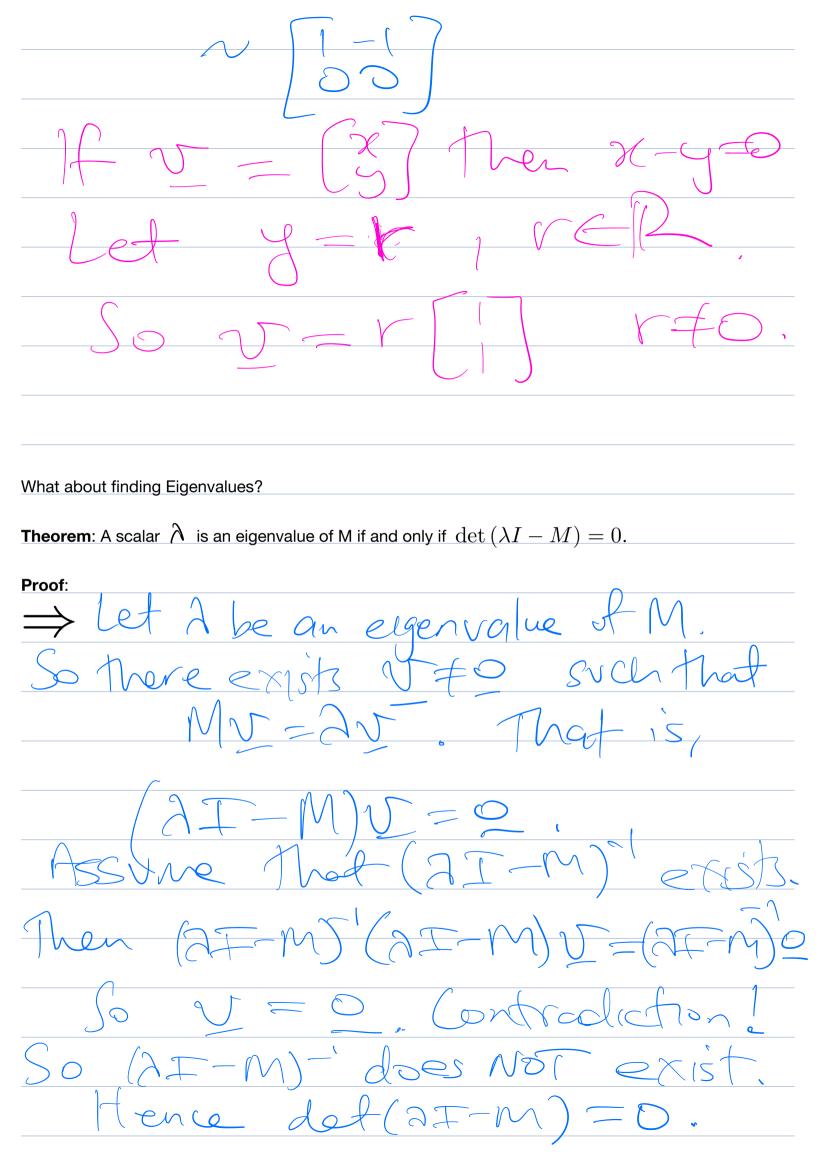
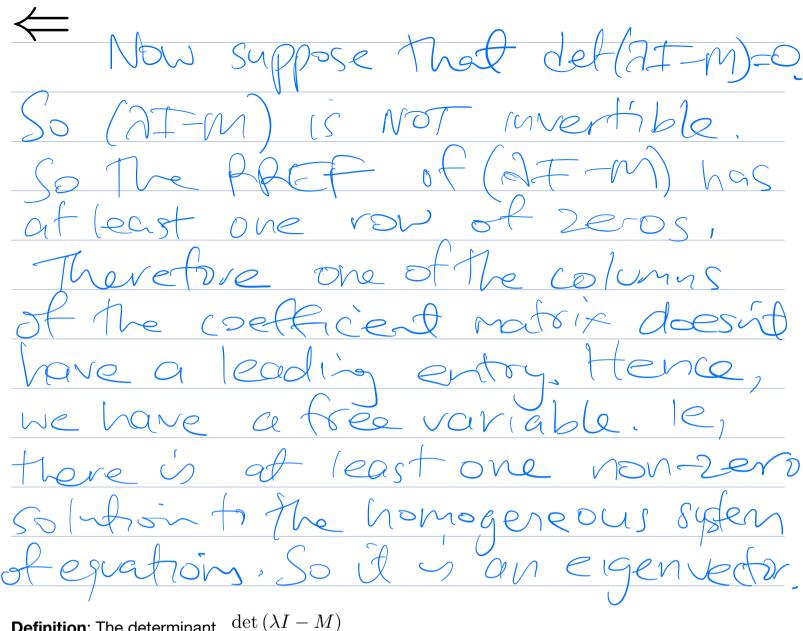
If γ is an eigenvalue, the eigenspace of M corresponding to γ is:
$\{\underline{v} \mid M\underline{v} = \lambda \underline{v}\}$
$= \{ \underline{v} \mid (\lambda I - M) \underline{v} = \underline{0} \}$
$= \{ \underline{v} \mid (\lambda I - M) \underline{v} = \underline{0} \}$ $= \{ \underline{v} \mid (\lambda I - M) \underline{v} = \underline{0} \}$ $= \{ \underline{v} \mid (\lambda I - M) \underline{v} = \underline{0} \}$
Example : Find the eigenspaces associated to 2, and 3, where $M = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.
$\lambda = 2$ 2I-M = 20 - 4-1 -2+1 02
-210 kets simplifie [-2]
Then $-2\pi t y = 0$ $1 + CR$
Ten -2x+====================================
$\mathcal{L} = \mathcal{L}$
So $\gamma = \begin{bmatrix} \zeta_1 \xi_1 \\ \xi_2 \end{bmatrix}$ or $s \begin{bmatrix} \zeta_1 \xi_1 \\ \xi_2 \end{bmatrix}$
$\lambda = 3$ where tell?
$\frac{307}{21} = -11$ $\frac{4}{21} = -12$





