

MATH2022 Week 06  
Worksheet

# MATH 2022 Week 6 Worksheet

Q1/ Put

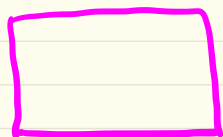
$$M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(a) Find

$$M \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

(b) By inspection, from the previous part, what are the eigenvalues?



(c) Find  $P$ ,  $P^{-1}$  and diagonal  $D$  such that  $M = P D P^{-1}$ .

$$P = \quad , \quad P^{-1} =$$

$$D =$$

(d) Find the characteristic polynomial

$$\chi(\lambda) = \det(\lambda I - M) = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix}$$
$$=$$

(e) What are the roots of  $\chi(\lambda)$ ?

$$\lambda = \boxed{\phantom{0000}}$$

(f) Find a general formula for  $M^n$ :

$$M^n = (P D P^{-1})^n = P D^n P^{-1}$$
$$=$$

(g) Thus find  $M^4 =$

Q2/ Put

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

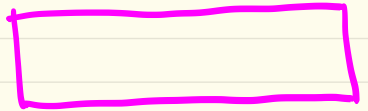
(a) Find

$$M \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} =$$

(b) By inspection, what are the eigenvalues?



(c) Find  $P$  and diagonal  $D$  such that  $M = P D P^{-1}$ .

$$P =$$

$$, D =$$

(d) Find  $P^{-1}$ :

(e) Find a general formula for  $M^n$ :

$$M^n =$$

(f) Thus find  $M^4 =$

Q3/ Put

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(a) Find surd expressions for the eigenvalues :

(b) What is the dominant eigenvalue to 3 decimal places?

$$\lambda_1 = \boxed{\phantom{000000}}$$

(c) What is the smaller eigenvalue to 3 decimal places?

$$\lambda_2 = \boxed{\phantom{000000}}$$

(d) Find

$$M^{-1} =$$

(e) Put  $\underline{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\underline{v}_{k+1} = M \underline{v}_k$  ( $k \geq 0$ )

$$r_k = \frac{\text{1st entry of } \underline{v}_k}{\text{1st entry of } \underline{v}_{k-1}}$$

$$s_k = \frac{\text{2nd entry of } \underline{v}_k}{\text{1st entry of } \underline{v}_k}$$

Complete the following table to 3 d.p.

$\underline{v}_0$	$\underline{v}_1$	$\underline{v}_2$	$\underline{v}_3$	$\underline{v}_4$	$\underline{v}_5$	$\underline{v}_6$	$\underline{v}_7$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 7 \\ 15 \end{bmatrix}$					
$r_k$	1	7					
$s_k$	3	2.143					

(f) Find  $M \begin{bmatrix} 1 \\ s_7 \end{bmatrix} =$

(g) Read off the dominant eigenvalue  $\approx$

(h) Put  $\underline{w}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\underline{w}_{k+1} = M^{-1} \underline{w}_k = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \underline{w}_k$

$$t_k = \frac{\text{1st entry of } \underline{w}_k}{\text{1st entry of } \underline{w}_{k-1}}$$

$$u_k = \frac{\text{2nd entry of } \underline{w}_k}{\text{1st entry of } \underline{w}_k}$$

Complete the following table to 3 d.p.

$\underline{w}_0$	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 1.5 \end{bmatrix}$	$\begin{bmatrix} 5.5 \\ -3.75 \end{bmatrix}$		
$t_k$	-2			
$u_k$	-0.75			

(i) Find  $M \begin{bmatrix} 1 \\ u_4 \end{bmatrix} =$

(j) Read off the negative eigenvalue of  $M \approx$



Q4/ Suppose that  $M$  is an invertible matrix with eigenvector  $\underline{v}$  corresponding to eigenvalue  $\lambda$ .

(a) Prove that  $\lambda \neq 0$ .

Proof:

(b) Prove that  $\underline{v}$  is an eigenvector for  $M^{-1}$  corresponding to eigenvalue  $\lambda^{-1}$ .

Proof: