

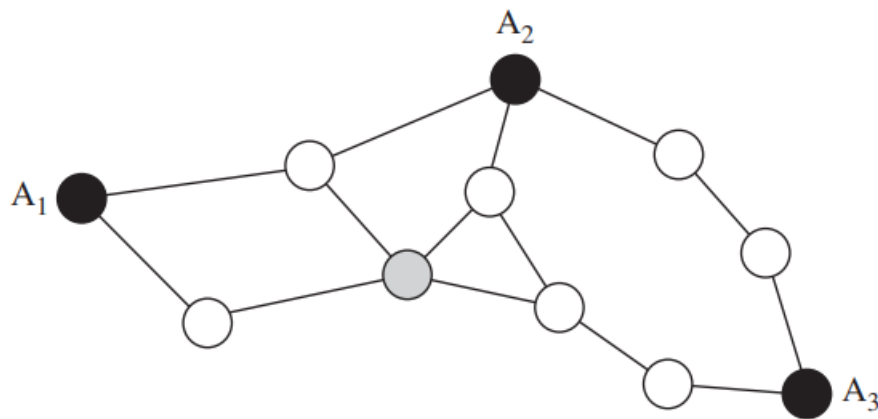
ECE 659/493: IOT Signal Processing and Intelligent Sensor Networks

Assignment 3 – Group 2

Instructor: Dr. Otman A. Basir

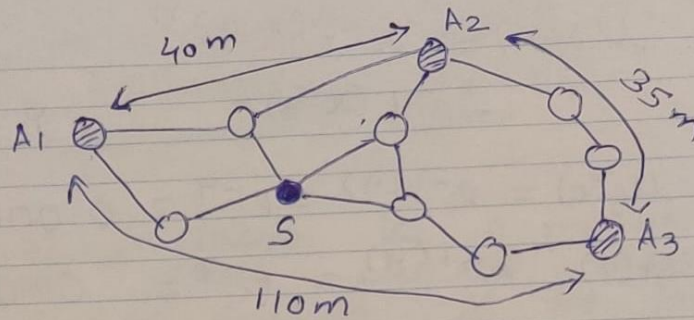
Student ID: Juhi Vasudev Bachani (20979706),
Akashdeep Singh Khehra (20988007),
Seoyoung Sim (20993717)

Q-1) The figure below shows a network topology with three anchor nodes. The distances between anchors A1 and A2, anchors A1 and A3, and anchors A2 and A3 are 40 m, 110 m, and 35 m, respectively. Use the Ad Hoc Positioning System to estimate the location of the gray sensor node (show each step of your process)



Ans)

Q-1)



- distance between A1 and A2 = 40m
A1 and A3 = 110m
A2 and A3 = 35m

- number of links (hop) between
A1 and A2 = 2
A1 and A3 = 5
A2 and A3 = 3

- correction = $\frac{\text{distance}}{\text{links}}$

- A1 computes the correction

$$= \frac{40 + 110}{2 + 5}$$

$$\left(\frac{\text{distance from A2} + \text{from A3}}{\text{links from A2} + \text{from A3}} \right)$$

$$= \frac{150}{7}$$

$$= 21.42$$

Similarly,

- A₂ Computes the correction = $\frac{40+35}{2+3}$

$$= \frac{75}{5} \quad \boxed{= 15}$$

- A₃ Computes the correction = $\frac{35+110}{3+5}$

$$= \frac{145}{8}$$

$$= 18.12$$

- we will calculate the minimum hop for from all anchors from S.

Minimum hop between A₁ and S = 2

A₂ and S = 2

A₃ and S = 3

- Generally minimum hop from all is 2, so we will use value of A₂ as it is closest to S.

So correction will use = 15.

- Distance to A₁ from S = $2 \times 15 = 30\text{m}$

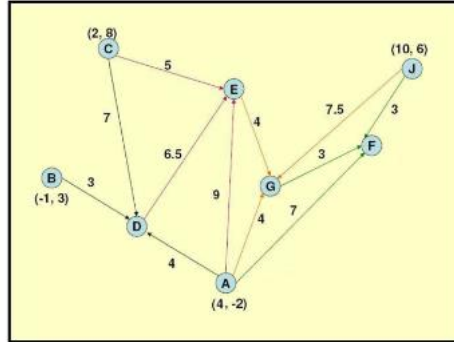
- Distance to A₂ from S = $2 \times 15 = 30\text{m}$

- Distance to A₃ from S = $3 \times 15 = 45\text{m}$

Kiloby

Q-2) For the IoT network given in the figure below.

