MATH1064 Assignment1

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Answer to question1

1.$p \to q$ (Premise)
2.$(r \lor s) \to (p \land \neg q)$ (Premise)
3.$\neg p \lor q$ (conditional to disjunction from 1)
4.$\neg(r \lor s) \lor (p \land \neg q)$ (conditional to disjunction from 2)
5.$\neg\neg(\neg p \lor q)$ (double negative from 3)
6.$\neg(p \land \neg q)$ (De Morgan's laws from 5)
7.$\neg(r \lor s)$ (Disjunctive Syllogism from 4&6)
8.$\neg r \land \neg s$ (De Morgan's laws from 7)
9.$\neg r$ (specialisation from 8)

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answer to question2

p	q	r	$\neg q$	$\neg r$	$p \wedge \neg q$	$p \wedge \neg q \wedge \neg r$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge \neg q \wedge \neg r) \vee \neg(p \vee q)$
T	T	T	F	F	F	F	T	F	F
T	T	F	F	T	F	F	T	F	F
T	F	T	T	F	T	F	T	F	F
T	F	F	T	T	T	T	T	F	T
F	T	T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	T	F	F
F	F	T	T	F	F	F	F	T	T
F	F	F	T	T	F	F	F	T	T

Table 1: Truth table for first proposition

p	q	r	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$r \to \neg(p \wedge \neg q)$	$(r \to \neg(p \wedge \neg q)) \wedge \neg q$
T	T	T	F	F	T	T	F
T	T	F	F	F	T	T	F
T	F	T	T	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	F
F	T	F	F	F	T	T	F
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

Table 2: Truth table for second proposition

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Simplify truth tables

p	q	r	$(p \wedge \neg q \wedge \neg r) \vee \neg(p \vee q)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

Table 3: simple truth table for first proposition, only contains proposition and p,q,r

p	q	r	$(r \rightarrow \neg(p \wedge \neg q)) \wedge \neg q$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

Table 4: simple truth table for second proposition, only contains proposition and p,q,r

From the truth table, it's evident that the two compound propositions are logically equivalent as they have the same truth values for all possible combinations of p, q, and r.

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answer to question3

(a) **True**
There are 2 possibilities:

case1 : If $k \geq 0$, then $Q(k)$ is true by defintion because k is a non-negative number. Thus, $Q(k) \vee Q(-k)$ is true since $Q(k)$ is true when $k \geq 0$.

case2 : If $k < 0$,then $-k \geq 0$,then $Q(-k)$ is true by defintion.Thus, $Q(k) \vee Q(-k)$ is true since $Q(-k)$ is true when $k < 0$.

Therefore, $\forall k \in \mathbb{Z}, Q(k) \vee Q(-k)$ is true.

The negation of this statement is $\exists k \in \mathbb{Z}, \neg Q(k) \wedge \neg Q(-k)$.

(b) **True**
There are 3 possibilities:

case1 : If $k1 \geq 0, k2 \geq 0$, then $Q(k1) \wedge Q(k2)$ is true by defintion.
In this case, $k1 * k2 \geq 0$ because $k1$ and $k2$ are none-negative integers , then $Q(k1 * k2)$ is true by definition.$Q(k1) \wedge Q(k2)$ and $Q(k1 * k2)$ are true in this case. Thus , $Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ is true when $k1 \geq 0, k2 \geq 0$.

case2 : If $k1 < 0, k2 < 0$, then $Q(k1) \wedge Q(k2)$ is false because $k1, k2$ are not non-negative integers. In this case , whether $Q(k1 * k2)$ is true or not, $Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ must be true. Thus , $Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ is true when $k1 < 0, k2 < 0$.

case3 : If $k1 < 0$ or $k2 < 0$, then $Q(k1)$ or $Q(k2)$ is false by definition . Then $Q(k1) \wedge Q(k2)$ is false. In this case , whether $Q(k1 * k2)$ is true or not, $Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ must be true. Thus , $Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ is true when $k1 < 0, k2 < 0$.

Therefore ,$\forall k1, k2 \in \mathbb{Z}, Q(k1) \wedge Q(k2) \rightarrow Q(k1 * k2)$ is true.
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(c) **False**
There are 3 possibilities:

case1 : If $k1 \geq 0, k2 \geq 0$, then $k1 * k2 \geq 0$, $Q(k1 * k2)$ is true by defintion.$Q(k1) \wedge Q(k2)$ is also true. Thus, $Q(k1 * k2) \rightarrow Q(k1) \wedge Q(k2)$ is true when $k1 \geq 0, k2 \geq 0$.

case2 : If $k1 < 0, k2 < 0$, then $k1 * k2 > 0$, $Q(k1 * k2)$ is true by defintion.$Q(k1) \wedge Q(k2)$ is false because $k1$ and $k2$ are not none-negative integers.Thus, $Q(k1 * k2) \rightarrow Q(k1) \wedge Q(k2)$ is false when $k1 < 0 and k2 < 0$.

case3 : If $k1 < 0$ or $k2 < 0$, Then $k1 * k2 < 0$,Q(k1 * k2) is false. Thus $Q(k1 * k2) \rightarrow Q(k1) \wedge Q(k2)$ is true when $k1 < 0$ or $k2 < 0$.

Therefore ,$\forall k1, k2 \in \mathbb{Z}, Q(k1 * k2) \rightarrow Q(k1) \wedge Q(k2)$ is false due to $Q(k1 * k2) \rightarrow Q(k1) \wedge Q(k2)$ is false when $k1 < 0, k2 < 0$

(d) **True**
There are 2 possibilities:

case1 : If $k1$ is even and $k2$ is divisible by 3,then $R(k1) \wedge R(k2)$ is true. Let $k1 = 2m, m \in \mathbb{Z}, k2 = 3n, n \in \mathbb{Z}$.Then $3k1 + 2k2$ can be written as $2 * 3 * (m + n)$.So that $3k1 + 2k2$ is a even number and also divisible by 3.Then $R(3k1 + 2k2) \wedge Q(3k1 + 2k2)$ is true.Thus , $R(k1) \wedge R(k2) \rightarrow R(3k1 + 2k2) \wedge S(3k1 + 2k2)$ is true.

case2 : If k1 is not even or k2 is not divisible by 3 , or k1 is not even and k2 is not divisible by 3.Then $R(k1) \wedge R(k2)$ must be flase. Thus ,$R(k1) \wedge R(k2) \rightarrow R(3k1 + 2k2) \wedge S(3k1 + 2k2)$ is true.

Therefore, $\forall k1, k2 \in \mathbb{Z}, R(k1) \wedge R(k2) \rightarrow R(3k1 + 2k2) \wedge S(3k1 + 2k2)$ is true.

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(e) **True**

There are 4 possibilities:

case1 : If $k1$ is even and $k2$ is divisible by 3,then $R(k1) \vee S(k2)$ is true. Let $k1 = 2m, m \in \mathbb{Z}, k2 = 3n, n \in \mathbb{Z}$.Then $3k1 + 2k2$ can be written as $2 * 3 * (m + n)$.So that $3k1 + 2k2$ is a even number and also divisible by 3.Then $R(3k1 + 2k2) \vee Q(3k1 + 2k2)$ is true.Thus , $R(k1) \vee S(k2) \rightarrow R(3k1 + 2k2) \vee S(3k1 + 2k2)$ is true.

case2 : If $k1$ is even but $k2$ is not divisible by 3,then $R(k1) \vee S(k2)$ is true. Let $k1 = 2n, n \in \mathbb{Z}$, then $3k1+2k2$ can be written as $2*(3n+k2)$,which is a even number.Then $R(3k1 + 2k2)$ is true by definition.Such that $R(3k1 + 2k2) \vee S(3k1 + 2k2)$ is true.Thus , $R(k1) \vee S(k2) \rightarrow R(3k1 + 2k2) \vee S(3k1 + 2k2)$ is true.

case3 : If $k1$ is not even but $k2$ is divisible by 3,then $R(k1) \vee S(k2)$ is true. Let $k2 = 3n, n \in \mathbb{Z}$, then $3k1 + 2k2$ can be written as $3(k1 + 2n)$,which is divisible by 3.Then $S(3k1+2k2)$ is true by definition.Such that $R(3k1+2k2) \vee S(3k1 + 2k2)$ is true.Thus , $R(k1) \vee S(k2) \rightarrow R(3k1 + 2k2) \vee S(3k1 + 2k2)$ is true.

case 4 : If $k1$ is not even and $k2$ is not divisible by 3,then $R(k1) \vee S(k2)$ is false. Then whether $R(3k1 + 2k2) \vee Q(3k1 + 2k2)$ is true or not , $R(k1) \vee S(k2) \rightarrow R(3k1 + 2k2) \vee S(3k1 + 2k2)$ is true.

(f) **False**

case 1 : If k1 is even,then $\neg R(k1)$ is false. This statement is false.

case 2 : If k1 is odd ,then $\neg R(k1)$ is true. But $3k1 + 2k2$ is odd ,such that $R(3k1 + 2k2)$ is false ,this statment is false.

Negation : $\forall k1, k2 \in \mathbb{Z}, \neg R(3k1 + 2k2) \vee R(k1)$

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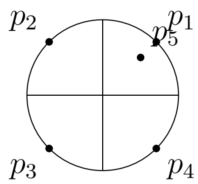
(g) **false**

Let $k = 6m, m \in \mathbb{N}$,then k is a even number and is divisible by 3.So that $R(k) \wedge S(k)$ is true.Assume that $l = 12m$,$\neg(R(l) \wedge S(l))$ is false.
Therefore , this statement is false.

answer to question4



As illustrated in the figure, the circle is divided into 4 equal sectors. Assume that the 5 points are not located at the intersections of the sectors (this can be achieved by adjusting the direction of the divisions).
Placing $\{p1, p2, p3, p4, p5\}$ into these 4 sectors,at least one sector must contain 2 of these points by the pigeonhole principle.

The distance between 2 points in the same sector is smaller than $\sqrt{2}$(because the diameter splits the circle into these sectors).
Therefore, by pigeonhole principle,$\exists pj, pk \in \mathbb{P} : (j \neq k) \vee (d(pj, pk) < \sqrt{2})$ is true.