Definition: The determinant $\det (\lambda I - M)$

is a polynomial in λ with leading term λ^{κ} , and is called the characteristic polynomial of M. We write $\chi\left(\lambda\right)=\chi_{\text{PM}}(\lambda)=\det\left(\lambda I-M
ight).$

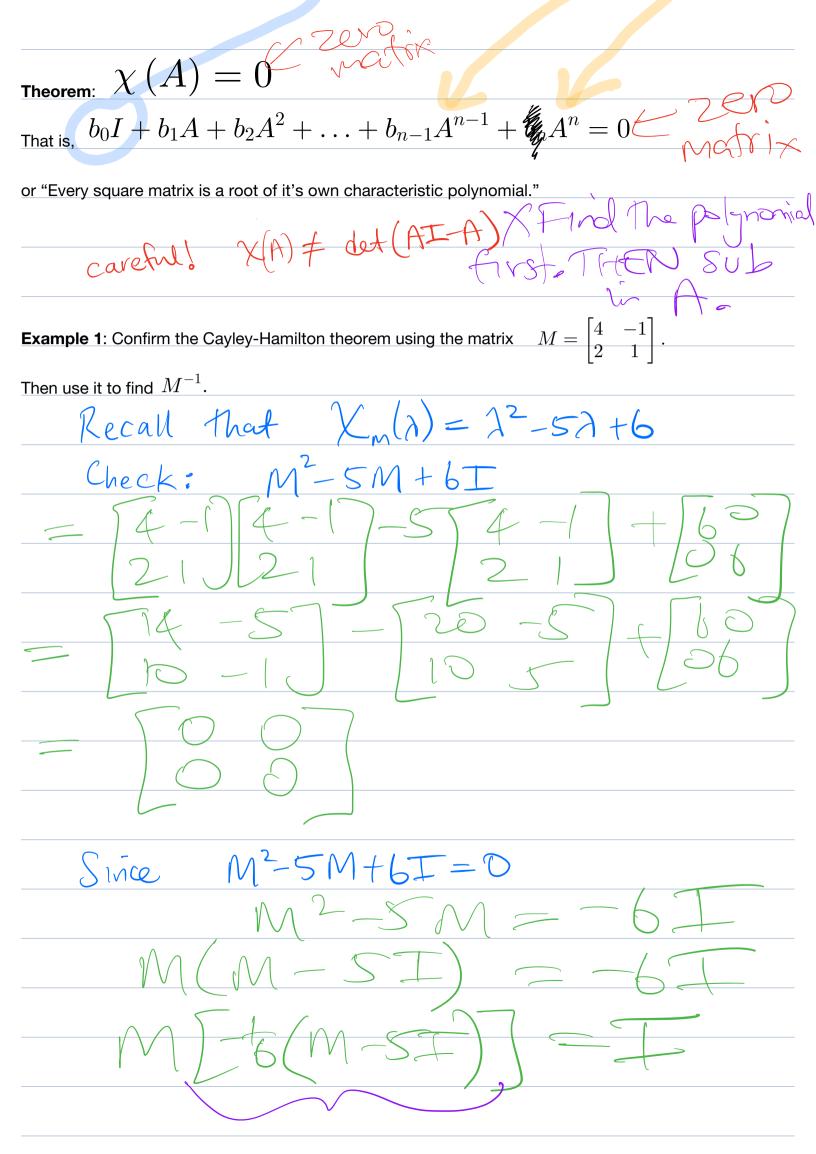
 $\det\left(M-\lambda I\right)$ Note: Some people call

