THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 1

MATH2022: Linear and Abstract Algebra — Semester 1, 2024

Lecturer: *Leah Neves* and *Sam Jeralds*

Due 11:59pm Sunday 14 April 2024.

This assignment contains four questions and is worth 5% of your total mark. It must be uploaded through the MATH2022 Canvas page `https://canvas.sydney.edu.au/courses/56888/assignments/518805`. Please include your SID but not your name, as anonymous marking will be implemented.

1. Let G be a finite group with an even number of elements. Show that there must be some element α in G, $\alpha \neq e$ (e the identity element) such that $\alpha^2 = e$.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$$

over $\mathbb{Z}_5$.

(a) Is A invertible over $\mathbb{Z}_5$? Justify your answer.

(b) How many solutions does the system of equations

$$\begin{aligned} x + 3y + z &= 2 \\ x + y + 2z &= 3 \\ 2y + 4z &= 4 \end{aligned}$$

have over $\mathbb{Z}_5$?

3. For each of the following statements, indicate whether they are true or false. If a statement is true, prove it. If a statement is false, give a counterexample.

(a) For two permutations α, β in the symmetric group $Sym(n)$, if α is odd, then the conjugation $\beta^{-1}\alpha\beta$ is also odd.

(b) For three square matrices A, B, and C of the same size, if $AC = BC$, then $A = B$.

(c) Let A be a 3×3 matrix over $\mathbb{R}$ such that $A^T = -A$ (where A^T is the transpose). Then $\det(A) = 0$.

4. Let A, B be two square matrices of the same size over a field F such that $AB = BA$. If $\vec{v}$ is an eigenvector of A, show that the product $(B \cdot \vec{v})$ is also an eigenvector of A with the same eigenvalue as $\vec{v}$.

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