



# Teacher salaries and teacher aptitude: An analysis using quantile regressions

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## ABSTRACT

This study investigates the relationship between salaries and scholastic aptitude for full-time public high school humanities and mathematics/sciences teachers. For identification, we rely on variation in salaries between adjacent school districts within the same state. The results indicate that teacher aptitude is positively correlated with teacher salaries with an elasticity point estimate of 0.132. However, using quantile regressions, we find the elasticity estimates form an inverted U-shape across the scholastic aptitude distribution, and that higher aptitude teachers are more profoundly affected by the percentage of students eligible for free lunch and local street crime, while lower aptitude teachers are more profoundly affected by local education support. Furthermore, studying mathematics/sciences teachers, we find that while the elasticity estimates maintain an inverted U-shape, scholastic aptitude is not correlated with changes in salaries for the lower 40 percentiles of the aptitude distribution.

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## 1. Introduction

An increased emphasis on teachers has been seen in the education reform debate following the passing of the No Child Left Behind Act of 2002 and the Race to the Top Fund of 2009, with particular concern over the quality of mathematics and sciences instruction. Hiring better qualified or higher aptitude teachers can be particularly difficult due to salary schedules which equalize pay across all teachers regardless of aptitude or teaching subject.<sup>1</sup> The literature that estimates the relationship between

financial incentives and measures of teacher aptitude all find positive correlation. The purpose of this paper is to empirically test whether this correlation is non-linear across the teacher aptitude distribution, and whether mathematics/sciences teachers are differentially affected.<sup>2</sup>

The teacher labor supply literature has shown that 'quality' teachers respond to financial incentives.<sup>3</sup> The empirical strategy of this literature is typically to control for teacher, school, district, and community characteristics and test whether changes in some financial incentives have a significant impact on whether either a certain type of teacher remains in teaching, or on some teacher-level characteristics. Studying local labor markets, Figlio (1997)

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<sup>1</sup> A salary schedule is a pay grade matrix that dictates teachers' salaries by years of experience and educational attainment. The schedule may also provide salary reductions due to probationary statuses or any additional pay to certain types of teachers, i.e., compensation to retain teachers in areas of shortage. The schedule may be self-imposed by schools or may be the result of bargaining between teachers' unions and school boards.

<sup>2</sup> Quantile regression analysis has become more commonplace in the education literature to investigate non-linear returns. Examples include Eide and Showalter (1998), Levin (2001), and Husain and Millimet (2009).

<sup>3</sup> For instance, see Levinson (1988), Ballou (1996), Figlio (1997), Podgursky, Monroe, and Watson (2004), Imazeki (2005), Clotfelter, Glennie, Ladd, and Vigdor (2008), Jackson (2009), and Player (2009).

finds that a 1% increase in a school district's salary is associated with 0.75% higher probability of recruiting a teacher from a selective college. Studying the national labor market, [Player \(2009\)](#) finds that teachers who graduate from the most selective undergraduate institutions have salaries that are between 7% and 14% higher than those who graduate from the least selective colleges. While these findings indicate that the elasticity of supply of teacher aptitude with respect to wages is positive, no study has yet to investigate the correlation along the aptitude distribution.<sup>4</sup> One natural empirical strategy to investigate these nonlinearities is using a quantile regression framework.

The main reason why the literature has avoided quantile regression analysis is due to the coarse measure of scholastic aptitude. Severe data limitations on teachers' actual scholastic aptitude scores, such as their ACT or SAT scores, force researchers to construct teacher aptitude proxies such as the selectivity of teachers' undergraduate college (see [Ballou, 1996](#); [Figlio, 1997](#); [Jackson, 2009](#); [Player, 2009](#)).<sup>5</sup> The selectivity of undergraduate college proxy does not take into consideration majors of these graduates in the aptitude ranking, nor does it differentiate aptitude beyond a 2–3 point Lickert scale. In this paper, we construct an extensive college-major aptitude index to use as a proxy aptitude measure for teachers. The aptitude index is based on percentile ranking in the distribution of college students' scholastic aptitude scores. This aptitude proxy refinement decreases measurement error and permits quantile regression analysis to be conducted.

In a related literature, the local teacher labor market literature has shown that when salary is rigid across teachers, such as one district in a large metropolitan area, teachers sort into schools based on school or district characteristics. Teachers with more experience and degrees from more competitive colleges sort into schools in communities with higher per capita income, by the percent of non-white students in the school, and by the percent of student eligible for free lunch (see [Ehrenberg & Brewer, 1994](#); [Lankford, Loeb, & Wyckoff, 2002](#); [Podgursky et al., 2004](#)). These results have been confirmed by [Jackson \(2009\)](#) who uses post-desegregation data to find that schools that repatriated black students incurred a significant decrease in teacher experience, level of teachers' degrees, and competitiveness of teachers' college. [Bonesrønning, Falch, and Strøm \(2005\)](#) studying Norway also find that when wages are rigidly structured, teachers sort themselves into schools by workload. Studying the labor supply of teachers, [Engel and Jacob \(2011\)](#) show that the demographic characteristics of schools have significant effects on the number of applicants per vacancy, with the expected direction. Interestingly, even the size of a district can have an effect on teacher sorting. [Heutel \(2009\)](#), using a tournament model, demonstrates that higher quality teachers will

accept equivalent pay to low quality teachers if they have higher probabilities of obtaining administrative positions. Research has also found that teachers sort across school types with better academic credentialed teachers sorting into private and charter schools rather than traditional public schools (see [Podgursky, 2008](#)).

Teacher aptitude would be of little concern if teachers had no effect on student achievement. In recent years, there has been an abundance of papers that demonstrate that teachers do, in fact, matter.<sup>6</sup> [Ferguson \(1991\)](#) and [Ferguson \(1996, chap. 8\)](#) finds significant positive effects of teacher test scores on student test scores in Texas and Alabama schools, respectively. Similarly, [Strauss and Sawyer \(1986\)](#) find that a 1% increase in the standardized test scores of teachers increases the pass rates of North Carolina high school students by 5% on math and reading proficiency tests. [Ehrenberg and Brewer \(1994\)](#), using national data, find that the quality of a teacher's undergraduate institution is highly related with student test outcomes and that a one category increase in the selectivity of a teacher's institution is associated with a 1–2% increase in student test scores.<sup>7</sup> [Aaronson, Barrow, and Sander \(2007\)](#) examine 9th-grade math teachers in Chicago Public Schools and find an effect between observable teacher characteristics and student outcomes, with one of the strongest effects coming from a teacher's undergraduate major. Math and science majors were found to have a positive effect on math scores while education majors had a negative effect.<sup>8</sup> For a general survey of the literature linking teacher academic ability and student achievement, [Greenwald, Hedges, and Laine \(1996\)](#) and [Hanushek \(1981, 1986\)](#) provide excellent summaries of previous findings. These surveys generally conclude that of all measurable school and teacher characteristics, academic ability of teachers has the largest effect on student outcomes. As [Hanushek \(1981\)](#) states, “[t]he only relatively consistent finding is that ‘smarter’ teachers seem to do better in terms of student achievement.”

To study the relationship between salaries and teacher aptitude, we focus on full-time public high school teachers who teach in either the humanities, mathematics, or the sciences. We focus on these particular subjects based on the above cited literature. To identify this relationship, we rely on between-district salary variation while controlling for as many school, district, and community characteristics as possible, and also use fixed effects to control for unobservables. The empirical results of this paper indicate that lifetime teacher salary is positively correlated with teacher aptitude with an elasticity point estimate of 0.132 across all teachers. However, using quantile regressions, we find that the elasticity estimates form an inverted U-shape across the scholastic aptitude distribution, with elasticity

<sup>4</sup> Some work, such as [Podgursky et al. \(2004\)](#), [Jackson \(2009\)](#), and [Gilpin \(2011\)](#), have investigated whether financial incentives have differential effects across teaching subjects, teachers of various race, and gender.

<sup>5</sup> One notable exception is [Podgursky et al. \(2004\)](#) who use teachers' actual ACT scores. However, their data is limited to one state, Missouri, with little variation in salary schedules across districts. Thus, they do not investigate the relationship between salaries and teacher aptitude.

<sup>6</sup> In the news, an Ohio mother was convicted of falsifying her residency records to enroll her child in a neighboring school district. Copley-Fairlawn School District indicated that she was cheating because her daughter received a quality education without paying taxes to fund it (see [Canning & Tanglao, 2011](#)).

<sup>7</sup> Further examples include [Hanushek, 2011](#); [Loeb & Page, 2000](#); [Rockoff, 2004](#); [Summers & Wolfe, 1977](#); [Winkler, 1975](#) who find similar positive effects on student achievement.

<sup>8</sup> Similar results are found in [Clotfelter, Ladd, and Vigdor \(2007\)](#).

estimates of 0.052, 0.152, and 0.108 for the 25th, 50th, and 75th percentiles. We also find that higher aptitude teachers are more profoundly affected by the percentage of students eligible for free lunch and local street crime, while lower aptitude teachers are more profoundly affected by local education support. Furthermore, in studying mathematics/sciences teachers, we find that while the elasticity estimates maintain an inverted U-shape, scholastic aptitude is not correlated with changes in salaries for those ranked in the lower 40 percentiles of aptitude.

The paper is organized as follows. Section 2 develops a theoretical model that frames the identification of the correlation between salaries and teacher quality. Section 3 provides information on the data construction and summary statistics while Section 4 provides the empirical strategy. Section 5 contains the main empirical results and Section 6 concludes.

## 2. The theoretical model

In order to provide an empirical framework in which to examine the relationship between salaries and teacher aptitude, we first analyze a theoretical model on the teacher staffing decisions of high schools. The basic structure utilized here is similar to that of Gilpin and Kaganovich (2012). A school's objective is to produce the highest per student education quality given their budget constraint.<sup>9</sup> For simplicity, we abstract from teaching/non-teaching input decisions, focusing solely on the teacher input decisions, and assume that schools combine humanities and mathematics/sciences teachers to educate students. Schools set a salary schedule for all teachers regardless of teaching subject, with agreement from the teachers' union if present, and choose the quantity of humanities and mathematics/sciences teachers to hire.<sup>10</sup> The salary schedule also imposes an additional implicit constraint: schools no longer bargain with individual teachers for their specific salary. Rather, schools hire bundles of humanities and mathematics/sciences teachers based on a set salary schedule. Instead of modeling the entire salary schedule structure, we simplify the analysis by assuming one salary,  $w$ , that represents the average salary of teachers in a particular school. The (teacher input) budget constraint of a school is

$$B = w(a_h)Q_h(a_h) + w(a_s)Q_s(a_s) \quad (1)$$

and the salary schedule constraint across subjects is

$$w(a_h) = w(a_s) = w \quad (2)$$

where  $w(a_k)$  are the market wage rates for aptitude  $a_k$  exogenously given to the school by the labor markets, while  $Q_k(a_k)$  is the quantity of teachers hired with scholastic aptitude  $a_k$  for  $k = \{h, s\}$ . The index,  $k$ , represents the teaching subject, where  $h$  is humanities subjects and  $s$  is mathematics/sciences subjects. The school's budget,  $B$ , is taken

as given. Schools recognize that the salary schedule will have differential effects across teaching subjects depending on the per subject wage elasticity of aptitude supplied. The salary schedule, through the budget constraint (1), dictates the *per subject* teacher aptitude for the school. Thus, by setting the salary schedule, schools implicitly choose the aptitude of humanities and mathematics/sciences teachers hired.

The school maximizes education quality by adjusting the resources expended on teachers in both subject areas. The teacher inputs can be increased either by hiring more teachers in a given subject of similar aptitude, or by hiring higher aptitude teachers. By postulating an explicit education production function, the effects of a salary schedule on the quantity and aptitude of humanities and mathematics/sciences teachers in a school can be recovered. The per student education quality production function is specified as<sup>11</sup>

$$E = (a_h^{\beta_h} Q_h^{1-\beta_h})^{\alpha_h} (a_s^{\beta_s} Q_s^{1-\beta_s})^{\alpha_s} \quad (3)$$

Education quality is assumed to be increasing but diminishing in all inputs. Assuming that maximizing education quality requires both humanities and mathematics/sciences teachers, e.g., the graduation rate or college attendance rate, this restricts  $\alpha_k$  for  $k = \{h, s\}$  to be greater than 0, i.e., the return on humanities and mathematics/sciences teachers (quantity and aptitude) are essential and somewhat non-substitutable. Second, there exists a quantity–aptitude production trade-off for both subjects.<sup>12</sup>

In addition to the school's hiring decisions, we also model individuals' employment choices. Individuals with aptitude  $\{a_h, a_s\}$  face a decision whether to teach or to work outside of teaching and, if they choose to teach, which subject and school. Thus, potential teachers face the following problem

$$\max \{U_j(w_j, l_j, v_j) \quad \forall j \in J, U_0\}$$

where  $U_j$  is the indirect utility of a teacher offered a job at school  $j$ , and  $J$  is the set of all offers. The indirect utility received from each teaching job is a function of the salary,  $w_j$ , and working conditions,  $X_j$ .  $U_0$  is the indirect utility received from working outside of teaching. Aggregating individuals' choices yield the wage elasticities of aptitude

<sup>9</sup> While the unit of analysis is the school level, little is lost by assuming a district-level analysis.

<sup>10</sup> It is anticipated that teachers' unions will push wages up for a given set of teachers. However, this effect is unlikely to be differential across teaching subjects.

<sup>11</sup> To retrieve analytic results, we assume the flexible Cobb–Douglas production function. Indeed, Uzawa (1962) makes clear to obtain analytic solutions, 4-factor CES production functions require either all elasticities between pairs of factors be identical, or if any differ, they all must equal each other and all remaining elasticities must be unity. To allow more flexibility on the factor returns, we assume a Cobb–Douglas functional form that allows the pairs of parameters to be different from each other.

<sup>12</sup> Much empirical education literature has analyzed the factors of student performance estimating the returns to either quality or quantity of teachers while attempting to control the other variable. For example, Aaronson et al. (2007), Clotfelter et al. (2007), Rivkin, Hanushek, and Kain (2005), and Goldhaber and Anthony (2007) estimate the effect of teacher characteristics on student achievement while Angrist and Lavy (1999), Krueger and Whitmore (2001), Krueger (1999), and Jepsen and Rivkin (2002) estimate how class size affects student achievement. All find positive effects of teacher quantity or quality on student achievement.

supplied for a school. For simplicity, we posit linear supply functions:

$$a_h = m_h w - b_h, \quad a_s = m_s w - b_s \quad (4)$$

where  $b_k$  and  $m_k$  for  $k = \{h, s\}$  are parameters. The parameters set the lower bound of aptitude supplied as well as the gradient. The supply-side of this matching model assumes the quantity supplied at each aptitude level is met or is in excess above the wage floor value  $b_k m_k^{-1}$ . This assumption is not critical to the analysis since the general labor markets dictate the supply curve and there is ample evidence that there is excess supply in the teacher labor market (see Hanushek, 2011). In what follows, we assume the salary level set by the schools are all above the wage floors.

Given the framework outlined above, the market for teachers is characterized by a complex set of decisions by school administrators and individuals. However, the two-sided job match model does allow some insight into the effects of imposing a uniform salary schedule across subjects. Given the wage elasticities of supply, we now define the school's maximization problem using Eqs. (1)–(4):

$$\max_{w, Q_s, Q_h} (a_h^{\beta_h} Q_h^{1-\beta_h})^{\alpha_h} (a_s^{\beta_s} Q_s^{1-\beta_s})^{\alpha_s} \quad (5)$$

s.t. (4) and

$$B = w(Q_h + Q_s)$$

The solution to this particular problem can be solved for analytically.<sup>13</sup> More insightful are the equilibrium hiring conditions between humanities and mathematics/sciences teachers:

$$Q_h = \frac{\alpha_h(1-\beta_h)}{\alpha_s(1-\beta_s)} Q_s \quad (6)$$

$$a_h = m_h m_s^{-1} a_s - b_h + b_s m_s^{-1} m_h$$

The above equations make it clear that the relative quantities of teachers between humanities and mathematics/sciences subjects are dependent on the education production parameters, and in particular, the return on investing an additional teacher resource in each subject,  $\alpha_k$ , and the relative returns to additional quantities of teachers in both subjects,  $1 - \beta_k$ . It is also clear from the above equations that the relative quantities of humanities and mathematics/sciences teachers are independent of the wage structure.<sup>14</sup> Secondly, due to the salary schedule equalizing salaries across subjects, parameter differences between the wage elasticities of aptitude supplied drive observed differences in the hiring of teachers across subjects with respect to aptitude. Specifically, the differences

in the relative parameter magnitudes of  $m_s/m_h$  and  $b_s/b_h$  drive differences in aptitude between subjects within a school. Lastly, the above equations also demonstrate that the salary schedule,  $w$ , is the school's demand for aptitude. Each school's demand for teacher aptitude is perfectly elastic, indicating that schools are willing to accept any aptitude for a given salary and permit the supply-side of the market to determine the aptitude of teachers hired in equilibrium.<sup>15</sup>

Eq. (4) also makes clear that depending on the salary and the wage elasticities of aptitude supplied, the aptitude of humanities teachers hired may be greater than, equal to, or less than that of mathematics/sciences teachers. Assuming that  $\varepsilon_{A_s, w_s} < \varepsilon_{A_h, w_h}$ , where  $\varepsilon_{A_s, w_s}$  is the wage elasticity of aptitude supplied for mathematics/sciences subjects and  $\varepsilon_{A_h, w_h}$  is the wage elasticity of aptitude supplied for humanities subjects, Eq. (6) indicates that the aptitude of mathematics/sciences teachers hired will be lower than the aptitude of humanities teachers hired.<sup>16</sup> This is graphically demonstrated in Fig. 1 for School 1 offering salary  $w_1$ . The *per subject* equilibrium aptitudes of teachers hired,  $\{a_{h_1}, a_{s_1}\}$ , are identified by the intersections of supply and demand in the humanities and mathematics/sciences teaching subjects' markets. Furthermore, administrators optimally hire mathematics/sciences teachers of lesser aptitude than those of humanities teachers.<sup>17</sup>

We now place this single school's maximization problem within an array of schools that compete in the general teacher labor market. Given identical work environments and model parameters, all schools will have similar, per student, quantities of humanities and mathematics/sciences teachers. Secondly, level differences in salaries across schools cause differences in the aptitude of teachers hired between schools as well as between teaching subjects. This is graphically depicted for two schools in Fig. 1. For School 1, offering  $w_1$  hires teachers with aptitudes  $a_{s_1}$  and  $a_{h_1}$ , respectively. Similarly, School 2 offering  $w_2$  hires teachers with aptitude  $a_{s_2}$  and  $a_{h_2}$ , respectively. As indicated by Fig. 1, between schools with similar working conditions, as salaries increase, the aptitude of all teachers hired rises, and the aptitude of humanities teachers rises faster than mathematics/sciences teachers. Lastly, the change in aptitude with respect to changes in salary may be non-linear depending on the shape of the supply curves.

This analytical framework provides empirically testable hypotheses concerning the elasticities of aptitude with respect to wages. The first is whether the correlation

<sup>13</sup> See Appendix B for full details.

<sup>14</sup> Given that class size and teaching loads are perfectly observable, it would seem that these analytical results on teacher quantity across teaching subject could be tested empirically. However, the enforcement of 'Equal Pay for Equal Work' typically constrains workloads to be equal across subjects regardless whether it is optimal to do so. Thus, empirically, it is difficult to identify whether quantities are similar due to the parameter values causing optimal quantities across subject to be similar or whether hiring practices are constrained due to institutional features. Indeed, initial regressions using class size and other workload characteristics indicate there is no statistically significant difference between humanities and mathematics/sciences teachers' workloads.

<sup>15</sup> Ballou (1996) emphasizes the lack of competitive pressures in public education permit applicants with weaker academic records to be hired. Our model reflects this with a perfectly elastic demand.

<sup>16</sup> While earnings tend to be larger for higher aptitude mathematics/sciences individuals than humanities individuals in the non-education workforce, this may not be true for individuals who have entered the education sector. Mathematics/sciences teachers may have fewer opportunities outside of teaching than humanities teachers given the type of skills to teach in mathematics/sciences versus the type of skills to teach in the humanities.

<sup>17</sup>  $b_k m_k^{-1}$  defines the lower bound for which the school may hire individuals of sufficient aptitude, i.e., the lower bound may be considered the salary required to successfully hire a teacher of sufficient quality such as a college graduate.

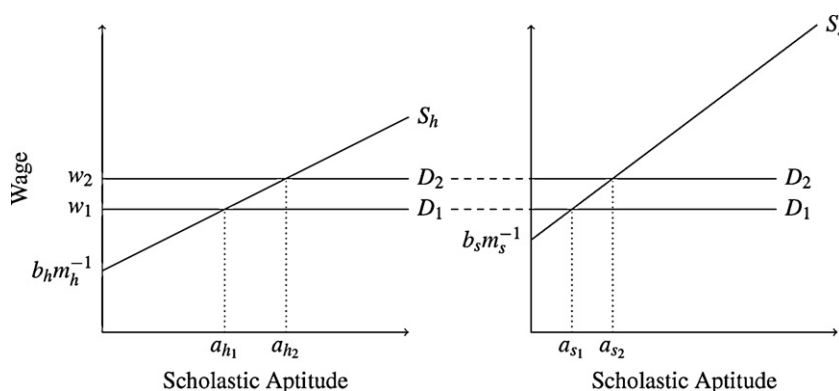


Fig. 1. Schools' optimal hiring strategies.

between teacher salaries and teach aptitude is non-linear. The second is whether mathematics/sciences teachers are differentially affected. By controlling for teacher, school, and community characteristics that shift the supply curve, we can identify the relationship between salary and aptitude through changes in the salary levels between schools. We acknowledge that identification of this relationship requires a stable supply curve and that there are many unobservable influences that may upward bias the elasticity estimate. We discuss this issue thoroughly along with the empirical specification in the ensuing sections.

### 3. Data

The primary data for this study comes from the 1999 to 2000, 2003 to 2004, and 2007 to 2008 restricted-access versions of the *Schools and Staffing Survey* (SASS) conducted by the *National Center for Education Statistics* (NCES). This survey incorporates questionnaires from roughly 50,000 teachers in 10,000 public schools every four years. The SASS data contain district-level salary schedule data along with several teacher, school, school district, and community characteristics.

The variable of interest is the scholastic aptitude of teachers. Scholastic aptitude is defined as a teacher's percentile in the overall scholastic aptitude distribution, among all college students, either ACT or SAT. To construct scholastic aptitudes, we first construct the college students' aptitude distribution by standardizing all undergraduate college students' ACT and SAT scores from six rounds (1989 to 1990, 1992 to 1993, 1995 to 1996, 1999 to 2000, 2003 to 2004, 2007 to 2008) of restricted-access versions of the *National Post-Secondary Aid Survey* (NPSAS).<sup>18</sup> The standardization of scholastic scores converts raw ACT and SAT scores into percentiles and forces the mean aptitude of all college students to be the 50th percentile. Each teacher's scholastic aptitude score is then converted to percentile within the aptitude distribution of all college students.

Since the SASS only provides teachers' majors as well as the undergraduate institutions and not their actual ACT and SAT scores, we impute their scores based on

the average scholastic aptitude of the college students who also majored with them at their institution.<sup>19</sup> As a measurement of scholastic aptitude, this imputed value provides better precision than using either the college selectivity or college ranking indices such as those found in the literature.<sup>20</sup> Thus, changes in teacher aptitude in high schools occur from teachers of lower ranking universities or majors being hired. By construction, our scholastic measurement has the advantage of not mixing scholastic scores across majors as there are significant differences in the aptitude of students across majors within the same university. Indeed, the average aptitude of college students majoring in education is 38.661 percentiles compared to the average aptitude of college students majoring in mathematics/sciences, 64.184 percentiles. While our measure of scholastic aptitude reduces measurement error by using university-major scholastic scores rather than university-wide scholastic scores, there is still substantial within university-major variation in our scores. It is possible that some of this variation is correlated to teachers' decisions to work in certain schools. Without teachers' actual scores, this issue remains.

Table 1 presents the scholastic scores of college students and full-time public high school teachers for the humanities and mathematics/sciences.<sup>21</sup> The average scholastic aptitude percentile of college students majoring in the humanities, mathematics, and the sciences is 58.140 among all college students. Those teaching in either the humanities, mathematics, or the sciences have an average scholastic aptitude percentile of 50.635, a –12.9% difference from their college peers. The lower average aptitude

<sup>19</sup> For double majors and second degree teachers, we take the arithmetic average of their scholastic aptitudes across their majors.

<sup>20</sup> The disadvantage of this approach is that university-major scores are detrended across years to obtain a sufficient sample of students within each of the 9452 university-majors. The literature on teacher aptitude also does not permit institutional quality to vary over time. We anticipate that quality of schools at various universities take long periods of time to change. Given that 74% of teachers have less than 20 years worth of experience and our data cover 20 years of college students, these proxy scores are representative of the major-university aptitude for the majority of teachers when they attended university.

<sup>21</sup> See Appendix B for the classification of college majors and teaching subjects.

<sup>18</sup> See Dorans (1999) for conversion details.



**Table 1**

Scholastic aptitudes of college students and high school teachers by subject.

	College students <sup>a</sup>		High school teachers <sup>b</sup>		Difference <sup>c</sup>
	Mean	Std. Dev.	Mean	Std. Dev.	
Humanities	53.294	32.857	50.847	16.817	–4.6%
Mathematics/sciences	64.184	27.265	50.371	16.788	–21.5%
All <sup>d</sup>	58.140	30.369	50.635	16.805	–12.9%

Notes: Scholastic aptitude is measured as the percentile in the distribution of all college students. See Appendix B for classification of college majors and teaching subjects.

<sup>a</sup> Includes only college students who majored in either the humanities, mathematics, or the sciences.

<sup>b</sup> Includes full-time public high school teachers whose primary teaching subject are either in the humanities, mathematics, or the sciences.

<sup>c</sup> In percentage change.

<sup>d</sup> Includes only those in the humanities, mathematics, or the sciences.

of teachers reflects almost entirely the lower average aptitude of mathematics/sciences teachers. As shown in Table 1, teachers in the humanities subjects are quite similar to their peers, while those in mathematics/sciences are –21.9% lower aptitude than their college peers.

We also construct a measure of lifetime teacher salary from the SASS data, defined as the current value of the expected future flow of salary over a forty year teaching career at the teacher's current school district. We measure this using

$$\text{permanent income}_{d,l} = \text{startpay}_{d,l} \times 40 + (\text{step}_{d,l} \times 40^2)/2 \quad (7)$$

where  $l = \{BA, MA\}$  and  $\text{startpay}_{d,l}$  is the starting pay of teachers for education attainment  $l$  in school district  $d$  with corresponding annual salary steps increase of  $\text{step}_{d,l}$ . We construct this for those that have a bachelor's degree (labeled BA) and for those that have a master's degree or beyond (labeled MA).<sup>22</sup> The advantage of this measure is that it is exogenous to teachers' experience. Thus, only shifts in the salary schedule are identified and not merely movements along the schedule (see Hanushek, Kain, & Rivkin, 1999 for similar analysis). The salary measure is significantly right-skewed. After logging the measure, it approximates a normal distribution.

We also include several community-level characteristics. First, using annual *Common Core of Data* (CCD), we measure school support as the percent of school expenditures from local versus state funds. Second, urban indicators are available from the SASS for 'rural', 'urban', and 'metro' areas defined by the *U.S. Census Bureau*. Third, using the *Bureau of Labor Statistics*, we include the local area unemployment rate surrounding each teacher's school. Lastly, using annual FBI's *Uniform Crime Reports*, we measure street crime surrounding the school as the rate of crime offenses per 1000 persons.<sup>23</sup>

<sup>22</sup> See Stern (1989), Heutel (2009), and Player (2009) for further analysis of salary schedules.

<sup>23</sup> Using factor analysis, we find that the community-level characteristics (community income and percent of adult population with at least a bachelor's degree) have high communalities (71.1% and 61.7%, respectively) with the school characteristics (percent of student body eligible for free lunch, percent of student body being a minority, whether the school is in a metro area, and whether the school is in a rural area) variables. Furthermore, both, community income and percent of adult population with

Summary statistics for the teacher, school, school district, and community characteristics are found in Table 2 for humanities/mathematics/sciences teachers, humanities teachers, and mathematics/sciences teachers, respectively. The data indicate that 45% of the total sample are mathematics/sciences teachers while 55% are humanities teachers. Almost 50% of all teachers have an advanced degree, and the majority of teachers are female and white. The average years of teaching experience is 14 years, and 71% of all teachers remain in the state where they earned their first undergraduate degree. Teachers' average class size is 22.3 students and they teach, on average, 5 classes. For schools, 34.3% of students are eligible for free lunch while, on average, 31.6% are non-white. 72.6% of schools have a union present. Percent of local school expenditures varies between 0% and 100%, and the average is 41.8%. Lastly, rate of street crime vary substantially from only 0.021 crimes per 1000 persons to 287.911 crimes per 1000 persons. The average street crime rate is 51.959 crimes per 1000 persons. Subsampling the data by teaching subjects, 48.9% of mathematics/sciences teachers are male compared to 40% of humanities teachers. Interestingly, the distribution of teachers' salary is identical for both subjects with a 14.506 average and 0.2 standard deviation.

#### 4. Empirical specification

The empirical strategy is based on the theoretical model. The theoretical model demonstrates that schools choose the salary schedule according to the limits of their budget constraint and the wages that teachers of various scholastic aptitudes are willing to accept. For their part, teachers accept an offer based on a combination of wages and non-pecuniary school characteristics as well as other idiosyncratic reasons. Given that a school's demand for teacher aptitude is perfectly elastic and equal to their offered salary, differences in salaries between schools can identify the correlation between salary and aptitude. The identification requires a stable supply curve across districts.

Prior to discussing our strategies to estimate a stable supply curve, we first specify the empirical model. Given that the demand curve is perfectly elastic, we estimate only

at least a bachelor's degree, had eigenvalues less than one which lends support to excluding them over the school characteristic variables.

**Table 2**  
Variable definitions and summary statistics.

Variable name	Variable description	Hum./Math./Sci. teachers				Hum. teachers		Math./Sci. teachers	
		Mean	Std. Dev.	Min.	Max.	Mean	Std. Dev.	Mean	Std. Dev.
<i>Dependent variable</i>									
Scholastic aptitude	Percentile in the aptitude distribution of college students	50.636	6.805	4.339	99.108	50.847	16.817	50.371	16.788
<i>Teacher characteristics</i>									
Advanced degree	Holds an advanced degree: 1	0.490	0.500	0	1	0.488	0.500	0.492	0.500
Male	Male: 1	0.439	0.496	0	1	0.400	0.490	0.489	0.500
Non-white	Non-white: 1	0.125	0.331	0	1	0.130	0.337	0.119	0.324
Non-white tchr/stu	Non-white teacher X % non-white students	7.931	23.991	0	100	7.947	23.807	7.912	24.219
Experience	Years of teaching experience <sup>a</sup>	13.912	10.462	0	46	14.009	10.541	13.792	10.363
Extracurricular	Involved in extracurricular school program: 1	0.626	0.484	0	1	0.631	0.482	0.619	0.486
Geographic preference	Teaching in state where earned 1st undergrad. deg.: 1	0.707	0.455	0	1	0.705	0.456	0.709	0.454
Under 27	Teacher under 27 years old: 1	0.093	0.290	0	1	0.089	0.285	0.096	0.295
Class size	Teacher's average class size	22.267	7.097	1	50	22.450	7.012	22.038	7.196
Number of classes	Number of teacher's classes	4.983	1.330	1	15	4.998	1.342	4.964	1.314
<i>School characteristics</i>									
% eligible for free lunch	Students eligible for free lunch <sup>b</sup>	34.136	25.088	0	100	33.916	25.078	34.411	25.098
% non-white	Non-white students <sup>b</sup>	31.609	31.705	0	100	31.640	31.686	31.570	31.730
IEP	Individual education plan students <sup>b</sup>	9.590	12.683	0	100	9.671	13.020	9.488	12.250
LEP	Limited-English proficiency students	3.655 <sup>b</sup>	10.762	0	100	3.749	10.875	3.539	10.618
<i>District characteristics</i>									
log(permanent income)	Lifetime teacher income <sup>c</sup>	14.506	0.198	12.556	15.186	14.506	0.199	14.506	0.197
Size of district	Number of schools in district	38.792	90.638	1	1164	38.782	90.361	38.805	90.989
Union	Teachers' union present: 1	0.726	0.446	0	1	0.731	0.443	0.720	0.449
<i>Community characteristics</i>									
School support	School expenditures from local funds <sup>b</sup>	41.827	19.645	0	100	42.077	19.636	41.956	19.724
Rural	School is located in rural area: 1	0.328	0.470	0	1	0.322	0.467	0.337	0.473
Metro	School is located in metro area: 1	0.224	0.417	0	1	0.226	0.418	0.221	0.415
Unemployment rate	Local unemployment rate <sup>b</sup>	5.040	1.949	0.7	25	5.019	1.943	5.066	1.955
Street crime	Crime rate in school's neighborhood <sup>d</sup>	51.959	27.500	0.021	287.911	52.077	27.304	51.811	27.745
Observations <sup>e</sup>		13,840				7700		6140	

<sup>a</sup> Maximum truncated to 4th largest observation as per data license restrictions.

<sup>b</sup> In percent.

<sup>c</sup> In logged 2007 dollars.

<sup>d</sup> Per 1000 persons.

<sup>e</sup> Rounded to the nearest 10 as per data license restrictions.

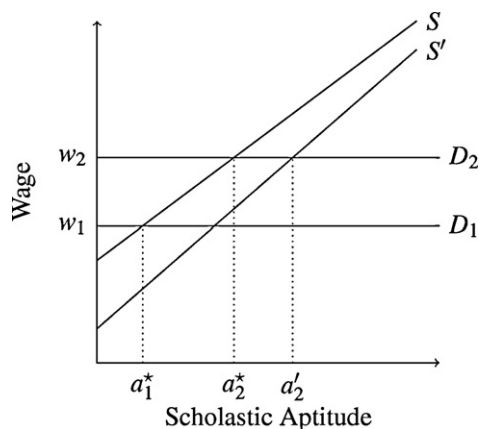


Fig. 2. Depiction of potential bias in salary effect on teacher aptitude.

a single equation model with teachers' scholastic aptitudes as the dependent variable. The specification is

$$A_{ij} = \beta_0 + X_i\beta_1 + X_j\beta_2 + X_m\beta_3 + \delta_k + \delta_s + \delta_t + \gamma_d + \epsilon_{ij} \quad (8)$$

where  $A_{ij}$  is the observed scholastic aptitude of teacher  $i$  hired in school  $j$ ,  $X_i$  represents vectors of teacher characteristics,  $X_j$  represents vectors of school/district characteristics including  $\log(\text{permanent income})$ ,  $X_m$  represents vectors of community characteristics surrounding the school,  $\delta_k$ ,  $\delta_s$  and  $\delta_t$  are teaching-subject, state, and time fixed effects, and  $\gamma_d + \epsilon_{ij}$  is the error term.<sup>24</sup> The error term is composed of two parts. The first,  $\gamma_d$ , accounts for any intragroup correlations within school district  $d$ . The second,  $\epsilon_{ij}$ , is the random component. Given that teachers are not paid compensating differentials for unfavorable school characteristics, salary and school environment characteristics can be included together without any endogeneity issues (see Gilpin, 2011; Imazeki, 2005; Levinson, 1988 for further details).<sup>25</sup> Furthermore, given our construction of teacher salary, teacher experience can also be included on the right-hand-side as it is uncorrelated with lifetime teacher income.

As discussed above, a stable supply curve is required to identify the correlation between salaries and scholastic aptitude without bias. The identification problem is depicted in Fig. 2 for two schools. Given that all supply-side factors are identical, the pairs  $(w_1, a_1^*)$  and  $(w_2, a_2^*)$  represent the wage-aptitude relationship in each school. The point  $(w_2, a_2')$  represents the wage-aptitude relationship in School 2 if the supply-side factors are not identical between these two schools. The true salary effect is  $a_2^* - a_1^*$ , and the

bias is  $a_2' - a_2^*$ . The bias occurs when the salary effect picks up other supply side factors as well. That is, any factor correlated with both the teacher's pay and work location will also be included in the salary effect if it is not controlled in the empirical model. Insufficient controls may generate a positive bias in the estimated salary coefficient. Thus, controlling the many factors that influence teachers' decisions of which schools to work in is key to identification of the correlation between salaries and teacher aptitude.

Our identification strategy is five pronged. First, we utilize teaching subject, state, and time fixed effects to control for time-invariant omitted variables.<sup>26</sup> State-fixed effects leave variation in variables between teachers, schools, districts, or communities (depending on the level of the variable in question) within the same state. Second, we utilize as many observable teacher, school, and neighborhood characteristics as possible. Third, we use district-level cluster robust standard errors to account for any unobservable intra-district correlations between teachers within the same district. Fourth, we build the results up from reduced models to show the bias and add factors progressively to identify which level(s) (teacher, school, district, and/or community) of characteristics identify the model. Fifth, we conduct sensitivity analysis investigating interactive terms between salary and the statistically significant school, district, and neighborhood characteristics.<sup>27</sup>

We also investigate the effects through quantile regressions. To do this, the analysis groups teachers based on their scholastic aptitudes, and then tests whether these groups are different with respect to salary and other characteristics. We do this since the correlation between salary and aptitude may be non-linear, e.g., higher aptitude teachers may have a different salary-aptitude correlation than lower aptitude teachers. Thus, Eq. (8) becomes

$$A_{i,j}(\tau) = \beta_0(\tau) + X_i\beta_1(\tau) + X_j\beta_2(\tau) + X_m\beta_3(\tau) + \delta_k(\tau) + \delta_s(\tau) + \delta_t(\tau) + \epsilon_{i,j}(\tau) \quad (9)$$

where the variables are as described in Eq. (8) with the addition of  $\tau$  representing a given percentile of scholastic aptitude.<sup>28</sup> We estimate the coefficients' standard errors using 200 bootstrap replications.

Prior to estimating the model, we investigate the magnitude of the intra-district correlation of teachers. The main concern of intra-district correlation is whether there is sufficient between-district variation in the sample to identify the correlation between teachers' salaries and teacher aptitude. If intra-district correlation is high, i.e., a majority of teachers come from a few districts, then the statistical power may be quite low to identify the correlation between salary and teacher aptitude. A Type II error may occur. This

<sup>24</sup> One of the advantages of a linear-log model specification is that elasticities can be estimated for the logged variables. The slope coefficient on the lifetime teacher income variable,  $\log(\text{permanent income})$ , is interpreted as a unit in the log of lifetime teacher income is correlated with a  $.01 * \beta_{\log(\text{permanent income})}$  change in teacher aptitude. This coefficient can be converted to an elasticity by dividing the coefficient by average teacher aptitude. The elasticity estimate is interpreted as a 1% change in lifetime teacher income is correlated with a  $\beta_{\log(\text{permanent income})}/\bar{A}$  percent change in teacher aptitude where  $\bar{A}$  is average teacher aptitude.

<sup>25</sup> Note, paying all teachers the same amount in all districts in a particular state is different from providing additional compensation to teachers working in unfavorable schools.

<sup>26</sup> As discussed above, salaries are set at the district level. Thus, district-level fixed effects cannot be included. However, we do include state-fixed effects since districts mostly hire individuals from within state, and scholastic aptitude may be highly correlated with in-state college selectivity.

<sup>27</sup> The interaction between salaries and various characteristics will identify shifts in the supply curve that occur due to differences between schools.

<sup>28</sup> Ma and Koenker (2006) provide an in-depth analysis and comparison of various methods on estimating quantile regressions.



does not seem to be the case. 37% of districts have two or less teachers in the sample while almost 80% of schools have six or less teachers. Given that we also investigate humanities and mathematics/sciences teachers separately, this implies that approximately 80% of the sample will have three or fewer teachers per subject for each district. Further evidence of small intraclass correlation is provided through the statistically significant intraclass correlation statistic of 0.13 for teachers' aptitude.

## 5. Empirical results

### 5.1. Estimation results on scholastic aptitude

Table 3 presents the empirical results of estimating Eq. (8) for humanities, mathematics, and sciences teachers jointly. We build up to the full model by successive regressions with additional controls to statistically test the sensitivity of the variable of interest,  $\log(\text{permanent income})$ . As discussed above, we include fixed effects and cluster-robust standard errors in all specifications.<sup>29</sup> Reduced Model 1 is the most reduced model including only teacher salary and fixed effects. In the subsequent regression, we also include teacher-level characteristics (Reduced Model 2). We then extend the controls to include teacher, school, and district-level characteristics (Reduced Model 3). Lastly, we include community-level characteristics (Reduced Model 4). In the full specification model, we extend the analysis by also including marginal teacher and school characteristics that have been shown in the literature to sometimes be significant (Full Model).

As reported in column Reduced Model 1, without any controls, the  $\log(\text{permanent income})$  coefficient is statistically significant at 1%, and at a maximum, 11.927. This corresponds to a 0.245 elasticity estimate, i.e., a 1% change in lifetime teacher income is positively correlated with a 0.245% change in teacher aptitude. The 90% confidence interval surrounding this elasticity estimate is 0.042. In Reduced Model 2, including teacher-level characteristics decreases the lifetime teacher salary coefficient estimate by 1.936–9.996, or equivalently, an elasticity estimate of 0.205 with a 90% confidence interval of 0.042. In addition, the explanatory power of the model increases from an adjusted  $R^2$  of 0.095–.141. The teacher factor coefficients have the expected signs and significances. Teachers with advanced degrees are positively correlated with teacher aptitude as well as teachers that are from out-of-state. Non-white teachers seem to be negatively correlated with teacher aptitude. However, this result becomes insignificant in the fully specified model. Lastly, the experience coefficient is negative. This may indicate that experience

is negatively correlated with teacher aptitude. However, since the aptitude measure is detrended, this result is tentative and requires further analysis.

In Reduced Model 3, all other coefficients remain nearly identical from Reduced Model 2 except for the  $\log(\text{permanent income})$  coefficient. The results indicate that the percent of the school's student body eligible for free lunch is positively correlated with the aptitude of teachers hired. Interestingly, percent of non-white students has no correlation with teacher aptitude while the correlation of unions and teacher aptitude is negative and significant (see Levinson, 1988 for similar results). However, in the full model that includes community characteristics, the magnitude of the union coefficient halves and becomes insignificant in the by teaching subject regressions. The  $\log(\text{permanent income})$  coefficient further decreases by 1.885–8.111, or an elasticity estimate of 0.167 with a 90% confidence interval of 0.043. The adjusted  $R^2$  increases only marginally to .145.

The coefficients on the community characteristics in Reduced Model 4 are as expected. Local support for schools and metro areas are both positively correlated with teacher aptitude. Communities with high rates of unemployment and street crime are negatively correlated with teacher aptitude, respectively. We suspect that higher aptitude teachers are generally attracted to metropolitan areas for culturally enriching activities and that these teachers recognize that they do not necessarily have to live in the communities where their employment is located (see Ballou, 1996 for similar results). Interestingly, the percent of students eligible for free lunch coefficient reduces by approximately 25% when the rate of street crime is included in the model. This result indicates that previous estimates, excluding the local crime rate, may have overestimated the effect of school poverty on teacher recruitment and retention. The coefficient of  $\log(\text{permanent income})$  further reduces by 1.720–6.390. This converts to an elasticity estimate of 0.131 with a 90% confidence interval of 0.048.

Given that the  $\log(\text{permanent income})$  coefficient continually declines through the successive reduced models, we include six additional factors to test stability of the  $\log(\text{permanent income})$  coefficient. The results of this specification are the last column in Table 3, the Full Model. Three out of the four teacher factors are significant, the interaction term between non-white teacher and percent of non-white students, an indicator variable whether the teacher is under age 27, and the number of classes a teacher is assigned. None of the additional school characteristics are significant. Jointly, through a  $F$ -test, they are significant at a 1% level. The inclusion of these variables increases the explanatory power, but does not have an effect on the  $\log(\text{permanent income})$  coefficient.<sup>30</sup>

The stability of the  $\log(\text{permanent income})$  coefficient between Reduced Model 4 and the Full Model provides evidence that the factors affecting the supply curve have

<sup>29</sup> To investigate how clustering impacts the significance of the coefficient estimates, we run each equation separately with homoscedastic, heteroscedastic robust, and cluster robust standard errors at the district level. The statistical significance of the standard errors remains the same across these three estimations even though the cluster robust standard errors are higher than the standard and robust standard errors. Another robustness check is to remove the state fixed effects to see how large the unobservable effects are on the  $\log(\text{permanent income})$  coefficient. While not reported in the output tables, unobservables effects are quite minimal.

<sup>30</sup> Robustness checks using the number of schools in a district squared and teacher experience squared in the full model are both insignificant and have no effect on the  $\log(\text{permanent income})$  coefficient.

**Table 3**

Coefficient estimates of scholastic aptitude regressions and elasticity estimates.

Variable	Reduced Model 1		Reduced Model 2		Reduced Model 3		Reduced Model 4		Full Model	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
<i>Teacher characteristics</i>										
Advanced degree			2.061***	0.280	1.915***	0.282	1.955***	0.299	2.139***	0.299
Male			−0.090	0.243	−0.107	0.249	−0.009	0.264	0.053	0.265
Non-white			−7.718***	0.525	−7.305***	0.533	−7.271***	0.551	−0.410	0.962
Non-white tchr/stu									−0.125***	0.017
Experience			−0.135***	0.013	−0.126***	0.013	−0.125***	0.014	−0.101***	0.014
Extracurricular			−0.125	0.264	−0.128	0.269	−0.241	0.282	−0.164	0.281
Geographic preference			−4.921***	0.339	−4.789***	0.345	−4.680***	0.366	−4.715***	0.364
Under 27									2.496***	0.481
Class size									−0.030	0.020
Number of classes									−0.185*	0.105
<i>School characteristics</i>										
% eligible for free lunch					−0.049***	0.007	−0.037***	0.008	−0.036***	0.008
% non-white					0.002	0.008	−0.005	0.008	0.014	0.009
IEP									−0.018	0.012
LEP									0.024	0.017
<i>District characteristics</i>										
log(permanent income)	11.927***	1.247	9.996***	1.238	8.111***	1.276	6.390***	1.407	6.405***	1.418
Size of district					0.007**	0.004	0.011**	0.005	0.013***	0.005
Union					−1.010***	0.302	−0.632**	0.314	−0.578*	0.312
<i>Community characteristics</i>										
school support							0.042***	0.011	0.040***	.011
Rural							−0.336	0.358	−0.361	0.359
Metro							0.837**	0.410	0.752*	0.410
Unemployment rate							−0.623***	0.119	−0.566***	0.117
Street crime							−0.015**	0.006	−0.017***	0.006
Elasticity estimates <sup>a</sup>	0.245 ± 0.042		0.205 ± 0.042		0.167 ± 0.043		0.131 ± 0.048		0.132 ± 0.048	
Teaching subject fixed effects	Yes		Yes		Yes		Yes		Yes	
State fixed effects	Yes		Yes		Yes		Yes		Yes	
Time fixed effects	Yes		Yes		Yes		Yes		Yes	
Adjusted R <sup>2</sup>	0.095		0.141		0.145		0.158		0.165	

Notes: \*  $p < .1$ , \*\*  $p < .05$ , \*\*\*  $p < .01$ . School district-level cluster robust standard errors. Includes teachers whose primary teaching subject are either in the humanities, mathematics, or the sciences.

<sup>a</sup> 90% confidence interval reported.

been sufficiently controlled and the correlation between teacher lifetime income and teacher aptitude can be identified without significant bias. The results of the full model indicate that a 1% change in lifetime teacher income is positively correlated with a 0.132 change in teacher aptitude. This estimate makes it clear that school, district, and community characteristics are important, and that even if the elasticity estimate was to be a causal estimate, only modest gains to teacher aptitude can be had by increasing teacher salary.<sup>31</sup>

<sup>31</sup> As a further robustness check, in separate regressions of the Full Model, we include interaction terms of log(permanent income) with quartile dummies for the percent eligible for free lunch, school support, size of district, and urban indicators, respectively. The results of these separate regressions indicate that the relationship between lifetime teacher income and teacher aptitude is independent of the percent of students eligible for free lunch, local community support for schools, and urban indicators, e.g., schools with few students eligible for free lunch have the same positive relationship between lifetime teacher income and teacher aptitude as schools with many students eligible for free lunch. However, these schools will have observed differences in teacher aptitude due to the direct effects of various school and community characteristics (see Table 3). The results for the regression including school district size interaction terms indicate that the positive relationship between log(permanent income) and teacher aptitude is dependant on school size, larger districts seem to be able to pay less than larger districts for similar amounts of

### 5.1.1. Estimation results on scholastic aptitude by teaching subject

We now separately estimate the Reduced Model 4 and Full Model specifications for humanities teachers and mathematics/sciences teachers. This, in essence, unconstrains all control variables. We estimate both models to show stability in the log(permanent income) coefficient. The estimation results are provided in Table 4. We find that the lifetime teacher income elasticity for humanities teachers is higher than mathematics/sciences teachers, 0.136 versus 0.111. Post-estimation testing using Chow tests fail to reject that lifetime teacher income and all other coefficients across regressions are equal. Thus, even though humanities teachers seem to have a higher elasticity estimate, humanities and mathematics/sciences teachers appear to behave similarly.

### 5.2. Quantile estimation results

Using a quantile regression framework, we study the correlation between salary and aptitude across the

teacher aptitude. While there is a statistical difference, the magnitude of difference is underwhelming small. These results are available upon request from the author.

**Table 4**

Coefficient estimates of scholastic aptitude regressions and elasticity estimates by teaching subject.

Variable	Humanities teachers				Mathematics/sciences teachers			
	Reduced Model 4		Full Model		Reduced Model 4		Full Model	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
<i>log(permanent income)</i>	6.939***	1.684	7.084***	1.699	5.587***	2.075	5.443***	2.085
Elasticity estimates <sup>a</sup>	0.136 ± 0.054		0.139 ± 0.055		0.111 ± 0.068		0.108 ± 0.068	

Notes: Regressions include all controls listed in Table 2, and state and time fixed effects. \* $p < .1$ , \*\* $p < .05$ , \*\*\* $p < .01$ . School district-level cluster robust standard errors.

<sup>a</sup> 90% confidence interval reported.

teachers' aptitude distribution. We provide the coefficient estimates from these regressions for the 10th, 25th, 50th, 75th, and 90th percentiles in Table 5 and graph the elasticities for quantile regressions every 2.5 percentiles in Fig. 3. The results of the quantile regressions indicate that the elasticity estimates in the above fixed effects models are not representative of the teacher aptitude distribution. The elasticity estimates form an inverted U-shape across the teacher aptitude distribution. At the lower part of the teacher aptitude distribution, the elasticity estimate is 0.052. This indicates that in schools with lower scholastic aptitude teachers, changes in *log(permanent*

income) do not have a strong positive relationship with changes in teacher aptitude after controlling for school, district, community, and unobservable characteristics. On the other hand, schools with teacher aptitude around the 50th percentile have a strong positive correlation between *log(permanent income)* and teacher aptitude, an elasticity of 0.152 with a 90% confidence interval of .052. Around the 65th percentile, the elasticity flattens to 0.108. We also find that higher aptitude teachers are more affected by the percentage of students eligible for free lunch as well as local street crime. Lower aptitude teachers are affected by local education support while higher aptitude teachers are not.

**Table 5**

Coefficient estimates of scholastic aptitude quantile regressions and elasticity estimates.

Quantile	10		25		50		75		90	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
<i>Teacher characteristics</i>										
Advanced degree	0.383*	0.212	0.937***	0.307	2.021***	0.350	1.335***	0.312	1.062***	0.376
Male	−0.021	0.159	−0.217	0.244	0.204	0.301	−0.186	0.251	−0.342	0.270
Non-white	−2.206	2.138	1.734*	0.974	1.404	1.105	0.426	0.941	1.508	1.189
Non-white tchr/stu	−0.165***	0.025	−0.165***	0.014	−0.155***	0.021	−0.116***	0.02	−0.091***	0.025
Experience	−0.033***	0.012	−0.064***	0.012	−0.086***	0.018	−0.048***	0.016	−0.026*	0.015
Extracurricular	−0.116	0.175	−0.036	0.242	−0.422	0.306	0.119	0.274	−0.240	0.279
Geographic preference	0.317	0.480	−2.345***	0.406	−4.092***	0.541	−6.209***	0.557	−11.974***	0.737
Under 27	0.805**	0.363	2.066***	0.526	2.256***	0.547	1.732***	0.569	1.047**	0.507
Class size	−0.011	0.013	−0.017	0.019	−0.0305	0.023	−0.016	0.021	−0.026	0.021
Number of classes	0.003	0.071	−0.128	0.089	−0.212*	0.12	−0.229*	0.119	−0.135	0.108
<i>School characteristics</i>										
% eligible for free lunch	−0.016***	0.006	−0.022***	0.006	−0.041***	0.009	−0.028***	0.009	−0.022***	0.008
% non-white	−0.002	0.005	−0.015**	0.006	0.017*	0.008	0.012	0.008	0.022**	0.010
IEP	−0.016*	0.009	−0.020**	0.005	−0.02	0.014	−0.003	0.013	−0.003	0.010
LEP	0.010	0.014	0.032	0.020	0.044**	0.019	0.048**	0.023	0.012	0.019
<i>District characteristics</i>										
<i>log(permanent income)</i>	1.316	0.989	1.940*	1.168	7.243***	1.503	6.434***	1.218	6.499***	1.751
Size of district	0.002	0.005	0.015**	0.006	0.020***	0.003	0.023***	0.006	0.014***	0.003
Union	−0.136	0.182	−0.317	0.275	−0.264	0.316	−0.378	0.305	−0.359	0.308
<i>Community characteristics</i>										
School support	0.018**	0.008	0.033***	0.009	0.035***	0.013	0.024**	0.011	0.016	0.012
Rural	0.272	0.210	−0.328	0.288	−0.227	0.387	−0.831**	0.358	−0.614**	0.304
Metro	0.366	0.292	1.096***	0.393	0.444	0.465	−0.533	0.456	0.099	0.448
Unemployment rate	−0.190**	0.087	−0.441***	0.095	−0.626***	0.155	−0.442***	0.147	−0.170	0.135
Street crime	0.001	0.004	−0.008	0.006	−0.018***	0.006	−0.024***	0.007	−0.024***	0.006
Scholastic aptitude	31.137		37.572		47.685		59.738		71.314	
Elasticity estimates <sup>a</sup>	0.042 ± 0.032		0.052 ± 0.051		0.152 ± 0.052		0.108 ± 0.034		0.091 ± 0.040	
Teaching subject fixed effects	Yes		Yes		Yes		Yes		Yes	
State fixed effects	Yes		Yes		Yes		Yes		Yes	
Time fixed effects	Yes		Yes		Yes		Yes		Yes	
Pseudo R <sup>2</sup>	0.170		0.121		0.113		0.104		0.169	