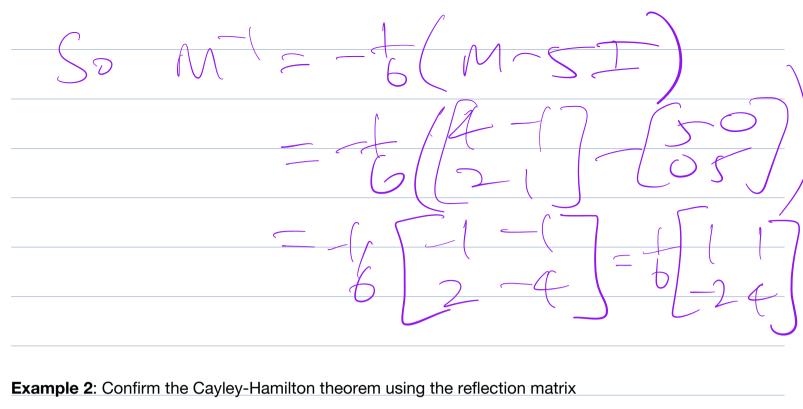
the characteristic polynomial, which is the same if n is even, and the negative if n is odd.

Example: Find the Eigenvalues of
$$M=\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$$
 .
$$\det\left(\lambda I-M\right)=$$

$= (\lambda + \chi \lambda - () + 2$
$= 3^2 - 53 + 6$ $= (3-2)(3-3)$
$S_0 A = 2 B_1$
Note: Not all real matrices have real Eigenvalues (example later). However, all complex matrices have complex Eigenvalues. Why?
The Fundamental Medicon
Afgebra Says That
every non-constant polynamial has a root in C.

Section 7.2 The Cayley-Hamilton Theorem	
Arthur Cayley (1821-1895)	William Hamilton (1805-1868
Arthur Cayley (1821-1895) lascovered abstract	discovered arithmetic of
group axioms	Villiam Hamilton (1805-1868) discovered avithmetic of yeaternions - extends complex numbers into higher dimensions
The Cayley-Hamilton Theorem is a remarkable connection by	
In a theoretical sense, it can help you simplify high powers of	of a matrix, find the inverse of a matrix and is
helpful in solving ordinary differential equations. It is used in	n many practical applications such as
computer programming, coding and engineering - any situa	tion where Eigenvalues are required. For
example, in structural analysis and signal processing. avoiding dangerous filtering/not resonance	se reduction
Consider a square nxn matrix $A = \{Q_i, \dots, j=1, \dots, j=$	ν with entries from a field, F. Then the
$\chi_{A}(\lambda) = \det(\lambda t - A) = \begin{vmatrix} \lambda - \alpha_{11} & -\alpha_{21} \\ -\alpha_{21} & \lambda - \alpha_{22} \end{vmatrix}$	$\frac{d_{12} - a_{1n}}{d_{22} - a_{2n}}$
$=b_0+b_1\lambda+1$	622+ ·-+ 6,12 1+2"
for some bo, b,,,, b	$b_2\lambda^2 + \cdots + b_{n-1}\lambda^{n-1} + \lambda^n$ $n \in \mathbb{F}$





$$M = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}.$$

$$\chi_{M}(\lambda) = \begin{bmatrix} \lambda & 0 & -\cos\theta \\ 0 & 1 & \cos\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - \cos\theta \\ -\sin\theta \\ -\sin\theta \end{bmatrix} + \cos\theta$$

$$= \begin{bmatrix} \lambda - \cos\theta \\ 1 + \cos\theta \\ -\cos\theta \end{bmatrix} - \sin\theta$$

$$= \begin{bmatrix} \lambda - \cos\theta \\ 1 + \cos\theta \\ -\cos\theta \end{bmatrix} - \sin\theta$$

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