Course: Probability and Statistics

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Module 3: Elements of Probability
Lesson 1: Introduction to Probability

Topic: Introduction to Probability

Reading Objectives:

In this reading, you will understand the concept of probability, the sample space, and the events of an experiment. You will also be introduced to the basic concepts of set theory used in probability.

Main Reading Section:

Introduction

The concept of the probability of a particular event of an experiment is subject to various meanings or interpretations. In the subjective interpretation, the probability of an outcome is not considered a property of the outcome but rather a statement about the beliefs of the person who is quoting the probability concerning the chance that the outcome will occur.

For instance, if you think that the probability that it will rain tomorrow is .3 and you feel that the probability that it will be cloudy but without any rain is .2, then you should feel that the probability that it will either be cloudy or rainy is .5 independently of your individual interpretation of the concept of probability.

1. Sample space and events

Definition 1: Consider an experiment whose outcome is not predictable with certainty in advance. Although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known. This set of all possible outcomes of an experiment is known as the **sample space** of the experiment and is denoted by S.

Example 1. If the outcome of an experiment consists in the determination of the sex of a newborn child, then S = {g, b} where the outcome g means that the child is a girl and b that it is a boy.

Example 2. If the experiment consists of running a race among the seven horses having post positions 1, 2, 3, 4, 5, 6, 7, then S = {all orderings of (1, 2, 3, 4, 5, 6, 7)}. The outcome (2, 3, 1, 6, 5, 4, 7) means, for instance, that the number 2 horse is first, then the number 3 horse, then the number 1 horse, and so on.

Definition 2: Any subset E of the sample space is known as an **event.** That is, an event is a set consisting of possible outcomes of the experiment.

Example 3. In Example 1, if E = {g}, then E is the event that the child is a girl. Similarly, if F = {b}, then F is the event that the child is a boy.

Properties

- For any two events, E and F, of a sample space S, we define the new event EUF, called the **union** of the events E and F, to consist of all outcomes either in E or in F or in both E and F.
- For any two events, E and F, we may also define the new event EF, sometimes written as E∩F, called the **intersection** of E and F, to consist of all outcomes in both E and F.
- ∅ refers to an event consisting of no outcomes, also known as a **null event**.
- If EF = ∅, implying that E and F cannot both occur, then E and F are said to be **mutually exclusive**.
- For any event E, we define the event E^c, referred to as the **complement** of E, to consist of all outcomes in the sample space S that are not in E.
- For any two events, E and F, if all of the outcomes in E are also in F, then we say that E is contained in F and write E ⊂ F (or equivalently, F ⊃ E). Thus, if E ⊂ F, then the occurrence of E necessarily implies the occurrence of F. If E ⊂ F and F ⊂ E, then we say that E and F are equal (or identical), and we write E = F.
- We can also define unions and intersections of more than two events. In particular, the union of the events $E_1, E_2, \ldots, E_n$, denoted either by $E_1 \cup E_2 \cup \ldots \cup E_n$ or by $\cup_1^n E_i$ is defined to be the event consisting of all outcomes that are in E_i for at least one i = 1, 2,...,n. Similarly, for the intersections.

2. **Venn diagrams and the algebra of events**

Definition 3: The **Venn diagram** is a graphical representation of events that is very useful for illustrating logical relations among them. The sample space S is represented as consisting of all the points in a large rectangle, and the events E, F, G, . . ., are represented as consisting of all the points in given circles within the rectangle.

The operations of forming unions, intersections, and complements of events obey certain rules not dissimilar to the rules of algebra.

1. Commutative law E ∪ F = F ∪ E and E∩F =F∩E
2. Associative law (E ∪ F) ∪ G = E ∪ (F ∪ G) and (E∩F) ∩G = E∩ (F∩G)
3. Distributive law (E ∪ F) ∩G = E∩G ∪ F∩G and E∩F ∪ G = (E ∪ G) ∩ (F ∪ G)
4. De Morgan's laws $(E \cup F)^c = E^c \cap F^c$ and $(E \cap F)^c = E^c \cup F^c$

Reading Summary

In this reading, you have learned the following:

- The sample space and events of an experiment.
- Various properties for different events.
- Algebra of events using some well-defined laws.