Report

I. Introduction

The Apollo program aims to land humans on the Moon and return them safely to Earth. This requires a thorough understanding of gravitational forces and rocket performance. This report provides an overview of the gravitational environment between Earth and the Moon and an analysis of the Saturn V rocket's first stage performance to refine mission planning.

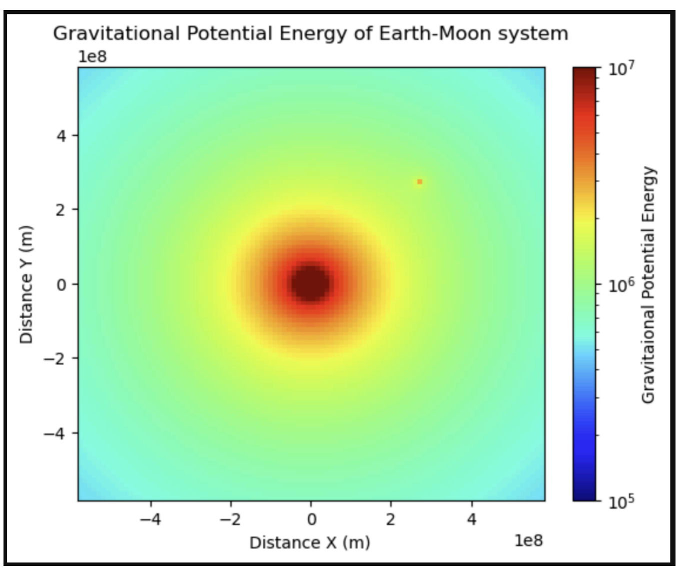
Because this report is intended for policymakers, the discussion prioritizes practical implications rather than complex mathematical derivations. While NASA engineers use precise equations to predict spacecraft behavior, this report conveys key concepts in an accessible manner.

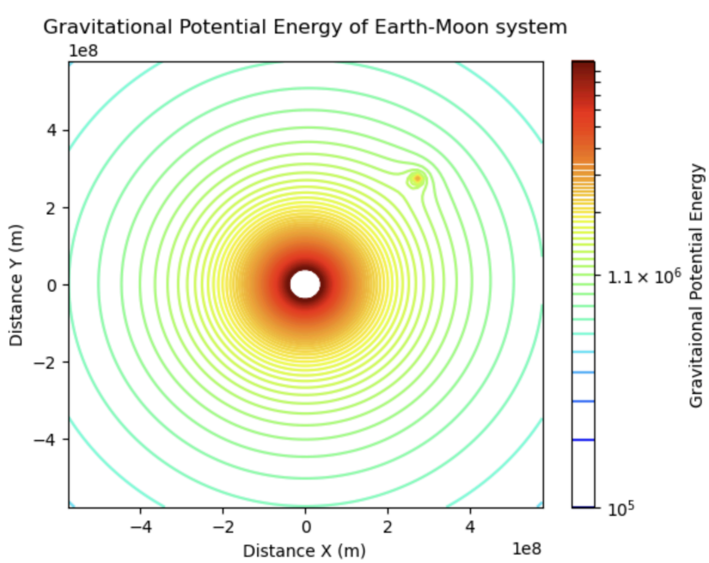
II. The Gravitational Potential of the Earth-Moon System

Gravity governs the motion of celestial bodies and spacecraft. The gravitational potential describes how much energy is needed to move between points in a gravitational field. Mathematically, it is given by:

$$\Phi(r) = \frac{-GM}{r}$$

where G is the gravitational constant, M is the mass of the body, and R is the distance. As illustrated in the graph below, when distance increases, the gravitational pull weakens.



