Extended Answer Section

There are **three** questions in this section, each with a number of parts.

Write your answers in the space provided below each part. There is extra space at the end of the paper.

1. (a) Compute the following definite integral
$\int_0^1 \frac{6x^2 + 6}{x^3 + 3x + 1} dx.$

Question 1 continues on the next page.

(b) Compute the following indefinite integral					
$\int x^2 \cos(x) dx.$					
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Question 1 continues on the next page.

(i)	Sketch the region \mathcal{R} .
(*)	
(ii)	Compute the area of \mathcal{R} . Show all necessary working.

(iii)	Compute the volume of the solid of revolution obtained by rotating \mathcal{R} around the x-axis. Show all necessary working.					

Page 16 of 28

Main Exam A Semester 1 2018

Question 2 begins on the next page

2. (a) (i)	Find the second order	r Taylor polynomial P_2 about $x = 0$, of the function
		$f(x) = \frac{1}{\sqrt{1+x^2}}.$

(ii) Using the second order polynomial as an approximation of the function f, find an approximate value of the integral

$$\int_{-1}^{1} \frac{dx}{\sqrt{1+x^2}} \, .$$

(iii)	Provide an	upper	bound	for	the	approximation	error

$$\left| \int_{-1}^{1} \frac{dx}{\sqrt{1+x^2}} - \int_{-1}^{1} P_2(x) \, dx \right| \, .$$

Question 2 continues on the next page.

(b) (i) Find the natural domain and range of the function $f: \mathbb{C} \to \mathbb{C}$ defined by

$$f(z) = 1 + \frac{1}{z - i}.$$

(ii) Sketch on the complex plane $\mathbb C$ the set $\{f(z) \mid |z-i| > 1\}$.

Page 20 of 28

Main Exam A Semester 1 2018

Question 3 begins on the next page

3. (a) Given the function $f(x) = 2x^3 - 3x^2 - 12x$, (i) Find the local maxima and minima and the intervals of increase or decrease

(ii) Find the points of inflection and the intervals of concavity.

Main	EXAM	Α	Semester.	1	2018
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PAGE	22	OF	28			
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(iii)	Use the information from parts (i) and (ii) to sketch the graph of f , showing the most important features.

Question 3 continues on the next page.

(b) Using any valid method and showing all the steps of your working, calculate the following limits or show that they do not exist.

(i)	lim	$3x^4 + 2x^2 + 3$
(1)	$x \rightarrow \infty$	$5x^4 - 7x - 2$

(ii)	lim	$e^x - x - 1$
$(\iota\iota)$	$x \rightarrow 0$	$\overline{x^2}$

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Page 24 of 28

(c) (i)	Use the binomial expansion $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ and the formula for $\sin \theta$ in terms of complex exponentials to find a formula for $\cos^4 \theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$.

(ii)	Use the result in Part (i) above to calculate the value of the following definite integral, $\int_0^{\pi/2} \cos^4\theta \ d\theta.$				

Main Exam A Semester 1 2018 Page 26 of 28