PAPER TITLE: **SIMULATION BASED SOLAR WATER HEATING SYSTEM OPERATION IN RESIDENTIAL ENVIRONMENT**

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SUBJECT: **APPENDIX 1 -** *Calculation Method*

In this paper, the observed system architecture consists of a solar collector with a working fluid running through it, as given in Figure 1 (left). When the fluid has gone through the collector, it enters a heat exchanger, where it transfers heat to cooler water. The water heats up and is stored in a tank. Also, there are two pumps to force the circulation of the working fluid and water. The tank is considered mixed during hours with the solar heat collection.
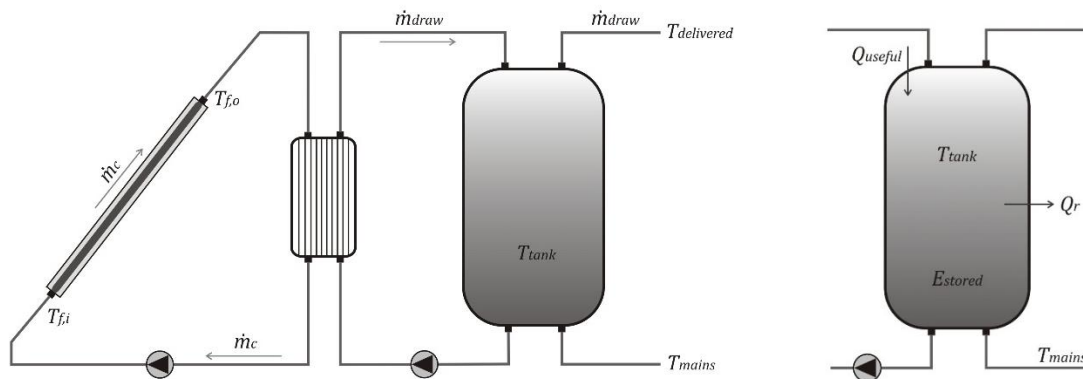

Figure 1. Solar Water Heating System (left) and Energy balance (right)

In order to define quantities of heat transfer in addition to energy storage, it is necessary to perform tank examination. Here, several ways of heat transfer occur. These are namely heat transfer to the room, from the tank to the cold incoming water, as well as energy received from the solar collector minus any losses from the collector to the outside, which when combined, represent $\dot{Q}_{useful}$. In this paper relations from [A1] are used to develop a simple model based on this system. Subsequently, the useful energy delivered to the water tank is the energy received by the collector minus losses from the collector to the environment. The energy gained across the collector could be defined as given in the equation (e.1):

$$\dot{Q}_{useful} = \dot{m}C_p[T_{f,o} - T_{f,i}] \tag{e.1}$$

Here *f,i* refers to "fluid inlet" and *f,o* to "fluid outlet", and the temperature rise across the collector $T_{f,o} - T_{f,i}$ depends on the incident radiation. Also, the minimal heat loss from the collector to the environment occurs when the overall collector is at the fluid inlet temperature since the fluid inlet is the coldest fluid temperature in the collector, and colder temperatures mean lower gradients between the collector and the environment. The minimal heat loss could be described as given in the equation (e.2):

$$\dot{Q}_{min,loss} = A_C U_L(T_{f,o} - T_{f,i}) \tag{e.2}$$

Here U_L is the overall collector heat loss coefficient and A_C is the collector area. The derivation of this coefficient is fairly involved in [A1] where representative values of U_L are tabulated for various collector configurations. Given the aforementioned, it will not be further discussed in this study. However, for a single-cover collector, U_L depends on the convection coefficient between the absorbing plate and glass cover, the radiation coefficient from the plate to the cover, and the radiation coefficient from the cover to the air. Using the minimal loss, the maximal gain across the solar collector could be defined as given in the equation (e.3):

$$\dot{Q}_{max,collected} = \dot{Q}_{solar} - \dot{Q}_{min,loss}$$
$$= A_C[I_{\tau\alpha} - U_L(\bar{T}_t - T_o)] \tag{e.3}$$

In equation (e.3), I represent the solar insolation, and $\tau\alpha$ is the transmittance-absorbance product which accounts for optical losses. Also, note that the collector inlet fluid temperature has been changed to the mean-tank temperature $\bar{T}_t$ to reflect that water entering the collector during the collection mode is coming from the tank [A2]. In the next step it is possible to define a modifier relating the useful energy gain to the maximal possible gain as given in the equation (e.4):

$$\dot{Q}_{useful} = F_R \dot{Q}_{max,collected}$$
$$= F_R A_C[I_{\tau\alpha} - U_L(T_{f,o} - T_{f,i})]$$
$$= F_R A_C[I_{\tau\alpha} - U_L(\bar{T}_t - T_o)] \tag{e.4}$$

The reason this is desirable is that the equation for useful energy now includes the temperature of the fluid entering the collector. It could be noted that the term F_R is introduced. F_R represents the collector heat removal factor and is derived by taking eq. (e.4) and substituting other expressions for the temperature terms with additional manipulating. The process is again involved, but a summary of the steps is to derive the useful heat flux per unit length between the tubes within the collector, derive the useful heat flux per unit length in the flow direction within the tubes, and apply these relations to eq. (e.4). This term is also updated to account for the heat exchanger, pipes, and flow rate [A1].

Referring to Figure 1 (right), it is assumed that the storage tank is located indoors, where the room is likely at a different temperature than the mean tank temperature, while the tank itself is not perfectly insulated, thus heat transfer occurs. The type of heat transfer which will occur in this case is assumed to be a combination of convection loss between moving tank water and the tank walls, conduction loss through the tank walls, radiation from the tank walls to the room, and convection between the room and tank. An overall tank heat loss coefficient U is used so that this complicated thermal circuit can be represented by the resistance between two nodes. Composite system heat transfer in this form can be represented by an expression analogous to Newton's Law of Cooling [A3] and given in the equation (e.5):

$$\dot{Q}_r = UA_t(\bar{T}_t - T_r) \tag{e.5}$$

Of note is that composite systems of this form use the end-points of the thermal circuit to compute the temperature difference, in this case, the mean tank temperature and the room temperature. The value of U either must be calculated based on physical properties and knowledge of temperatures or by choosing a representative value based on previous experience or technical specification.

Furthermore, as cold water enters the bottom of the tank, the enthalpy of the water in the tank decreases. Recall the conservation of energy for an open system with no kinetic or potential energy changes [A3] given in the equation (e.6):

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e \tag{e.6}$$

Having in mind that there is no source of work in the tank, this term equals zero and is no longer considered. Also, if assumed that the mass flow rate entering the tank is the same as leaving the tank, it

is possible to generate equation (e.7), upon which basic thermodynamic relations for liquids could be applied to further simplify the enthalpy expression as given in the equation (e.8).

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_{draw}[h_i - h_e] \tag{e.7}$$

$$h_i - h_e = C_p[T_i - T_e] \tag{e.8}$$

The eq. (e.8) could be slightly modified since the inlet temperature is determined from the cold water entering the tank, and the exit temperature is assumed to be the mean tank temperature such as given in the equation (e.9):

$$h_i - h_e = C_p[T_{mains} - \bar{T}_t] \tag{e.9}$$

Given the previously mentioned, it means that the eq. (e.6) could be written as (e.10):

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_{draw} C_p[\bar{T}_t - T_{mains}] \tag{e.10}$$

Assuming that the energy of the control volume is the internal energy of the water, enables generation of equation (e.11):

$$E_{cv} = \rho V_t C_p \bar{T}_t \tag{e.11}$$

Lastly, it is possible to form the equation (e.12) as given below:

$$\frac{d\bar{T}_t}{dt} = \frac{\dot{Q}_{cv} - \dot{m}_{draw} C_p[\bar{T}_t - T_{mains}]}{\rho V_t C_p} \tag{e.12}$$

Here it was assumed that the density, specific heat, and volume of the water in the tank are constant and clearly this is a linear, first-order differential equation of the form:

$$\frac{dT}{dt} = f(t, T) \tag{e.13}$$

The Time-Marching Method introduction

Although it is possible to choose from a vast selection of time-marching methods [A4], for simplicity and stability purposes approximation of all differential equations could be ensured by using the implicit-Euler method as given in the form of equation (e.1.1). However, this is only a first-order accuracy method, and due to many simplification steps previously taken, it doesn't make sense to try and achieve higher-order approximations.

$$T_{i+1} = T_i + \Delta t f(t_{i+1}, T_{i+1}) \tag{e.1.1}$$

The implicit-Euler method is the first-order accuracy in time, unconditionally stable, but as in the case of all implicit methods, it requires solving a linear system for each time step. Applying this numerical method to (e.12), it is possible to generate the following expression:

$$\bar{T}_t = T_{t,p} + \Delta t \left(\frac{\dot{Q}_{cv} - \dot{m}_{draw} C_p[\bar{T}_t - T_{mains}]}{\rho V_t C_p} \right) \tag{e.1.2}$$

Here, the heat transfer rate is derived from Fig. 1 (left) and summing each heat term by assuming heat coming in is positive and heat going out is negative as given in the equation (e.1.3):

$$\dot{Q}_{cv} = \dot{Q}_{useful} - \dot{Q}_{room} \tag{e.1.3}$$

By introducing the expression for the heat transfer one could get the following:

$$\left(\bar{T}_t - T_{t,p}\right) = \frac{\dot{Q}_{useful}\Delta t - \dot{Q}_{room}\Delta t - \dot{m}_{draw}C_p[\bar{T}_t - T_{mains}]\Delta t}{\rho V_t C_p} \tag{e.1.4}$$

Eq. (e.1.4) is the model equation for the implicit method, while if the expressions for each heat term are introduced, it is possible to get expression as given in the equation (e.1.5):

$$\begin{aligned} &\left(\bar{T}_t - T_{t,p}\right) \\ &= \frac{F_R A_C [I_{\tau\alpha} - U_L(\bar{T}_t - T_o)]\Delta t - UA_t(\bar{T}_t - T_r)\Delta t - \dot{m}_{draw}C_p[\bar{T}_t - T_{mains}]\Delta t}{\rho V_t C_p} \end{aligned} \tag{e.1.5}$$

The eq. (e.1.5) is the equation that must be solved at every time step to map the evolution of temperature. Since the derived model is linear, it is possible to generate a solution for a closed-form solution for $\bar{T}_t$, by expanding all the terms given in (e.1.4), bringing all $\bar{T}_t$ terms to one side, factoring, and dividing through to get expression as given in the equation (e.1.6):

$$\bar{T}_t = \frac{T_{t,p}\rho V_t C_p + \left(F_R A_C [I_{\tau\alpha} + U_L T_o] + UA_t T_r + \dot{m}_{draw}C_p T_{mains}\right)\Delta t}{\rho V_t C_p + \left(F_R A_C U_L + UA_t + \dot{m}_{draw}C_p\right)\Delta t} \tag{e.1.6}$$

A simple step towards validating this model is to check that the dimensions are consistent by proving that each side equates to a unit of temperature as given below:

$$[K] = \frac{\left[Kkg\frac{J}{kgK}\right] + \left[m^2\frac{Ws}{m^2}\right] + \left[m^2\frac{Ws}{m^2K}K\right] + \left[\frac{Ws}{m^2K}m^2K\right] + \frac{kg}{s}\frac{J}{kgK}Ks}{\left[kg\frac{J}{kgK}\right] + \left[m^2\frac{Ws}{m^2K}\right] + \left[\frac{Ws}{m^2K}m^2\right]} = \left[\frac{J}{\frac{J}{K}}\right] = [K] \tag{e.1.7}$$

Discharging mode (Introduction of Stratified-Tank Mode)

Opposite to charging mode, the second operational mode occurs during discharging, where the storage tank is assumed to consist of two variable volume nodes, one of hot water, and one of incoming cold water from the mains [A5]. This section aims to explain the derivation of the tank temperatures during the stratified mode, for the water tank illustrated in Figure 2.
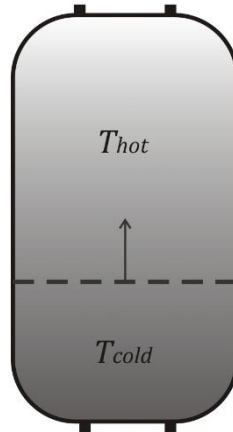

Figure 2. Stratified Tank

At the beginning two assumptions fundamental to this mode are necessary. The first one assumes that no conduction occurs between the hot and cold water in the tank, which is not physically correct. Nevertheless, a back-of-the-envelope calculation suggests that the conduction is only about five percent of the total energy transfer for a typical case and more importantly, it is not difficult to integrate conduction to the model in the future. In addition, the second assumption or better said view is that when the heat is exiting the system, it is assumed to be negative, whereas it is positive if entering the system. This is the opposite of the work convention, which assumes work done by the system as positive.

