BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

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Probability and Statistics

Week 3 Live Session

Dr. Pushpendra Gupta

Today's Focus

- 🧠 Recap
- 🎯 Chebyshev's Inequality in Practice
- ⚖️ Visualization Techniques
- 📊 Comparing quartile methods
- 💻 Hands-on Visualization using Google Colab

Week 1 Recap

- 📊 Central Tendency
- 📏 Variability Measures
- 📈 Data Visualization
- 🎯 Distribution Analysis

Week 2 Recap

- 📊 Quartile Deviation and IQR
- 📏 5 Point Summary
- 📈 Box Plot
- 🎯 Infer from Statistical Summary

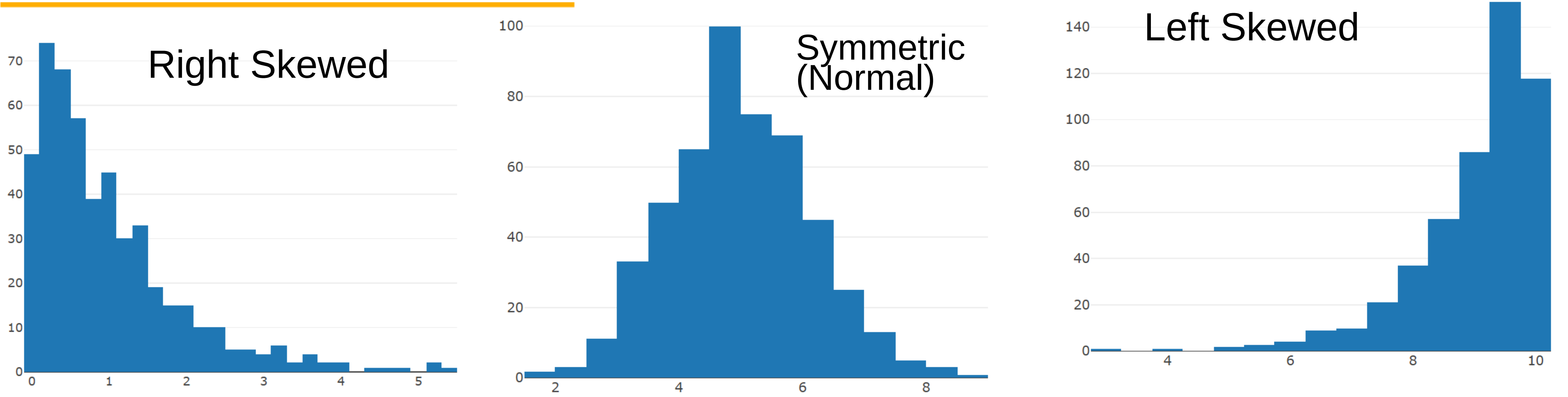
Week 3 Recap

📊 Element of Probability

Combinations

Finding probabilities

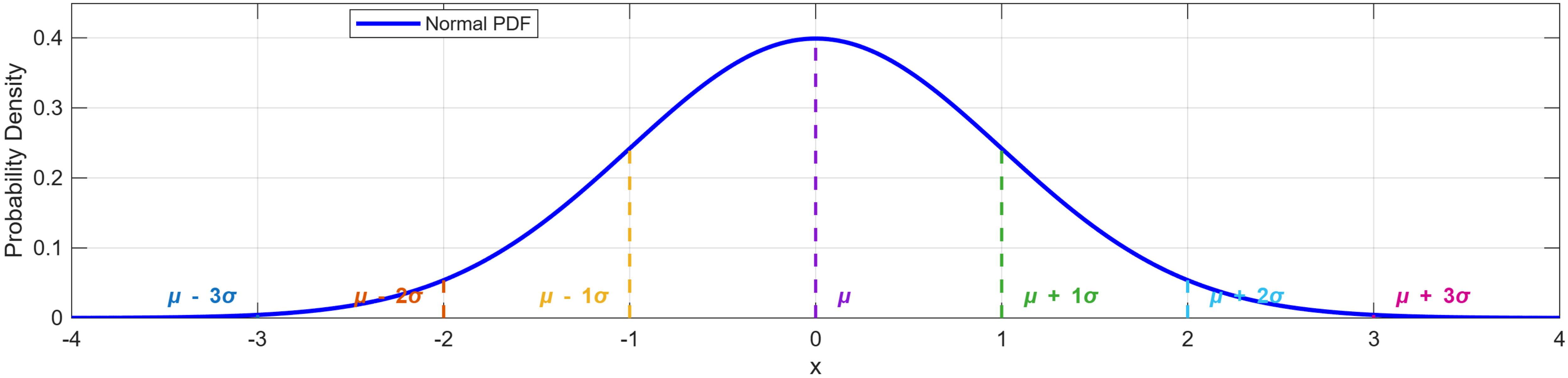
Chebyshev's Inequality in Practice



For ANY dataset, at least $\left(1 - \frac{1}{k^2}\right)$ of data lies within k standard deviations

at most $\frac{1}{k^2}$ of the data will be outside k standard deviations from the mean.

Chebyshev's Inequality in Practice



k	At most outside $k\sigma$	At least within $k\sigma$	Normal comparison
1.5	44.4%	55.6%	86.6% within
2	25%	75%	95% within
2.5	16%	84%	98.8% within
3	11.1%	88.9%	99.7% within

Two Forms of Chebyshev's Inequality

Form 1: Upper Bound (for outliers)

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Interpretation: At most $\frac{1}{k^2}$ of the data will be outside k standard deviations from the mean.

Form 2: Lower Bound (for central data)

$$P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

Interpretation: At least $\left(1 - \frac{1}{k^2}\right)$ of the data will be within k standard deviations from the mean.

Chebyshev's Inequality in Practice

A payment system processes transactions with mean Rs. 500 and standard deviation Rs. 150. Without knowing the distribution, what can we say about transactions outside normal ranges?

using Chebyshev with $k = 2$:

- Range: $\mu \pm 2\sigma = 500 \pm 300 = [200, 800]$

- Chebyshev guarantees: At most 25% of transactions fall outside [200, 800]
- Equivalently: At least 75% of transactions are between 200 and 800

This 25% is an upper bound on outliers, not the probability that any specific flagged transaction is fraudulent!

$$P(\text{transaction outside } [200, 800]) \leq 0.25$$

Chebyshev's Inequality in Practice

A manufacturing process produces widgets with mean weight 100g and standard deviation 5g. What percentage of widgets must weigh between 85g and 115g?

This range is $\mu \pm 3\sigma$ ($k = 3$) By Chebyshev's inequality:

At least $1 - 1/9 = 88.9\%$ of widgets fall in this range.

Chebyshev's Inequality in Practice

Serial No.	Student Transactions (Rs.)	Business Transactions (Rs.)
1	210	11,200
2	280	8850
3	190	33,100
...	...	...
100	320	9950

Assume Student data has:

Mean (μ) = Rs. 250, Standard Deviation (σ) = Rs. 50

$$\mu \pm 2.5\sigma = 250 \pm 2.5 \times 50 = [Rs. 125, Rs. 375]$$

Chebyshev's Inequality in Practice

Assume Student data has:

Mean (μ) = Rs. 250, Standard Deviation (σ) = Rs. 50

$$\mu \pm 2.5\sigma = 250 \pm 2.5 \times 50 = [Rs.125, Rs.375]$$

84% of transactions will be within [Rs. 125, Rs. 375]. (use $1 - 1/k^2$)
At most 16% (≤ 16 transactions out of 100) fall outside this range.

But What If Data Is Normal?

Actual coverage for $\mu \pm 2.5\sigma$ jumps to $\sim 98.76\%$
Only $\sim 1.24\%$ $(\approx 1\ transaction)$ would be outside [Rs. 125, Rs. 375].

Chebyshev's Inequality in Practice

Consider 100 coin flips with p = 0.5. What’s the probability of getting 70 or more heads?

$n = 100, p = 0.5,$

Mean $= np = 50$, Standard deviation $= \sqrt{(np(1-p))} = 5$

70 heads is 4 standard deviations from the mean.

Chebyshev's Bound $P(|X - 50| \geq 4 \times 5) \leq 1/4^2 = 0.0625$

Meaning $P(X \leq 30 \; or \; X \geq 70) \leq 6.25\%$

Chebyshev's Inequality in Practice

Consider 100 coin flips with p = 0.5. What's the probability of getting 70 or more heads?

However Exact probability can only be found using Binomial

$$P(X >= 70) = \frac{100!}{70!\,30!} \times\ 0.5^{70} \times\ (1 - 0.5)^{30} \approx 0.000003$$

Chebyshev gives a conservative upper bound (6.25%) while the actual probability is much smaller (0.0003%).

Frequency Tables

A frequency table organizes data by showing how often each value or category appears.

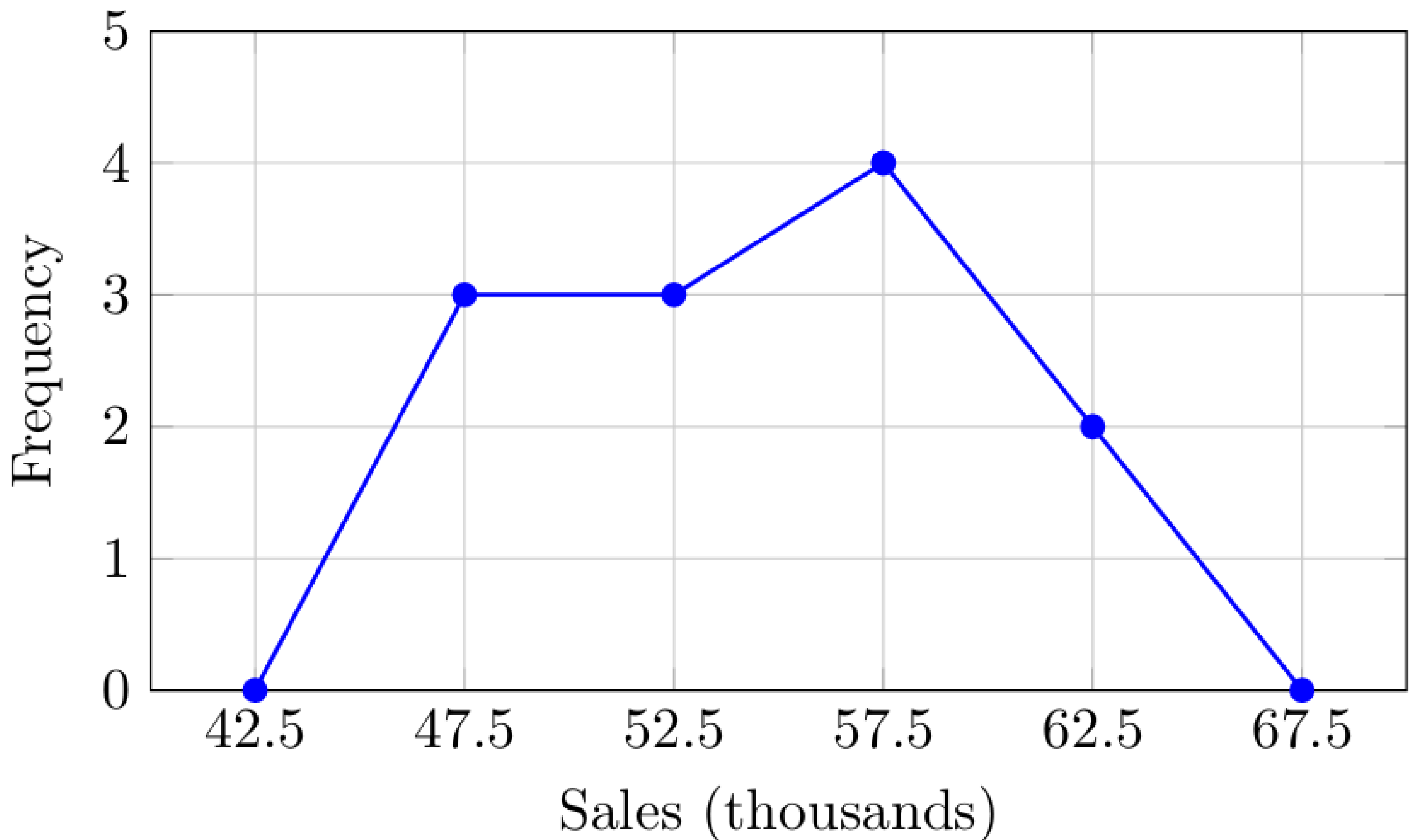
(Monthly Sales Data). Consider monthly sales (in thousands) from a small retail store over 12 months: 45, 52, 48, 55, 62, 58, 51, 49, 56, 60, 53, 57

Sales Range	Frequency	Relative Frequency
45-49	3	0.25
50-54	3	0.25
55-59	4	0.33
60-64	2	0.17

Frequency Polygon - Continuous Class

Continuous Classes have no gaps between successive intervals. The upper limit of one class equals the lower limit of the next class.

Sales Range	Frequency	Midpoint Calculation
45-50	3	(45+50)/2= 47.5
50-55	3	52.5
55-60	4	57.5
60-65	2	62.5



Frequency Polygon- Discontinuous Class

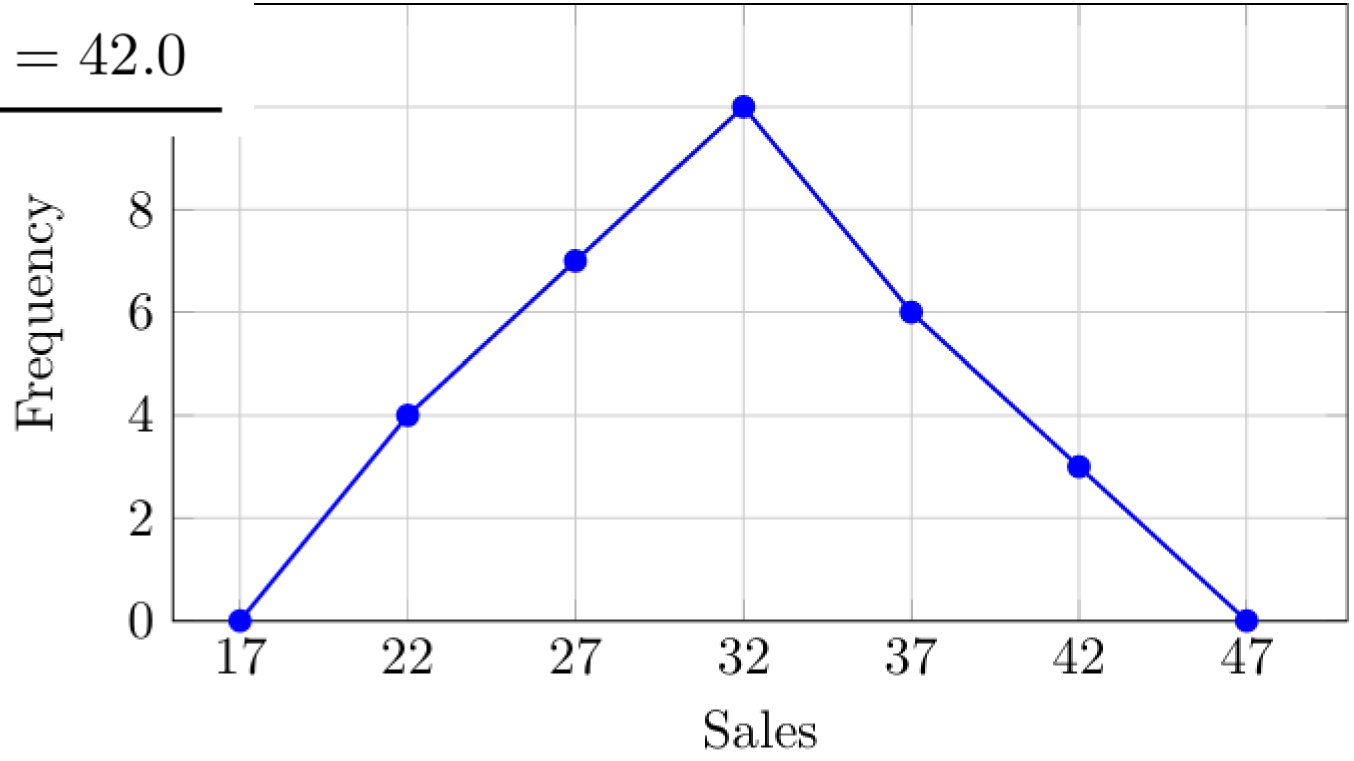
Discontinuous classes have gaps between successive intervals.
e.g., classes like 20-24, 25-29, 30-34 have a gap of 1 unit (24 to 25)

