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1.(a)

Characteristic equation: $m^2+m=0$

$\therefore m_1=0,\ m_2=-1$

The general solution:

$y(x) = C_1 + C_2 e^{-x}, \quad C_1, C_2 \in R$

$y(0)=1$, we have $C_1+C_2=1$.

$y'(0)=1$, we have $C_2=-1$.

Thus $\begin{cases} C_1=2 \\ C_2=-1 \end{cases}$

$y(x) = 2 - e^{-x}$

1.(b)

(i) $\frac{dz(x)}{dx} = \frac{d}{dx}\left(e^{-x}y(x)\right) = -e^{-x}y(x) + \frac{dy(x)}{dx}e^{-x}$

$= e^{-x}\left(\frac{dy(x)}{dx} - y(x)\right)$

(ii) $\frac{d^2z(x)}{dx^2} = \cancel{\frac{d}{dx}\left(e^{-x}\left(\frac{d^2y}{d}\right.\right.} \frac{d}{dx}\left(\frac{dz(x)}{dx}\right) = \frac{d}{dx}\left(e^{-x}\frac{dy(x)}{dx} - e^{-x}y(x)\right)$

$= -e^{-x}\frac{dy(x)}{dx} + e^{-x}\frac{d^2y(x)}{dx^2} + e^{-x}y(x) - e^{-x}\frac{dy(x)}{dx}$

$= e^{-x}\left(\frac{d^2y(x)}{dx^2} - 2\frac{dy(x)}{dx} + y(x)\right)$

(iii)

(A) $\int \frac{d^2z(x)}{dx^2}\,dx = \int 1\,dx$

$\frac{dz(x)}{dx} = x + C_2$

$\int \frac{dz(x)}{dx}\,dx = \int (x+C_2)\,dx$

$z(x) = \frac{x^2}{2} + C_1x + C_2$

Thus, the general solution is $z(x) = \frac{x^2}{2} + C_1x + C_2$

(iii)

(B)

Since $z(x) = \frac{x^2}{2} + C_1 x + C_2$

We have $e^{-x} y(x) = \frac{x^2}{2} + C_1 x + C_2$

$y(x) = e^x \frac{x^2}{2} + C_1 e^x x + C_2 e^x$

2.

(a)

(i) Known that $x^2+y^2+z^2=3$

We have $z = \pm\sqrt{3-x^2-y^2}$

Since it contains $(1,1,-1)$.

$z = f(x,y) = -\sqrt{3-x^2-y^2}$

The natural domain is given by $3-x^2-y^2 \geq 0$.

~~由~~ Thus:

$$\{(x,y) \in R^2 \mid x^2+y^2 \leq 3\}$$

(ii) plug in $(1,1,-1)$, we have $x^2+y^2+1=3$.

Thus, the level curve equation is:

$$x^2+y^2=2$$

(b)

Known that $f(x,y) = -\sqrt{3-x^2-y^2}$

$f_x(1,1) = \frac{x}{\sqrt{3-x^2-y^2}} = 1$

$f_y(1,1) = \frac{y}{\sqrt{3-x^2-y^2}} = 1$

$z = x-1+y-1-1 = x+y-3$

Therefore, the equation of the level curve containing the point $(1,1)$ is:

$$x+y-z=3$$

(C.)

The Line L contains the point $(1,1,-1)$:

$$\begin{cases} 1 = 1+t \\ 1 = 1+t \\ -1 = -1+2t \end{cases}$$

Thus, when $t=0$, the line L passes through the point $(1,1,-1)$.

Intersects the Z-axis.

$$\begin{cases} 1+t=0 \\ 1+t=0 \\ -1+2t=k \end{cases}$$

we have $t=0$, $k=-3$

So, when $t=-1$, the line intersects the z-axis at the point $(0,0,-3)$

Does not intersect the y-axis.

For the line to intersect the y-axis, we need $x=0$ but $y \neq 0$. However, from the parametric equations, we see that $x(t)=y(t)$.

Therefore, L does not intersect the y-axis.

(D) Know that the tangent plane at $(1,1,-1)$ is:

$$x+y-z=3$$

plug in parametric equations.

$$(1+t)+(1+t)-(-1+2t)$$

$$= t+t-2t+1+1+1$$

$$= 3$$

Thus, L in (C) sits on the plane P in (b)