- Find out the location of each node based on multilateration with the information of anchor node coordinates and the distance between nodes given in the figure.
- Show how the DV-HOP ad-hoc positioning technique can be used to estimate the location of each node.



Ans)

The location of anchor nodes and the distance between anchor hop and other nodes are already given. Based on DV-HOP ad-hoc positioning technique, we only consider the node which is the closest (in this case, 1 hop) in location calculation. Since we already know the distance and location of anchor nodes, we can derive the location of other nodes by using multilateration.

Let the location of $D = (x_D, y_D)$, $E = (x_E, y_E)$, $G = (x_G, y_G)$, $F = (x_F, y_F)$.

Using multilateration,

$$2 \begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix} \begin{bmatrix} x_U \\ y_U \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\ (r_2^2 - r_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \end{bmatrix}$$

1) Location of D

$$(x_A, y_A) = (4, -2), (x_B, y_B) = (-1, 3), (x_C, y_C) = (2, 8), r_A = 4, r_B = 3, r_C = 7$$

$$2 \begin{bmatrix} -2 & 10 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} 15 \\ 18 \end{bmatrix}$$

$$\therefore (x_D, y_D) = (1.3125, 1.0125) \approx (1.3, 1.0)$$

2) Location of E

$$(x_A, y_A) = (4, -2), (x_D, y_D) = (1.3, 1.0), (x_C, y_C) = (2, 8), r_A = 9, r_D = 6.5, r_C = 5$$

$$2 \begin{bmatrix} -2 & 10 \\ 0.7 & 7 \end{bmatrix} \begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} 104 \\ 40.50875 \end{bmatrix}$$

$$\therefore (x_E, y_E) = (-15.013, 4.39479) \approx (-15.0, 4.4)$$

3) Location of G

$$(x_A, y_A) = (4, -2), (x_E, y_E) = (-15.0, 4.4), (x_J, y_J) = (10, 6), r_A = 4, r_E = 4, r_J = 7.5$$

$$2 \begin{bmatrix} 6 & 8 \\ 25 & 1.6 \end{bmatrix} \begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} 75.75 \\ 49.38 \end{bmatrix}$$

$$\therefore (x_G, y_G) = (0.719118, 4.19504) \approx (0.7, 4.2)$$

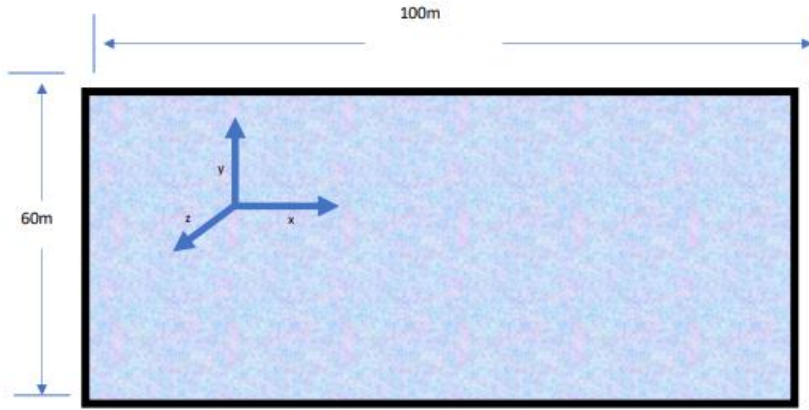
4) Location of F

$$(x_A, y_A) = (4, -2), (x_G, y_G) = (0.7, 4.2), (x_J, y_J) = (10, 6), r_A = 7, r_G = 3, r_J = 3$$

$$2 \begin{bmatrix} 6 & 8 \\ 9.3 & 1.8 \end{bmatrix} \begin{bmatrix} x_F \\ y_F \end{bmatrix} = \begin{bmatrix} 156 \\ 119.2375 \end{bmatrix}$$

$$\therefore (x_F, y_F) = (5.29167, 5.78125) \approx (5.3, 5.8)$$

Q-3) Consider the case of the interior of a building of a rectangular shape 100mx60mx5m.