The gap creates ambiguity. Where would a value of 24.5 fall? To resolve this, we create class boundaries that make the intervals continuous.

Sales Range	Frequency	Lower Boundary	Upper Boundary	Midpoint
20-24	4	$20 - 0.5 = 19.5$	$24 + 0.5 = 24.5$	$(19.5 + 24.5)/2 = 22.0$
25-29	7	$25 - 0.5 = 24.5$	$29 + 0.5 = 29.5$	$(24.5 + 29.5)/2 = 27.0$
30-34	10	$30 - 0.5 = 29.5$	$34 + 0.5 = 34.5$	$(29.5 + 34.5)/2 = 32.0$
35-39	6	$35 - 0.5 = 34.5$	$39 + 0.5 = 39.5$	$(34.5 + 39.5)/2 = 37.0$
40-44	3	$40 - 0.5 = 39.5$	$44 + 0.5 = 44.5$	$(39.5 + 44.5)/2 = 42.0$

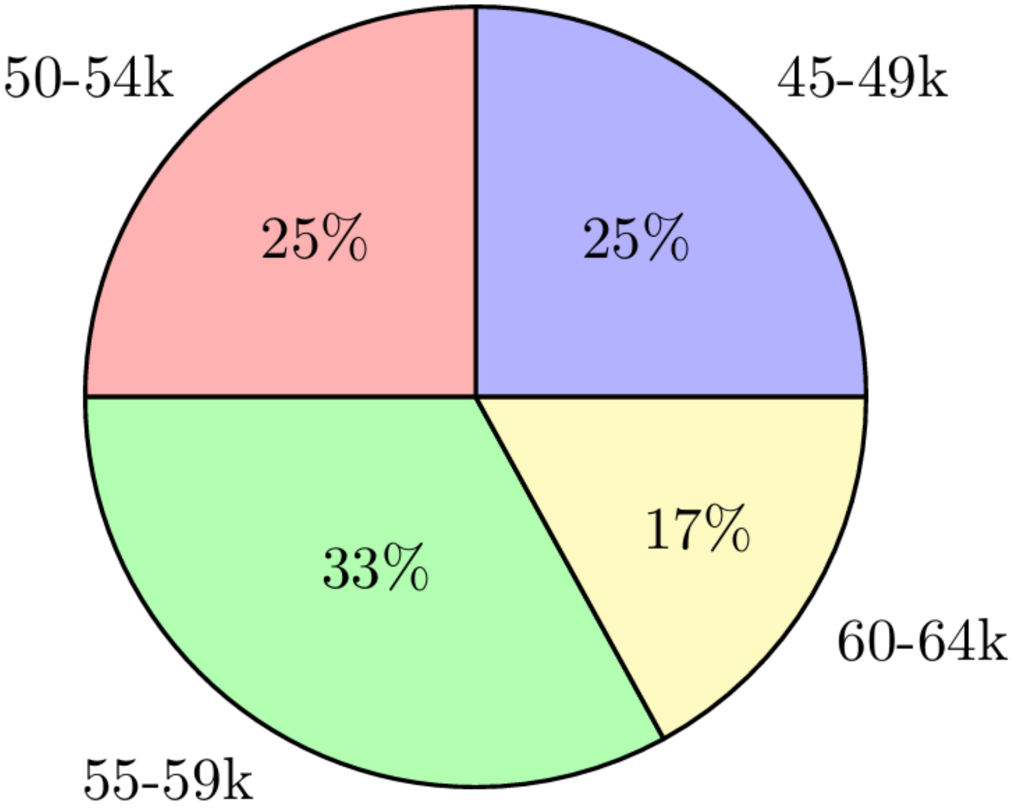
Frequency Polygon- Discontinuous Class

Sales Range	Frequency	Midpoint
20-24	4	$(19.5 + 24.5)/2 = 22.0$
25-29	7	$(24.5 + 29.5)/2 = 27.0$
30-34	10	$(29.5 + 34.5)/2 = 32.0$
35-39	6	$(34.5 + 39.5)/2 = 37.0$
40-44	3	$(39.5 + 44.5)/2 = 42.0$



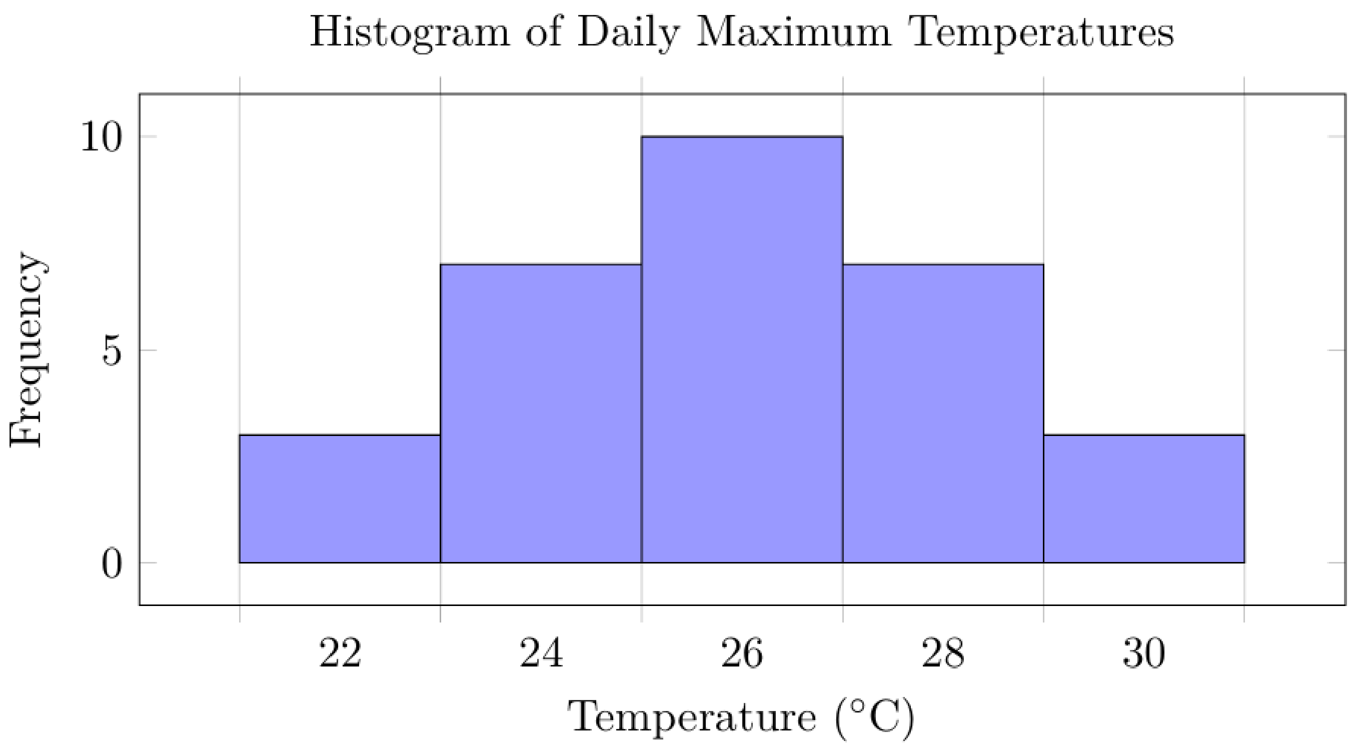
Pie Charts

Pie charts display proportions effectively for categorical data.



