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## Probability and Statistics

Week 4 Live Session

Dr. Pushpendra Gupta

# Recap

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Chebyshev Inequality



Distribution Analysis



Data Visualization







Choosing the Right “Average”

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# Today's Focus

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-  Recap
-  Probability
-  Conditional Probability
-  Hands-on Visualization using Google Colab



# What is Probability?

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Probability measures how likely an event is to occur.

Expressed as a number between 0 and 1:

- ☐ 0: Impossible event
- ☐ 1: Certain event
- ☐ 0.5: Equal chance (e.g., coin flip)

# Core Concept

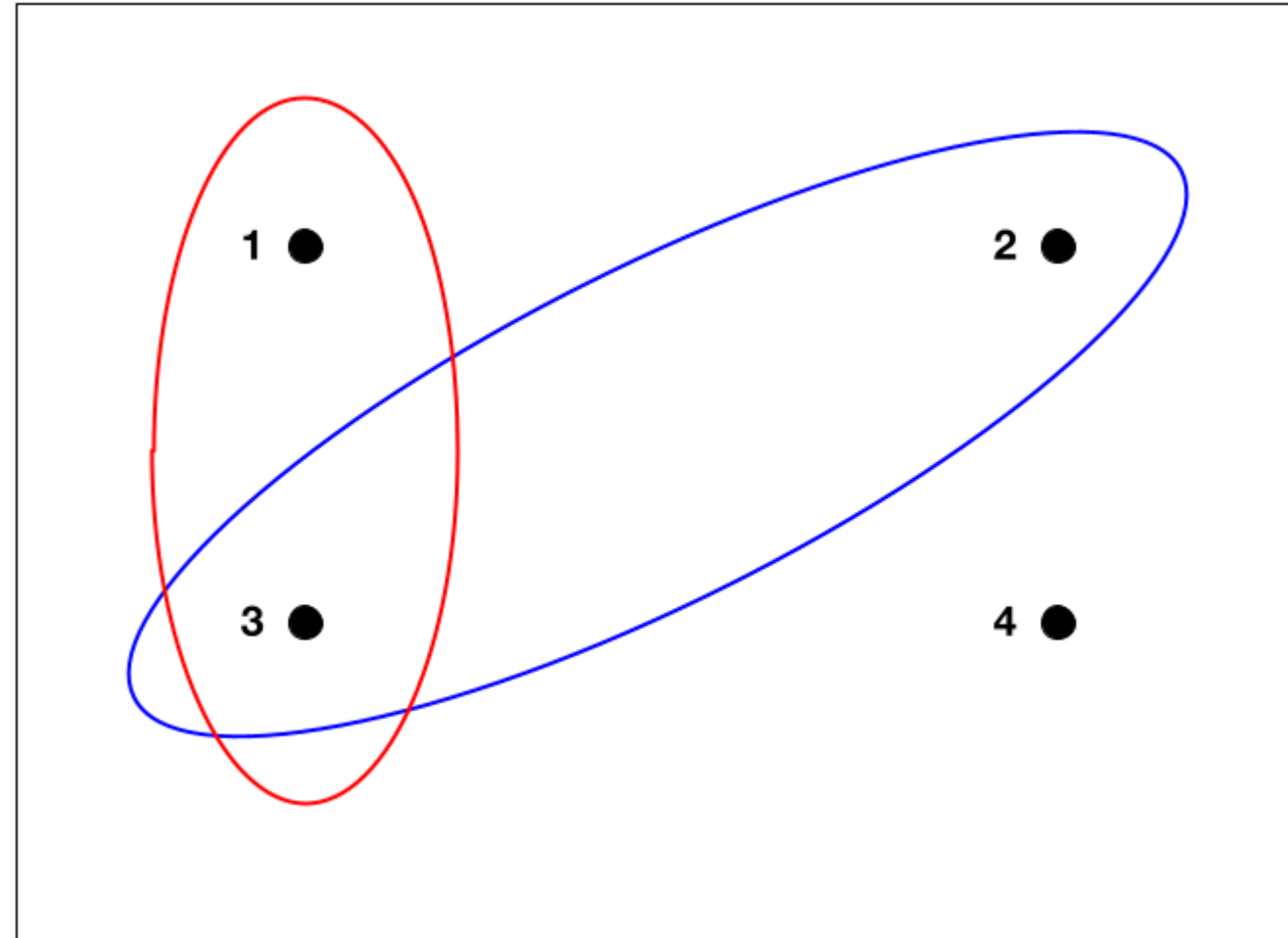
$$P(A) = \frac{\text{Number of Favorable}}{\text{Number of Possible Outcomes}}$$

**Experiment:** Action with observable results (e.g., rolling a die).

**Sample Space (S):** All possible outcomes (e.g., {1,2,3,4,5,6} for a die).

**Event (A):** Subset of sample space (e.g., rolling an even number).

**Favorable Outcome:** Results that satisfy the event.



[https://en.wikipedia.org/wiki/Sample\\_space](https://en.wikipedia.org/wiki/Sample_space)

# Simple Events: Rolling a Die

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$$P(A) = \frac{\text{Number of Favorable}}{\text{Number of Possible Outcomes}}$$

Experiment: Roll a fair six-sided die.

Sample Space: {1, 2, 3, 4, 5, 6} (Total outcomes: 6).

We'll calculate probabilities for specific events.



# Rolling Greater Than 4

Event: Roll a number  $> 4$ .

Favorable Outcomes: {5, 6} (2 outcomes).

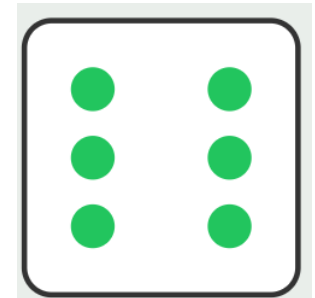
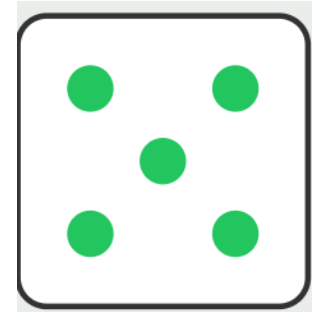
$$P(> 4) = \frac{2}{6} = \frac{1}{3} \approx 0.3333 \text{ (33.33\%)}$$

```
favorable_outcomes = 2
```

```
total_outcomes = 6
```

```
probability = favorable_outcomes / total_outcomes
```

```
print(f"Probability: {probability}")
```





# Rolling an Even Number or Rolling $\leq 3$

Event: Roll an even number.

Favorable Outcomes: {2, 4, 6} (3 outcomes).

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2} \approx 0.5 \text{ (50\%)}$$

Event: Roll a number  $\leq 3$

Favorable Outcomes: {1, 2, 3} (3 outcomes).

$$P(\leq 3) = \frac{3}{6} = \frac{1}{2} \approx 0.5 \text{ (50\%)}$$

# Probability of 1 to 6

Event: Roll a number between 1 and 6 (inclusive).

Favorable Outcomes: {1, 2, 3, 4, 5, 6} (6 outcomes).

$$P(1 \text{ to } 6) = \frac{6}{6} = 1 \text{ (100\%)}$$

# Part B – Probability Using Combinations

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Use combinations for selecting items (when order does not matter)

$$C(n, r) = C_r^n = \binom{n}{r} = \frac{n!}{r! (n - r)!}$$

$C(5, 2) = 10$  meaning there are 10 ways to choose 2 from 5.