It is desired that objects moving around inside this building be located by means of the wireless signals propagating inside the building. These signals are produced by three wireless devices: one device is located at $(x=0.0\text{m}, y=30.0, z=3.0\text{m})$, one device is located at $(x=50.0\text{m}, y=60.0\text{m}, z=4.0\text{m})$, one device is located at $(x=100.0\text{m}, y=30.0\text{m}, z=3.0\text{m})$. The floor of the building is a grid of equal size square-tiles. It suffices to locate the device roaming inside the building in terms of a tile index.

RSSI Model: Please refer to the article “Indoor Positioning Algorithm Based on the Improved

RSSI Distance Model”, posted on the course site.

Common propagation path-loss models include the free space propagation model, the logarithmic distance path-loss model, etc. Studies have shown that the channel fading characteristic follows a lognormal distribution. RSSI distance measurement generally uses the logarithmic distance path-loss model [32–34]. It is expressed as

$$RSSI = 10n \lg(d) + A + X_s, \quad (1)$$

where d is the distance between the transmitter and the receiver, and n is a path-loss parameter related to the specific wireless transmission environment. The more obstacles there are, the larger n will be. A is the RSSI with distance d_0 from the transmitter. X_S is a Gaussian-distribution random variable with mean 0 and variance σ^2 .

For convenience of calculation, d_0 usually takes a value of 1 meter. Since X_S has a mean of 0, the distance-loss model can be obtained with

$$RSSI = 10n \log(d) + A, (2)$$

where A is the average measured RSSI when the received node is 1 meter away from the transmit node which is related to the RF circuits of Bluetooth nodes. By gathering the RSSI values for Bluetooth beacons at different distances and using the least squares algorithm to fit the parameters, we can obtain the RSSI distance model.

Ans)

We are already given in the question that pathloss is 4db which is denoted by 'n'. The value of standard deviation is 5.1db. Locations A, B, and C are signal-producing wireless devices, and A' represents the average RSSI as measured at a distance of one metre from the transmitter.

- 1. Using the model in Equation 2, generate an RSSI profile as a function of the distance d, for d =1 to 140m.**

Solution-:

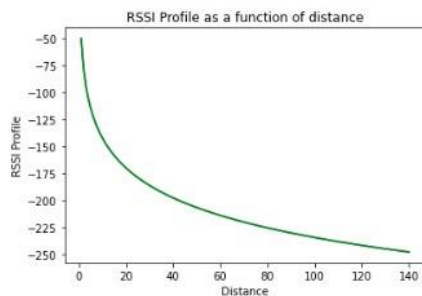
The mean RSSI value can be measured using the equation- $RSSI' = -10n * \log(d) + A'$

Hence, after applying the equation the Output comes out to be:-


```
In [40]: 1 RSSI_profile=[]
2 d= list(range(1,141))
3 for i in range (1,141):
4     RSSI_profile.append(RSSI(i))
5 RSSI_profile=np.array(RSSI_profile)
6 print(RSSI_profile)
7 print("Mean: ",mean(RSSI_profile))
8 print("Standard deviation: ",statistics.pstdev(RSSI_profile))
9 #Plotting the RSSI profile
10 plt.plot(d,RSSI_profile)
11 plt.title("RSSI Profile as a function of distance")
12 plt.xlabel("Distance")
13 plt.ylabel("RSSI Profile")
14 plt.plot(d,RSSI_profile,color='Green')

[ -50.    -77.726  -93.944 -105.452 -114.378 -121.67  -127.836 -133.178
-137.889 -142.103 -145.916 -149.396 -152.598 -155.562 -158.322 -160.904
-163.329 -165.615 -167.778 -169.829 -171.781 -173.642 -175.42  -177.122
-178.755 -180.324 -181.833 -183.288 -184.692 -186.048 -187.359 -188.629
-189.86  -191.054 -192.214 -193.341 -194.437 -195.503 -196.542 -197.555
-198.543 -199.507 -200.448 -201.368 -202.266 -203.146 -204.006 -204.848
-205.673 -206.481 -207.273 -208.05  -208.812 -209.559 -210.293 -211.014
-211.722 -212.418 -213.101 -213.774 -214.435 -215.085 -215.725 -216.355
-216.975 -217.586 -218.188 -218.78  -219.364 -219.94  -220.507 -221.067
-221.618 -222.163 -222.7    -223.229 -223.752 -224.268 -224.778 -225.281
-225.778 -226.269 -226.754 -227.233 -227.706 -228.174 -228.636 -229.093
-229.545 -229.992 -230.434 -230.872 -231.304 -231.732 -232.155 -232.574
-232.988 -233.399 -233.805 -234.207 -234.605 -234.999 -235.389 -235.776
-236.158 -236.538 -236.913 -237.285 -237.654 -238.019 -238.381 -238.74
-239.096 -239.448 -239.797 -240.144 -240.487 -240.827 -241.165 -241.5
-241.832 -242.161 -242.487 -242.811 -243.133 -243.451 -243.767 -244.081
-244.392 -244.701 -245.008 -245.312 -245.614 -245.914 -246.211 -246.506
-246.799 -247.09  -247.379 -247.666]
Mean:  -208.63434285714285
Standard deviation:  37.553796959784705

Out[40]: [<matplotlib.lines.Line2D at 0x1e49a9cddb0>]
```



