Course: Probability & Statistics

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Module 5: Random Variables
Lesson 1: Random Variables

Reading Objectives:

After going through this reading, you will be able to define random variables and learn about the types of random variables with examples. You will also be able to introduce the concept of expectation and its properties.

Main Reading Section:

1. Random Variable

When a random experiment is performed, we are often not interested in all of the details of the experimental result but only in the value of some numerical quantity determined by the result. For instance, in tossing dice, we are often interested in the sum of the two dice and are not really concerned about the values of the individual dice. That is, we may be interested in knowing that the sum is 7 and not be concerned over whether the actual outcome was (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), or (6, 1). These quantities of interest that are determined by the result of the experiment are known as **random variables**.

Example 1. Suppose that an individual purchases two electronic components, each of which may be either defective or acceptable. In addition, suppose that the four possible results—(d, d), (d, a), (a, d), and (a, a)—have respective probabilities 0.09, 0.21, 0.21, and 0.49 [where (d, d) means that both components are defective, (d, a) means that the first component is defective and the second acceptable, and so on]. If we let X denote the number of acceptable components obtained in the purchase, then X is a random variable taking on one of the values 0, 1, and 2 with respective probabilities

$P\{X = 0\} = 0.09$

$P\{X = 1\} = 0.42$

$P\{X = 2\} = 0.49$

If we were mainly concerned with whether there was at least one acceptable component, we could define the random variable I as follows:

$$I = \{1 \ if \ X = 1 \, or \, 2 \ 2 \ if \ X = 0$$

2. Types of Random Variables

- Random variables whose set of possible values can be written either as a finite sequence x_1, x_2,....... x_n or as an infinite sequence x_1, x_2,....., are said to be **discrete**.
- However, there also exist random variables that take on a continuum of possible values. These are known as **continuous** random variables.

Definition 1: For a discrete random variable *X*, we define the **probability mass function** p(a) of *X* by p(a) = P{*X* = a}. The probability mass function p(a) is positive for at most a countable number of values of a. That is, if *X* must assume one of the values $x_1, x_2, \ldots$, then

p(x_i) > 0, for *i* = 1, 2,...

p(*x*) = 0, for all other values of *x*.

Since *X* must take on one of the values x_i, we have $\sum_{i=1}^{\infty} p(x_i) = 1$

Expectation

Definition 2:

- If *X* is a discrete random variable taking on the possible values x_1, x_2..., then the expectation or expected value of X, denoted by *E*[*X*], is defined by

$$E[X] = \sum_i x_i P\{X = x_i\}$$

- Suppose that *X* is a continuous random variable with probability density function *f*. Since, for a small value of *dx*, *f*(*x*)*dx* ≈ P{*x* < *X* < *x*+*ds*}. Hence, it is natural to define the expected value of *X* by

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

Properties of the expected value

- If a and b are constants, then $E[a X + b] = a E[X] + b$
- $E[X^n] = \{\sum_x x^n p(x), \quad if\ X\ is\ discrete\ \int_{-\infty}^{\infty} x^n f(x)\,dx, \quad if\ X\ is\ continuous$

Reading Summary

In this reading, you have learned about:

- Random variables and their types
- Expectation of the random variables with some properties