# Drawing Colored Balls

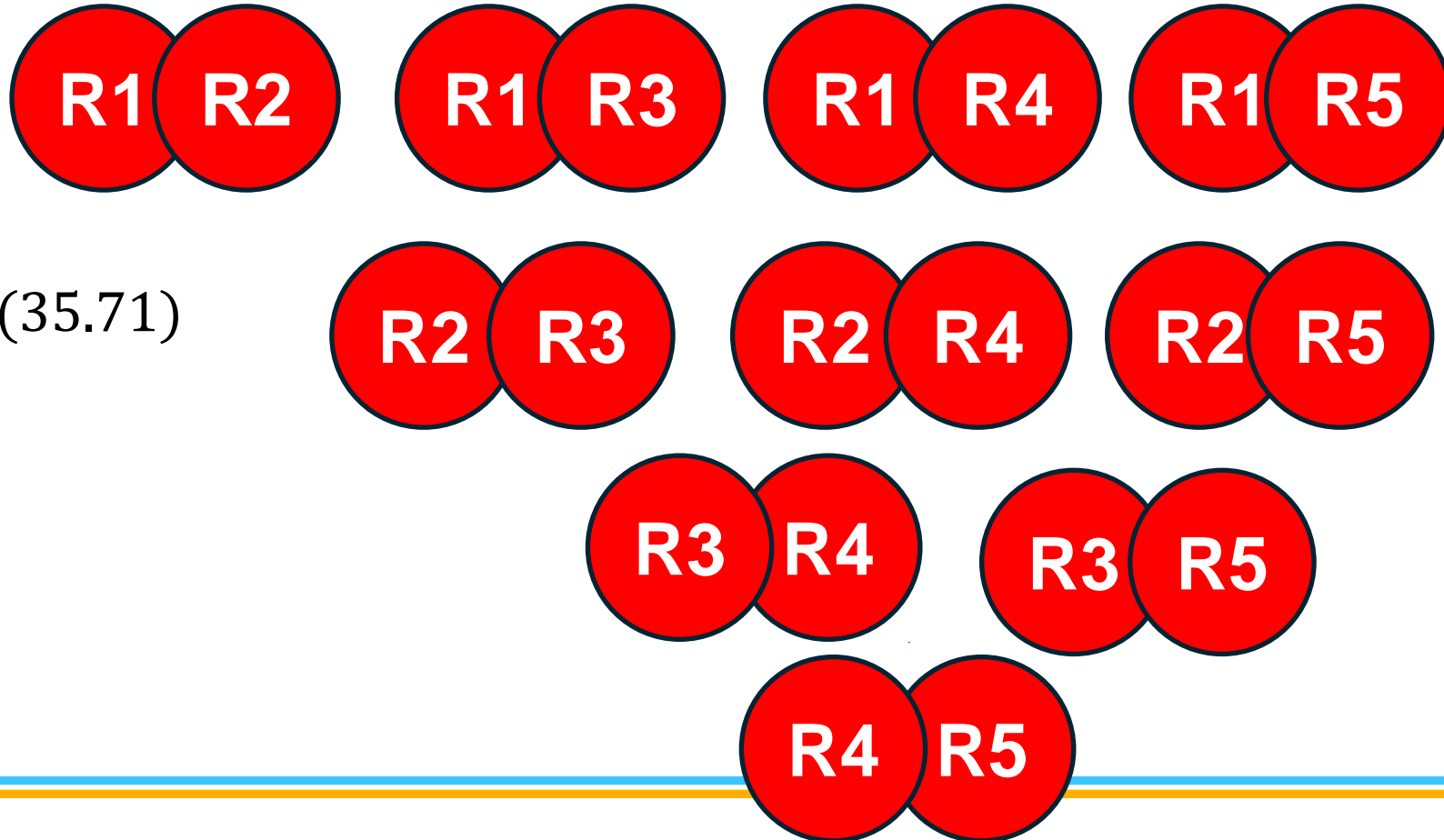
Bag: 5 red balls + 3 blue balls (Total: 8)

Event: Draw 2 balls at random. Probability both red?

Total ways:  $C(8,2) = 28$ .

Favorable  $C(5,2) = 10$ .

$$P(\text{both red}) = \frac{10}{28} = \frac{5}{14} \approx 0.3571 \text{ (35.71\%)}$$



# Conditional Probability

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Conditional probability is the probability of an event occurring given that another event has already occurred. We write this as

$$P(A|B),$$

which reads as "the probability of A given B."

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

# The Fruit Basket Problem

Imagine a basket contains 10 pieces of fruit:

2 Red Apples  
3 Green Apples  
1 Green Banana  
4 Yellow Bananas

Total: 10 fruits

Question 1: What is the probability of picking an apple?

$$\begin{aligned} P(\text{Apple}) &= \text{Number of Apples} / \text{Total Fruits} \\ &= (2 + 3) / 10 \\ &= 5/10 \\ &= 0.5 \text{ or } 50\% \end{aligned}$$

There's a 50% chance of picking an apple.



# The Fruit Basket Problem

Question 1: What is the probability of picking an apple?

$$P(\text{Apple}) = \frac{5}{10} = 0.5 \text{ or } 50\%$$

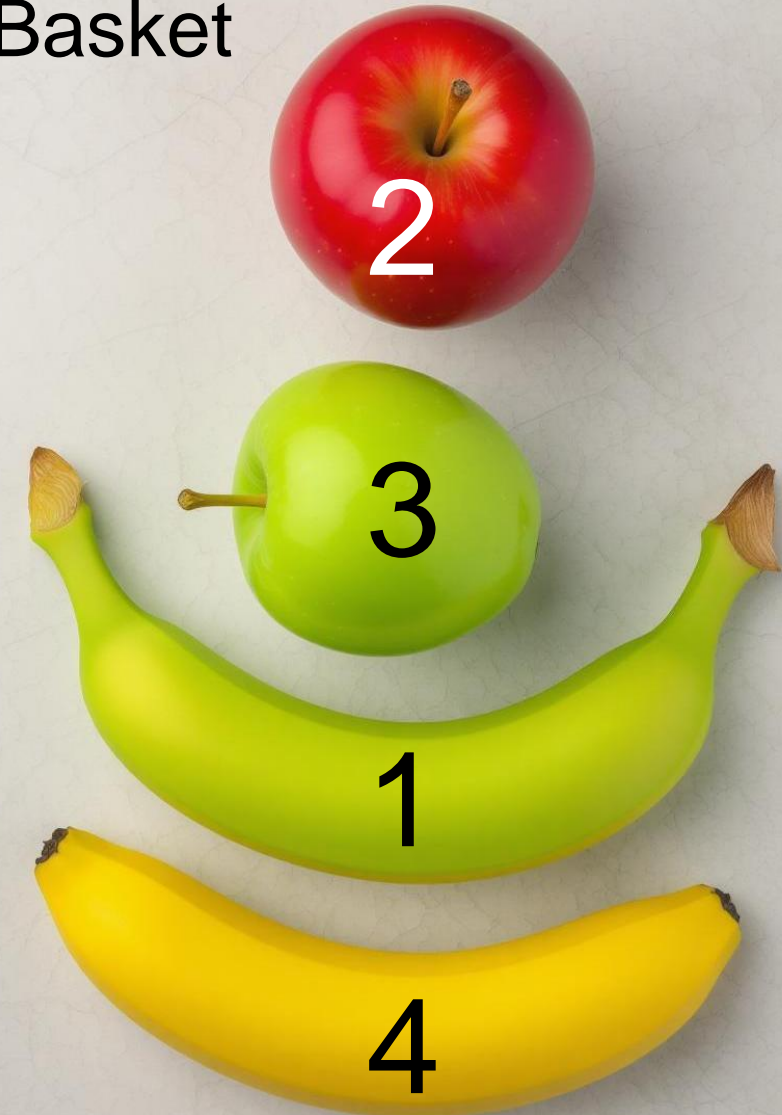
Question 2: Now You Know It's Green! Now, what is the probability that it's an apple?

