THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Calculus Exercises 2 (Week 2)

MATH1062/MATH1023: Mathematics 1B (Calculus) Semester 2, 2024

Material covered

(1) Separable equations

Assumed Knowledge

Factorisation of expressions. Techniques of integration. Trigonometric identities and properties of ln.

Objectives

1. Recognise a differential equation as a separable equation.
2. Solve a separable equation by separation of variables.

Exercises

Questions marked with * are harder questions.

1. Check which of the following first order differential equations are separable. Write the ones that are separable in form $dy/dx = f(x)g(y)$. For the ones that are not separable, justify your answer.

(a) $x\dfrac{dy}{dx} = x^3 + \cos(2x)$

(b) $\dfrac{1}{y}\dfrac{dy}{dx} = x + 1$

(c) $\dfrac{dy}{dx} = xy + 1$

*(d) $\dfrac{dy}{dx} - 2y(\ln|x| + 1) + 1 = \ln|x|\ln|y| + \ln|\dfrac{y}{x}|$
Hint: Look up properties of ln.

*(e) $\dfrac{dy}{dx} = \sin(x + y) + \sin(x - y + \pi/2)$
Hint: Look up trigonometric identities.

(f) $\dfrac{dy}{dx} - x^4 y = 3x + 7$

2. Find the general solutions of

(a) $\dfrac{dy}{dx} = y\cos x$

(b) $(1 + x)\dfrac{dy}{dx} + y^2 = 0$

(c) $xy^2\dfrac{dy}{dx} = x + 1$

(d) $\dfrac{dy}{dx} = \dfrac{x}{y}$

3. (a) Given $y = A\sqrt{x^2+1}$, where A is an arbitrary constant, show by substitution that it satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{xy}{x^2+1}$.

(b) The general solution of a first-order differential equation depends on one arbitrary constant. It follows that the solution given in part (a) is the general solution of $\dfrac{dy}{dx} = \dfrac{xy}{x^2+1}$. Find the particular solution satisfying the initial condition $y(0) = 1$.

***4.** Consider a particle with mass $m = 1$ moving in a straight line and subject to a force $f = -v\sin(t)$ in the direction of the movement.

(a) Using Newton's second law of mechanics ($f = ma$, where $a = dv/dt$), write a differential equation for the velocity v of the particle.

(b) Find the general solution of the differential equation.

(c) Find the particular solution for the case in which at time $t = 0$ the particle has velocity $v = 1$.

(d) Write down a differential equation for the position s of the particle.
Hint: The velocity $v(t)$ of a particle equals the rate of change of its position $s(t)$.

(e) Without solving the equation, discuss what happens with $s(t)$ in the limit $t \to \infty$ for different values of v at $t = 0$.

Short answers to selected exercises

1. (a) Yes (b) Yes (c) No (d) Yes (e) Yes (f) No

2. (a) $y = Ae^{\sin x}$