The University of Sydney
School of Mathematics and Statistics

Calculus Tutorial 6 (Week 7)

MATH1062/MATH1023: Mathematics 1B (Calculus) Semester 2, 2024

Questions marked with * are harder questions.

Material covered

(1) More general differential equations
(2) Practice questions from single variable calculus

Summary of essential material

Systems of linear equations with constant equations have a strict algorithm to their solutions.

For the second half of the sheeet, recall what you learned about single variable calculus.

Questions to complete during the tutorial

1. (a) Find the general solution to $y'' - 8y' + 16y = 0$.

(b) Find a particular solution to $y'' - 8y' + 16y = e^{4x}$. (Hint: try $y_p(x) = u(x)e^{4x}$ and then find $u(x)$.)

(c) Find the general solution to $y'' - 8y' + 16y = e^{4x}$.

2. Consider the following system of differential equations

$$\dot{x} = 2x,$$
$$\dot{y} = x - 3y.$$

(a) Find $x(t)$ and $y(t)$ by solving the equations in the system successively.

(b) Eliminate x to obtain a 2nd-order differential equation for y, and then find $y(t)$ and $x(t)$.

3. Recall that the derivative of $f(x)$ is by definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. Use this definition to find $f'(x)$ for $f(x) = x^3$.

4. Find the equation of the tangent line to the curve $y = xe^{-x}$ at $x = 2$.

5. Compute the following limits.

(a) $\lim_{x\to 0}(e^{\sin(2x)} - 2 + \sqrt{x^2+4})$

(b) $\lim_{x\to 3} \dfrac{x^2+2x-3}{x^2-4}$

(c) $\lim_{x\to 0} \dfrac{\sin^2 x}{2x^2}$

(d) $\lim_{x\to 0} x\cos\left(\dfrac{1}{x^2}\right)$

6. Is

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$$

continuous at $x = 2$?

7. Find the global minimum and maximum values of $f(x) = x^3 + 2x^2 - 4x + 4$ on the closed interval $[-3, 3]$.

Short answers to selected exercises

1. (a) $y(x) = (Ax + B)e^{4x}$ (b) $y(x) = \dfrac{1}{2}x^2 e^{4x}$ (c) $y(x) = \left(\dfrac{x^2}{2} + Ax + B\right) e^{4x}$

2. (a) $x(t) = Ae^{2t}$, $y(t) = \dfrac{A}{5}e^{2t} + Ce^{-3t}$ (b) $x(t) = 5Be^{2t}$, $y(t) = Ae^{-3t} + Be^{2t}$

3. $f'(x) = 3x^2$.

4. $y = e^{-2}(4 - x)$.

5. (a) 1 (b) $\dfrac{12}{5}$ (c) $\dfrac{1}{2}$ (d) 0

6. No.

7. On $[-3, 3]$, $f(x)$ has global minimum value $\dfrac{68}{27}$ and global maximum value 37.