The University of Sydney
School of Mathematics and Statistics

Practice Quiz

MATH1062: Mathematics 1B — Semester 2, 2024

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Information about the quiz

- The actual quiz will have a total of 16 questions in two parts (calculus and statistics) of 8 questions each.
- All questions will be multiple-choice questions with four answer options and exactly one correct answer. Note that most questions below do not fit this question type. However, we believe that these are excellent questions to help you study for the quiz.
- Answers to the questions below will be posted closer to the date of the quiz.
- The questions given below will likely take longer to solve than the questions you will encounter in the quiz.

Part A: Calculus

1. Find the general solution of the differential equation

$$\frac{dy}{dx} = x\cos x.$$

2. Which of the following differential equations are linear? In the cases that are linear, write the differential equations in standard form for linear equations:

$$(a)\ \frac{dy}{dx} = x^2 + y^2, \qquad (b)\ \frac{dy}{dx} = x + y,$$
$$(c)\ e^x\left(\frac{dy}{dx} + 2y\right) = 1, \qquad (d)\ \frac{dy}{dx} = x^3y^3.$$

3. Solve the linear differential equations from the previous question. In each case, give the integrating factor and find y explicitly as a function of x.

4. Which of the following differential equations are separable? In the cases that are separable, write the differential equations in standard form for separable equations:

$$(a)\ \frac{dy}{dx} = x + xy^2, \qquad (b)\ \frac{dz}{dx} = x + z,$$
$$(c)\ \frac{dw}{dt} = 1 + w^2\sin t, \qquad (d)\ \frac{dv}{du} = \cos(u+v) + \cos(u-v).$$

5. Solve the separable differential equations from the previous question.

6. The general solution of the logistic equation $\frac{dP}{dt} = P - P^2$ is $P(t) = \frac{1}{1+Ce^{-t}}$. Find the particular solution for the initial population size $P(0) = 5$.

7. An isolated colony of animals has a birth rate and a death rate that are proportional to the total population. The population doubles every 30 years. At time $t = 0$, the population is 1000 animals. Calculate the population after 40 years and the time taken for the population to increase tenfold.

8. A pie is removed from a refrigerator and placed straight in an oven at $\bar{T} = 200°C$. Assuming Newton's law of heating, the temperature T of the pie, in degrees Celsius, is described by the differential equation

$$\frac{dT}{dt} = -k(T - \bar{T}).$$

where t is measured in minutes and k is the heating rate measured in 1/minute. Find the particular solution $T(t)$ with $T(0) = 5°C$ and $k = 0.01$/minute. How long does it take for the pie to reach 5 times its initial temperature (to the nearest minute)?

Part B: Statistics

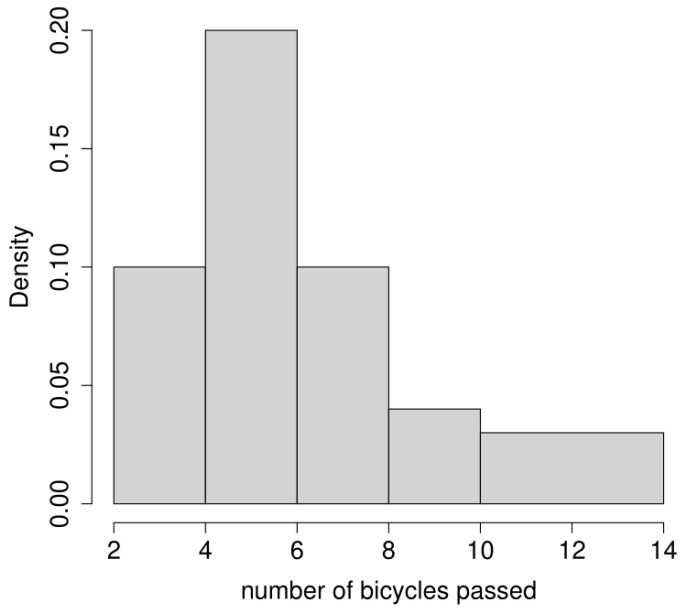
9. **(Mean and SD)** In a dataset of size 8, the mean is 7 and standard deviation is 4. We add 4 to each observation in the dataset. The new mean and SD are respectively

(a) 7 and 4
(b) 11 and 8
(c) 7 and 8
(d) 11 and 4

10. **(Probability)** Select the correct statement from below.

