The University of Sydney
School of Mathematics and Statistics

Calculus Tutorial 7 (Week 8)

MATH1062/MATH1023: Mathematics 1B (Calculus) Semester 2, 2024

Questions marked with * are harder questions.

Material covered

(1) Curves and surfaces in 3-dimensional space

Summary of essential material

Equation of an ellipse with semi-axes of lengths a (in the x direction) and b (in the y direction):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Parametric equations of an ellipse with semi-axes of lengths a and b:

$$x = a\cos t, \quad y = b\sin t \quad \text{for } t \in \mathbb{R}.$$

Questions to complete during the tutorial

1. Sketch the following parametric curves in the xy-plane, assuming that t takes all values in $\mathbb{R}$:

(a) $x = \cos t$, $y = \sin t$

(b) $x = \cos t$, $y = 4\sin t$

(c) $x = 2 + \cos t$, $y = -1 + \sin t$

(d) $x = t^2$, $y = t$

2. Find the natural domains for the functions of two variables given by the following formulas, and sketch the domains as subsets of $\mathbb{R}^2$. Find the ranges of each function, assuming in each case that the domain is the natural domain.

(a) $\ln(x + 2y)$ (b) $\sqrt{x + 1 - y^2}$ (c) $\cos\left(x/\sqrt{y}\right)$

3. Sketch the level curves of the function $f(x, y) = x^2 + 2y^2$ corresponding to the function values $c = 0$, $c = 1$ and $c = 2$. Use these to help sketch the graph of this function, that is, the surface with equation $z = x^2 + 2y^2$. (Note that there are no level curves for $c < 0$. Why?)

4. In each case, sketch and describe the given surface in $\mathbb{R}^3$. In addition, write down two points in $\mathbb{R}^3$ that are on the surface.

(a) $x^2 + y^2 + (z - 1)^2 = 4$

(b) $(x - 3)^2 + (y - 1)^2 + (z + 1)^2 = 1$

(c) $x^2/4 + y^2/9 + z^2 = 1$

5. Sketch the following parametric curve in the xy-plane: $x = 2\cos t$, $y = 4\sin t$.

On your sketch indicate the direction of increasing t. Find the point (x, y) corresponding to $t = 0$, and mark it on your sketch. Repeat for $t = \pi$, for $t = 2\pi$ and for $t = -\pi$.

*6. Describe the curve in three-dimensional space given by $x = 2\cos t$, $y = 4\sin t$, $z = t$. Sketch it, indicating the direction of increasing t, and mark in the points corresponding to $t = n\pi$ for all $n \in \{-1, 0, 1, 2, 3\}$.

*7. What does the curve $(x, y, z) = (2\cos t, 4\sin t, t^2)$ look like? Sketch it.

Short answers to selected exercises

1. (a) Circle: centre at $(0, 0)$, radius 1.

(b) Ellipse, centre at $(0, 0)$.

(c) Circle centred at $(2, -1)$, with radius 1.

(d) Sideways-opening parabola with vertex at $(0, 0)$.

4. (a) Sphere: radius 2 and centre $(0, 0, 1)$.

(b) Sphere: raius 1 and centre $(3, 1, -1)$.

(c) Ellipsoid: centre $(0, 0, 0)$.

(d) Paraboloid: vertex at $(0, 0, 0)$.

(e) Cone: vertex at $(0, 0, 0)$.