**Course: Probability and Statistics**

**Faculty: Dr. Kota Venkata Ratnam**
**Module 3: Elements of Probability**
**Lesson 2: Axioms of Probability, Sample Spaces Having Equally Likely Outcomes**

**Reading Objectives:**

In this reading, you will gain insights into the axioms of probability with some propositions. You will also be introduced to the basic concepts of sample spaces having equally likely outcomes.

**Main Reading Section:**

## 1. Axioms of Probability

We shall suppose that for each event E of an experiment having a sample space S, there is a number, denoted by P(E), that is in accord with the following three axioms:

Axiom 1. $0 \leq P(E) \leq 1$

Axiom 2. $P(S) = 1$

Axiom 3. For any sequence of mutually exclusive events $E_1, E_2, \ldots\ldots$ (that is, events for which $E_i \cap E_j = \varnothing$ when $i \neq j$),

$$P\left(\cup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P\left(E_i\right), \qquad n = 1, 2, \ldots\ldots, \infty$$

We call P(E) the probability of the event E.

**Proposition 1**. $P(E^c) = 1 - P(E)$

**Proposition 2**. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

**Example 1**. A total of 28 percent of males living in Nevada smoke cigarettes, 6 percent smoke cigars, and 3 percent smoke both cigars and cigarettes. What percentage of males smoke neither cigars nor cigarettes?

**Solution**: Let E be the event that a randomly chosen male is a cigarette smoker, and let F be the event that he is a cigar smoker. Then, the probability this person is either a cigarette or a cigar smoker is

$P(E \cup F) = P(E) + P(F) - P(E \cap F) = .28 + .06 - .03 = .31$

Thus, the probability that the person is not a smoker is 1 − .31 = .69, implying that 69 percent of males smoke neither cigarettes nor cigars.

2. **Sample spaces having equally likely outcomes**

For a large number of experiments, it is natural to assume that each point in the sample space is equally likely to occur. That is, for many experiments whose sample space S is a finite set, say S = {1, 2,...,N}, it is often natural to assume that

$P(\{1\}) = P(\{2\}) = \cdots = P(\{N\}) = p$ (say)

Now it follows from Axioms 2 and 3 that

$1 = P(S) = P(\{1\}) + \cdots + P(\{N\}) = Np$

which shows that $P(\{i\}) = p = 1/N$

From this, it follows from Axiom 3 that for any event E, $P(E) = \frac{\textit{Number of points in E}}{N}$

In words, if we assume that each outcome of an experiment is equally likely to occur, then the probability of any event E equals the proportion of points in the sample space contained in E.

**Definition 1: Basic principle of counting**

Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2, then together, there are mn possible outcomes of the two experiments.

**Example 2.** Two balls are "randomly drawn" from a bowl containing 6 white and 5 black balls. What is the probability that one of the drawn balls is white and the other black?

Solution: If we regard the order in which the balls are selected as being significant, then as the first drawn ball may be any of the 11 and the second any of the remaining 10, it follows that the sample space consists of $11 \cdot 10 = 110$ points. Furthermore, there are $6 \cdot 5 = 30$ ways in which the first ball selected is white and the second black, and similarly, there are $5 \cdot 6 = 30$ ways in which the first ball is black and the second white. Hence, assuming that "randomly drawn" means that each of the 110 points in the sample space is equally likely to occur, then we see that the desired probability is

$$\frac{30+30}{110} = \frac{6}{11}$$

**Definition 2: Generalized basic principle of counting**

If r experiments that are to be performed are such that the first one may result in any of $n_1$ possible outcomes, and if for each of these $n_1$ possible outcomes, there are $n_2$ possible outcomes of the second experiment, and if for each of the possible outcomes of the first two experiments, there are $n_3$ possible outcomes of the third experiment, and if, ..., then there are a total of $n_1 \bullet n_2 \cdots n_r$ possible outcomes of the r experiments.

**Example 3.** Mr. Jones has 10 books that he is going to put on his bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Jones wants to arrange his books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

**Solution:** There are 4! 3! 2! 1! arrangements such that the mathematics books are first in line, then the chemistry books, then the history books, and then the language book. Similarly, there are 4! 3! 2! 1! possible arrangements for each possible ordering of the subjects. Hence, as there are 4! possible orderings of the subjects, the desired answer is 4! 4! 3! 2! 1! = 6912.

**Reading Summary**

In this reading, you have learned the following:

- The axioms of probability with some propositions.
- The sample space has equally likely outcomes and the basic principle of counting.