ECE 657 Assignment 4 – Group 80

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Problem 1

In the given problem, fuzzy set is F and below are the membership functions.

(i) "fast speed" is denoted by $\mu_F(v)$

(ii) "very fast speed" is denoted by $\mu_F(v - v_0)$, where $v_0 > 0$.

(iii) "presumably fast speed" is denoted by $\mu_F^2(v)$

1.a) Appropriateness of the use of membership functions:

The membership function $\mu_F(v - v_0)$ is appropriate to represent the linguistic hedge "very fast speed". It means speeds upto a certain limit of v_0, will not be considered as "very high speed". And, the elements of the set after v_0, would be considered as high speed.

The membership function $\mu_F^2(v)$ is <u>not appropriate</u> to represent the linguistic hedge "presumably fast speed". Because by applying the function $\mu_F^2(v)$, it results in <u>contraction</u> of the membership function. The membership function may represent the linguistic hedge "very" or "too" but it cannot represent the linguistic hedge "presumably fast speed".

1.b) $F = \left\{ \frac{0.1}{10}, \frac{0.3}{20}, \frac{0.6}{30}, \frac{0.8}{40}, \frac{1.0}{50}, \frac{0.7}{60}, \frac{0.5}{70}, \frac{0.3}{80}, \frac{0.1}{90} \right\}$

Discrete universe $V = \{0, 10, 20, 30 \ldots\ldots 190, 200\}$ rev/s

and $V_0 = 50$ rev/s.

<u>Membership function of "very fast speed"</u> $\mu_F(v - v_0)$

We can calculate the membership function $\mu_F(v - v_0)$ by deducting $v_0 = 50$ rev/s from the elements of the discrete universe V. Any elements in the universe V, if it is < 50 then value $v - v_0$ would be < 0. Thus we can assume that for the elements $\{0, 10, 20 \ldots 50\}$ in the universe

V, the membership value would be 0.

Now we can calculate the rest of the values as follows,

$$\mu(60) = \mu(60-50) = \mu(10) \Rightarrow \mu(60) = \frac{0.1}{60}$$

Since we are deducting 50, from 60 thus we are getting element 10. for element 10 in the ~~univers~~ universe V the degree of membership is given as 0.1.

Thus, we can get the rest of the values as follows,

$$\mu(70) = \mu(70-50) = \mu(20) = \frac{0.3}{70}$$

$$\mu(80) = \mu(80-50) = \mu(30) = \frac{0.6}{80}$$

$$\mu(90) = \mu(90-50) = \mu(40) = \frac{0.8}{90}$$

$$\mu(100) = \mu(100-50) = \mu(50) = \frac{1.0}{100}$$

$$\mu(110) = \mu(110-50) = \mu(60) = \frac{0.7}{110}$$

$$\mu(120) = \mu(120-50) = \mu(70) = \frac{0.5}{120}$$

$$\mu(130) = \mu(130-50) = \mu(80) = \frac{0.3}{130}$$

$$\mu(140) = \mu(140-50) = \mu(90) = \frac{0.1}{140}$$

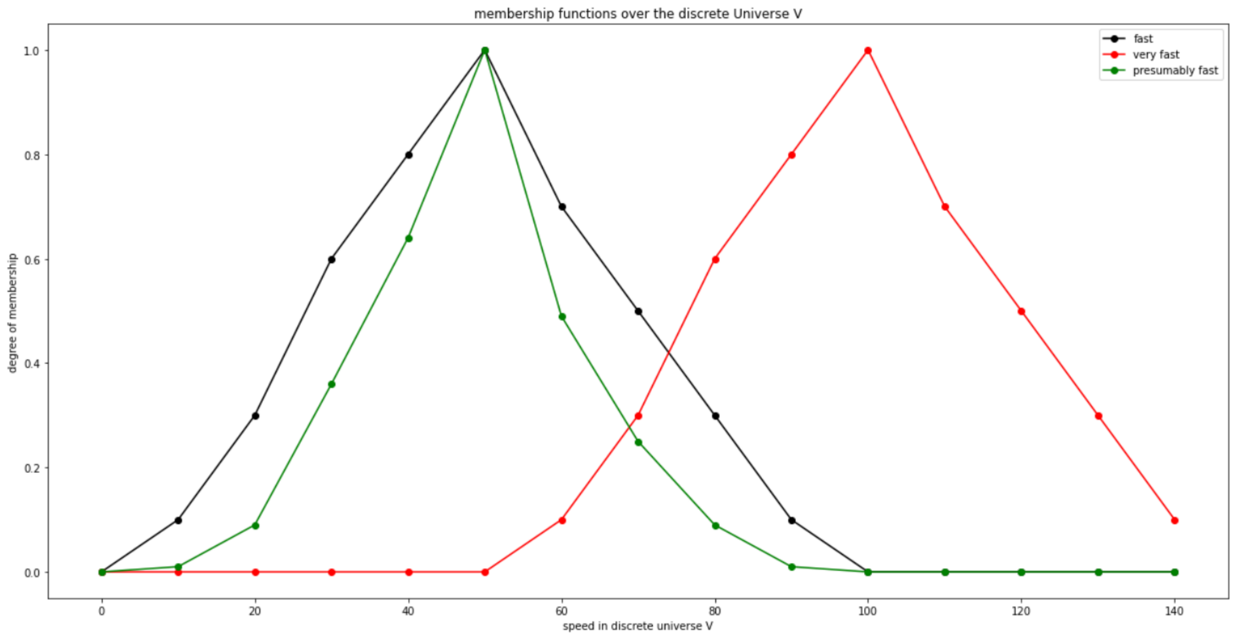
Thus $\mu(v-v_0) = \left\{\frac{0.1}{60}, \frac{0.3}{70}, \frac{0.6}{80}, \frac{0.8}{90}, \frac{1.0}{100}, \frac{0.7}{110}, \frac{0.5}{120}, \frac{0.3}{130}, \frac{0.1}{140}\right\}$

