MATH2022 Week 06

Worksheet

MATH 2022 Week 6 Worksheet

Q1/ Put $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(a) Find

$M \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} =$

$M \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$

(b) By inspection, from the previous part, what are the eigenvalues? $\boxed{}$

(c) Find P, P^{-1} and diagonal D such that $M = PDP^{-1}$.

$P =$, $P^{-1} =$

$D =$

(d) Find the characteristic polynomial

$$\chi(\lambda) = \det(\lambda I - M) = \begin{vmatrix} \lambda-2 & -1 \\ -1 & \lambda-2 \end{vmatrix}$$

$=$

(e) What are the roots of $\chi(\lambda)$?

$$\lambda = \boxed{}$$

(f) Find a general formula for M^n:

$$M^n = (PDP^{-1})^n = PD^nP^{-1}$$

$=$

(g) Thus find $M^4 =$

Q2/ Put $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$.

(a) Find

$$M \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} =$$

(b) By inspection, what are the eigenvalues?

$\boxed{}$

(c) Find P and diagonal D such that $M = PDP^{-1}$:

$P =$, $D =$

(d) Find P^{-1}:

(e) Find a general formula for M^n:

$M^n =$

(f) Thus find $M^4 =$

Q3/ Put

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(a) Find surd expressions for the eigenvalues :

(b) What is the dominant eigenvalue to 3 decimal places ?

$\lambda_1 = \boxed{}$

(c) What is the smaller eigenvalue to 3 decimal places ?

$\lambda_2 = \boxed{}$

(d) Find

$M^{-1} =$

(e) Put $\underset{\sim}{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\underset{\sim}{v}_{k+1} = M \underset{\sim}{v}_k \quad (k \geq 0)$

$$r_k = \frac{\text{1st entry of } \underset{\sim}{v}_k}{\text{1st entry of } \underset{\sim}{v}_{k-1}}$$

$$s_k = \frac{\text{2nd entry of } \underset{\sim}{v}_k}{\text{1st entry of } \underset{\sim}{v}_k}$$

Complete the following table to 3 d.p.

$\underset{\sim}{v}_0$	$\underset{\sim}{v}_1$	$\underset{\sim}{v}_2$	$\underset{\sim}{v}_3$	$\underset{\sim}{v}_4$	$\underset{\sim}{v}_5$	$\underset{\sim}{v}_6$	$\underset{\sim}{v}_7$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 7 \\ 15 \end{bmatrix}$					
r_k	1	7					
s_k	3	2.143					

(f) Find $M \begin{bmatrix} 1 \\ s_7 \end{bmatrix} =$

(g) Read off the dominant eigenvalue $\approx$ $\boxed{}$

(h) Put $\underset{\sim}{w}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\underset{\sim}{w}_{k+1} = M^{-1}\underset{\sim}{w}_k = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \underset{\sim}{w}_k$

$$t_k = \frac{\text{1st entry of } \underset{\sim}{w}_k}{\text{1st entry of } \underset{\sim}{w}_{k-1}}$$

$$u_k = \frac{\text{2nd entry of } \underset{\sim}{w}_k}{\text{1st entry of } \underset{\sim}{w}_k}$$

Complete the following table to 3 d.p.

$\underset{\sim}{w}_0$	$\underset{\sim}{w}_1$	$\underset{\sim}{w}_2$	$\underset{\sim}{w}_3$	$\underset{\sim}{w}_4$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 1.5 \end{bmatrix}$	$\begin{bmatrix} 5.5 \\ -3.75 \end{bmatrix}$		
t_k	-2			
u_k	-0.75			

(i) Find $M \begin{bmatrix} 1 \\ u_4 \end{bmatrix} =$

(j) Read off the negative eigenvalue of $M \approx$ $\boxed{}$

Q4/ Suppose that M is an invertible matrix with eigenvector $\underset{\sim}{v}$ corresponding to eigenvalue λ.

(a) Prove that $\lambda \neq 0$.

Proof:

(b) Prove that $\underset{\sim}{v}$ is an eigenvector for M^{-1} corresponding to eigenvalue λ^{-1}.

Proof: