ELLIPTIC CURVES
Homework 3

8. Show that the plane curve $X^2 + Y^2 + Z^2 = 0$ is a nontrivial twisted form of the projective line $\mathbf{P}^1_{\mathbf{Q}}$ and determine an explicit cocycle representing its class in $H^1(\mathrm{Gal}(\overline{\mathbf{Q}}|\mathbf{Q}), \mathrm{Aut}(\mathbf{P}^1_{\overline{Q}}))$.

[*Hint:* This is a similar, but much easier, calculation to that in the example of a nontrivial element in a Ш(E) seen in class. You may want to simplify your life by working with the affine equation $x^2 + y^2 + 1 = 0$ and the affine line over **Q**.]

9. Let E be an elliptic curve over the finite field $\mathbf{F}_q$ of order q and let ℓ be a prime number not dividing q. Show that the group $H^1(\mathbf{F}_q, \overline{E}[\ell])$ is finite and in fact trivial for all but finitely may ℓ.

10. Let E be an elliptic curve defined over a finite extension $K|\mathbf{Q}_p$, and let $\ell \neq p$ be a prime number. Show that if the inertia subgroup I in $\mathrm{Gal}(\overline{K}|K)$ fixes some element in the Tate module $T_\ell(E)$, then E has good or multiplicative reduction.

Remark: The converse also holds. In fact, it E has good or multiplicative reduction, it can be shown that I acts on $T_\ell(E)$ by unipotent matrices (i.e. upper triangular 2×2 matrices with 1 in the diagonal). In these cases the standard terminology is that E has *semistable* reduction.