Having this in mind, during the tank examination only the heat transfer to the room from the hot water in the tank, and heat transfer to the cold water from the room could be spotted, due to the first assumption that no heat conduction from the hot to cold water is present in this simple model.

In this case, heat transfer occurs to the room in the same manner as in the mixed-tank mode, with the exception of two nodes necessary to consider as given in the equations (e.2.1) and (e.2.2):

$$\dot{Q}_{r,cold} = UA_{cold}(T_r - T_{cold}) \tag{e.2.1}$$
$$\dot{Q}_{r,hot} = UA_{hot}(T_r - T_{hot}) \tag{e.2.2}$$

Furthermore, during discharging, the volume of hot water decreases due to the user demand profile. Here, cold water enters the bottom of the tank from the mains to replenish the water. The model equations regarding the volume of the hot and cold water could be as given in the equations (e.2.3) and (e.2.4):

$$V_{hot} = V_{hot,p} - \frac{\dot{m}_{draw}\Delta t}{\rho} \tag{e.2.3}$$
$$V_{cold} = V_t - V_{hot} \tag{e.2.4}$$

Calculating these volumes is critical at every time step because they allow the calculation of mass in each node. Of note is that the initial condition when switching from collection mode is $V_{hot} = V_t$ and $V_{cold} = 0$, while mass is simply calculated as $m = \rho V$.

Subsequently, since the volume and hence mass of hot water vary dynamically, it is necessary to consider this when formulating the energy balance [A6]. The left-hand side of the energy balance takes the form as given in the equation (e.2.5):

$$\frac{d(m_{hot}C_pT_{hot})}{dt} = m_{hot}C_p\frac{dT_{hot}}{dt} + T_{hot}C_p\frac{dm_{hot}}{dt} \tag{e.2.5}$$

Here it is apparent that the product rule applies due to both the temperature and mass of the hot water varying in time. The mass flow rate of the hot water exiting the control volume is equal to the amount of water being drawn by the user indicating that the mass flow rate out of the hot node is negative.

$$\dot{m}_{hot} = -\dot{m}_{draw} \tag{e.2.6}$$

Assuming perfectly stratified behavior there will be no mass entering the hot control volume and the energy balance takes the form as given in the equations (e.2.7) - (e.2.9):

$$-\dot{m}_{draw}T_{hot}C_p + m_{hot}C_p\dot{T}_{hot} = \dot{Q}_{r,hot} + \dot{m}_{draw}[-h_e] \tag{e.2.7}$$
$$m_{hot}C_p\dot{T}_{hot} = \dot{Q}_{r,hot} - \dot{m}_{draw}T_{hot}C_p + \dot{m}_{draw}T_{hot}C_p \tag{e.2.8}$$
$$\dot{T}_{hot} = \frac{-UA_{hot}[T_{hot} - T_r]}{m_{hot}C_p} \tag{e.2.9}$$

Implicitly, by approximation of the differential equation results in:

$$T_{hot} = \frac{T_{hot,p}m_{hot}C_p + \Delta t U A_{hot} T_r}{m_{hot}C_p + \Delta t U A_{hot}} \tag{e.2.10}$$

Here, A_{hot} is the outer surface area of the portion of the tank that contains hot water, which corresponds to the top surface area.

$$H_{hot} = \frac{V_{hot,p}}{\pi r_t^2} \tag{e.2.11}$$

$$A_{hot} = \pi r_t^2 + 2\pi r_t H_{hot} \tag{e.2.12}$$

Where H_{hot} is the variable height of hot water.

As with the hot water, it is necessary to account for the variable volume of the cold node:

$$\frac{d(m_{cold}C_p T_{cold})}{dt} = m_{cold}C_p \frac{dT_{cold}}{dt} + T_{cold}C_p \frac{dm_{cold}}{dt} \tag{e.2.13}$$

The mass flow rate of the cold water should be roughly equal to the amount of water entering the system, which has been approximated as equal to the user draw:

$$\dot{m}_{hcoldot} = \dot{m}_{draw} \tag{e.2.14}$$

The energy balance takes the form:

$$\dot{m}_{draw}T_{cold}C_p + m_{cold}C_p \dot{T}_{cold} = UA_{cold}(T_r - T_{cold}) + \dot{m}_{draw}h_i \tag{e.2.15}$$

$$m_{cold}C_p \dot{T}_{cold} = UA_{cold}(T_r - T_{cold}) + \dot{m}C_p(T_{mains} - T_{cold}) \tag{e.2.16}$$

$$\dot{T}_{cold} = \frac{UA_{cold}(T_r - T_{cold}) + \dot{m}C_p(T_{mains} - T_{cold})}{m_{cold}C_p} \tag{e.2.17}$$