2. Generate a fingerprint for the tile grid. Each grid tile fingerprint is the RSSI readings from the three devices measured at that particular tile. Use the centre of the tiles for your calculations.

Solution:-

For generating a fingerprint we are given sensors at (x=0.0m, y=30.0m, z=3.0m), one at (x=50.0m, y= 60.0m, z=4.0m) and one at (x=100.0m, y=30.0m, z=3.0m) we have created a function **def fp rint** to determine the fingerprint of a specific tile location w.r.t the router positions.

The output in this case comes out to be:-

```

Tile at position 0 is [0.5, 0.5, 0]
Tile at position 1 is [0.5, 1.5, 0]
Tile at position 2 is [0.5, 2.5, 0]
Tile at position 3 is [0.5, 3.5, 0]
Tile at position 4 is [0.5, 4.5, 0]
Tile at position 5 is [0.5, 5.5, 0]
Tile at position 6 is [0.5, 6.5, 0]
Tile at position 7 is [0.5, 7.5, 0]
Tile at position 8 is [0.5, 8.5, 0]
Tile at position 9 is [0.5, 9.5, 0]
Tile at position 10 is [0.5, 10.5, 0]
Tile at position 11 is [0.5, 11.5, 0]
Tile at position 12 is [0.5, 12.5, 0]
Tile at position 13 is [0.5, 13.5, 0]
Tile at position 14 is [0.5, 14.5, 0]
Tile at position 15 is [0.5, 15.5, 0]
Tile at position 16 is [0.5, 16.5, 0]
Tile at position 17 is [0.5, 17.5, 0]
Tile at position 18 is [0.5, 18.5, 0]
Tile at position 19 is [0.5, 19.5, 0]
Tile at position 20 is [0.5, 20.5, 0]
Tile at position 21 is [0.5, 21.5, 0]
Tile at position 22 is [0.5, 22.5, 0]
Tile at position 23 is [0.5, 23.5, 0]
Tile at position 24 is [0.5, 24.5, 0]
Tile at position 25 is [0.5, 25.5, 0]
...
Tile at position 5970 is [99.5, 30.5, 0]
Tile at position 5971 is [99.5, 31.5, 0]
Tile at position 5972 is [99.5, 32.5, 0]
Tile at position 5973 is [99.5, 33.5, 0]
Tile at position 5974 is [99.5, 34.5, 0]
Tile at position 5975 is [99.5, 35.5, 0]
Tile at position 5976 is [99.5, 36.5, 0]
Tile at position 5977 is [99.5, 37.5, 0]
Tile at position 5978 is [99.5, 38.5, 0]
Tile at position 5979 is [99.5, 39.5, 0]
Tile at position 5980 is [99.5, 40.5, 0]
Tile at position 5981 is [99.5, 41.5, 0]
Tile at position 5982 is [99.5, 42.5, 0]
Tile at position 5983 is [99.5, 43.5, 0]
Tile at position 5984 is [99.5, 44.5, 0]
Tile at position 5985 is [99.5, 45.5, 0]
Tile at position 5986 is [99.5, 46.5, 0]
Tile at position 5987 is [99.5, 47.5, 0]
Tile at position 5988 is [99.5, 48.5, 0]
Tile at position 5989 is [99.5, 49.5, 0]
Tile at position 5990 is [99.5, 50.5, 0]
Tile at position 5991 is [99.5, 51.5, 0]
Tile at position 5992 is [99.5, 52.5, 0]
Tile at position 5993 is [99.5, 53.5, 0]
Tile at position 5994 is [99.5, 54.5, 0]
Tile at position 5995 is [99.5, 55.5, 0]
Tile at position 5996 is [99.5, 56.5, 0]
Tile at position 5997 is [99.5, 57.5, 0]
Tile at position 5998 is [99.5, 58.5, 0]
Tile at position 5999 is [99.5, 59.5, 0]

