



#### **Probability and Statistics**

Week 2 Live Session

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### Today's Focus



- Hands-on Visualization using Google Colab
- Recap
- **©** Chebyshev's Inequality in Practice
- Choosing the Right 'Average'
- Skewness: What is your data is really saying

#### Week 1 Recap



- Central Tendency
- Variability Measures
- Data Visualization
- **o** Distribution Analysis

#### Week 2 Recap



- Quartile Deviation and IQR
- 5 Point Summary
- Box Plot
- of Infer from Statistical Summary



#### **Objective** Chebyshev's Inequality in Practice



The Universal Guarantee:

For ANY dataset, at least  $\left(1 - \frac{1}{k^2}\right)$  of data lies within k standard deviations

No assumptions about distribution shape needed!

k = 2: At least 75% of data within  $2\sigma$ 

k = 3: At least 89% of data within  $3\sigma$ 

Perfect for AI because:

Real-world data is often non-normal

New data distributions are unknown





#### **©** Chebyshev's Inequality in Practice



The Universal Guarantee:

For ANY dataset, at least  $\left(1 - \frac{1}{k^2}\right)$  of data lies within k standard deviations

at most  $\frac{1}{\nu^2}$  of the data will be outside k standard deviations from the mean.

$\overline{k}$	At most outside $k\sigma$	At least within $k\sigma$	Normal comparison
1.5	44.4%	55.6%	86.6% within
2	25%	75%	95% within
2.5	16%	84%	98.8% within
3	11.1%	88.9%	99.7% within



#### **©** Chebyshev's Inequality in Practice



A payment system processes transactions with mean Rs. 500 and standard deviation Rs. 150. Without knowing the distribution, what can we say about transactions outside normal ranges?

using Chebyshev with k = 2:

- Range:  $\mu \pm 2\sigma = 500 \pm 300 = [200, 800]$
- Chebyshev guarantees: At most 25% of transactions fall outside [200, 800]
- Equivalently: At least 75% of transactions are between 200 and 800

This 25% is an upper bound on outliers, not the probability that any specific flagged transaction is fraudulent!

P(transaction outside [200, 800])  $\leq$  0.25

A manufacturing process produces widgets with mean weight 100g and standard deviation 5g. What percentage of widgets must weigh between 85g and 115g?

This range is  $\mu \pm 3\sigma$  (k = 3) By Chebyshev's inequality:

At least 1-1/9 = 88.9% of widgets fall in this range.



## Chebyshev in Action - Anomaly Detection



**fraud Detection System:** 

Monitor transaction amounts for each user No assumptions about spending patterns Use Chebyshev bounds to flag unusual transactions

Implementation:

Calculate mean and  $\sigma$  for user's historical data Flag transactions beyond  $2.5\sigma$  as "suspicious" Guaranteed to catch extreme anomalies

Business Impact:

Reduce false positives (incorrectly flagged as fraudulent) Catch 84%+ of extreme cases automatically



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