Where:

$$H_{cold} = \frac{V_{cold,p}}{\pi r_t^2} \tag{e.2.18}$$

$$A_{cold} = \pi r_t^2 + 2\pi r_t H_{cold} \tag{e.2.19}$$

Applying implicit Euler:

$$T_{cold} = \frac{T_{cold,p}m_{cold}C_p + \Delta t(UA_{cold}T_r + \dot{m}_{draw}C_p T_{mains})}{m_{cold}C_p + \Delta t(UA_{cold} + \dot{m}_{draw}C_p)} \tag{e.2.20}$$

Transmitted Irradiance and Energy Delivered

The incident irradiation is determined depending on the application of the preferable sky-model. The vastly known sky models are isotropic, HDKR, and Perez, which are summarized in [A1, A7-A8]. However, the calculation of incident irradiation starts by summarizing beam, diffuse, and ground reflected radiation:

$$I_i = I_b + I_d + I_g \tag{e.3.1}$$

Where i, b, d, g stand for "incident", "beam", "diffuse", and "ground" respectively, while the amount of irradiation transmitted is highly dependent on the angle of incidence indicating that each component of radiation must be modified as given in the equation (e.3.2):

$$I_i = K_{\tau\alpha,b}I_b + K_{\tau\alpha,d}I_d + K_{\tau\alpha,g}I_g \tag{e.3.2}$$

Here the $K_{\tau\alpha}$ terms are known as incidence angle modifiers which reflect how much radiation is transmitted at a certain angle relative to how much radiation would be transmitted when the radiation is directly normal to the collector [A1]. A general expression commonly used to estimate these modifiers at angles less than or equal to $60°$ is given below.

$$K_{\tau\alpha} = 1 - b_0\left(\frac{1}{cos(\theta) - 1}\right) \tag{e.3.3}$$

Here b_0 is called the incidence angle modifier coefficient. For the beam component, the angle in (e.3.3) is simply the angle of incidence while in the case of the diffuse and ground components, the angles used (in degrees) are:

$$\theta_d = 59.7 - 0.1388\beta + 0.001497\beta^2 \tag{e.3.4}$$
$$\theta_g = 90 - 0.5788\beta + 0.002693\beta^2 \tag{e.3.5}$$

Here β is the collector slope measured in radians. At angles larger than $60°$, the approximation of (e.2.3) breaks down as the relation between $K_{\tau\alpha}$ and the incidence angle becomes more non-linear [A1]. For angles between $60 - 90°$, the incidence angle modifiers could be calculated as follows:

$$K_{\tau\alpha} = (1 - b_0)(\theta - 90)\frac{\pi}{180} \tag{e.3.6}$$

This is a simple straight line which has the property of being equal to 0 when $\theta = 90°$ and equal to 1 $- b_0$ when $\theta = 60°$.

Lastly, the energy delivered in the solar collection process is calculated at the output of the solar storage tank as:

$$Q_{delivered} = \dot{m}_{draw}C_p(T_{delivered} - T_{mains}) \tag{e.3.7}$$

Besides, the auxiliary energy required to bring the water to the set temperature after solar collection could be calculated as follows:

$$Q_{aux} = \eta_{aux}\dot{m}_{draw}C_p(T_{set} - T_{delivered}) \tag{e.3.8}$$

Where η_{aux} is the efficiency of the auxiliary tank and is assumed to equal one for an electric source. The amount of auxiliary energy that would be required if solar water heating was not used would be:

$$Q_{aux,only} = \eta_{aux}\dot{m}_{draw}C_p(T_{set} - T_{mains}) \tag{e.3.9}$$

The amount of energy saved is how much less auxiliary was required due to solar water heating minus the amount of pump power required to drive the collector and tank flow:

$$Q_{saved} = Q_{aux,only} - Q_{aux} - P_{pump} \tag{e.3.10}$$

Finally, the solar fraction, a ratio of how much energy is saved by solar heating to how much energy is required in total could be approximated as:

$$F_{net} = \frac{Q_{saved}}{Q_{aux,only}} \tag{e.3.11}$$

This representation also includes the effect of parasitic power to the pump.

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