```

We have created a “fprint_grid.csv” file for the same which shows the entire data.

3. Consider a roaming device, placed at the center of the indexed by (30, 45, 0). Estimate the RSSI readings using the same model.

```

In [44]: ▶ 1 roam_device_tile = [30,45,0]
          2 roam_rssi = fprint(roam_device_tile, loc_A, loc_B, loc_C)
          3 print(roam_rssi)

[-194.8313, -186.9294, -215.6513]

```


4. Compute the tile location of the roaming device by matching its RSSI readings as in 3 above, with that stored in the grid fingerprint. Repeat this ten times and compute the mean location value.

```
File locs for the tiles:
[[21.5, 57.5, 0], [22.5, 51.5, 0], [35.5, 39.5, 0], [25.5, 51.5, 0], [20.5, 54.5, 0], [19.5, 47.5, 0], [22.5, 50.5, 0], [22.5, 51.5, 0], [37.5, 37.5, 0], [26.5, 52.5, 0]]

RSSI values for tiles 10 iterations:
[[-203.6291 -180.356 -230.4951]]

[[-188.1664 -183.5359 -216.3332]]

[[-193.84 -177.811 -218.5642]]

[[-189.3804 -178.5552 -228.5913]]

[[-195.0466 -186.0789 -219.0109]]

[[-183.3975 -189.1207 -213.2581]]

[[-190.6088 -181.1972 -218.3709]]

[[-188.1664 -183.5359 -216.3332]]

[[-188.631 -180.0337 -221.6849]]

[[-183.5985 -182.2143 -224.3024]]

Ques 3.4
The mean loc value using RSSI is [25.4 49.4 0.]
```

5. Compute the location error, i.e, the distance between the true tile location and the mean location value.

Solution:-

Therefore, mean location error value using RSSI is as follows:-

The estimated location using the triangulation method is:-

```
Ques 3.5
The mean loc err value using RSSI is 6.3655321851358195
Estimated loc using the triangulation method is [23.57426167 48.18948349 3.60631612]
```

6. Using the estimated readings in 3 above and the RSSI model, compute the distance between the roaming device and each wireless anchor. Use a trilateration technique to estimate the three dimensional location of the roaming device. Compare that to true location.

Solution:-

Therefore, the mean location value using trilateration is 0.029223750499847

```
Ques 3.6
The mean loc err value using trilateration is 8.029223750499847
```

Q-4) Suppose we have two sensors with known (and different) variances v_x and v_y , but unknown (and the same) mean μ . Suppose we observe n_x observations from the first sensor and n_y observations from the second sensor. Call these D_x and D_y . Assume all distributions are Gaussian.

1. What is the posterior $p(\mu|D_x, D_y)$, assuming a non-informative prior for μ ? Give an explicit expression for the posterior mean and variance. Hint : uses Bayesian updating twice, once to get from $p(\mu) \rightarrow p(\mu|D_x)$ (starting from a non-informative prior, which we can simulate using a precision of 0), and then again to get from $p(\mu|D_x) \rightarrow p(\mu|D_x, D_y)$.

$$\text{Density of Gaussian distribution : } f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\text{non-informative prior : } \pi(\mu) = \mu$$

$$\text{when we find the likelihood of } L(\mu|D_x), L(D_x|\mu) = \prod f(D_x) = \prod \frac{1}{\sqrt{2\pi v_x}} e^{-\frac{(D_x-\mu)^2}{2v_x}}.$$

$$\text{Then, the posterior is } p(\mu|D_x) \propto L(D_x|\mu)\pi(\mu) = \mu e^{-\frac{(D_x-\mu)^2}{2v_x}}.$$

$$\text{Similarly, when we find the likelihood of } L(\mu|D_x, D_y), L(D_x, D_y|\mu) = \prod f(D_x, D_y) = \prod \frac{1}{\sqrt{2\pi v_x}} e^{-\frac{(D_x-\mu)^2}{2v_x}} \frac{1}{\sqrt{2\pi v_y}} e^{-\frac{(D_y-\mu)^2}{2v_y}}.$$

$$\text{Then, the posterior is } p(\mu|D_x, D_y) \propto L(D_x, D_y|\mu)p(\mu|D_x) = \mu e^{-\frac{(D_x-\mu)^2}{2v_x}} e^{-\frac{(D_y-\mu)^2}{2v_y}}.$$

$$\text{The posterior mean is } E(\mu) = \int_{-\infty}^{\infty} \mu p(\mu|D_x, D_y) d\mu = \frac{v_x}{v_x+v_y} D_x + \frac{v_y}{v_x+v_y} D_y.$$

$$\text{The posterior variance is } Var(\mu) = \int_{-\infty}^{\infty} \mu^2 p(\mu|D_x, D_y) d\mu - (E(\mu))^2 = \frac{v_x v_y}{v_x+v_y} = \left(\frac{1}{v_x} + \frac{1}{v_y}\right)^{-1}.$$

2. Suppose the y sensor is very unreliable. What will happen to posterior mean estimate Give a simplified approximate expression.

If y sensor is very unreliable, then in the progress of calculating posterior mean, we can ignore D_y part, so the posterior mean is $\frac{v_x}{v_x+v_y} D_x$.

Q-5) Given that the sensors provide the following assessment in the form of mass functions as in the table below, use DS evidence fusion to compute the evidence on each potential identity. Calculate the conflict factor.

Identity	Sensor D ₁	Sensor D ₂
F	0.3	0.4
M	0.15	0.10
A	0.03	0.02
Animal	0.42	0.45
Unknown	0.10	0.03
Total Mass	1.00	1.00

Ans)

Q-5)

2 Sensors D_1 and D_2

		F	M	A	Animal	Unknown
		0.4	0.10	0.02	0.45	0.03
F	0.3	$= 0.3 \times 0.4$ $= 0.12$	α	α	α	$= 0.3 \times 0.03$ $= 0.009$
M	0.15	α	$= 0.15 \times 0.10$ $= 0.015$	α	α	$= 0.15 \times 0.03$ $= 0.0045$
A	0.03	α	α	$= 0.03 \times 0.02$ $= 0.0006$	α	$= 0.03 \times 0.03$ $= 0.0009$
Animal	0.42	$= 0.42 \times 0.4$ $= 0.168$	$= 0.42 \times 0.10$ $= 0.042$	α	$= 0.42 \times 0.45$ $= 0.189$	$= 0.42 \times 0.03$ $= 0.0126$
Unknown	0.10	$= 0.10 \times 0.4$ $= 0.04$	$= 0.10 \times 0.10$ $= 0.01$	$= 0.10 \times 0.02$ $= 0.002$	$= 0.10 \times 0.45$ $= 0.045$	$= 0.10 \times 0.03$ $= 0.003$

- ~~Female~~ F can be in unknown set so taken two values one of F and other Unknown
- For ~~M~~ M, taken value of M and Unknown
- For A, values are considered of A and Unknown
- Animal, animal can be F, M, Animal and Unknown.
- For Unknown, unknown has possibility of being F, M, A, Animal and Unknown.