Method 1: Now we only consider GREEN fruits: 3 Green Apples and 1 Green Banana

Total Green Fruits: 4

$$\begin{aligned} P(\text{Apple} \mid \text{Green}) &= \text{Number of Green Apples} / \text{Total Green Fruits} \\ &= 3 / 4 = 0.75 \text{ or } 75\% \end{aligned}$$

Basket





# The Fruit Basket Problem

Question 1: What is the probability of picking an apple?

$$P(\text{Apple}) = 50\%$$

Question 2: Now You Know It's Green! Now, what is the probability that it's an apple?

$$\text{Method 1: } P(\text{Apple} \mid \text{Green}) = \frac{3}{4} = 75\%$$

Method 2: Using Bayes'  $P(A|B) = P(A \cap B) / P(B)$

Where:

$A$  = Picking an Apple

$B$  = Picking a Green fruit

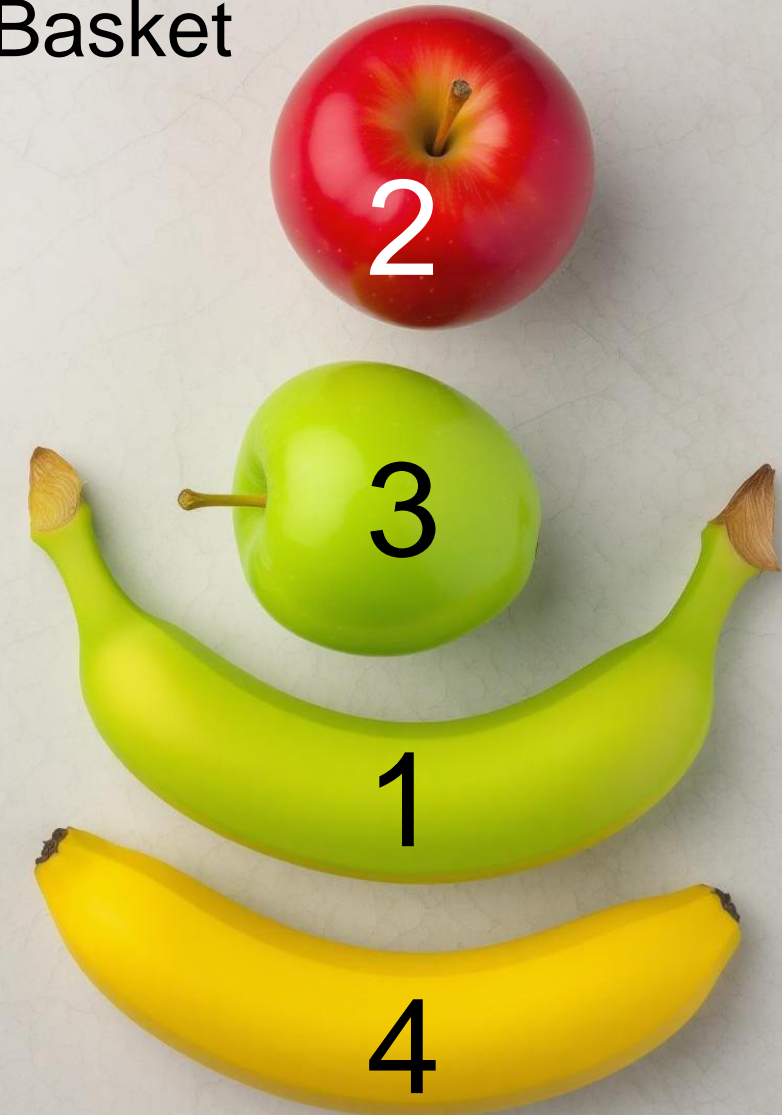
$A \cap B$  = Picking a Green Apple

$$P(A \cap B) = P(\text{Green Apple}) = 3/10 = 0.3$$

$$P(B) = P(\text{Green fruit}) = (3 + 1)/10 = 4/10 = 0.4$$

$$P(A|B) = 0.3 / 0.4 = 3/4 = 0.75 \text{ or } 75\%$$

Basket





# The Fruit Basket Problem

Question 1: What is the probability of picking an apple?

$$P(\text{Apple}) = 50\%$$

Question 2: Now You Know It's Green! Now, what is the probability that it's an apple?

