Lab 2 Report by Benjamin Ackerman

I. Introduction

As you all know, our company stands at the forefront of the world's mining industry with one of the deepest mines in the world, roughly 4 km deep. As a company we wanted to test to see the exact vertical depth of the shaft by dropping a 1kg test mass down the shaft and timing when it hits the bottom. In order to do this however we needed a model of how the test mass would react in a shaft that was exactly 4 km.

When making this model there were multiple physics concepts taken into account that require a brief introduction. The concepts include the force of drag on the object, the Coriolis force, and variable gravity. The force of drag (air resistance) is a force caused by the earth's atmosphere that directly opposes the motion of the object. This is to say that it causes the object to stop accelerating at a speed known as terminal velocity. The Coriolus force causes the object to appear to move horizontally in the shaft. It is important to note that the object does not actually move horizontally, instead the earth's rotation catches up to the falling object. Finally we must take into account a variable force of gravity. As you go further into the earth the force of gravity on the object changes which can affect the motion of the object.

Python was used heavily throughout the bulk of this project as a way to simulate the effects listed above.

II. Fall Time

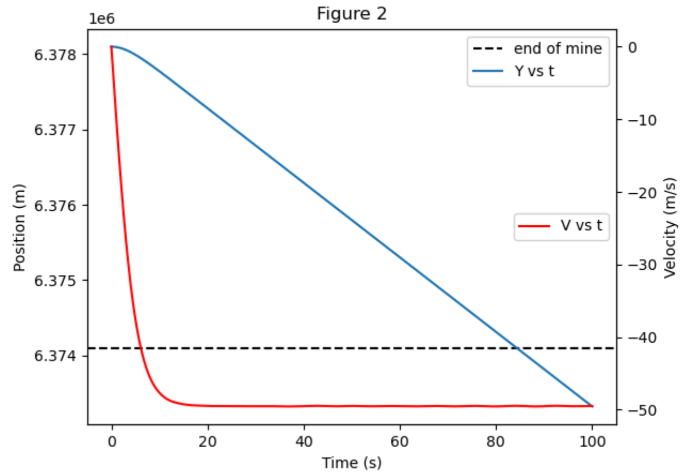
We first simulated how long it would take for the test mass to fall down the mine shaft if it was in perfect free fall, nothing but the force of gravity affects it, to get a basic understanding of the simplest case. Using simple physics equations derived by Newton's laws we found that the mass will reach the bottom of the mine in 28.6s.We later confirmed this using a function in python known as solve_Ivp that was programmed with a broader differential equation. The difference between the two values were negligible so we greenlighted the use of the solve_ivp function in further calculations. The broad differential equation used in the solve_ivp function is:

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$$

Where $\frac{d^2y}{dt^2}$ is the acceleration of the object, g is consonant gravitational acceleration, α is a drag coefficient $\frac{dy}{dt}$ is velocity and γ which is the speed dependence of drag. For the purposes of this calculation α was 0. Using this equation we then checked to see the time it would take the mass to reach the bottom if we replaced constant g with a g that changes with radius denoted by:

$$g(r) = g_o(1 - (\frac{r}{R_E})^2)$$

Where g_o is constant gravitational acceleration r is the radius mass is currently at and Re is the radius of the earth. Using a varying g the mass reaches the bottom of the mine in 28.6s (0.009s slower). While varying gravity did not have a major effect, drag force did as shown by Figure 2 on the right. Unlike the other two cases, when drag is taken into account the mass does not accelerate indefinitely. When using a drag coefficient (***α***) of 0.004 it took the mass 84.3s to reach the bottom of the mine. In the graph that is shown by the blue line crossing the dashed black line.



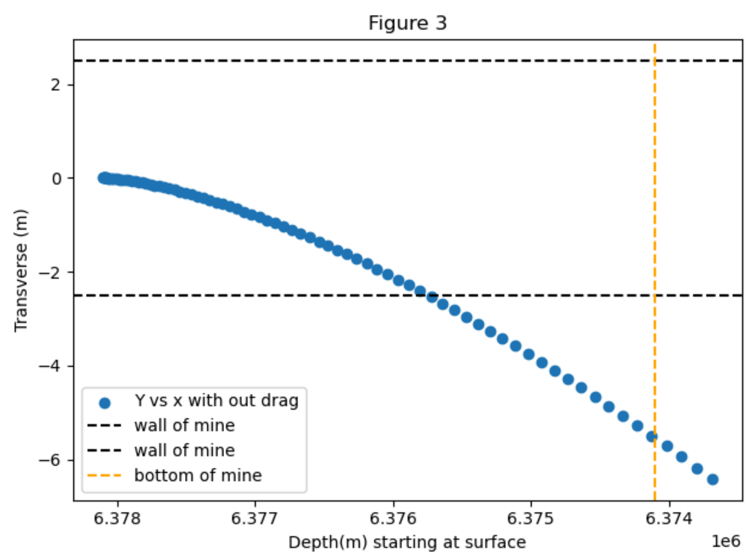
III. Depth Measurement

As we further the realism of the simulation we must also take into account the coriolis force. As mentioned above the coriolis force is not an actual force, instead it is the Earth's rotation catching up with the falling object's motion. We simulated this by adding the equations:

$$F_{cx} = +2m\Omega v_y$$

$$F_{cx} = -2m\Omega v_x$$

To the differential equation above where Ω is Earth's rotational frequency m is the mass of the object and v is velocity in the noted direction. The addition of the Coriolis force added some unnecessary issues with our plan of measuring the mine shaft. As shown in Figure 3 on the right, the mass will hit the wall of our 5m diameter mine shaft after 21.9s without drag force. With the addition of drag force, the mass hits the wall around 8 seconds later. This means that our experiment will not work as expected as the mass will never reach the bottom. We will not be able to continue the experiment.



IV. Crossing Times

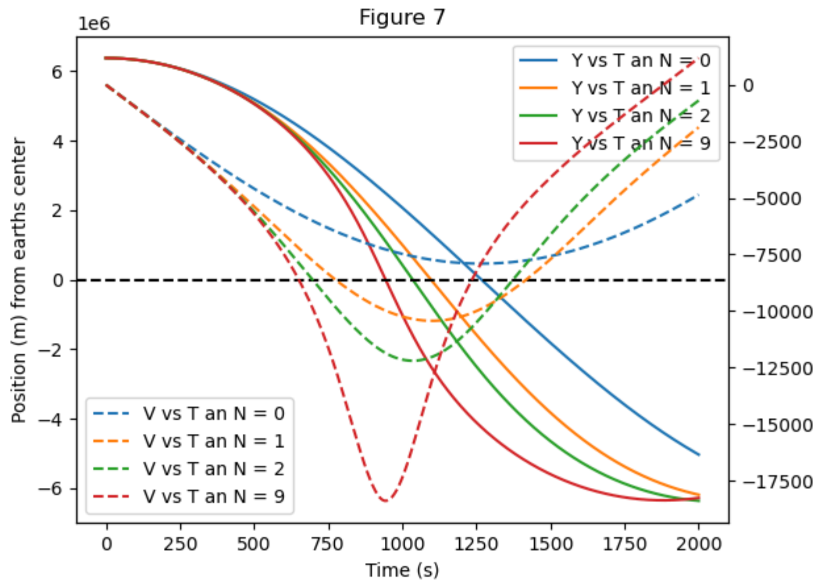
In light of the failure of the initial experiment, we decided it would be beneficial for the company to better understand the mechanics of inner earth travel at long distances. When doing this type of calculation/experiment it is common for people to simplify the situation by saying that the earth has a uniform density/mass as you get closer to the center. When doing this, if we were to drop the test mass down the shaft with no drag or Coriolis force it would reach the center of the earth in 1266.2s and all the way through in 2533.15s. We found by studying both the Earth and the Moon that in terms of density the time it takes for an object to fall all the way through a planet can be shown as:

$$T_{fall} = \frac{T_{orb}}{2} \propto \sqrt{\frac{1}{\rho}}$$

Where ρ is the uniform density of the object. In practice however this is not the case, we need to vary the density as we go down. The equation used to model the actual density distribution of the earth is:

$$\rho(r) \;=\; \rho_n(1 - \frac{r^2}{R_e^{\,2}})^n$$

Where ρ_n is the density normalization factor that varies based on the value of n to make sure the mass of the Earth stays constant in calculations. When n = 0 it is as if the earth has uniform density, and when n is at an extreme of n = 9 it means that the density is incredibly non-uniform.



Since the density distribution affects the distribution of the mass, the earth must remain the same mass and volume, a non-uniform density causes the motion of our test mass to change greatly. As shown in Figure 7 on the right.

We used four test cases of n and their respective ρ_n to show how varying density can in turn change the way an object will travel through the Earth. Let's take a look at the two extreme values of n = 0 and n = 9, for reference Earth is actually at around n = 2. N = 0 was mentioned above as being uniform density and is shown by the blue lines in the graph. N = 9 is an extremely non-uniform density and is shown as the lines in red on the graph. To display exactly how the density can affect the motion of the mass we compared the time it took for the test mass to reach the center of the Earth in each case. When n = 0 it took the mass 1267.2s to reach the center of the Earth, around the same as above. When n = 9 however, it took the test mass 943.9s to reach the center of the earth.

V. Discussion of Future Work

Over the course of this experiment we made key assumptions which allowed us to make the math and models much simpler for our purposes. Two of the largest assumptions are that the earth is a sphere and that density was constant (during the section on our mine). For future work we should recalculate our work with the maximum radius of the Earth changing as it does in real life. Also We would like to look into different ways to measure the depth of our mines.