

---

**MAIN EXAM 2020 - Extended Answer Questions**

---

1. (a) Let  $z = 1 + i\sqrt{3}$  and  $w = -\sqrt{3} - i$ .
- (i) Calculate the modulus and principal argument of  $z$  and  $w$ .
  - (ii) Write down the polar exponential form of  $z$  and  $w$ .
  - (iii) Calculate the polar exponential form of  $\frac{z}{w}$ .
  - (iv) Determine the principal argument of  $\frac{z}{w}$ .
- (b) (i) Calculate  $\cos^3(2\theta)$  by using the binomial theorem and the complex form of  $\cos \theta$ .
- (ii) Find  $\int \cos^3(2\theta) d\theta$ .
2. (a) (i) Calculate the following limits or show that they do not exist.
- (A)
- $$\lim_{x \rightarrow -2^+} \frac{x+2}{|x+2|}$$
- (B)
- $$\lim_{x \rightarrow 0} x^3 \sin\left(\frac{3\pi}{x}\right)$$
- (C)
- $$\lim_{x \rightarrow +\infty} (1+x)^{\frac{2}{x}}$$
- (ii) Find the 5th order Taylor polynomial  $P_5(x)$  for the function  $\cos(x)$  about  $x = 0$ .
- (iii) Use the Taylor polynomial that you have found in part (ii) to approximate the integral  $\int_0^1 \cos(x^3) dx$ . (You do not need to calculate the error in this approximation.)
3. (a) Given the function  $f(x) = \frac{x^2 + x - 1}{x^3}$
- (i) Find the natural domain and vertical asymptotes, if any. Justify your answers.
  - (ii) Find horizontal asymptotes, if any. Justify your answers.
  - (iii) Calculate the first derivative  $f'(x)$ .
  - (iv) Find the critical points and intervals of increase/decrease of  $f$ .
  - (v) Calculate the second derivative  $f''(x)$ .
  - (vi) Find the points of inflection and intervals of concavity of  $f$ .
  - (vii) Find the global maximum and global minimum of  $f(x)$  on the interval  $[3, 6]$ .