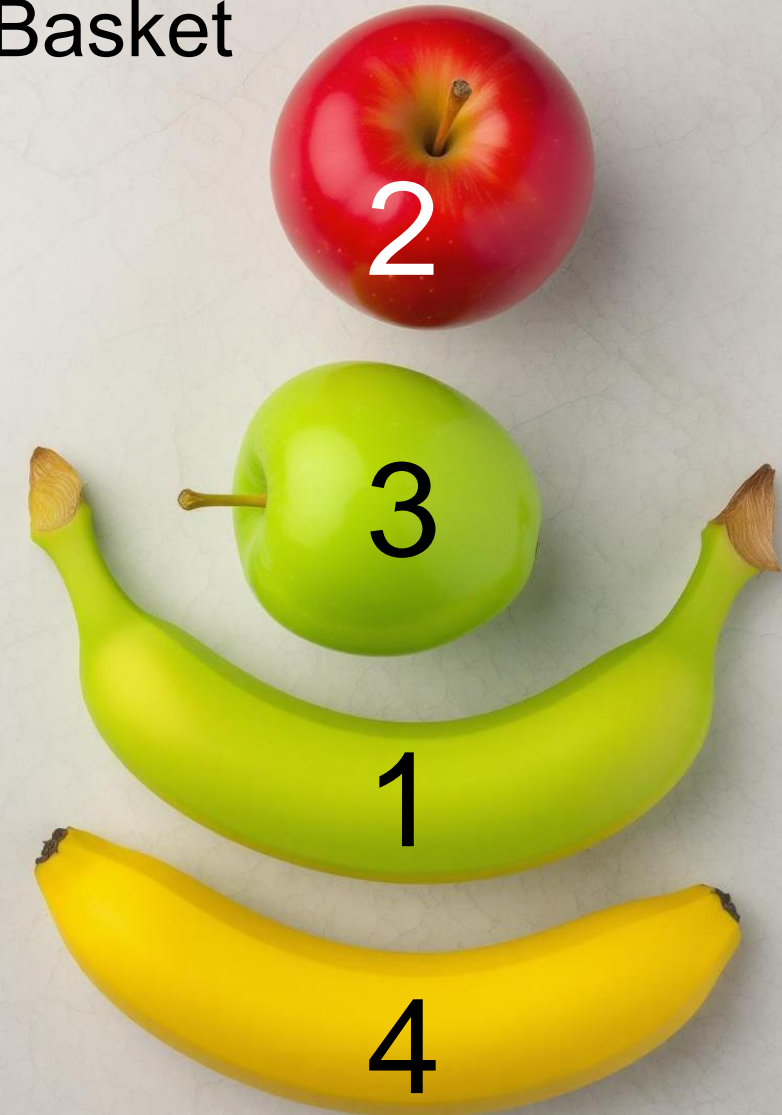
$$\text{Method 1: } P(\text{Apple} \mid \text{Green}) = \frac{3}{4} = 75\%$$

$$\text{Method 2: Using Bayes' } P(A|B) = P(A \cap B) / P(B)$$
$$P(A|B) = 0.3 / 0.4 = 3/4 = 0.75 \text{ or } 75\%$$

Before knowing the color:  $P(\text{Apple}) = 50\%$   
After knowing it's green:  $P(\text{Apple} \mid \text{Green}) = 75\%$

The probability **INCREASED** from 50% to 75%!  
**Why?** Because green apples are more common than green bananas in our basket. The information "it's green" makes it **MORE** likely to be an apple.

Basket



# The Fruit Basket Problem

Question 1: What is the probability of picking an apple?

$$P(\text{Apple}) = 50\%$$

Question 2: Now You Know It's Green! Now, what is the probability that it's an apple?

$$P(\text{Apple} \mid \text{Green}) = \frac{3}{4} = 75\%$$

The Power of Conditional Probability:

New information can **INCREASE** probabilities (apples became more likely- from **50% to 75%**)

New information can **DECREASE** probabilities (bananas became less likely- from **50% to 25%**)

# Example

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You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability 0.8; with water, it will die with probability 0.15. You are 90 percent certain that your neighbor will remember to water the plant.

- a. What is the probability that the plant will be alive when you return?
- b. If it is dead, what is the probability your neighbor forgot to water it?"

