

MATH2022 Assignment 1

SID:530157791

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1 Q1

In group G , for each element $\alpha \neq e$, there is a unique inverse α^{-1} such that $\alpha\alpha^{-1} = e$ (*Inverse Element*). Since G has an even number of elements, the total number of non-identity elements is odd. Which means that if we try to partition them into pairs like (α, α^{-1}) , there must be at least 1 element can not be paired with a distinct element. The unpaired element must therefore be its own inverse, implying $\alpha = \alpha^{-1}$. For this α , it follows that $\alpha^2 = \alpha\alpha = e$

2 Q2

(a)

$$\begin{aligned}\det(A) &= -2 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \\ &= -2 + 4 \cdot (-2) \\ &= -10 \\ &= 0\end{aligned}$$

Therefore, Matrix A is not invertible over Z_5

(b)

$$\begin{aligned}& \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & 4 \end{array} \right] \\ &= \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 4 \\ 0 & 2 & 4 & 4 \end{array} \right] \\ &= \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]\end{aligned}$$

Since z can take any value in $Z_5 = 0, 1, 2, 3, 4$, and for each z , there is exactly 1 solution for x and y . Thus, there are 5 solutions for this system.

3 Q3

(a)

True. To do the conjugation $\beta^{-1}\alpha\beta$, we just simply replace every α_i in each cycle with $\alpha_i\beta$, it doesn't change the number of elements in a permutation.

(b)

False. If $C = \mathbf{0}$, the statement "If $AC = BC$, then $A = B$ " still holds true when $A \neq B$.

(c)

True. We have $\det(A) = \det(A^T)$. If $A^T = -A$, then $\det(A) = \det(-A)$, and since $\det(-A) = (-1)^n \det(A)$ for an $n \times n$ matrix, where $n = 3$ in this case, it follows that $\det(A) = -\det(A)$ which indicates that $\det(A) = 0$.

4 Q4

Assume that the eigenvalue of eigenvector \mathbf{v} is λ , so that by the description above, we have:

$$A\mathbf{v} = \lambda\mathbf{v}$$

To prove that $B\mathbf{v}$ is also an eigenvector of A with the same eigenvalue λ as \mathbf{v} , we need to show that $A(B\mathbf{v}) = \lambda(B\mathbf{v})$.

Proof

$$\begin{aligned} A(B\mathbf{v}) &= (AB)\mathbf{v} \\ &= B(A\mathbf{v}) \\ &= B\lambda\mathbf{v} \\ &= \lambda(B\mathbf{v}) \end{aligned}$$