THE UNIVERSITY OF SYDNEY

Student number:

Room number:

Seat number:

ANONYMOUSLY MARKED
(Please do not write your name on this exam paper)

CONFIDENTIAL EXAM PAPER

This paper is not to be removed from the exam venue.

Computer Science

EXAMINATION

Semester 2 – Practice, 2024

COMP2022 Models of Computation

EXAM WRITING TIME: 120 minutes
READING TIME: 10 minutes

EXAM CONDITIONS:
This is a RESTRICTED OPEN book exam — specified materials permitted.
Writing is not permitted during reading time.

MATERIALS PERMITTED IN THE EXAM VENUE:
Only one A4 sheet of handwritten and/or typed notes, double-sided
(No electronic devices of any kind are permitted.)

MATERIALS TO BE SUPPLIED TO STUDENTS:
This exam paper booklet.

INSTRUCTIONS TO STUDENTS:
DO NOT ATTACH EXTRA PAGES TO THIS EXAM BOOKLET. **Neatly** write your student ID in the box at the top right of this front page. Write your answers in the spaces provided in the exam paper booklet using blue or black ink. If you run out of space in the space provided right after each problem statement, write "CONTINUED", and continue your answers on pages 15-20, **clearly indicating what is rough work and what are solutions**. Please tick this box to confirm that your examination paper is complete ☐

For examiner use only:

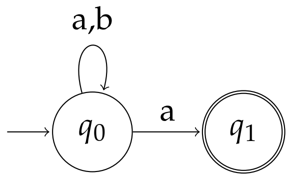
Problem	1	2	3	4	5	6	7	8	9	10	Total
Marks											
Out of	15	4	4	4	3	4	4	4	3	5	50

Problem 1. Each question has only one correct answer: score $+1$ for selecting only the correct answer (like so ■), and 0 otherwise.

(1) Which of the following statements is NOT true about the regular languages:

- ☐ They are closed under union.
- ☐ They are closed under intersection.
- ☐ They are closed under concatenation.
- ☐ They are closed under intersection with a context-free language.

(2) Consider the following NFA.



Which of the following strings is accepted by it?

- ☐ ϵ
- ☐ aa
- ☐ aab
- ☐ $abab$

(3) Suppose we have an NFA, with epsilon transitions, with 5 states. We apply the construction to remove the epsilon transitions (without further simplification). How many states will the resulting NFA have?

- ☐ 5
- ☐ 11
- ☐ 25
- ☐ 32

(4) Consider the regular expression (allowing shorthand) over the alphabet $\Sigma = \{a, b\}$: $\Sigma^* a \Sigma^* b \Sigma^* b \Sigma^* a \Sigma^*$. What is the language of this regular expressions?

- ☐ All strings
- ☐ All strings with at least two as and at least two bs
- ☐ The set of strings that contain $abba$, in that order, but not necessarily consecutively.
- ☐ The set of strings that contain a palindrome as a substring.

(5) Consider the grammar:

$$S \to SS \mid aaS \mid bbS \mid \epsilon$$

Which of the following is a leftmost derivation in this grammar?

- ☐ $S \implies SS \implies SaaS \implies Saa \implies bbSaa \implies bbaa$
- ☐ $S \implies SS \implies aaSS \implies aaS \implies aabbS \implies aabb$
- ☐ $S \implies aaS \implies aaSS \implies aabb$
- ☐ $S \implies bSb \implies bSSb \implies bSbSbb \implies bSbbb \implies bbbb$

(6) Which of the following types of rules are not allowed for CFGs in CNF?

- ☐ $A \to a$
- ☐ $A \to AA$
- ☐ $A \to BC$
- ☐ $A \to aB$

(7) Consider the grammar:

$$S \to 0S \mid 0S1S \mid \epsilon$$

Which of the following strings is NOT generated by it?

- ☐ ϵ
- ☐ 001
- ☐ 010
- ☐ 011

(8) Let A, B be recognisable languages over an alphabet Σ. Which of the following statements does NOT follow?

- ☐ $\Sigma^* \setminus A$ is recognisable.
- ☐ AB is recognisable.
- ☐ $A \cap A$ is recognisable.
- ☐ $A \cup B$ is recognisable.

(9) Which of the following languages is NOT Turing-decidable?

- ☐ The set of encodings of DFAs D, E such that $L(D) = L(E)$.
- ☐ The set of encodings of DFAs D, E such that $L(D) \cap L(E)$ is empty.
- ☐ The set of encodings of CFGs G such that $L(G)$ is empty.
- ☐ The set of encodings of TMs M such that $L(M)$ is empty.

(10) What is the definition of **P**?