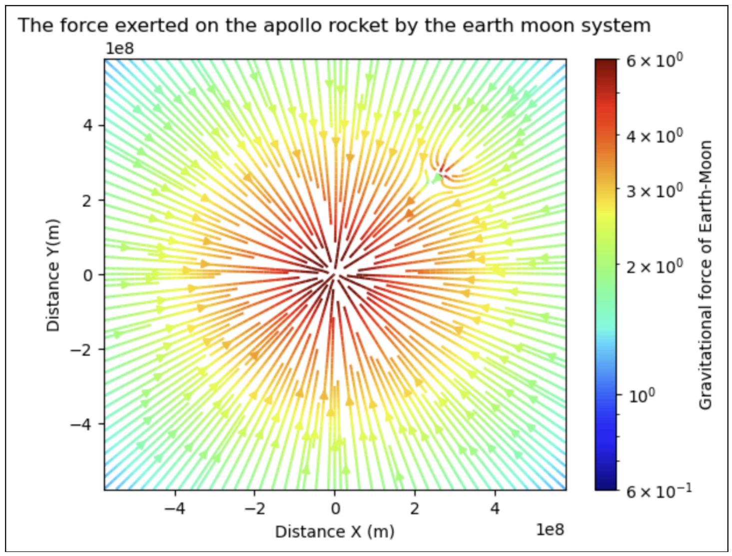


III. The Gravitational Force of the Earth-Moon System

Gravitational force is the direct attraction between two masses. It determines the strength and direction of the pull exerted by Earth and the Moon on the spacecraft. The force experienced by the spacecraft due to gravity follows:

$$F = \frac{GM_1 m_2}{R^2}$$

where R is the distance between two masses. M1 is the mass of the body and m2 is the mass of rocket. This relationship allows us to model how the spacecraft transitions from Earth's dominant gravitational influence to the Moon's as it travels. Understanding this force is essential for designing efficient trajectories.



We generated visualizations of gravitational potential and force, identifying key points such as Lagrange points, where the gravitational forces of Earth and the Moon balance. These locations could be useful for future space missions requiring stable orbital positions. We can also see the direction of the force with the arrow vectors on the plot on the right.

IV. Projected Performance of the Saturn V Stage 1

The Saturn V rocket propels the Apollo spacecraft by expelling exhaust gases at high speed. The change in velocity follows the Tsiolkovsky rocket equation:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

where v_e is the exhaust velocity, m_o is the initial wet mass, m(t) accounts for the burning on the fuel, and g accounts for gravity.

Using this equation and the equation for burn time of $T = \frac{m_o - m_f}{m}$ where m_f is the dry mass and m is the turn rate of the fuel, we estimated that the first stage of Saturn V burns for approximately 157 seconds and reaches an altitude of about 74 km before stage separation. However, NASA's test data reports a burn time of 160 seconds and an altitude of 70 km. The small discrepancy likely arises from our simplified assumptions.

V. Discussion of Approximations and Future Work
Several key approximations affect the accuracy of our calculations:

- **Neglecting Atmospheric Drag:** In reality, air resistance slows the rocket, requiring additional thrust. Our model assumes vacuum conditions, leading to overestimations of altitude.
- **Constant Gravity Approximation:** We treated Earth's gravity as uniform, but in reality, it decreases slightly with altitude. This simplification introduces minor errors in trajectory calculations.
- **Ignoring Multi-Stage Effects:** The first stage is only part of the full mission. The later stages contribute additional thrust and even a lower mass as parts of the rocket detach. This would require separate analysis.
- **No Consideration of Guidance Adjustments:** The Apollo rocket did not travel straight up,it followed a precise trajectory controlled by guidance systems. This was not accounted for.

Future refinements should integrate these factors to improve accuracy. Including aerodynamic drag, real-time gravity variations, and detailed stage-by-stage modeling will produce better predictions. Additionally, comparing with more test data will help refine our approach.

The Apollo program requires precise calculations to ensure mission success. Our analysis provides an initial estimate of the forces acting on the spacecraft and the performance of Saturn V’s first stage. While simplified, our results demonstrate the fundamental physics behind Apollo’s mission design. Further refinements will enhance accuracy, ensuring safe and efficient lunar missions. Continued investment in NASA’s research and development will enable future improvements and the success of upcoming space exploration initiatives.