# The University of Sydney
## School of Mathematics and Statistics

# MATH1002
## Linear Algebra

June 2019 Lecturers: Z. Afsar, A. Aksamit, D. Badziahin, A. Fish, A. Kerschl, B. Pauwels.

Time Allowed: **Writing - one and a half hours; Reading - 10 minutes**

Exam Conditions: This is a closed-book examination — no material permitted. Writing is not permitted at all during reading time.

Family Name: ............................................... SID: ............................

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Please check that your examination paper is complete (21 pages) and indicate by signing below.
I have checked the examination paper and affirm that it is complete.

Signature: ................................................... Date: ..........................

**This examination has two sections: Multiple Choice and Extended Answer.**

The Multiple Choice Section is worth 50% of the total examination.
There are 20 questions. The questions are of equal value.
All questions should be attempted.

Answers to the Multiple Choice questions must be entered on the Multiple Choice Answer Sheet before the end of the examination.

The Extended Answer Section is worth 50% of the total examination.
There are 3 questions. The questions are of equal value.
All questions should be attempted. Working must be shown.

**THE QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM.**

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## Multiple Choice Section

*For each question, choose exactly one option.*

**Your answers must be entered on the Multiple Choice Answer Sheet.**

**1.** Let $\mathbf{u} = [1, \alpha, 0]$, $\mathbf{v} = [5, -4, 15]$ and $\mathbf{w} = [-1, \beta, -3]$. Find the values of $\alpha$ and $\beta$ so that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal and $\mathbf{v}$ and $\mathbf{w}$ are parallel.

(a) $\alpha = \frac{5}{4}, \beta = \frac{4}{5}$ (b) $\alpha = \frac{4}{5}, \beta = \frac{5}{4}$ (c) $\alpha = \frac{5}{4}, \beta = -\frac{4}{5}$

(d) $\alpha = -\frac{5}{4}, \beta = \frac{4}{5}$ (e) $\alpha = -\frac{4}{5}, \beta = \frac{5}{4}$

**2.** What is the area of the triangle inscribed by the vectors $[2, -1, 3]$ and $[-1, 1, 0]$?

(a) $\sqrt{19}$ (b) $\frac{19}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{\sqrt{19}}{2}$ (e) $\sqrt{7}$

**3.** Which one of the following augmented matrices is in row echelon form?

(a) $\left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 1 & 6 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{array}\right]$ (b) $\left[\begin{array}{cc|c} 2 & 4 & 3 \\ 0 & 0 & 1 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{array}\right]$

(d) $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$ (e) None of the above.

**4.** The determinant of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 2 \\ 2 & 4 & -2 \end{bmatrix}$ is equal to

(a) 16 (b) $-16$ (c) 0 (d) 4 (e) $-4$

**5.** Consider the following system of equations:

$$\begin{array}{rcrcrcrcl} x & + & y & + & z & - & w & = & 0 \\ & & -y & + & 2z & - & w & = & 0 \\ & & & & 3z & - & 2w & = & 0 \end{array}$$

Which one of the following statements about this system is true?

(a) There is no solution.

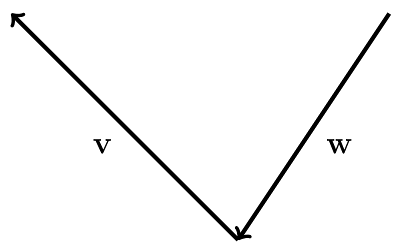
(b) The general solution is expressed using exactly 1 parameter.

(c) The general solution is expressed using exactly 2 parameters.

(d) The general solution is expressed using 3 or more parameters.

(e) There is a unique solution.

**6.** Suppose $\mathbf{v}$ and $\mathbf{w}$ are two non-zero vectors lying in this page:




Which of the following is true?

(a) $(\mathbf{v} \times \mathbf{w}) \times \mathbf{v}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

(b) $(\mathbf{w} \times \mathbf{v}) \times (\mathbf{v} \times \mathbf{w})$ is parallel to $\mathbf{w}$ but not $\mathbf{v}$.

(c) $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{w}$ is a non-zero scalar.

(d) $\mathbf{v} \times \mathbf{w}$ points upwards, towards the ceiling.

(e) $\mathbf{w}$ and $\mathbf{v} \times \mathbf{w}$ are parallel.

**7.** The two lines given by the respective parametric equations

$$\left.\begin{array}{rcl} x & = & 3 + t \\ y & = & -5 + 2t \\ z & = & -5 - t \end{array}\right\} \; t \in \mathbb{R} \qquad \text{and} \qquad \left.\begin{array}{rcl} x & = & -3 - 2s \\ y & = & -2 - 4s \\ z & = & 1 + 2s \end{array}\right\} \; s \in \mathbb{R}$$

(a) do not intersect.

(b) intersect at the point $(-3, -2, 1)$.

(c) intersect at the point $(7, 3, -9)$.

(d) intersect at the point $(-2, -15, 0)$.

(e) coincide.

**8.** The cosine of the angle between the vectors $[1, 3, -5]$ and $[3, 2, 1]$ is equal to

(a) $\dfrac{7\sqrt{10}}{4}$ (b) $\dfrac{11}{15}$ (c) $\dfrac{2}{7\sqrt{10}}$ (d) $\dfrac{4}{7\sqrt{10}}$ (e) $\dfrac{2}{245}$

**9.** Consider the three planes with equations

$$\begin{array}{rrcrcrcl} \mathcal{P}_1: & x & + & 2y & - & z & = & 1, \\ \mathcal{P}_2: & 2x & - & 5y & + & 3z & = & 1, \\ \mathcal{P}_3: & 2x & - & 4y & + & z & = & 10. \end{array}$$

Which of the following is true?

(a) $\mathcal{P}_1$ and $\mathcal{P}_2$ are parallel to each other but not parallel to $\mathcal{P}_3$.

(b) $\mathcal{P}_1$ and $\mathcal{P}_3$ are parallel to each other but not parallel to $\mathcal{P}_2$.

(c) $\mathcal{P}_2$ and $\mathcal{P}_3$ are parallel to each other but not parallel to $\mathcal{P}_1$.

(d) None of the planes are parallel to each other.

(e) All of the planes are parallel to each other.

**10.** Which one of the following sequences of row operations, when applied to the matrix $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, produces the matrix $\begin{bmatrix} d-a & e-b & f-c \\ 2a & 2b & 2c \end{bmatrix}$?

(a) First $R_1 \to R_1 - R_2$, then $R_2 \to 2R_2$, then $R_1 \leftrightarrow R_2$.

(b) First $R_1 \to 2R_1$, then $R_1 \leftrightarrow R_2$, then $R_1 \to R_1 - R_2$.

(c) First $R_2 \to R_2 - R_1$, then $R_1 \leftrightarrow R_2$, then $R_1 \to 2R_1$.

(d) First $R_1 \leftrightarrow R_2$, then $R_1 \to 2R_1$, then $R_1 \to R_1 - R_2$.

(e) First $R_1 \leftrightarrow R_2$, then $R_1 \to R_1 - R_2$, then $R_2 \to 2R_2$.

**11.** Suppose that $\mathbf{v}$ and $\mathbf{w}$ are two linearly independent vectors in $\mathbb{R}^n$. Which one of the following statements is true?

(a) $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

(b) If $a\mathbf{v} + b\mathbf{w} = c\mathbf{v} + d\mathbf{w}$, where $a, b, c, d \in \mathbb{R}$, then $a = c$ and $b = d$.

(c) $\mathbf{v}$ and $\mathbf{w}$ are parallel.

(d) The vectors $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ are linearly dependent.

(e) None of the above.

**12.** Which one of the following statements is true?

(a) Any set of three vectors from $\mathbb{R}^2$ must span $\mathbb{R}^2$.

(b) There are three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ such that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.

(c) If $\text{span}\{\mathbf{u}, \mathbf{v}\} = \mathbb{R}^2$, then $\{\mathbf{u}, \mathbf{v}\}$ is a basis for $\mathbb{R}^2$.

(d) For any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$, there is a subset of $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ that is a basis for $\mathbb{R}^2$.

(e) The set $\{\mathbf{u}, \mathbf{v}, \mathbf{0}\}$ is a basis for $\mathbb{R}^2$ only if $\{\mathbf{u}, \mathbf{v}\}$ is a basis for $\mathbb{R}^2$.

**13.** Which one of the following statements is true, given that $A$ is a matrix of size $3 \times 3$, $B$ is a matrix of size $3 \times 2$, and $C$ is a matrix of size $2 \times 3$?

(a) $ACB$ is defined.

(b) $2A + CB$ is defined.

(c) $(BC)^2$ is a $2 \times 2$ matrix.

(d) $B(A - BC)$ is defined.

(e) $A^2 + BC$ is a $3 \times 3$ matrix.

**14.** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, which one of the following is true?

(a) $A$ is not invertible.

(b) $A$ is invertible and $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -3 & 1 & 0 \end{bmatrix}$.

(c) $A$ is invertible and $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$.

(d) $A$ is invertible and $A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix}$.

(e) None of the above.

**15.** Let $A = P\,D_1\,P^{-1}$, and $B = P\,D_2\,P^{-1}$ where $D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $D_2 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, and $P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Then $(AB)^5$ is

(a) $\begin{bmatrix} 2^5 & 3^5 - 2^5 \\ 0 & 3^5 \end{bmatrix}$.
(b) $\begin{bmatrix} -3^5 & 2^5 - 3^5 \\ 0 & -2^5 \end{bmatrix}$.
(c) $\begin{bmatrix} 3^5 & 0 \\ 0 & 2^5 \end{bmatrix}$.
(d) $\begin{bmatrix} 2^5 & 1 \\ 0 & 3^5 \end{bmatrix}$.
(e) $\begin{bmatrix} -2^5 & -3^5 + 2^5 \\ 0 & -3^5 \end{bmatrix}$.

**16.** Let $A$ and $B$ be $n \times n$ diagonalisable matrices. Which one of the following is always true?

(a) $A + B$ is diagonalisable.
(b) $AB$ is diagonalisable.
(c) $A^2$ and $B^2$ are diagonalisable.
(d) $AB = BA$.
(e) none of the above.

**17.** Let $B$ be a $4 \times 4$ matrix and suppose that $\det(B) = 3$. Then $\det\left(-\frac{1}{\sqrt{3}}B\right)$ is

(a) $1/3$ (b) $3$ (c) $-1/\sqrt{3}$ (d) $-\sqrt{3}$ (e) $-1$

**18.** Suppose a $3 \times 3$ matrix $A$ has 3 distinct eigenvalues $\lambda_1$, $\lambda_2$ and $\lambda_3$. Which one of the following is NOT necessarily true?

(a) $\det(A) = \lambda_1\lambda_2\lambda_3$.
(b) The characteristic polynomial of $A$ has 3 distinct roots.
(c) $A$ is invertible.
(d) If $B$ is any $3 \times 3$ invertible matrix then $BAB^{-1}$ has eigenvalues $\lambda_1$, $\lambda_2$ and $\lambda_3$.
(e) There is a $3 \times 3$ invertible matrix $P$ so that $PAP^{-1} = \begin{bmatrix} \lambda_2 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_1 \end{bmatrix}$.

**19.** Let $P$ be a transition matrix of a Markov chain on $n$ states. Which of the following is NOT necessarily true.

(a) $P^2$ is a transition matrix for a Markov chain.

(b) $P$ is an $n \times n$ matrix.

(c) If $P$ is invertible, then $P^{-1}$ is a transition matrix for a Markov chain

(d) If $Q$ is another transition matrix for a Markov chain on $n$ states, then $PQ$ is a transition matrix for a Markov chain.

(e) If $Q$ is another transition matrix for a Markov chain on $n$ states, then $\frac{1}{2}(P+Q)$ is a transition matrix for a Markov chain

**20.** Which one of the following is true for all $n \times n$ matrices $A$ and $B$?

(a) if $A^2 = B^2$ then $A = \pm B$

(b) if $AB = 0$ then $A = 0$ or $B = 0$

(c) $(A+B)^2 = A^2 + 2AB + B^2$

(d) if $A$ is invertible then $AB$ is invertible

(e) if $AB$ is invertible then $A$ and $B$ are invertible

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## Extended Answer Section

*There are* **three** *questions in this section, each with a number of parts. Write your answers in the space provided. If you need more space there are extra pages at the end of the examination paper.*

**1.** (a) Let $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$ be three planes in three dimensional space given by the equations

$$\mathcal{P}_1 \;:\; x + 3y + 4z = 5; \quad \mathcal{P}_2 \;:\; 2x + 5y + z = b; \quad \mathcal{P}_3 \;:\; -x + ay + 3z = 0,$$

where $a, b \in \mathbb{R}$ are parameters.