Conflict factor α

$$= (0.3 \times 0.10) + (0.3 \times 0.02) + (0.3 \times 0.45) \\ + (0.15 \times 0.4) + (0.15 \times 0.02) + (0.15 \times 0.45) \\ + (0.03 \times 0.4) + (0.03 \times 0.10) + (0.03 \times 0.45) \\ + (0.42 \times 0.02)$$

$$= 0.03 + 0.006 + 0.135 + \\ 0.06 + 0.003 + 0.0675 + \\ 0.012 + 0.003 + 0.0135 + \\ 0.0084$$

$$= 0.3384$$

$$1 - \alpha = 0.6616$$

$$D_1 + D_2 \{F\} = 0.12 + 0.009 + 0.168 \\ + 0.04 \\ = 0.337$$

To Normalize we will divide by $1 - \alpha$

$$= \frac{0.337}{0.6616}$$

Normalized

$$D_1 + D_2 \{F\} = 0.5093$$

$$D_1 + D_2 \{M\} = 0.015 + 0.0045 + 0.042 \\ + 0.01$$

$$= 0.0715$$

$$= 0.1080$$

$$D_1 + D_2 \{A\} = 0.0006 + 0.0009 + 0.002$$

$$= \frac{0.0035}{0.6616}$$

$$= 0.0052$$

$$D_1 + D_2 \{Animal\} = 0.168 + 0.042$$

$$+ 0.189 + 0.0126$$

$$+ 0.045$$

$$= \frac{0.4566}{0.6616}$$

$$= 0.6901$$

$$D_1 + D_2 \{Unknown\} = 0.009 + 0.0045 +$$

$$0.0009 + 0.0126 + 0.003$$

$$+ 0.04 + 0.01 + 0.002$$

$$+ 0.045$$

$$= \frac{0.127}{0.6616}$$

$$= 0.1919$$

Q-6) Compressed Sensing:

- Create a sparse vector of 512 random sensory values. Plot this vector
- Create a random measurement matrix to compress the sensory vector in a) to a compressed version consisting of 128 values. Plot this vector.
- Use the MATLAB function `l1eq_pd` function to recover the 512 sensory data. Plot the recovered signal and compare to the original one in (a) by computing the correlation factor between the two signals).

Ans) The Code and output is attached below:

```

Q_6Final.m
1      clc;
2      clear;
3      L = 512;
4      m = 128;
5      N_z = 128;
6      x = zeros(L,1);
7
8      % -----| a) -----
9
10     A = randperm(L);
11     A=A(1:N_z);
12     x(A) = sign(randn(1, N_z));
13     figure;
14     subplot(3,1,1);
15     plot(x);
16     axis([0 L -2 2]);
17     title('Data');
18
19     % ----- b) -----
20
21     A = randn(N_z, L);
22     b = A * x + 0.005 * randn(m,1);
23     subplot(3,1,2);
24     plot(1:m, b, 'g');
25     title('Compressed data');
26
27     % ----- c) -----
28     Uncompressed_data = l1eq_pd(x, A,[], b);
29     subplot(3,1,3);
30     plot(1:L, Uncompressed_data, 'r');
31     axis([0 512 -2 2]);
32     title('Recovered Data');
33
34
35     function l1 = l1eq_pd(x0, A, A1, b)
36         t_cg = 0.00000001;
37         m_cg = 200;
38         t_pd = 0.001;
39         m_pd = 50;
40         if (nargin < 5), t_pd = t_pd; end
41         if (nargin < 6), m_pd = m_pd; end
42         if (nargin < 7), t_cg = t_cg; end
43         if (nargin < 8), m_cg = m_cg; end
44
45         N = length(x0);
46         y_0 = zeros(N,1);
47         y_1 = ones(N,1);
48         beta = 0.5;
49         U = 10;
50         g = [y_0; y_1];
51

```



```

52 % Checking initial point
53 if (isa(A,'function_handle')) & (norm(A*x0-b)/norm(b) > t_cg)
54     disp('hard to find initial point');
55 elseif (isa(A,'function_handle')) & (r_cg > 1/2)
56     disp('no initial point');
57     l1 = x0;
58     return;
59 elseif (norm(A*x0-b)/norm(b) > t_cg)
60     disp('Hard to find intial point');
61     opts.POSDEF = true; opts.SYM = true;
62     [w, con] = linsolve(A*A', b, opts);
63     if (con < 1e-14)
64         disp('cant find initial point');
65         l1 = x0;
66         return;
67     end
68 end

