The University of Sydney
School of Mathematics and Statistics

Calculus Tutorial 9 (Week 10)

MATH1062/MATH1023: Mathematics 1B (Calculus) — Semester 2, 2024

Questions marked with * are harder questions.

Material covered

(1) Applications of partial derivatives

Summary of essential material

The differential of function $f(x,y)$ is given by $df = f_x(x,y)dx + f_y(x,y)dy$. Recall that it can be used to approximate functions near "easy-to-compute" function values.

Questions to complete during the tutorial

1. Find the differentials of the following functions:

(a) $f(x,y) = xe^y$

(b) $g(x,y) = \ln(xy^2)$

(c) $h(p,q) = p^5q^3$

(d) $m(a,b) = abc$

2. Find the differential of $f(x,y) = \sqrt{20 - x^2 - 7y^2}$. Use this to approximate the value of $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ when $x = 1.95$ and $y = 1.08$.

3. Let $f(x,y) = \sin(xy - x^2)$. Use the differential to approximate $f(0.98, 1.03)$ and $f(1.04, 1.02)$, and $f(0.99, 1)$.

4. Use the formula for the implicit derivative to calculate dy/dx, where the given equation defines y implicitly as a function of x.

(a) $x^2 - xy + y^3 = 8$

(b) $\cos(x - y) = xe^y$

5. Use the chain rule to calculate $\dfrac{dz}{dt}$ where

(a) $z = x^2 + y^2 + xy$, $x = \sin(t)$, $y = \cos(t)$

(b) $z = \cos(x + 4y)$, $x = t^2$, $y = t^3$

***6.** Suppose that $z = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Calculate $\partial z/\partial r$ and $\partial z/\partial\theta$ in terms of $\partial z/\partial x$ and $\partial z/\partial y$.

Hence, show that the Cauchy–Riemann equations for differentiable functions $u(x,y)$ and $v(x,y)$,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

become

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

and

$$\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r},$$

in cylindrical polar coordinates (r, θ).

Hint: For the last part, start with expressions for $\dfrac{\partial u}{\partial r}$ and $\dfrac{\partial v}{\partial r}$.