- ☐ All decision problems that are decided by a deterministic Turing machine.
- ☐ All decision problems that are decided by a non-deterministic Turing machine.
- ☐ All decision problems that are decided by a deterministic Turing machine that runs in polynomial time.
- ☐ All decision problems that are decided by a non-deterministic Turing machine that runs in polynomial time.

(11) Let Σ be the input alphabet. Let M be a non-deterministic Turing machine that halts on all its inputs. Then $L(M) = \Sigma^*$ exactly when:

- ☐ For every string in Σ^*, every branch of its computation tree has an accepting configuration.
- ☐ For every string in Σ^*, some branch of its computation tree has an accepting configuration.
- ☐ For every string in Σ^*, every branch of its computation tree has a rejecting configuration.
- ☐ For every string in Σ^*, some branch of its computation tree has a rejecting configuration.

(12) The propositional formula $\neg(q \vee (p \wedge q))$ is equivalent to:

- ☐ q
- ☐ $\neg q$
- ☐ p
- ☐ $\neg p$

(13) Which of the following statements about propositional formulas F, G is true?

- ☐ If F is satisfiable and G is valid then $F \rightarrow G$ is valid.
- ☐ If F is valid and G is satisfiable then $F \rightarrow G$ is valid.
- ☐ If F is valid and G is satisfiable then $F \wedge G$ is valid.
- ☐ If F is satisfiable and G is satisfiable then $F \vee G$ is valid.

(14) Which of the following is true about the formula $(p \leftrightarrow q) \wedge p$:

- ☐ It is satisfiable.
- ☐ It is valid.
- ☐ It is equivalent to $p \rightarrow q$.
- ☐ It is equivalent to p.

(15) What is the meaning of the formula $\forall x(P(x) \to Q(x))$?

- ☐ Every element satisfies P.
- ☐ Every element satisfies P and Q.
- ☐ Every element that satisfies P also satisfies Q.
- ☐ Some element satisfies P and Q.

Problem 2. This question is about Regular Languages over the alphabet $\Sigma = \{0, 1\}$. [4 marks] Provide a regular expression for the set of strings not equal to the string 011. Briefly explain your answer.

Problem 3. This question is about Regular Languages over the alphabet $\Sigma = \{a\}$. Prove that the following language is not regular: $\{a^{n^2+1} : n \geq 1\}$. [4 marks]

Problem 4. This question is about Context-free Languages. Consider the CFG: [4 marks]

$$
\begin{aligned}
S &\to AU \mid BV \mid a \mid b \mid \epsilon \\
T &\to AU \mid BV \mid a \mid b \\
U &\to TA \\
V &\to TB \\
A &\to a \\
B &\to b
\end{aligned}
$$

Fill in the following CYK table for this grammar and the string *baaab*. No additional explanation is needed.

1	2	3	4	5

Problem 5. This question is about Context-free Languages over the alphabet $\Sigma = \{a, b\}$. [3 marks]
Provide a context-free grammar for the language of strings of the form uv where $|u| = |v|$ and u contains at least one a. Briefly explain your answer.

Problem 6. This question is about Turing Machines. Consider the following nondeterministic Turing Machine over input alphabet $\{a, b, c\}$ with initial state 0: [4 marks]

0	a	b	r	0
0	b	c	r	0
0	c	a	r	0
0	a	b	l	1
0	b	c	l	2
0	c	a	l	3
1	$*$	$*$	l	1
1	a	$*$	$*$	halt-accept
2	$*$	$*$	l	1
2	b	$*$	$*$	halt-accept
3	$*$	$*$	l	1
3	c	$*$	$*$	halt-accept

Provide a clear and concise description of its language. Briefly explain your answer.

Problem 7. This question is about Turing Machines. Argue that the language [4 marks]

$$L = \{Source_M, w : L(M) \text{ does not accept } w\}$$

is not recognisable.

Problem 8. This question is about Propositional Logic. Draw a truth table for the formula $(p \to \neg q) \land (\neg q \to \neg p)$. Include all subformulas. No additional explanation is needed. [4 marks]

Problem 9. This question is about Propositional Logic. Give a polynomial time algorithm that takes a CNF formula F as input and decides if F is valid. Briefly explain your answer. [3 marks]

Problem 10. These questions are about Predicate Logic. Consider the domain of countries, that includes Australia, New Zealand, and China, and the following predicates:

- $near(x, y)$ is a binary predicate expressing that x is near y.
- $island(x)$ is a unary predicate expressing that x is an island.

Use this to write the following statements in predicate logic. No additional explanation is needed.

a) China is not an island. [1 mark]

b) Every country that is near Australia is near New Zealand. [1 mark]

c) If country x is near country y, then country y is near country x. [1 mark]

d) Every country that is not an island is near an island. [2 marks]

(exam questions end here. use the rest of the space for rough work and/or solutions)

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