```

```

69 x = x0;
70 u = (0.95)*abs(x0) + (0.10)*max(abs(x0));
71
72 %first iteration
73 f1 = x - u;
74 f2 = -x - u;
75 l_1 = -1./f1;
76 l_2 = -1./f2;
77 if (isa(A,'function_handle'))
78     v = -A(l_1-l_2);
79     A1v = A1(v);
80     perim = A(x) - b;
81 else
82     v = -A*(l_1-l_2);
83     A1v = A'*v;
84     perim = A*x - b;
85 end
86
87 gap = -(f1'*l_1 + f2'*l_2);
88 t = U*2*N/gap;
89 dual = g + [l_1-l_2; -l_1-l_2] + [A1v; y_0];
90 percen = [-l_1.*f1; -l_2.*f2] - (1/t);
91 r_norm = norm([dual; percen; perim]);
92 pr = 0;
93 Output = (gap < t_pd) | (pr >= m_pd);
94 while (~Output)
95
96     pr = pr + 1;
97
98     s1 = -l_1./f1 - l_2./f2;
99     s2 = l_1./f1 - l_2./f2;
100     sx = s1 - s2.^2./s1;
101     e1 = -1/t*(-1./f1 + 1./f2) - A1v;
102     e2 = -1 - 1/t*(1./f1 + 1./f2);

```

```

103     e3 = -perim;
104
105     if (isa(A,'function_handle')) & (r_cg > 1/2)
106         e_0 = e3 - A(e1./sx - e2.*s2./(sx.*s1));
107         hpfun = @(z) -A(1./sx.*A1(z));
108         [dv, r_cg, i_cg] = cgsolve(hpfun, e_0, t_cg, m_cg, 0);
109
110         disp('Cant find solution.');
```

$$l1 = x;$$

```

112         dx = (e1 - e2(s2)./s1 - A1(dv))./sx;
113         Adx = A*dx;
114         A1dv = A1*dv;
115         return
116     else
117         e_0 = -(e3 - A*(e1./sx - e2.*s2./(sx.*s1)));
118         hp = A*(sparse(diag(1./sx))*A');
119         [dv,con] = linsolve(hp, e_0); % ----- Quan Wang
120         if (con < 1e-14)
121             disp('Previous iteration.');
```

$$l1 = x;$$

```

123         return
124     end
125     dx = (e1 - e2.*s2./s1 - A'*dv)./sx;
126     Adx = A*dx;
127     A1dv = A'*dv;
128 end

129
130     du = (e2 - s2.*dx)./s1;
131
132     d1 = (l_1./f1).*(-dx+du) - l_1 - (1/t)*1./f1;
133     d2 = (l_2./f2).*(dx+du) - l_2 - 1/t*1./f2;
134
135     % m
136     i1 = find(d1 < 0); i2 = find(d2 < 0);
137     s = min([1; -l_1(i1)./d1(i1); -l_2(i2)./d2(i2)]);
138     i1 = find((dx-du) > 0); i2 = find((-dx-du) > 0);
139     s = (0.99)*min([s; -f1(i1)./(dx(i1)-du(i1)); -f2(i2)./(-dx(i2)-du(i2))]);

```

```

141     back1 = 0;
142     while (1)
143         l1 = x + s*dx; up = u + s*du;
144         l_1p = l_1 + s*d1; l_2p = l_2 + s*d2;
145         f1p = l1 - up; f2p = -l1 - up;
146
147         r3 = perim + s*Adx;
148
149         s = beta*s;
150         back1 = back1 + 1;
151         if (back1 > 32)
152             disp('Stuck..')
153             l1 = x;
154             return
155         end
156     end
157
158
159
160     %next iteration
161     x = l1; u = up;
162     A1v = A1l2;
163     l_1 = l_1p; l_2 = l_2p;
164     f1 = f1p; f2 = f2p;
165
166     gap = -(f1'*l_1 + f2'*l_2);
167     t = U*2*N/gap;
168     perim = r3;
169     percen = [-l_1.*f1; -l_2.*f2] - (1/t);
170     dual = g + [l_1-l_2; -l_1-l_2] + [A1v; y_0];
171     r_norm = norm([dual; percen; perim]);
172
173     Output = (gap < t_pd) | (pr >= m_pd);
174 end
175
176 end

```

OUTPUT:

