

## Calculus Tutorial 10 (Week 11)

MATH1062/MATH1023: Mathematics 1B (Calculus)

Semester 2, 2024

Questions marked with \* are harder questions.

### Material covered

(1) Directional derivatives and gradient vector

### Summary of essential material

Recall that the gradient of  $f(x, y)$  is given by  $\nabla f(x, y) = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j}$ , where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the coordinate directions. The directional derivative of  $f(x, y)$  at point  $(a, b)$  in direction  $\mathbf{u}$  is then given by  $D_{\mathbf{u}}f(a, b) = \nabla f(a, b) \cdot \hat{\mathbf{u}}$ , where  $\hat{\mathbf{u}}$  is the unit vector in  $\mathbf{u}$ -direction.

### Questions to complete during the tutorial

1. Find the gradient vectors of the following functions.

(a)  $f(x, y) = x^2y^3$

(b)  $f(x, y) = e^{xy-y^2}$

2. Calculate the directional derivative in the direction of  $\mathbf{u}$  at the given point  $P$ . (Remember to find unit vectors in the direction of  $\mathbf{u}$ .)

(a)  $f(x, y) = x^2y^3$ ,  $\mathbf{u} = \mathbf{i} + \mathbf{j}$ ,  $P = (2, 1)$

(b)  $f(x, y) = e^{xy-y^2}$ ,  $\mathbf{u} = 12\mathbf{i} + 5\mathbf{j}$ ,  $P = (2, 2)$

3. Let  $f(x, y) = \sqrt{5x - 4y}$ .

(a) Find  $\nabla f(x, y)$ . Hence find  $\nabla f(4, 1)$ .

(b) Find the directional derivative of  $f(x, y)$  at the point  $(4, 1)$  in the direction given by the vector  $\sqrt{3}\mathbf{i} - \mathbf{j}$ .

(c) Find the direction of steepest slope at the point  $(4, 1)$ , and the slope in that direction.

\*4. Suppose that you are climbing a 1000m hill whose shape is given

$$z = 1000 - 0.01x^2 - 0.02y^2,$$

and you are standing at the point with coordinates  $(60, 100, 764)$ . The coordinate system has been chosen so that the positive  $x$ -axis points East, the positive  $y$ -axis points North, and the positive  $z$ -axis points up.

(a) In which direction should you face initially in order to proceed upwards via the steepest route?

(b) With the climbing gears you have, you are able to climb vertical slopes of at most  $70^\circ$ . Are you able to proceed upwards via the steepest route?

(c) In which direction should you travel initially if you want to maintain your height above sea level at 764 metres?

5. If  $f(x, y) = x - y^2$ , find  $\nabla f(3, -1)$  and use it to find the equation of the tangent line to the level curve  $f(x, y) = 2$  at the point  $(3, -1)$ . Sketch the level curve, the tangent line and the gradient vector.

### Short answers to selected exercises

2. (b)  $\frac{14}{13}$
3. (b) 0.79
4. (c)  $\pm(4\mathbf{i} - 1.2\mathbf{j})$