Given Information:

$$P(\text{plant dies} \mid \text{no water}) = 0.8$$

$$P(\text{plant dies} \mid \text{watered}) = 0.15$$

$$P(\text{neighbor remembers to water}) = 0.9$$

$$\text{Therefore, } P(\text{neighbor forgets}) = 0.1$$

# Part (a)

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a. What is the probability that the plant will be alive when you return?

We need to find  $P(\text{plant survives})$ .

Step 1: Identify the scenarios

The plant can survive in two ways:

- Neighbor remembers AND plant survives with water
- Neighbor forgets AND plant survives without water

# Part (a)

a. What is the probability that the plant will be alive when you return?

## Step 2: Calculate probabilities

**If watered:**  $P(\text{survives} \mid \text{watered}) = 1 - 0.15 = 0.85$

**If not watered:**  $P(\text{survives} \mid \text{no water}) = 1 - 0.8 = 0.2$

## Step 3: Use law of total probability

$$\begin{aligned} &P(\text{survives}) \\ &= P(\text{survives} \mid \text{watered}) \times P(\text{watered}) + P(\text{survives} \mid \text{not watered}) \times P(\text{not watered}) \\ &= (0.85 \times 0.9) + (0.2 \times 0.1) \\ &= 0.765 + 0.02 \\ &= \mathbf{0.785 \text{ or } 78.5\%} \end{aligned}$$

# Part (b)

If the plant is dead, what is the probability your neighbor forgot to water it?

We need to find  $P(\text{forgot} \mid \text{dead})$ . This is a Bayes' theorem problem.

Step 1: Use Bayes' theorem:  $P(\text{forgot} \mid \text{dead}) = P(\text{dead} \mid \text{forgot}) \times P(\text{forgot}) / P(\text{dead})$

Step 2: Find  $P(\text{dead})$

$$P(\text{dead}) = 1 - P(\text{survives}) = 1 - 0.785 = 0.215$$

Or calculated directly:

$$\begin{aligned} P(\text{dead}) &= P(\text{dead} \mid \text{watered}) \times P(\text{watered}) + P(\text{dead} \mid \text{not watered}) \times P(\text{not watered}) \\ P(\text{dead}) &= (0.15 \times 0.9) + (0.8 \times 0.1) \\ P(\text{dead}) &= 0.135 + 0.08 = 0.215 \end{aligned}$$

# Part (b)

If the plant is dead, what is the probability your neighbor forgot to water it?

Step 3: Calculate  $P(\text{forgot} \mid \text{dead})$

$$P(\text{forgot} \mid \text{dead}) = P(\text{dead} \mid \text{forgot}) \times P(\text{forgot}) / P(\text{dead})$$

$$P(\text{forgot} \mid \text{dead}) = (0.8 \times 0.1) / 0.215$$

$$P(\text{forgot} \mid \text{dead}) = 0.08 / 0.215$$

$$P(\text{forgot} \mid \text{dead}) = 0.372 \text{ or } 37.2\% \text{ (approximately } 16/43)$$

Answers

a) Probability plant is alive: 0.785 or 78.5%

b) Probability neighbor forgot (given plant is dead): 0.372 or 37.2%

Even though the plant is dead, there's only a 37.2% chance the neighbor forgot. Why? Because the neighbor remembers 90% of the time, and even when watered, the plant still has a 15% chance of dying. So most of the time when the plant dies, it's actually because it died despite being watered, not because the neighbor forgot.

# How to draw Conclusions/Insights

Suppose I am getting the following answer related to example shown during recorded session on Probability of cancer.

$$P(\text{cancer}) = 0.70$$

$$P(\text{cancer} \mid \text{elevated PSA}) = 0.822 \text{ or } 82.2\%$$

$$P(\text{cancer} \mid \text{normal PSA}) = 0.664 \text{ or } 66.4\%$$

What is the insight from the result?

If the test shows elevated PSA, the probability of cancer increases from 70% to 82.2%. If the test shows normal PSA, the probability of cancer decreases from 70% to 66.4%, but it's still quite high! This demonstrates why the PSA test is considered **unreliable**. Even with a normal test result, there's still a 66.4% chance of cancer in this case. The test provides some information but doesn't dramatically change the probability either way.



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