

**Solution 4.1.** The idea is that each state in  $Q'$  not only stores the state in  $Q$  but also tracks the current letter position (1st, 2nd, 3rd). Specifically, we extend each state  $q \in Q$  to  $\{(q, 0), (q, 1), (q, 2)\}$ , representing every first, second, and third letter respectively. For the first and second characters, we only update the number in the tuple. For example,  $(q_0, 0)$  becomes  $(q_0, 1)$  after receiving the first character. When the third letter is received (the number in the tuple is 2), the number resets to 0, and  $q_0$  transitions according to the original transition function in  $\delta$ . For example:

$$(q_0, 2) \xrightarrow{a} (q_1, 0)$$

The final states are set to  $\{(q_f, 0), (q_f, 1), (q_f, 2)\}$  for all  $q_f \in F$ .

**Solution 4.2.** For DFA B, we have DFA  $B = (Q', \Sigma, q'_0, \delta', F')$ :

- $Q' = Q \times \{0, 1, 2\}$
- $q'_0 = (q_0, 0)$
- $\delta'((q, n), l) = \begin{cases} (q, n+1) & n = 0, 1 \\ (\delta(q, l), 0) & n = 2 \end{cases}, q \in Q, l \in \Sigma$
- $F' = (q, n), q \in F, n \in \{0, 1, 2\}$