(a) If A and B are mutually exclusive events, then A and B are independent.
(b) If we take a sample of points of size 10 from a scatterplot with correlation coefficient 1, then the sampled points lie on a straight line.
(c) The slope of a regression line relating height and age among boys is 1.3 times higher than the slope of a regression line relating the height and age among girls. Therefore, the correlation coefficient between height and age among boys is 1.3 times higher than correlation coefficient between height and age among girls.
(d) More than one of the other listed options is correct.

11. **(Histogram)** Each day for a 25-day period, Jimmy counted the number of bicycles he passed on his drive to work. A histogram of his counts is plotted below. How many days did he pass less than 6 bicycles? (Assume each bin is of the left-closed and right-open form $[a, b)$.)



(a) 8 (b) 10
(c) 12 (d) 15

12. (Normal distribution) A study conducted at the University of Sydney shows that the average height of the female staff members is 165.52cm with a SD of 5.59cm. Consider this the full population of the female staff members of the University of Sydney and that the height can be described by a normal model. Which one is the correct R code for calculating the percentage of female staff members with height between 152.40cm and 167.64cm?

(a) qnorm$(167.64, 165.52, 5.59)$-qnorm$(152.4, 165.52, 5.59)$
(b) pnorm$(167.64, 165.52, 5.59^2)$-pnorm$(152.4, 165.52, 5.59^2)$
(c) qnorm$(167.64, 165.52, 5.59^2)$-qnorm$(152.4, 165.52, 5.59^2)$
(d) pnorm$(167.64, 165.52, 5.59)$-pnorm$(152.4, 165.52, 5.59)$

13. (Normal distribution) The weight of the box is normally distributed with a mean of 5 kilograms and a SD of 2 kilograms. The box is going to be shipped to a country where weights are commonly reported in pounds. To convert from kilograms to pounds, one must use the formula lb $= 2.205 \times$ kg, where kg is the weight in kilograms and lb is the weight in pounds. What is the probability that the weight of the box in pounds is greater than 8 pounds, which is the weight limit for packages being shipped to that country? (Round to 3 d.pl.) You may find the following R output useful:

```
> pnorm(0.343)
[1] 0.6342008
> pnorm(0.566)
[1] 0.7143031
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> pnorm(0.686)
[1] 0.7536434
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(a) 0.754 (b) 0.714
(c) 0.634 (d) 0.246

14. (Causation) In Australia, in 2010 there were 143,473 deaths from all causes, compared to 161,300 in 2020, which is a 12.43% increase from 2010 to 2020. Which of the following statements is definitely correct?

(a) The public health got worse over the period 2010-2020 in Australia.

(b) The increase is caused by the spread of COVID19.

(c) The data given are not enough to identify the reason for this increase.

(d) The increase is just random.

15. (Correlation) Consider the following table with pairs of values for x and y,

x	y
2	2
2	1
1	1

Now we add a 4th pair of values to make the correlation coefficient 0. By inspection, the added values for x and y are

(a) 0 and 0
(b) 2 and 1
(c) 1 and 2
(d) There is not enough information to tell.

16. (Causation) During the Great Depression of 1929-1933, better educated people tended to have shorter spells of unemployment. Does education protect you against unemployment?

(a) We can conclude that education protects against unemployment.

(b) We cannot make causal conclusions because there may exist other confounding factors.

(c) Given that these are historical observations, we can conclude that education protects against unemployment.

(d) We cannot conclude that education protects against unemployment because data are too old.