After this tutorial you should be able to:

1. Express in English the language of a DFA/NFA.
2. Design DFAs/NFAs for languages specified in English.
3. Convert RE to NFAs
4. Form a DFA for the union or intersection of two DFAs using the product construction.
5. Devise algorithms for solving certain decision problems about DFAs/NFAs

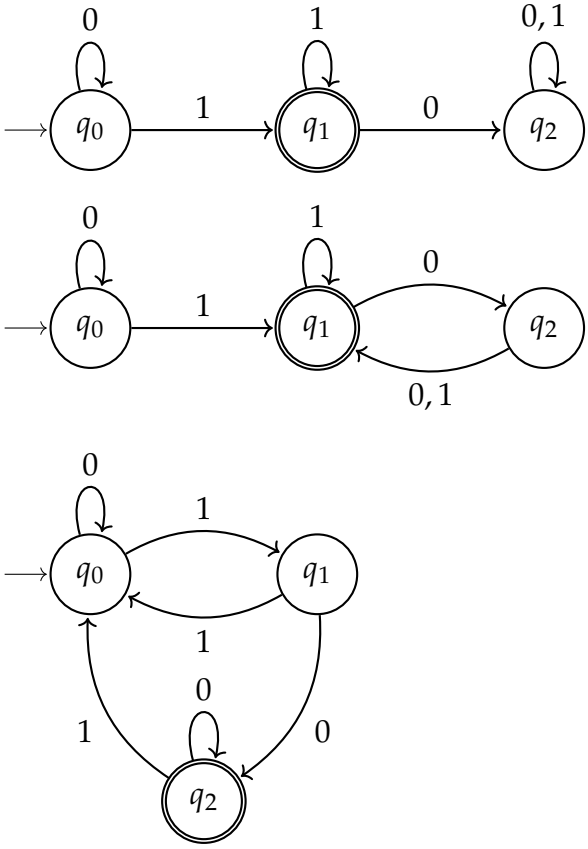
In your tutorial, I suggest you do one or two questions from each of the four sections: DFA, NFA, from RE to NFA, Algorithms. Use the rest of the problems and subproblems as practice, and to check you have mastered the material.

1 DFA

Problem 1. For each of the following DFAs:

(a) Give $(Q, \Sigma, \delta, q_0, F)$ for the automaton.

(b) Give a precise and concise English description of the language recognised by this automaton.



Problem 2. Draw the following automaton and give the language that it recognises:

- $Q = \{q_1, q_2, q_3\}$ is the set of states.
- $\Sigma = \{b, c, d\}$ is the alphabet,
- q_1 is the start state.
- $F = \{q_2\}$ is the set of accept states,

and the $\delta : Q \times \Sigma \to Q$ is given by the following table:

	b	c	d
q_1	q_2	q_1	q_1
q_2	q_3	q_1	q_3
q_3	q_3	q_3	q_3

Problem 3. For each of the following languages, draw a DFAs which recognise them. The alphabet for all the languages is $\{a, b, c\}$.

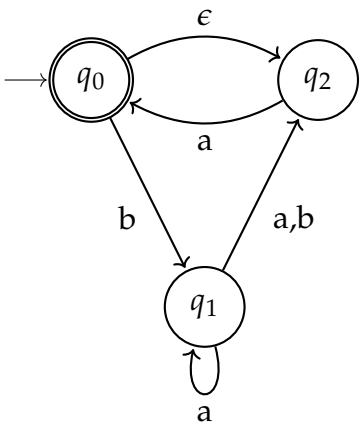
1. Strings that start with b
2. Strings that end with b
3. Strings that contain exactly one b
4. Strings that contain at least one b and are of length exactly 2

Problem 4.

1. Construct a DFA which accepts strings of 0s and 1s, where the number of 1s modulo 3 equals the number of 0s modulo 3.
2. Construct a DFA which accepts binary numbers that are multiples of 3, scanning the most significant bit (i.e. leftmost) first. Hint: let each state 'remember' the remainder of the string read so far.
3. Using the previous automaton, construct an automaton which accepts binary numbers which are NOT multiples of 3.