Membership function of "presumably fast speed" $\mu_F^2(v)$

We can calculate this membership function by squaring the degree of membership value of for each elements of the universe V. Thus the elements which are of very less speed would have a membership value closer to 0 and the speed that is fast, would have a degree of membership value closer to 1.

Thus we can get the membership function as follows,

$$\mu_F^2(v) = \left\{ \frac{0.01}{10}, \frac{0.09}{20}, \frac{0.36}{30}, \frac{0.64}{40}, \frac{1.0}{50}, \frac{0.49}{60}, \frac{0.25}{70}, \frac{0.09}{80}, \frac{0.01}{90} \right\}$$



The membership functions of "very fast speed" and "presumably fast speed" has been displayed over the discrete universe V. It is evident from the graph that "very fast speed" membership function represents fast speed elements of the universe V. Thus the function is appropriate to represent the linguistic hedge.

Whereas, the membership function $\mu_F^2(V)$ has contracted the graph. Thus, the function is not appropriate to represent the linguistic hedge "presumably fast speed".

Problem: 2)

Sketch the membership function

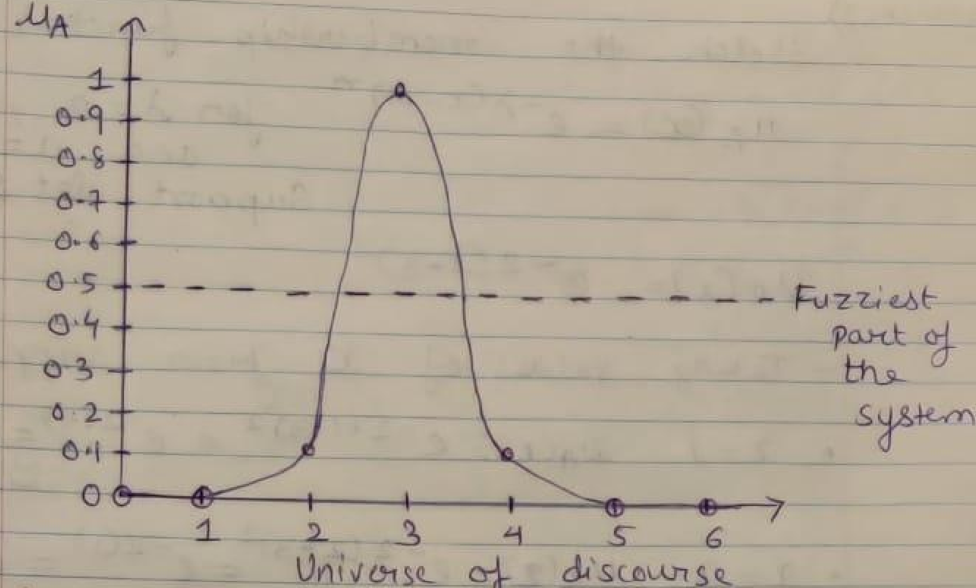
$\mu_A(x) = e^{-\lambda(x-a)^n}$ for $\lambda = 2$, $n=2$ and $a=3$

Support Set $S = [0,6]$

$\mu_A(x) = e^{-2(x-3)^2}$

- Taking values of x from support set

- $x=1$ $\mu_A(1) = e^{-2(1-3)^2} = e^{-2(4)} = 0.0003 \approx 0$
- $x=2$ $\mu_A(2) = e^{-2(2-3)^2} = e^{-2(1)} = 0.135$
- $x=3$ $\mu_A(3) = e^{-2(3-3)^2} = e^0 = 1$
- $x=4$ $\mu_A(4) = e^{-2(4-3)^2} = e^{-2} = 0.135$
- $x=5$ $\mu_A(5) = e^{-2(5-3)^2} = e^{-2(4)} = 0.0003 \approx 0$
- $x=6$ $\mu_A(6) = e^{-2(6-3)^2} = e^{-18} = 1.522 \times 10^{-8} \approx 0.0000\ldots \approx 0$
- $x=0$ $\mu_A(0) = e^{-2(0-3)^2} = e^{-18} = 1.522 \times 10^{-8} \approx 0$



On the sketch seperately show the shaded areas that represent the following fuzziness measures given by M_1, M_2 and M_3.

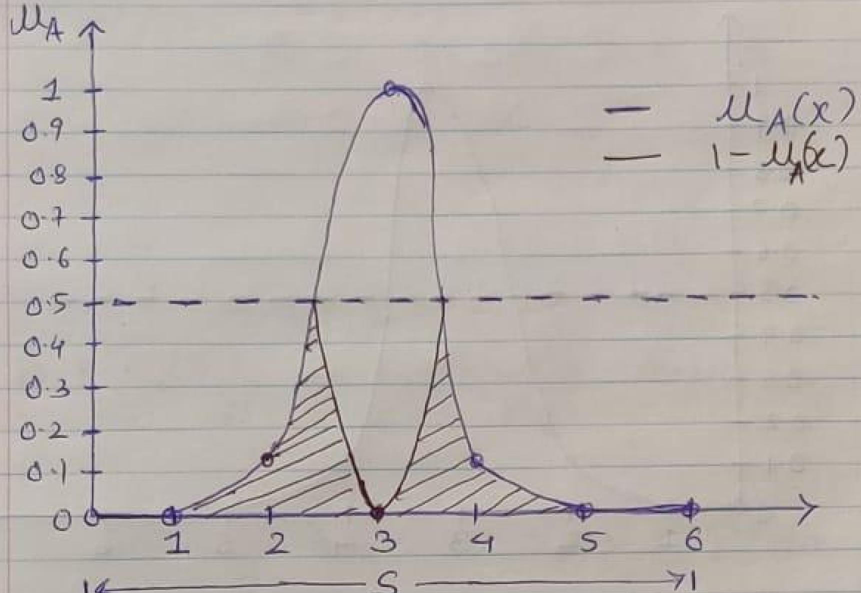
a) $M_1 = \int_S f(x)\,dx$ — (1)

$$\text{where } f(x) = \mu_A(x) \qquad \mu_A(x) \leq 0.5$$
$$= 1 - \mu_A(x) \qquad \mu_A(x) > 0.5$$

As calculated above,

- $x=0$ $\quad \mu_A(0) \approx 0 < 0.5 \quad f(0) = 0$
- $x=1$ $\quad \mu_A(1) = 0.0003 \approx 0 < 0.5 \quad f(1) = 0$
- $x=2$ $\quad \mu_A(2) = 0.135 < 0.5 \quad f(2) = 0.135$
- $x=3$ $\quad \mu_A(3) = 1 > 0.5 \quad f(3) = 1 - \mu_A(3)$
$= 1 - 1$
$= 0$

- $x = 4$ $\quad \mu_A(4) = 0.135 < 0.5 \quad f(4) = 0.135$
- $x = 5$ $\quad \mu_A(5) \approxeq 0 < 0.5 \quad f(5) = 0$
- $x = 6$ $\quad \mu_A(6) \approxeq 0 < 0.5 \quad f(6) = 0$



Graph for M1

b) $M_2 = \int_S |\mu_A(x) - \mu_{A_{1/2}}(x)| \, dx \quad -(2)$

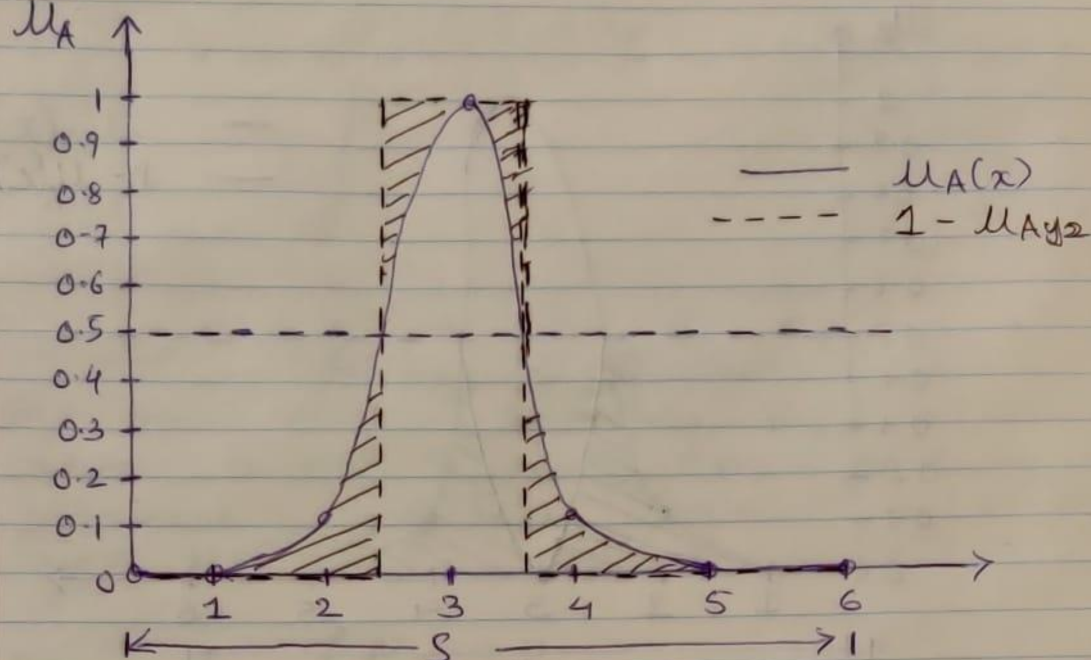
where $\mu_{A_{1/2}}$ is the α-cut of $\mu_A(x)$ for $\alpha = 1/2$

- $\mu_{A_{1/2}} = 1/2$ cut of A

this gives the distance of membership function from its $1/2$-cut.

