$ , defined for all real  $x \in (-\infty, \infty)$ , having the property that for any set  $B$  of real numbers

$$P(X \in B) = \int_B f(x) dx$$

The function  $f(x)$  is called the **probability density function** (PDF) of the random variable  $X$ .

Since  $X$  must assume some value,  $f(x)$  must satisfy  $\int_{-\infty}^{\infty} f(x) dx = 1$ .

- $P\{a \leq X \leq b\} = \int_a^b f(x) dx$
- $P\{X = a\} = 0$

The relationship between the cumulative distribution  $F(\cdot)$  and the probability density  $f(\cdot)$  is expressed by  $F(a) = P\{X \in (-\infty, a]\} = \int_{-\infty}^a f(x) dx$ . This also implies  $\frac{d}{da} F(a) = f(a)$ .

**Variance and Standard deviation**

**Definition 2:** If  $X$  is a random variable with mean  $\mu$ , then the variance of  $X$ , denoted by  $\text{Var}(X)$ , is defined by

$$\text{Var}(X) = E[(X - \mu)^2] \quad \text{or} \quad \text{Var}(X) = E[X^2] - E[X]^2$$

**Definition 3:** The quantity  $\sqrt{\text{Var}(X)}$  is called the standard deviation of  $X$ .

**Properties of variance:**

- $\text{Var}(aX + b) = a^2 \text{Var}(X)$
- $\text{Var}(aX) = a^2 \text{Var}(X)$

**Example 1:** Let  $X$  be a random variable with PDF given by

$$f(x) = \begin{cases} \frac{3}{2}x^2, & |x| \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

1. Find  $E[X]$  and  $\text{Var}(X)$ .
2.  $P\left(X \geq \frac{1}{2}\right)$

**Solution: 1.**  $E[X] = \int_{-1}^1 w f(w) dw = \frac{3}{2} \int_{-1}^1 w^3 dw = 0$

$$\text{Var}(X) = E[X^2] - E[X]^2 = \int_{-1}^1 w^2 f(w) dw = \frac{3}{2} \int_{-1}^1 w^4 dw = \frac{3}{5}$$

$$2. P\left(X \geq \frac{1}{2}\right) = \frac{3}{2} \int_{1/2}^1 w^2 dw = \frac{7}{16}$$

### Reading Summary

In this reading, you have learned about:

- The probability distribution function of continuous random variables
- Mean, variance, and standard deviation of continuous random variables