2 NFA

Problem 5. Consider the following NFA:



Does the NFA accept:

1. ϵ
2. a
3. baba
4. aaab

Problem 6. Draw an NFA for each of the following languages. The alphabet is $\Sigma = \{a, b, c\}$. You may use epsilon transitions.

1. Strings that end with *aa* (use only 3 states)
2. Strings that contain the substring 'bbc' (use only 4 states)
3. Strings that start with a or end in c (use only 4 states)
4. Strings that start with a and end in c (use only 3 states)

3 From RE to NFA

Problem 7. Use the construction demonstrated in lectures to construct an NFA for each of the following REs:

1. $0^\star 110^\star$
2. $(((00)^\star(11)) \mid 10)^\star$

4 Algorithms

Problem 8. In this problem you will show that the regular languages are closed under union. That is, if M_1, M_2 are DFAs, then so is $L(M_1) \cup L(M_2) = L(M)$ for some DFA M.

Hint. Simulate both automata at the same time using pairs of states.

1. Argue why the construction is correct.
2. Use the construction to draw a DFA for the language of strings over alphabet $\{a, b\}$ consisting of an even number of a's or an odd number of b's.

Show that the regular languages are closed under intersection. How would you change your union construction to do this?

Problem 9. The *non-emptiness problem for DFAs* is the following decision problem: given a DFA M, return "Yes" if $L(M) \neq \emptyset$ and "No" otherwise.

Devise an algorithm for solving this problem. What is the worst-case time complexity of your algorithm?

How would you adapt your algorithm to solve the non-emptiness problem for NFAs?

Problem 10. The *equivalence problem for DFAs* is the following decision problem: given DFAs M_1, M_2, return "Yes" if $L(M_1) = L(M_2)$, and "No" otherwise.

Devise an algorithm for solving this problem. What is the worst-case time complexity of your algorithm? What about space complexity (i.e., amount of memory needed)?

Hint. Use the facts that the regular languages are closed under the Boolean operations (i.e., complement, union, intersection), and the fact that the emptiness problem for DFAs is decidable.

Problem 11. (Exam 2022) Fix $\Sigma = \{0, 1\}$. Consider the operation *delzero* that maps a string u to the string in which every 0 is deleted. So, e.g., $\mathit{delzero}(0) = \epsilon$, $\mathit{delzero}(01101) = 111$, $\mathit{delzero}(1) = 1$, and $\mathit{delzero}(\epsilon) = \epsilon$. For a language $L \subseteq \Sigma^*$, let $\mathit{delzero}(L) = \{\mathit{delzero}(u) : u \in L\}$. Explain why if L is regular, then $\mathit{delzero}(L)$ is regular.

Problem 12. (Hard) A fundamental problem in string processing is to tell how close two strings are to each other. A simple measure of closeness is their Hamming distance, i.e., the number of position in which the strings differ, written $H(u, v)$. For instance, $H(aaba, abbb) = 2$ since the strings disagree in positions 2 and 4 (starting the count at 1). Positions at which the the strings differ are called *errors*.

Let $\Sigma = \{a, b\}$. Show that if $A \subseteq \Sigma^*$ is a regular language, then the set of strings

of Hamming distance at most 2 from some string in A, i.e., the set of strings $u \in \Sigma^*$ for which there is some $v \in A$ such that $len(u) = len(v)$ and $H(u, v) \leq 2$, is also regular.

Hint. Take a DFA M for A, say with state set Q, and build an NFA with state set $Q \times \{0, 1, 2\}$. The second component tells you how many errors you have seen so far. Use nondeterminism to guess the string $u \in A$ that the input string is similar to, as well as where the errors are.

How would you change the construction to handle Hamming distance 3 instead of 2?

How would you change the construction to handle that case that we don't require $len(u) = len(v)$? e.g., $H(abba, aba) = 2$.

p.s. The problem of telling how similar two strings are comes up in a variety of settings, including computational biology (how close are two DNA sequences), spelling correction (what are candidate correct spellings of a given misspelled word), machine translation, information extraction, and speech recognition. Often one has other distance measures, e.g., instead of just substituting characters, one might also allow insertions and deletions, and have different "costs" associated with each operation. The standard way of computing similarity between two given strings is by Dynamic Programming, an algorithmic technique you will delve into in `COMP3027:Algorithm Design`.