So,
$\mu_{1/2} = 1$ when $\mu_A(x) \geq 0.5$

$\mu_{1/2} = 0$ when $\mu_A(x) < 0.5$

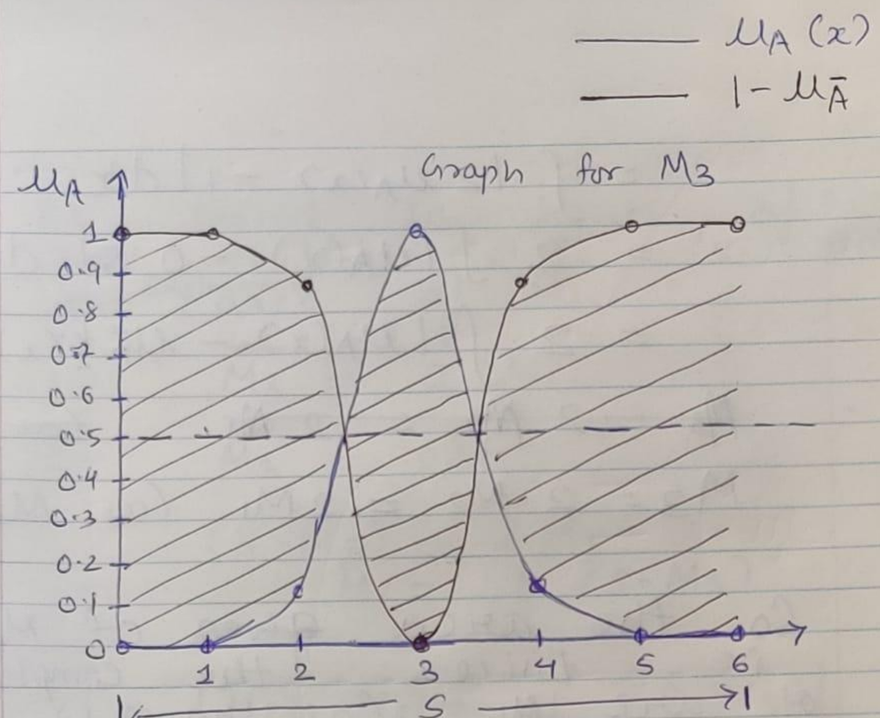


Graph for M_2

c) $M_3 = \int_S |\mu_A(x) - \mu_{\bar{A}}(x)| \, dx$

$\bar{A}$ = complement of fuzzy set A

M_3 is difference between membership function and its inverse



i) Establish relationship between M_1, M_2 and M_3.

$$M_2 = \int_S |\mu_A(x) - \mu_{A_{1/2}}(x)|\, dx$$

$$= \int_S f(x)\, dx$$

where $f(x) = |\mu_A(x) - \mu_{A_{1/2}}(x)|$

$$= M_1$$

(as equation's are identical)

$$M_3 = \int |\mu_A(x) - \bar{\mu}_A(x)|\, dx$$

$$= \int |\mu_A(x) - (1 - \mu_A(x))\, dx$$

As shown in figure of M_3, the shaded area is twice the complement of area of M_1.

$$\overline{M_3} = 2\ M_1$$

$$\overline{M_3} = 2M_1$$

$$(S - M_3) = 2M_1$$

$$M_1 = \frac{1}{2}(S - M_3)$$

$$\boxed{M_1 = M_2 = \frac{1}{2}(S - M_3)}$$

S = support set length.

ii) Indicate how these measures can be used to represent the degree of fuzziness of a membership function.

- Degree of fuzziness is a measure to know which element belongs within or outside of the fuzzy set.
- If $\mu_A(x) < 0.5$, element is outside the fuzzy set.

$\mu_A(x) > 0.5$, element is inside the fuzzy set.

$\mu_A(x) = 0.5$, most fuzzy element.

- M_1, M_2 and M_3 has continuous membership function but can also have possibility of being discrete membership.

- Usually fuzziness are calculated on per mode (peak)

- So if there is high - level of fuzziness then only high level fuzzy system can tolerate it which has multi-level fuzzy nodes.

- For multi-modal cases,

  expressions should be divided by number of modal points in A.

- higher the fuzzy resolution, lower the degree of fuzziness.

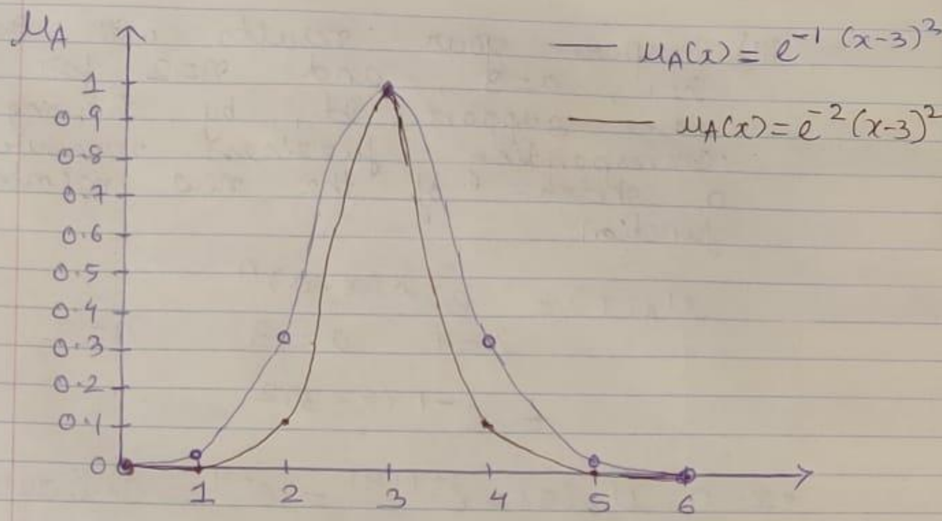
iii) Compare your results with the case $\lambda = 1$, $a = 3$, and $n = 2$ for the same support set, by showing the corresponding fuzziness measures on a sketch of the new membership function.