Notes: \*  $p < .1$ , \*\*  $p < .05$ , \*\*\*  $p < .01$ . Standard errors estimated using 200 bootstrap replications.

<sup>a</sup> 90% confidence interval reported.

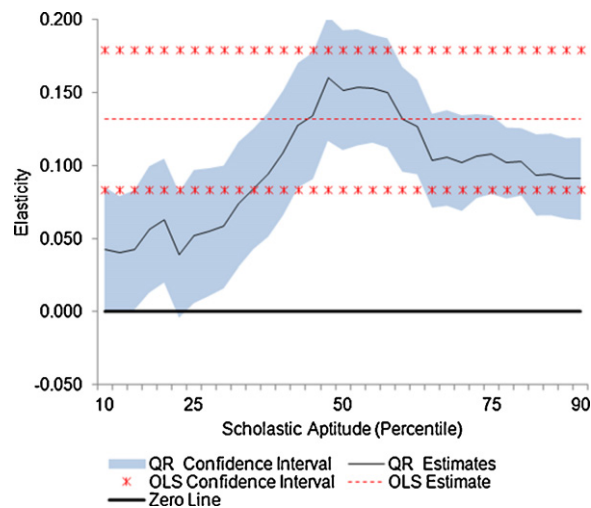


Fig. 3. Elasticity estimates for humanities/mathematics/sciences teachers.

We also conduct similar quantile regressions for humanities teachers and mathematics/sciences teachers, respectively. The abbreviated results are provided in Table 6 with corresponding elasticity estimates depicted in Fig. 4a. These results also indicate that the fixed effect elasticity estimates do not represent the distribution of mathematics/sciences teachers. While humanities teachers' estimates are similar to that of all teachers reported in Table 5, mathematics/sciences teachers are quite different. As observed in Fig. 4b, the bottom 40 percentiles in the mathematics/sciences teacher aptitude distribution is unresponsive to changes in lifetime teacher income after controlling for teacher, school, district, community, and unobserved characteristics. Furthermore, after the 50th percentile in the aptitude distribution, the elasticity estimate is 0.125 until approximately the 85th percentile where it decreases to 0.057. *F*-tests for each percentile rejects that the elasticities are equal between humanities and mathematics/sciences teachers for the lower quartile in the teacher aptitude distribution.

Table 6

Coefficient estimates of scholastic aptitude quantile regressions and elasticity estimates by teaching subject.

Quantile	10		25		50		75		90	
	Quantile regression $\log(\text{Permanent Income})$ estimates <sup>a</sup>									
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Humanities	2.401**	1.430	2.201	1.716	7.679***	2.158	6.996***	1.639	5.256**	2.213
Mathematics/sciences	−0.236	1.249	1.592	1.644	6.068**	2.583	6.002**	2.500	4.201	2.631
Scholastic aptitude										
Humanities	31.768		38.006		48.890		61.972		73.757	
Mathematics/sciences	31.341		37.774		48.517		61.395		73.198	
Elasticity estimates <sup>b</sup>										
Humanities	0.076 ± 0.063		0.058 ± 0.027		0.157 ± 0.087		0.113 ± 0.052		0.071 ± 0.059	
Mathematics/sciences	−0.008 ± 0.078		0.042 ± 0.085		0.125 ± 0.104		0.098 ± 0.080		0.057 ± 0.070	

<sup>a</sup> Quantile regressions include all controls listed in Table 2, and state and time fixed effects. \*  $p < .1$ , \*\*  $p < .05$ , \*\*\*  $p < .01$ . Standard errors estimated using 200 bootstrap replications.

<sup>b</sup> 90% confidence interval reported.

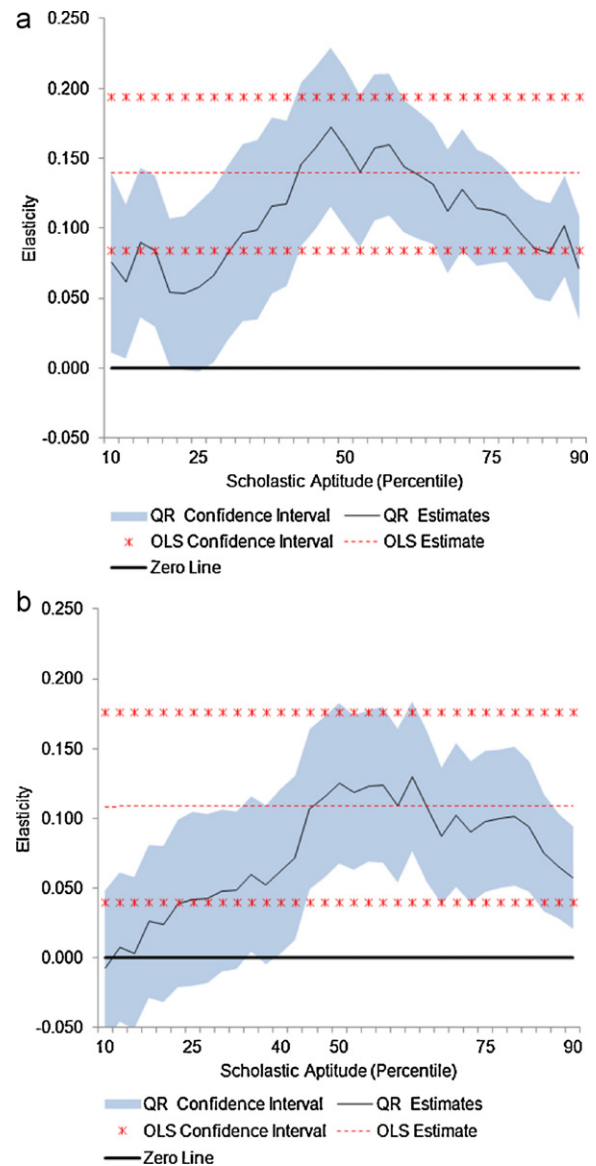


Fig. 4. (a) Elasticity estimates for humanities teachers and (b) elasticity estimates for mathematics/sciences teachers.

## 6. Conclusion and policy implication

This study examines the correlation between lifetime teacher salary and teacher aptitude. The empirical results find statistical evidence that teachers, schools, districts and community characteristics are correlated with teachers' aptitude. Most importantly, lifetime teacher salary and teacher aptitude is positively correlated with an elasticity of approximately 0.132 after controlling for teacher, school, district, community, and unobservable characteristics. This correlation is found to be robust across many different school types. Furthermore, using quantile regressions, the analysis indicates that the point estimate is not representative of the teacher aptitude distribution. The elasticity estimates form an inverted U-shape across the teacher aptitude distribution. Conducting regressions for humanities teachers and mathematics/sciences teachers, respectively, the analysis finds that the correlation between lifetime teacher income for humanities teachers is quite similar to that of all teachers, while mathematics/sciences teachers are different. The bottom 40 percentiles in the mathematics/sciences teacher aptitude distribution are unresponsive to changes in lifetime teacher income. We also find that higher aptitude teachers are more affected by the percentage of students eligible for free lunch as well as local street

crime. Lower aptitude teachers are affected by local education support while higher aptitude teachers are not.

These are key findings which highlights the non-linearity of relationships across the teacher aptitude distribution. If the lifetime teacher salary elasticity estimates on teacher aptitude were taken as causal, school districts' attempts to raise mathematics/sciences teachers' aptitude by simply increasing salaries across-the-board would not be effective for 40% of the aptitude distribution. The low positive correlation between salary and aptitude should not be taken that the scholastic aptitude of teachers cannot be raised, but rather it would require large increases in lifetime teacher salary to raise teacher aptitude. The results in this paper indicate that the school and community environment in which the teacher works are important determinants of teacher aptitude.

### Appendix A.

See [Tables A1, A2](#).

### Appendix B.

This appendix provides the solution to the school's per student education quality maximization problem Eq. (5).

**Table A1**

College majors by subject.

Humanities		Education
Communications or Journalism	Agricultural	Autism
Economics	Art	Counseling and Guidance
English Literature or Composition	Bilingual	Curriculum and Instruction
French	Business	Deaf and Hard-of-hearing
German	Cross-cultural	Developmentally Delayed
History	Early Childhood	Early Childhood Special
Humanities	Elementary	Administration
Latin	English/Language Arts	Psychology
Law	ESL	Industrial Arts
Library and Information Science	Foreign Languages	Learning Disabilities
Multi- or Inter-disciplinary Studies	Health	Mentally Retarded
Native American Studies	Home Economics	Mildly or Moderately Disabled
Other Area or Ethnic Studies	Kindergarten	Orthopedically Impaired
Other Languages	Mathematics	Other
Other Social Sciences	Music	Other Special
Philosophy	Native American	Severely Disabled
Political Science and Government	Physical	Social Studies
Psychology	Pre-kindergarten	Special
Public Administration or Service	Reading	Speech or Language Impaired
Religion or Theology	Religious	Trades and Industry
Russian	Science	Traumatically Brain Injured
Sociology	Secondary	Visually Impaired
Spanish	Emotionally Disturbed Behavior Disorders	
Mathematics/Sciences		All other majors
Ag. and Nat'l Resources	Art, Fine and Applied	All other areas
Biology/Life Science	Drama or Theater	Architecture
Chemistry	Music	Business and Management
Computer Science	Visual/Performing Arts	Environmental Design
Engineering		Family Consumer Science
Geology/Earth Science		General Studies
Mathematics		Health Professions
Other Natural Sciences		Military Science
Physics		
Statistics		



**Table A2**

High school classes by subjects.

Mathematics/Sciences	Humanities	Other
Algebra, Elementary	Literature	Accounting
Algebra, Intermediate	Civics	Agriculture or Natural Resources
Algebra, Advanced	Composition/Creative Writing	Arts and Crafts
Analytic Geometry	Economics	Band
Basic and General Mathematics	English as a Second Language	Business/Office
Biology or Life Science	French	Career Education
Business and Applied Mathematics	Geography	Child Care
Calculus	German	Chorus
Chemistry	History	Communications Technologies
Geology/Earth Science	Journalism	Computer Awareness/Applications
Geometry	Latin	Computer Programming
Integrated Mathematics	Other English/Language Arts Courses	Consumer Science
Integrated Science	Other Foreign Languages	Cosmetology
Other Mathematics	Other Social Science	Drama/Theater/Dance
Other Natural Science	Philosophy	Driver Education
Other Physical Science	Political Science/Government	Filmmaking and Photography
Physics	Psychology	Food Services
Pre-algebra	Reading	Health Education
Pre-calculus	Religion	Health Occupations
Space Science	Russian	Home Economics
Statistics and Probability	Social Studies	Keyboarding
Trigonometry	Sociology/Social Organization	Music
	Spanish	Other Computer Science
	World Civilization	Other Courses not Elsewhere Classified
		Other Visual/Performing Arts
		Other Vocational-technical Education
		Physical Education
		Trades and Industry
		Vocational Family

Instead of solving the problem with the school choosing  $\bar{w}$ ,  $Q_s$ , and  $Q_h$ , we transform the problem so that schools choose  $a_s$ ,  $Q_s$ , and  $Q_h$ . To transform problem Eq. (5), we first establish the relationship between  $a_s$  and  $a_h$  by equalizing salaries across teaching subjects using Eq. (4):

$$a_h = m_h m_s^{-1} a_s - b_h + b_s m_s^{-1} m_h \quad (\text{A.1})$$

The above condition must hold in equilibrium and makes clear that  $a_h$  will deviate from  $a_s$  dependent on the relative parameters of the wage elasticities of aptitude supplied, i.e.,  $a_h$  differs from  $a_s$  due to the relative difference between  $m_s$  and  $m_h$  as well as the difference between  $b_s$  and  $b_h$ . Substituting Eqs. (A.1) and (4) into the school's budget constraint, Eq. (1), yields the transformed budget constraint

$$B = (m_s^{-1} a_s + b_s m_s^{-1})(Q_s + Q_h) \quad (\text{A.2})$$

Likewise, we can substitute Eq. (A.1) into Eq. (3) to obtain the transformed per student education quality production function:

$$E = (a_s^{\beta_s} Q_s^{1-\beta_s})^{\alpha_s} ((m_h m_s^{-1} a_s - b_h + b_s m_s^{-1} m_h)^{\beta_h} Q_h^{1-\beta_h})^{\alpha_h} \quad (\text{A.3})$$

Given Eqs. (A.1) and (A.2), we can restate the education quality optimization problem (5) as

$$\max_{a_s, Q_s, Q_h} (a_s^{\beta_s} Q_s^{1-\beta_s})^{\alpha_s} ((m_h m_s^{-1} a_s - b_h + b_s m_s^{-1} m_h)^{\beta_h} Q_h^{1-\beta_h})^{\alpha_h} \quad (\text{A.4})$$

subject to (A.2)

Combining the first order necessary conditions for the above problem Eq. (A.4) yield the following equilibrium conditions:

$$Q_h = \frac{\alpha_h(1-\beta_h)}{\alpha_s(1-\beta_s)} Q_s \quad (\text{A.5})$$

$$\frac{\beta_s m_s w Q_s}{a_s(1-\beta_s)} + \frac{\beta_h E w Q_h}{a_h(1-\beta_h)} = (Q_s + Q_h) \quad (\text{A.6})$$

Substituting Eq. (A.5) in Eq. (A.6) yields:

$$\alpha_s m_s \beta_s w a_s^{-1} + \alpha_h m_h \beta_h w a_h^{-1} = \alpha_s(1-\beta_s) + \alpha_h(1-\beta_h) \quad (\text{A.7})$$

Combining this with Eq. (4) we obtain

$$\alpha_s \beta_s b_s a_h + \alpha_h \beta_h b_h a_s = (\alpha_s(1-2\beta_s) + \alpha_h(1-2\beta_h)) a_s a_h \quad (\text{A.8})$$

which using Eq. (A.1) obtains an equation solely as a function of  $a_s$

$$\alpha_s \beta_s b_s [m_h m_s^{-1} a_s - b_h + b_s m_s^{-1} m_h] + \alpha_h \beta_h b_h a_s = J [m_h m_s^{-1} a_s^2 - b_h a_s + b_s m_s^{-1} m_h a_s] \quad (\text{A.9})$$

$$J m_h m_s^{-1} a_s^2 + (J b_s m_s^{-1} m_h - J b_h - \alpha_s \beta_s b_s m_h m_s^{-1} - \alpha_h \beta_h b_h) a_s + \alpha_s \beta_s b_s (b_h - b_s m_s^{-1} m_h) = 0$$

where  $J = \alpha_s(1-2\beta_s) + \alpha_h(1-2\beta_h)$ . The above equation is

quadratic yielding unique non-negative solution:

$$a_s = (b_h m_h^{-1} m_s - b_s + J^{-1} \alpha_s \beta_s b_s + J^{-1} \alpha_h \beta_h b_h m_h^{-1} m_s) 2^{-1} \\ + [(b_s - b_h m_h^{-1} m_s - J^{-1} \alpha_s \beta_s b_s - J^{-1} \alpha_h \beta_h b_h m_h^{-1} m_s)^2 \\ - 4 J^{-1} \alpha_s \beta_s b_s (b_h m_h^{-1} m_s - b_s)]^{1/2} 2^{-1} \quad (\text{A.10})$$

and combining this with Eq. (A.1) yields:

$$a_h = (b_s m_s^{-1} m_h - b_h + J^{-1} \alpha_h \beta_h b_h + J^{-1} \alpha_s \beta_s b_s m_s^{-1} m_h) 2^{-1} \\ + [(b_h - b_s m_s^{-1} m_h - J^{-1} \alpha_h \beta_h b_h - J^{-1} \alpha_s \beta_s b_s m_s^{-1} m_h)^2 \\ - 4 J^{-1} \alpha_h \beta_h b_h (b_s m_s^{-1} m_h - b_h)]^{1/2} 2^{-1} \quad (\text{A.11})$$

Using Eq. (A.10), the equilibrium quantities of math/science and humanities teachers are identified by substituting Eq. (A.5) into Eq. (A.2)

$$Q_s = \frac{\delta_s B}{\bar{w}}, \quad Q_h = \frac{\delta_h B}{\bar{w}} \quad (\text{A.12})$$

where  $\delta_s = (\alpha_s(1 - \beta_s))/(\alpha_s(1 - \beta_s) + \alpha_h(1 - \beta_h))$ ,  $\delta_h = (\alpha_h(1 - \beta_h))/(\alpha_s(1 - \beta_s) + \alpha_h(1 - \beta_h))$ , and recalling that  $\bar{w} = m_s^{-1} a_s + b_s m_s^{-1}$ .

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