

Solution 5.1. Assume that the language is decidable. Then there exists a Turing machine D that can decide whether the given input M, x is in the language or not. For the input M, x , we can construct a new Turing machine M' with the following behavior:

- M' simulates M on the input string x ;
- If M halts during the simulation, M' will continue running until exactly $77n$ steps have been taken;
- If M does not halt, M' will continue running.

By constructing M' , we can solve the Halting Problem. If D accepts M', x , it means that M halts on x , and M' halts in exactly $77n$ steps. If D rejects, it means that M diverges on input x . Therefore, if such a Turing machine D existed that could decide whether M' halts in exactly $77n$ steps, we would be able to use it to solve the Halting Problem, which is known to be undecidable. This leads to a contradiction because the existence of D would imply that the Halting Problem is decidable. Hence, the language $\{M, x : M \text{ halts on } x \text{ in exactly } 77n \text{ steps for some integer } n > 0\}$ must be undecidable.