$$\mu_A(x) = e^{-\lambda(x-a)^n}$$

$$\lambda = 1 \quad a = 3 \quad n = 2$$

$$= e^{-1(x-3)^2}$$

- $x = 0 \quad \mu_A(0) = e^{-1(9)} = e^{-9} = 0.00012 \approx 0$
- $x = 1 \quad \mu_A(1) = e^{-1(4)} = e^{-4} = 0.018$
- $x = 2 \quad \mu_A(2) = e^{-1(-1)^2} = e^{-1} = 0.367$
- $x = 3 \quad \mu_A(3) = e^{-1(0)} = e^{0} = 1$
- $x = 4 \quad \mu_A(4) = e^{-1(1)^2} = e^{-1} = 0.367$
- $x = 5 \quad \mu_A(5) = e^{-1(4)} = e^{-4} = 0.018$
- $x = 6 \quad \mu_A(6) = e^{-1(9)} = e^{-9} = 0.00012 \approx 0$



- so here we can see as value of λ is decreased the graph expands horizontally.

a) $M_1 = \int_S f(x)\,dx$

where $f(x) = \mu_A(x)$ for $\mu_A(x) \leq 0.5$

$= 1 - \mu_A(x)$ for $\mu_A(x) > 0.5$

$x=0 \quad \mu_A(0) = 0 < 0.5 \quad f(0) = 0$

$x=1 \quad \mu_A(1) = 0.018 < 0.5 \quad f(1) = 0.018$

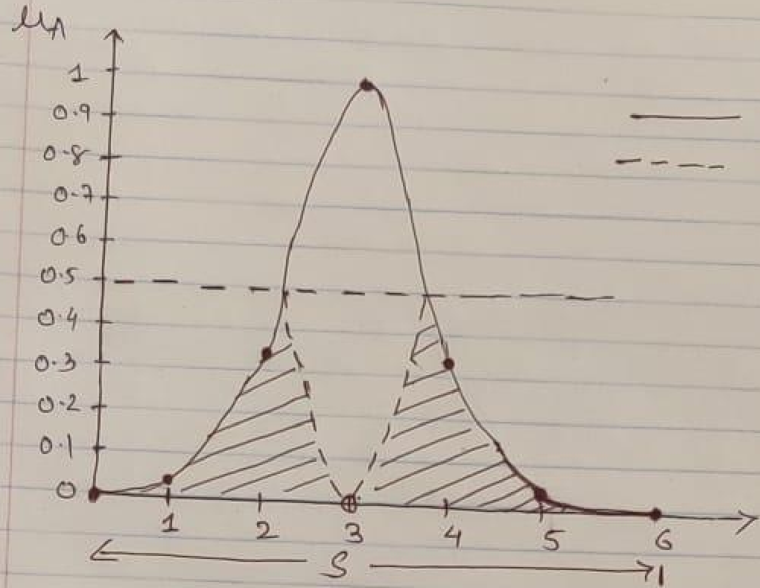
$x=2 \quad \mu_A(2) = 0.367 < 0.5 \quad f(2) = 0.367$

$x=3 \quad \mu_A(3) = 1 > 0.5 \quad f(3) = 1-1 = 0$

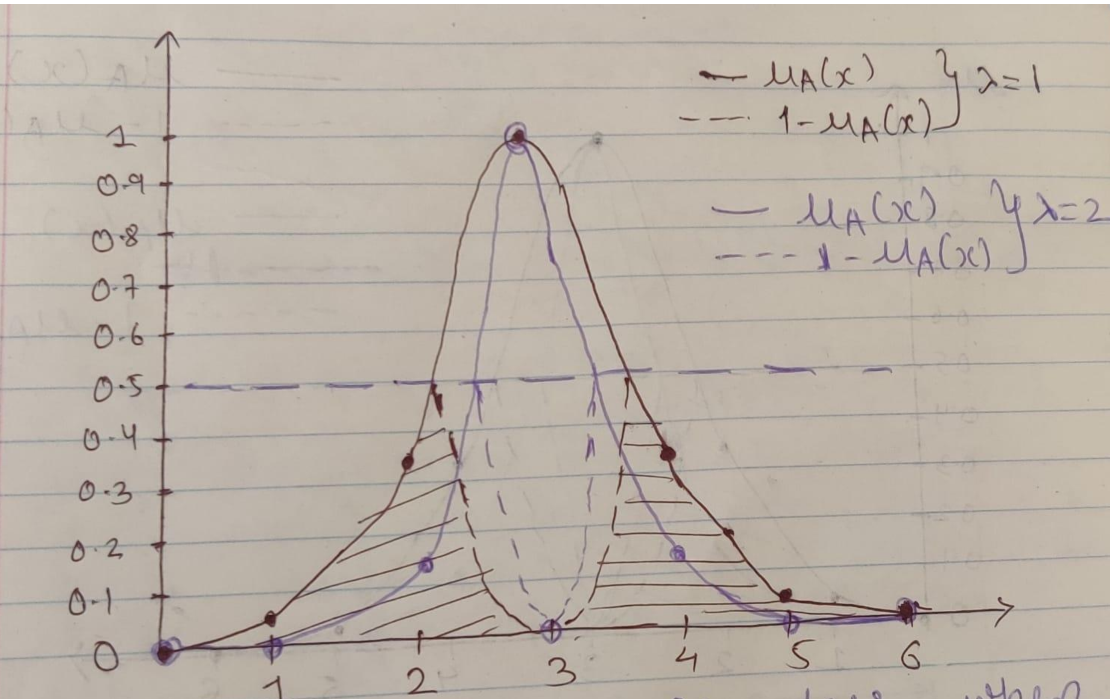
$x=4 \quad \mu_A(4) = 0.367 < 0.5 \quad f(4) = 0.367$

$x=5 \quad \mu_A(5) = 0.018 < 0.5 \quad f(5) = 0.018$

$x=6 \quad \mu_A(6) = 0.00012 < 0.5 \quad f(6) \approx 0$

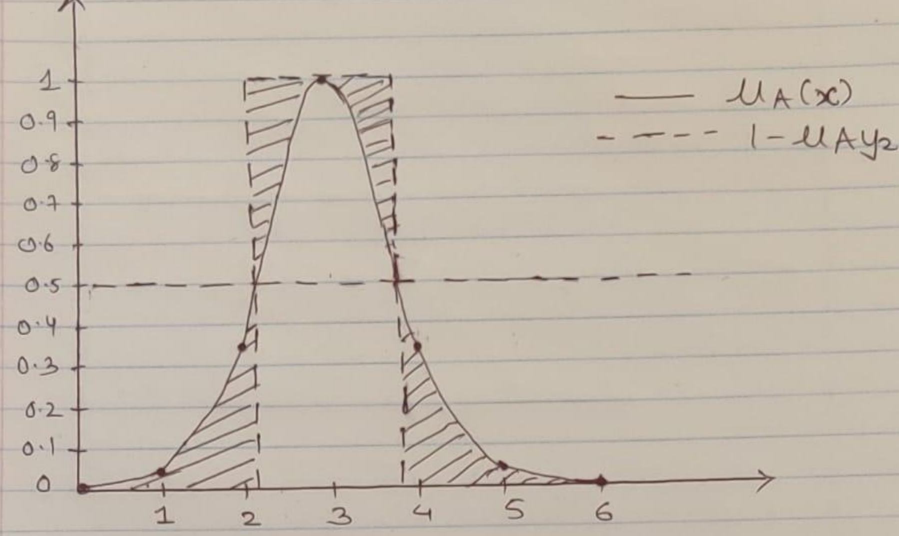


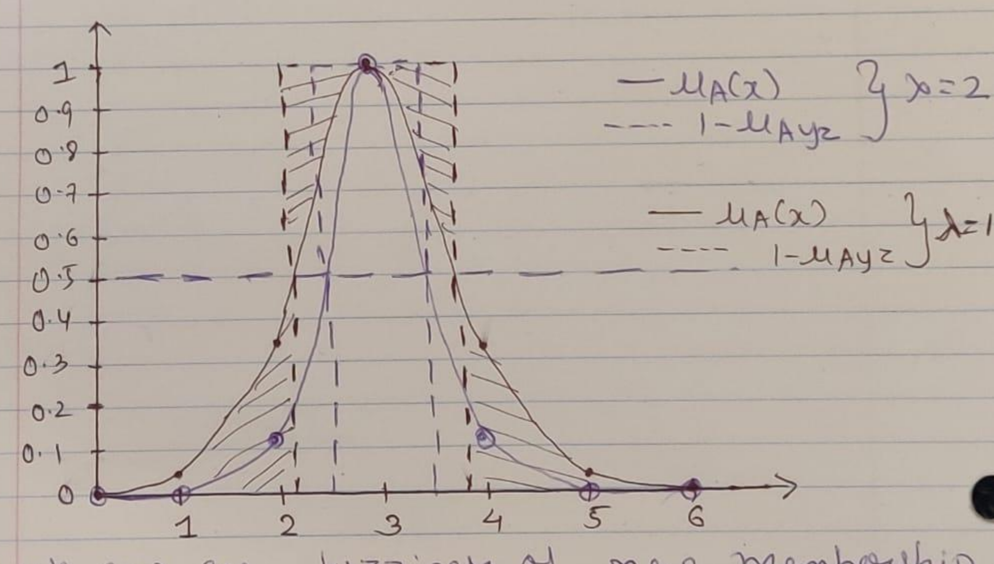
Graph for M_1 when $\lambda = 1$



Fuzziness in M1 is less when $\lambda = 1$ than $\lambda = 2$.