Histograms

Histograms group continuous data into bins and display frequencies as rectangular bars. (Temperature Data). Daily maximum temperatures (◦C) for 30 days: 22, 24, 23, 26, 28, 25, 27, 29, 24, 26, 31, 30, 28, 25, 27, 23, 24, 26, 28, 29, 27, 25, 24, 26, 30, 28, 27, 25, 23, 26



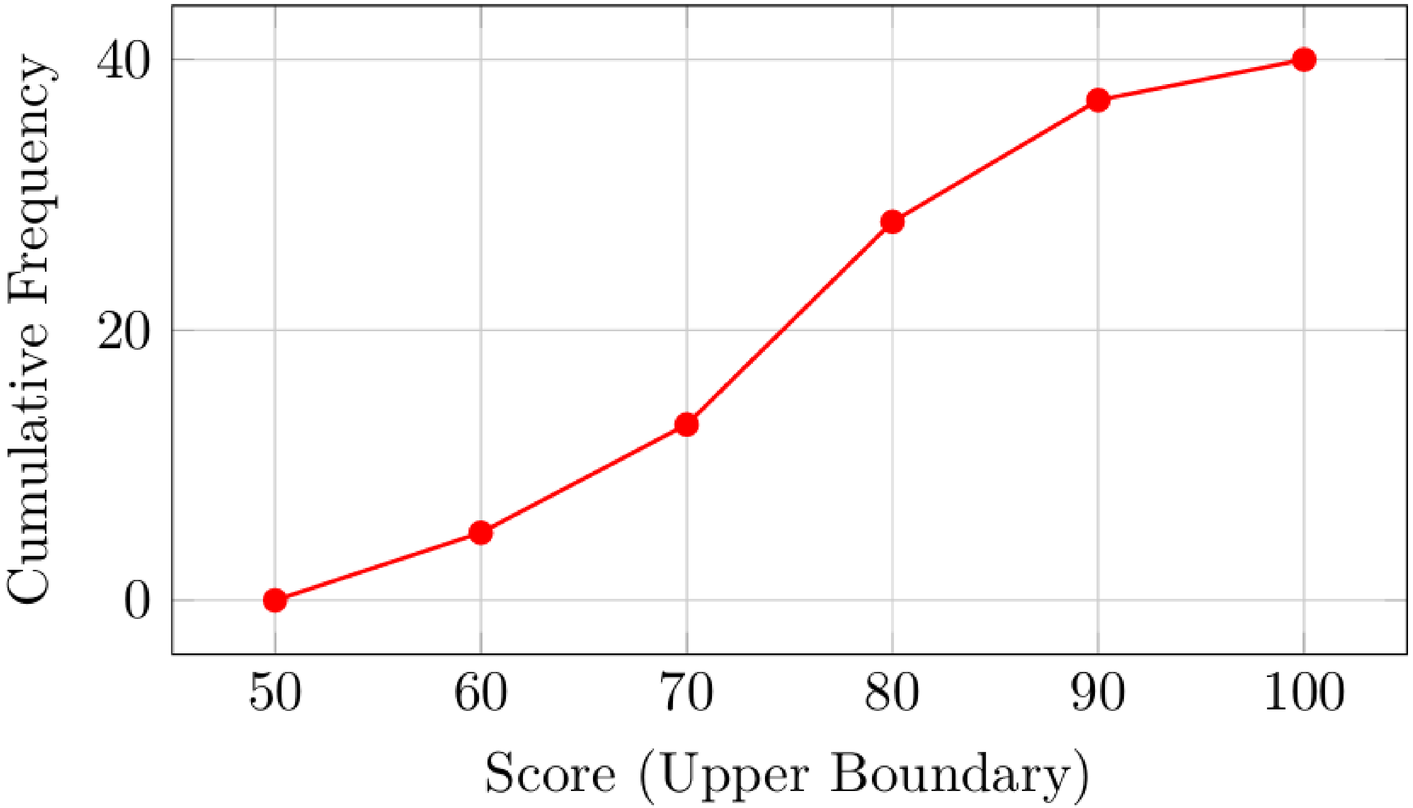
Ogives

An ogive displays cumulative frequencies. For a "less than" ogive, plot upper class boundaries on the x-axis and cumulative frequencies on the y-axis, then connect points with lines.

Score Range	Frequency	Upper Boundary	Cumulative Freq.
50-60	5	60	5
60-70	8	70	13
70-80	15	80	28
80-90	9	90	37
90-100	3	100	40

Ogives: An ogive displays cumulative frequencies.

Score Range	Frequency	Upper Boundary	Cumulative Freq.
50-60	5	60	5
60-70	8	70	13
70-80	15	80	28
80-90	9	90	37
90-100	3	100	40



Comparing Quartile Methods

np Method: n: total number of observations, p: desired quantiles
Calculate $n \times p$ where p is the quantile (0.25, 0.50, 0.75)
If $n \times p$ is an integer, take the average of the values at that position and the next one.
If $n \times p$ is not an integer, round up to the next integer.

Position Method:
Position =

$$\frac{p(n+1)}{100}$$

Where p is the percentile (25, 50 ,75)

Comparing with Odd n

Test scores for 11 students (sorted): 45, 52, 58, 63, 67, 71, 75, 79, 84, 88, 92

Position Method:

- Q_1 position: $\frac{25(11+1)}{100} = 3 \rightarrow Q_1 = 58$ (3rd value)
- Q_2 position: $\frac{50(11+1)}{100} = 6 \rightarrow Q_2 = 71$ (6th value)
- Q_3 position: $\frac{75(11+1)}{100} = 9 \rightarrow Q_3 = 84$ (9th value)

np Method:

- Q_1: $11 \times 0.25 = 2.75$ (not integer) $\rightarrow$ round up to 3 $\rightarrow Q_1 = 58$
- Q_2: $11 \times 0.50 = 5.5$ (not integer) $\rightarrow$ round up to 6 $\rightarrow Q_2 = 71$
- Q_3: $11 \times 0.75 = 8.25$ (not integer) $\rightarrow$ round up to 9 $\rightarrow Q_3 = 84$

Both Methods give **identical results** for this odd n case!

Comparing with Even n

12 students (Even): 45, 52, 58, 63, 67, 71, 75, 79, 84, 88, 92, 96

Position Method:

- Q_1 position: $\frac{25(12+1)}{100} = 3.25 \to$ interpolate
  $Q_1 = 58 + 0.25(63 - 58) = 59.25$
- Q_2 position: $\frac{50(12+1)}{100} = 6.5 \to$ interpolate
  $Q_2 = 71 + 0.5(75 - 71) = 73$
- Q_3 position: $\frac{75(12+1)}{100} = 9.75 \to$ interpolate
  $Q_3 = 84 + 0.75(88 - 84) = 87$

np Method:

- Q_1: $12 \times 0.25 = 3$ (integer!)
  $Q_1 = \frac{58+63}{2} = 60.5$
- Q_2: $12 \times 0.50 = 6$ (integer!)
  $Q_2 = \frac{71+75}{2} = 73$
- Q_3: $12 \times 0.75 = 9$ (integer!)
  $Q_3 = \frac{84+88}{2} = 86$

Methods give different results

Should method should you use?

Data Science Application

Which method should you use?

- The **np method** is what NumPy and most statistical software use by default
- The **position method** is commonly taught in statistics textbooks
- For large datasets ($n > 30$), the differences become negligible
- Always document which method you use for reproducibility

What we have covered

- ✅ Chebyshev's inequality as AI safety net
- ✅ Visualization Techniques
- ✅ Comparing different quartile methods

Thank You