($i$) Find $\mathbf{n}_1 \times \mathbf{n}_2$, where $\mathbf{n}_1$ is a normal vector for $\mathcal{P}_1$, and $\mathbf{n}_2$ is a normal vector for $\mathcal{P}_2$.

**Solution**
We have $\mathbf{n}_1 = [1, 3, 4]$ and $\mathbf{n}_2 = [2, 5, 1]$. Then $\mathbf{n}_1 \times \mathbf{n}_2 = [3-20, 8-1, 5-6] = [-17, 7, -1]$.

($ii$) Find the direction vector of the line of intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$.

**Solution**
The direction vector of the line of intersection is parallel to both planes $\mathcal{P}_1$ and $\mathcal{P}_2$. Hence it is orthogonal to both normal vectors $\mathbf{n}_1, \mathbf{n}_2$. Therefore the direction vector $\mathbf{d}$ is $\mathbf{n}_1 \times \mathbf{n}_2 = [-17, 7, -1]$.

$(iii)$ Find the parameters $a$ and $b$ such that three planes $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$ have empty intersection.

**Solution**
In other words we need to find the parameters $a$ and $b$ such that the system of linear equationsfor $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$ is inconsistent. We reduce the corresponding augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 2 & 5 & 1 & b \\ -1 & a & 3 & 0 \end{array}\right] \longrightarrow \begin{bmatrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{bmatrix} \longrightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 0 & -1 & -7 & b-10 \\ 0 & a+3 & 7 & 5 \end{array}\right]$$

$$\longrightarrow [R_3 \to R_3 + (a+3)R_2] \longrightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 0 & -1 & -7 & b-10 \\ 0 & 0 & -7(a+2) & 5+(b-10)(a+3) \end{array}\right]$$

From here we see that for $a \neq -2$ the system is consistent. For $a = -2$ the last inequality becomes $0 = b - 5$, thus the system is unconsistent for $b \neq 5$.

The final answer is $a = -2, b \neq 5$.

(b) $(i)$ Find the projection of the vector $[3,2,1]$ onto the line spanned by the vector $[1,2,3]$.

**Solution**
We have $\text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \dfrac{3+4+3}{1+4+9}[1,2,3] = \dfrac{10}{13}[1,2,3]$.

It can also be written as

$$\left[\frac{10}{13}, \frac{20}{13}, \frac{30}{13}\right].$$

Both answers are correct.

$(ii)$ By use of part (i) or otherwise, find the distance between the point $P(3,2,1)$ and the line with the parametric equations

$$\left.\begin{array}{l} x = t \\ y = 2t \\ z = 3t \end{array}\right\} \quad t \in \mathbb{R}.$$

**Solution**
Let $\mathbf{u} = \overrightarrow{OP}$ and $\mathbf{v} = [1,2,3]$ be the direction vector of the line. Then the distance from $P$ to the line will be the length of $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$

$$= \left[\frac{29}{13}, \frac{6}{13}, \frac{-17}{13}\right].$$

Its length equals to

$$\sqrt{\frac{29^2+6^2+17^2}{13^2}} = \frac{\sqrt{1166}}{13}.$$

The students do not need to simplify this answer any further.

(c) Show that any three non-zero vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^3$ which are orthogonal one to each other are linearly independent.

**Solution**

Assume that $a\mathbf{u}+b\mathbf{v}+c\mathbf{w} = 0$ for some scalars $a, b$ and $c$. Compute $(a\mathbf{u}+b\mathbf{v}+c\mathbf{w})\cdot\mathbf{u}$. On the one hand it is the zero vector because one of the multipliers is the zero vector. On the other hand we have

$$(a\mathbf{u} + b\mathbf{v} + c\mathbf{w}) \cdot \mathbf{u} = a\mathbf{u} \cdot \mathbf{u} + b\mathbf{v} \cdot \mathbf{u} + c\mathbf{w} \cdot \mathbf{u} = a\mathbf{u} \cdot \mathbf{u}.$$

Therefore $a = 0$. By analogy, we take the dot product of $a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$ with $\mathbf{v}$ and $\mathbf{w}$ to get that $b = c = 0$. Hence $\mathbf{u}\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

**2.** (a) Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

**Solution**

(b) Use your answer to (a), or another valid method, to write vectors $\mathbf{e}_2 = [0, 1, 0]$ and $\mathbf{e}_3 = [0, 0, 1]$ as linear combinations of vectors $\mathbf{v}_1 = [0, 1, 1]$, $\mathbf{v}_2 = [1, 0, 1]$, $\mathbf{v}_3 = [2, 0, 1]$.

**Solution**

(c) For which value of the parameter $a$, the set $S_a = \{[x, y] \in \mathbb{R}^2 \mid x + axy = 0\}$ is a subspace of $\mathbb{R}^2$? Provide a proof of your answer.

**Solution**

**3.** (a) Let

$$P = \begin{bmatrix} 0 & 0 & 1/2 \\ 1/2 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_0 = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix},$$

where $P$ is the transition matrix for a Markov chain with three states, and $\mathbf{x}_0$ is the initial state vector for the population.

(*i*) Compute $\mathbf{x_1}$ and $\mathbf{x_2}$.

**Solution**

(*ii*) What is the probability of moving from state 3 to state 1 in 2 transitions?

**Solution**

$(iii)$ Find $\lim_{n \to \infty} P^n \mathbf{x_0}$. (You can assume that the limit exists)

**Solution**

(b) Let $A$ and $B$ be $n \times n$ matrices.

$(i)$ Let $\lambda \neq 0$. Show that $\lambda$ is an eigenvalue of $AB$ if and only if it is also an eigenvalue of $BA$.

**Solution**

$(ii)$ Show that $I_n + AB$ is invertible if and only if $I_n + BA$ is invertible, where $I_n$ is the identity $n \times n$ matrix.

**Solution**

THERE ARE NO MORE QUESTIONS.

**More space is available on the next page.**

**This blank page may be used if you need more space for your answers.**

**End of Extended Answer Section.**

**End of Examination**

Write your SID here ⟶

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The University of Sydney
School of Mathematics and Statistics

**MATH1002 Linear Algebra**

**Code your SID into the columns below each digit, by filling in the appropriate oval.**

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## Answers ⟶

Attempt every question. You will **not** be awarded negative marks for incorrect answers.

Fill in exactly one oval per question.

If you make a mistake, draw a cross (X) through any mistakenly filled in oval(s) and then fill in your intended oval.

An answer which contains two or more filled in (and uncrossed) ovals will be awarded no marks.

| | **a** | **b** | **c** | **d** | **e** | | **a** | **b** | **c** | **d** | **e** |
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| **Q1** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q11** | ▭ | ▭ | ▭ | ▭ | ▭ |
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| **Q3** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q13** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q4** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q14** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q5** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q15** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q6** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q16** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q7** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q17** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q8** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q18** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q9** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q19** | ▭ | ▭ | ▭ | ▭ | ▭ |
| **Q10** | ▭ | ▭ | ▭ | ▭ | ▭ | **Q20** | ▭ | ▭ | ▭ | ▭ | ▭ |

## Correct Responses to the Multiple Choice Section of

**MATH1002 Linear Algebra**

*Semester 1 Main, 2019*

Q1 $\longrightarrow$ a
Q2 $\longrightarrow$ d
Q3 $\longrightarrow$ b
Q4 $\longrightarrow$ d
Q5 $\longrightarrow$ b
Q6 $\longrightarrow$ d
Q7 $\longrightarrow$ a
Q8 $\longrightarrow$ d
Q9 $\longrightarrow$ d
Q10 $\longrightarrow$ e
Q11 $\longrightarrow$ b
Q12 $\longrightarrow$ c
Q13 $\longrightarrow$ e
Q14 $\longrightarrow$ c
Q15 $\longrightarrow$ a
Q16 $\longrightarrow$ c
Q17 $\longrightarrow$ a
Q18 $\longrightarrow$ c
Q19 $\longrightarrow$ c
Q20 $\longrightarrow$ e