

Solution 3.1. Let $x_n = a^n$ for every n . If $i > j$, then x_i and x_j are distinguishable by $z = b^i$. Specifically, $x_i z = a^i b^i \in L$ because $a^i b^i = a^i f(a^i)$, while $x_j z = a^j b^i \notin L$ since $a^j b^i \neq a^j f(a^i)$. Therefore, we can conclude that for the injective function f , the language $\{x f(x) : x \in \Sigma^*\}$ is not regular.

Solution 3.2. Let x_n be the set of strings that have the same length l . It is known that for the injective function $f(x)$, if $m \neq n$, then $f(m) \neq f(n)$. Let x_i be any string of length l , and let x_j be another string of length l with $x_i \neq x_j$. Then x_i and x_j are distinguishable by $z = f(x_i)$. Specifically, $x_i z = x_i f(x_i) \in L$, while $x_j f(x_i) \notin L$. Therefore, we can conclude that the language $x f(x) : x \in \Sigma^*$ is not regular.