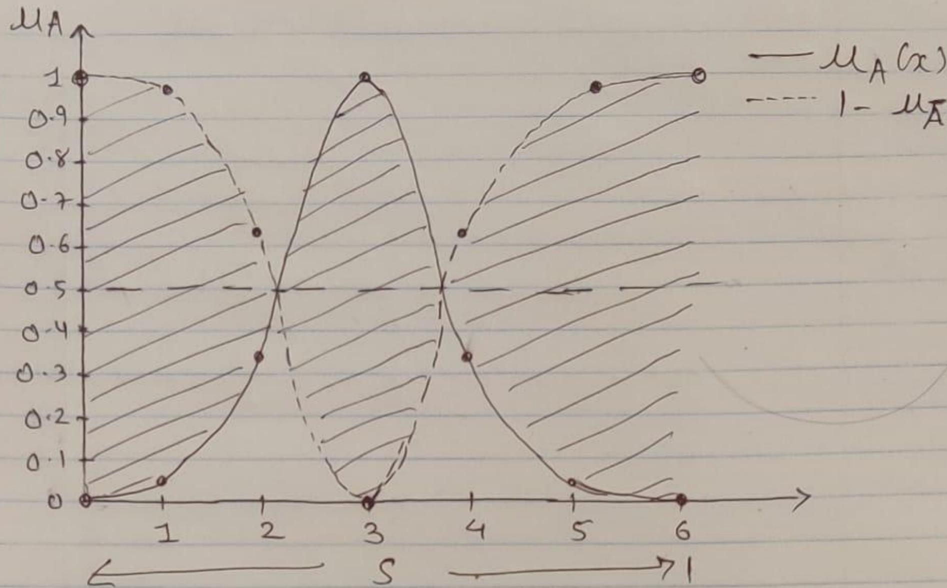
b) For $M_2 = \int |\mu_A(x) - \mu_{A_{1/2}}(x)| \, dx$



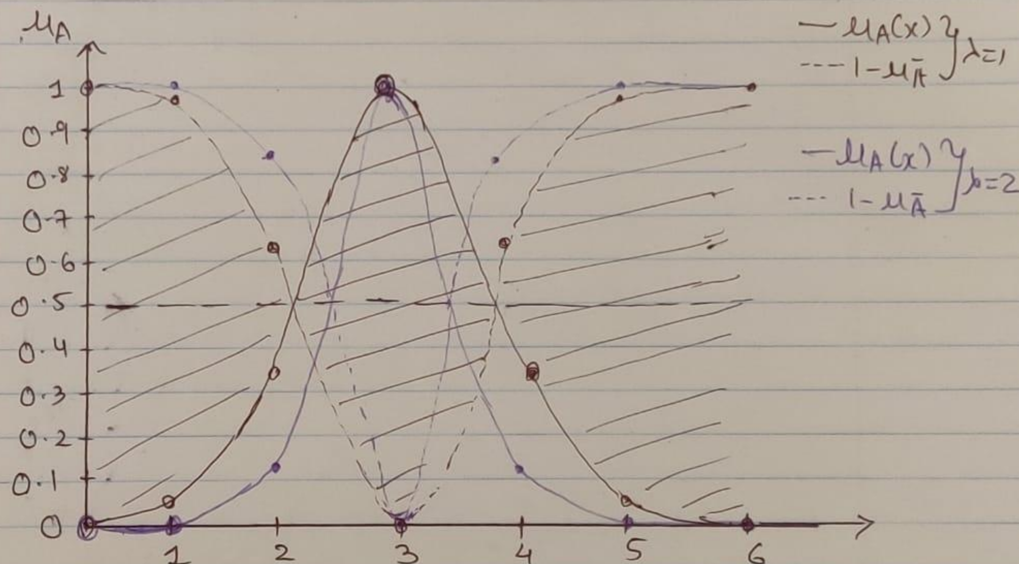


- We can see fuzziness of new membership is less for values less than 0.5 and more for values greater than 0.5.

For $M_3 = \int_S |\mu_A(x) - \mu_{\bar{A}}(x)| \, dx$



M_3 for new membership function



Comparision of both membership functions for M_3

- We can see the change in shaded area.

Problem 3

1. By definition $A' = 1 - A'$

$$X_{A'}(x) = 1 \text{ if } x \in A' = 1 - 0 \text{ if } x \in A' = 1 - 0 \text{ otherwise}$$
$$= 0 \text{ if } x \in A = 1 - 1 \text{ if } x \in A = 1 - 1 \text{ if } x < A$$
$$= 1 - X_A(x)$$

2. For the case where $\chi_A(x) \leq \chi_B(x)$

$\chi_A(x) \leq \chi_B(x) \Leftrightarrow A \subseteq B, \forall x \in X$

$A \cup B = B$ Hence $\chi_{A \cup B} = \chi_B$

For case where $\chi_A(x) \geq \chi_B(x)$

$\chi_A(x) \geq \chi_B(x) \Leftrightarrow B \subseteq A, \forall x \in X$

$A \cup B = A$ Hence $\chi_{A \cup B} = \chi_A$

Hence $\chi_{A \cup B} = \max(\chi_A, \chi_B)$

3. For the case where $\chi_A(x) \leq \chi_B(x)$

$\chi_A(x) \leq \chi_B(x) \Rightarrow A \subseteq B, \forall x \in X$

$A \cap B = A$ Hence $\chi_{A \cap B} = \chi_A$

For the case where $\chi_A(x) \geq \chi_B(x)$

$\chi_A(x) \geq \chi_B(x) \Rightarrow B \subseteq A, \forall x \in X$

$A \cap B = B$ Hence $\chi_{A \cap B} = \chi_B$

Hence $\chi_{A \cap B} = \min(\chi_A, \chi_B)$

4. $\chi_{A \to B}(x, y) = \min[1, \{\chi_A(x) + \chi_B(y)\}]$

$$= \min[1, \max(0, \chi_B(y))]$$

~~if B and A are in the same universe~~ As, $A \subseteq B$

$$\chi_{A \to B}(x, y) = \min(1, \max(0, 1)) = 1$$

~~if B and A are in different universe~~

~~$\chi_{A \to B}(x, y)$~~

5. Since binary logic and set theory ~~interchangeable~~ are convertible, they can be used in simplifying a logical knowledge base without knowing truth values of propositions

