

Extended Answer Section

There are **three** questions in this section, each with a number of parts.
Write your answers in the space provided below each part. There is extra space at the end
of the paper.

1. (a) Compute the following definite integral

$$\int_0^1 \frac{6x^2 + 6}{x^3 + 3x + 1} dx.$$

Question 1 continues on the next page.

$$\int x^2 \cos(x) \, dx.$$

Question 1 continues on the next page.

Question 2 begins on the next page

- $$f(x) = \frac{1}{\sqrt{1+x^2}}.$$

Question 2 continues on the next page.

- (ii) Using the second order polynomial as an approximation of the function f , find an approximate value of the integral

$$\int_{-1}^1 \frac{dx}{\sqrt{1+x^2}}.$$

- (iii) Provide an upper bound for the approximation error

$$\left| \int_{-1}^1 \frac{dx}{\sqrt{1+x^2}} - \int_{-1}^1 P_2(x) dx \right|.$$

Question 2 continues on the next page.

- (b) (i) Find the natural domain and range of the function $f : \mathbb{C} \rightarrow \mathbb{C}$ defined by

$$f(z) = 1 + \frac{1}{z - i}.$$

- (ii) Sketch on the complex plane \mathbb{C} the set $\{f(z) \mid |z - i| > 1\}$.

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Question 3 begins on the next page

3. (a) Given the function $f(x) = 2x^3 - 3x^2 - 12x$,

- (i) Find the local maxima and minima and the intervals of increase or decrease of f .

- (ii) Find the points of inflection and the intervals of concavity.

Question 3 continues on the next page.

- (iii) Use the information from parts (i) and (ii) to sketch the graph of f , showing the most important features.

Question 3 continues on the next page.

- (b) Using any valid method and showing all the steps of your working, calculate the following limits or show that they do not exist.

(i) $\lim_{x \rightarrow \infty} \frac{3x^4 + 2x^2 + 3}{5x^4 - 7x - 2}$

(ii) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$

Question 3 continues on the next page.

Question 3 continues on the next page.

- $$\int_0^{\pi/2} \cos^4 \theta \, d\theta.$$

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End of Extended Answer Section