Extended Answer Section

There are **three** *questions in this section, each with a number of parts. Write your answers in the space provided below each part. There is extra space at the end of the paper.*

1. (a) Use integration by parts to compute

$$\int_1^2 t^2 \ln t \, dt.$$

Question 1 continues on the next page.

(b) Use integration by substitution to compute

$$\int \tfrac{1}{2}x\sqrt{x+1}\,dx.$$

Question 1 continues on the next page.

(c) Consider the region $\mathcal{R}$ on the xy-plane bounded by

$$x = 0, \quad y = 1 + 3x^2, \quad y = 2x, \quad x = 1.$$

Sketch the region $\mathcal{R}$, and calculate the volume of revolution obtained by rotating $\mathcal{R}$ around the vertical line $x = 0$.

Question 1 continues on the next page.

(d) Show that the function

$$f(x) = \begin{cases} \dfrac{1}{x} - \cot x, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$.

Question 1 continues on the next page.

Question 2 begins on the next page

2. (a) Consider the sets:

$$A = \{x \in \mathbb{R} \mid -3 \leq x \leq 4\} \quad \text{and} \quad B = \{x \in \mathbb{R} \mid -1 \leq x \leq 3\}.$$

Plot the sets A, B, and $A \setminus B$ on the real number line.

Question 2 continues on the next page.

(b) Plot the following set on the complex plane:

$$\{z \in \mathbb{C} \,|\, |z-1| \leq 1\} \cup \{z \in \mathbb{C} \,|\, \mathrm{Re}(z) > 1\}$$

Question 2 continues on the next page.

(c) Given that $z = 1 + i$ is one solution, solve the equation

$$z^3 + \underset{3}{\cancel{2}}z^2 - 8z + 10 = 0.$$

Question 2 continues on the next page.

(d) Evaluate the limit

$$\lim_{x\to\infty} \frac{5x^9 - 4x^7 + 3}{10x^9 + 7x^8 - 12x^4}.$$

Explain your working carefully.

Question 2 continues on the next page.

(e) Consider the function

$$f : \mathbb{R} \setminus \{1\} \to \mathbb{R}, \quad f(x) = \frac{e^{x+1}(x^2 + 5x - 6)}{x - 1}.$$

Find a rule for a function $g : \mathbb{R} \to \mathbb{R}$ so that $g(x) = f(x)$ for all $x \neq 1$ and $g(x)$ is continuous at $x = 1$. Justify your choice carefully.

Question 2 continues on the next page.

(f) Find the global maximum and global minimum for the function $f(x) = |(x+1)(x-5)|$ over the interval $[1, 6]$.

Question 3 begins on the next page

3. Consider the function $\theta(t)$ which is defined implicitly by the equation

$$t + c = \int_0^{\theta(t)} \frac{1}{\sqrt{4 + 2\cos u}}\, du,$$

where $c \in \mathbb{R}$ is a constant.

(a) Use upper and lower Riemann sums with 3 subintervals to estimate the value of c if $\theta(0) = \pi$.

Question 3 continues on the next page.

(b) State the Fundamental Theorem of Calculus part I and use it to show that

$$\begin{aligned}\theta'(t) &= \sqrt{4 + 2\cos(\theta(t))},\\ \theta''(t) &= -\sin(\theta(t))\end{aligned}$$

Question 3 continues on the next page.

(c) Using the fact that $\theta(0) = \pi$, show that the 3rd order Taylor polynomial of $\theta(t)$ about $t = 0$ is given by

$$P_3(t) = \pi + \sqrt{2}t + \frac{1}{3\sqrt{2}}t^3$$

Question 3 continues on the next page.

(d) Give an approximate value of $\theta(1)$ using $P_3(t)$ and use the Taylor remainder theorem to show that the error from the true value of $\theta(1)$ of the approximation is less than or equal to $\dfrac{7}{4!}$.

End of Extended Answer Section