Problem 4

Prove non-decreasing property

From $\max[0, x+y-1]$ we know that
$a=0, b=0, c=x+y-1,$ assuming $d = x+y-1+\delta$
where $\delta \in \mathbb{R}$

~~$a^Tc = b^Td$~~

Case 1 – $a=b$, $c=d$ where $\delta = 0$
$a^Tc = \max(a,c) = \max(0, x+y-1)$ $b^Td = \max(b,d) = \max(0, x+y-1)$
$a^Tc = b^Td$

Case 2 – $a=b$, $c<d = x+y-1+\delta$, where $\delta > 0$
$a^Tc = \max(a,c) = \max(0, x+y-1)$ $b^Td = \max(b,d) = \max(0, x+y-1+\delta)$
if $x+y-1>0$, then $a^Tc = x+y-1$ and $b^Td = x+y-1+\delta$
we can conclude that $a^Tc < b^Td$

if $x+y-1<0$ then $a^Tc = 0$ and b^Td can be either
$x+y-1+\delta \geq 0$ or 0 depending on the value of δ

if $x+y-1+\delta \geq 0$, then $b^Td = x+y-1+\delta \geq 0$
if $x+y-1+\delta < 0$, then $b^Td = 0$
Since b^Td is either greater then or equals to 0
we can conclude that $a^Tc \leq b^Td$

Non-decreasing property holds for all cases

Prove commutativity property

Case 1 — $a = 0$, $b = x + y - 1 > 0$

$a^T b = \max(0, x + y - 1) = x + y - 1$

$b^T a = \max(x + y - 1, 0) = x + y - 1$

$\because a^T b = b^T a$ $\therefore$ commutativity holds

Case 2 — $a = 0$, $b = x + y - 1 \leq 0$

$a^T b = \max(0, x + y - 1) = 0$

$b^T a = \max(x + y - 1, 0) = 0$

$\because a^T b = b^T a$ $\therefore$ commutativity holds

Commutativity holds for all cases

Prove associativity property

Case 1 — $a = 0$ $b = x + y - 1 > 0$ $c = z > b$

$\cancel{(a^T b)^T c = \max(\max(0, x + y - 1)}$

$((a^T b)^T c) = \max(\max(0, x + y - 1), z) = z$

$a^T (b^T c) = \max(0, \max(x + y - 1, z)) = z$

$\cancel{(a^T b)^T c = a^T (b^T c)}$ ~~holds~~

Case 2 — $a = 0$ $b = x + y - 1 \leq 0$ $c = z > b$, $z > a$

$(a^T b)^T c = \max(\max(0, x + y - 1), z) = z$

$a^T (b^T c) = \max(0, \max(x + y - 1, z)) = z$

Case 3 — $a = 0$ $b = x + y - 1 \leq 0$ $c = z < b$, $z < a$

$(a^T b)^T c = \max(\max(0, x + y - 1), z) = 0$

$a^T (b^T c) = \max(0, \max(x + y - 1, z)) = 0$

Case 4 — $a = 0$ $b = x + y - 1 > 0$ $c = z < b$ and $z > a$

$(a^T b)^T c = \max(\max(0, x + y - 1), z) = x + y - 1$

$a^T (b^T c) = \max(0, \max(x + y - 1, z)) = x + y - 1$

Case 5 — $a = 0$ $b = x + y - 1 > 0$ $c = z < b$ and $z \leq a$

$(a^T b)^T c = x + y - 1$ $a^T (b^T c) = x + y - 1$

Case 6 — $a = 0$ $b = x + y - 1 \neq 0$ $c = z = a$

$(a^T b)^T c = 0$ $a^T (b^T c) = 0$

$\therefore (a^T b)^T c = a^T (b^T c)$ holds for all cases boundary conditions

$a = 0 \quad b = x + y - 1 = 0$

~~$a T 0 = \max(0, 0) = 0$~~

$a^{T} 0 = \max(0, \cancel{x} + y - 1) = 0$

T-conorm

According to DeMorgan's Law

$a^{S} b = 1 - (1 - a) T (1 - b)$

Direct substitution:

$a^{S} b = 1 - \max[(1-0), (1 - x + y - 1)]$

$= \cancel{0} 1 - \cancel{\max}(1 - 0) = 0 \quad \text{if } 0 \geq x + y - 1$

$= 1 - (1 - (x + y - 1)) = 1 - 1 + (x + y - 1)$

$= x + y - 1$

$= \min(0, x + y - 1) \quad \text{if } x + y - 1 < 0$

Hence $a^{S} b = \min(0, x + y - 1)$