

Solution 4.1. The idea is that each state in Q' not only stores the state in Q but also tracks the current letter position (1st, 2nd, 3rd). Specifically, we extend each state $q \in Q$ to $(q, 0)$, $(q, 1)$, and $(q, 2)$, representing every first, second, and third letter. For the first and second characters, we only update the number in the tuple. For example, $(q_0, 0)$ becomes $(q_0, 1)$ after receiving the first character. When the third letter is received (i.e., the number in the tuple is 2), the number resets to 0, and q_0 transitions according to the original transition function in δ . For example:

$$(q_0, 2) \xrightarrow{a} (q_1, 0)$$

The accept states are set to $(q_f, 0)$, $(q_f, 1)$, and $(q_f, 2)$ for all $q_f \in F$.

Solution 4.2. For DFA B, we have DFA $B = (Q', \Sigma, q'_0, \delta', F')$:

- $Q' = Q \times \{0, 1, 2\}$
- $q'_0 = (q_0, 0)$
- $\delta'((q, n), l) = \begin{cases} (q, n+1) & n = 0, 1 \\ (\delta(q, l), 0) & n = 2 \end{cases}, q \in Q, l \in \Sigma$
- $F' = (q, n), q \in F, n \in \{0, 1, 2\}$