The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1062: Mathematics 1B — Semester 2, 2024

Lecturers: Tiangang Cui, Jack Freestone, Brad Roberts, and Zhou Zhang

This **individual** assignment is due by **11:59pm Sunday 25 August 2024**, via Canvas. Late assignments will receive a penalty of 5% per calendar day until the closing date. After 10 days, the mark will be down to 0. The submission should include your SID. Please make sure you review your submission carefully. What you see is exactly how the marker will see your assignment. Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall mark to your assignment using the following criteria:

Rubric

Criteria	Ratings					Pts
Correct solutions to the questions	**4 pts** **Excellent** Excellent work, answering all parts correctly. There are at most only minor or trivial errors or omissions.	**3 pts** **Very good work** Making very good progress but with one or two substantial errors, misunderstandings or omissions throughout the assignment.	**2 pts** **Good work** Making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.	**1 pts** **Fair work** A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.	**0 pts** **No Marks** No credit awarded.	4 pts
Clear explanations, diagrams and working shown	**1 pts** **Full Marks** Criteria met.		**0 pts** **No Marks** No clear explanations.			1 pts
						Total Points: 5

Submission Instructions

Solutions to Part A must be prepared in handwritten form, and uploaded as a single pdf file to https://canvas.sydney.edu.au/courses/59770/assignments/553027. Although we allow iPad/tablet writing for assignments, it's highly recommended for students to rely on traditional pen and paper writing to mimic the exam situation.

Solutions to Part B must be prepared as a single html file and submitted to https://canvas.sydney.edu.au/courses/59770/assignments/553028. You should use the following file to complete Part B.

- An R Markdown worksheet `Assignment1Worksheet.Rmd` at https://canvas.sydney.edu.au/courses/59770/files/38709347.

You need to write your solutions as either embedded R code or text answers in the provided worksheet, and then generate the html file using `Knit` in R Studio.

Part A: Calculus Questions

Please justify your answers by showing all relevant works. Correct answers without adequate justification will not receive full marks.

1. Write the following differential equations for the unknown function, $y(x)$ with the variable x, in the standard form and illustrate the direction field by drawing the tangent intervals at the four points: $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$ in the (x, y)-plane.

For each differential equation, find the general solution, then find the particular solution satisfying $y(0) = 1$, finally sketch the solution curve in the above illustration of direction field. *All drawing and sketch must be done manually by hand.*

(a)
$$x\frac{dy}{dx} - 2x^2e^{x^2} = 0.$$

(b)
$$y\frac{dy}{dx} + x = 0.$$

2. Consider a cubic water tank with the side length 4, placed horizontally. There is a hole at the bottom. Due to a pump, the water will leave the tank according to the following differential equation:
$$\frac{dV}{dt} = -(1 + \sin t)\sqrt{V}$$
where $V(t)$ is the volume of water remaining in the tank at the time t.

(a) Find the general solution of this differential equation.

(b) Find the particular solution if the tank is full to start with, that is, $V(0) = 4^3 = 64$.

(c) For the solution from (b), let T be the time when the tank becomes empty, that is $V(T) = 0$. Is $T > 14$ or $T < 14$? Explain your choice.

Part B: Statistics Questions

Part B questions are provided in the R Markdown worksheet at `https://canvas.sydney.edu.au/courses/59770/files/38709347`. You should answer these questions using the worksheet and use the `Knit` button in R Studio to generate the html file. **We can only mark the html file.**