The University of Sydney
School of Mathematics and Statistics

Assignment 2

MATH1062: Mathematics 1B — Semester 2, 2024

Lecturers: Tiangang Cui, Jack Freestone, Brad Roberts, and Zhou Zhang

This **individual** assignment is due by **11:59pm Sunday 13 Oct 2024**, via Canvas. Late assignments will receive a penalty of 5% per day until the closing date. Your answers must be compiled in two separate documents, and uploaded in Canvas to two different submission boxes, as outlined in the submission instructions below. Both documents should include your SID. Please make sure you review your submissions carefully. What you see is exactly how the marker will see your assignment. Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 5% + 5% = 10% of your final assessment for this course. Your answers should be neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. The marker will give you feedback and allocate an overall mark to your assignment using the following criteria:

Rubric

Criteria	Ratings							Pts
Correct solutions to the questions	**8 pts** **Excellent** Excellent work, answering all parts correctly. There are at most only minor or trivial errors or omissions.	**7 pts** **Very good work** Making very good progress but with one or two substantial errors, misunderstandings or omissions throughout the assignment.	**6 pts** **Good work** Making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.	**5 pts** **Fair work** A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.	**3 pts** **Poor** Some attempt, with limited progress made.	**1 pts** **Extremely poor** Extremely limited attempt.	**0 pts** **No Marks** No credit awarded.	8 pts
Clear explanations, diagrams and working shown	**2 pts** **Full Marks** Criteria met.		**1 pts** **Partial marks** Some explanations given.		**0 pts** **No Marks** No clear explanations.			2 pts
								Total Points: 10

Submission instructions

Solutions to calculus questions (Part A) must be prepared in written form, and uploaded as a single pdf file to https://canvas.sydney.edu.au/courses/59770/assignments/553029.
Solutions to Part B must be prepared as a single **knitted** html file and submitted to https://canvas.sydney.edu.au/courses/59770/assignments/553030.

Part A: calculus questions

Please justify your answers by showing all relevant works. Correct answers without adequate justification will not receive full marks.

1. (a) Find the particular solution for the following differential equation for the unknown function $y(x)$,

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0, \quad y(0) = y'(0) = 1.$$

(b) Consider the following differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$$

for the unknown function $y(x)$ by following the steps below.

(i) For $z(x) = e^{-x}y(x)$, show that

$$e^{-x}\left(\frac{dy(x)}{dx} - y(x)\right) = \frac{dz(x)}{dx}.$$

(ii) Still for $z(x) = e^{-x}y(x)$, using (i) or not, show that

$$e^{-x}\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y\right) = \frac{d^2z(x)}{dx^2}.$$

(iii) Assuming (ii), the original differential equation is transformed to

$$\frac{d^2z(x)}{dx^2} = 1.$$

(A) Obtain the general solution for this new differential equation for the unknown function $z(x)$.

(B) Using $z(x) = e^{-x}y(x)$, obtain the general solution for the original differential equation for the unknown function $y(x)$.

2. In $\mathbb{R}^3$, there is the following sphere centred at the origin,

$$S = \{(x, y, z) \mid x^2 + y^2 + z^2 = 3\}.$$

We consider the point $(1, 1, -1)$ on S in the following.

(a) Consider the part of the sphere S near $(1, 1, -1)$ as the graph of $z = f(x, y)$.

(i) What is the natural domain of the function $f(x, y)$?

(ii) What is the equation of the level curve containing the point $(x, y) = (1, 1)$?

(b) Calculate the equation for the tangent plane P of the sphere S at the point $(1, 1, -1)$.

(c) Show that the line L,

$$x(t) = 1 + t,\ y(t) = 1 + t,\ z(t) = -1 + 2t,\ t \in \mathbb{R}$$

contains the point $(1, 1, -1)$, intersects with the z-axis and doesn't intersect with the y-axis.

(d) Show that the line L in (c) sits on the plane P in (b).

Part B: statistics questions

Solutions must be prepared as a single html file and submitted to https://canvas.sydney.edu.au/courses/59770/assignments/553030. You should use the following files to complete the assignment.

- A data file `AutumnCleaned.csv` at https://canvas.sydney.edu.au/courses/59770/files/39409879. This data file is needed to knit the worksheet and complete your assignment questions.
- An R Markdown worksheet `Assignment2Worksheet.Rmd` at https://canvas.sydney.edu.au/courses/59770/files/38324259. Questions are provided in this R Markdown worksheet.

You need to write your solutions as either embedded R code or text answers in the provided worksheet, and then generate the html file using `Knit` in R Studio. **We can only mark the knitted html file.** The generated html file should contain all your work, including code.