Curling scientific

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Chapter 1

Rock trajectory

1.1 Applying the 'Denny' model

Mark Denny published a model in [Den98], see A.1. To apply it for simulation we need to

- compute the ice properties from the draw-to-tee time and curl.
- transform from $t_0 = 0$ to $t_0 > 0$
- transform the equations from rock-coordinates to world-coordinates
- compute the initial speed of a rock from the hog-to-hog time and y_0

1.1.1 Basic equations

After substituting eq. (A.5) the basic equations are:

$$x(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\,\mu\,t^3\,(3\,g\,\mu\,t - 4\,v_0)}{48\,\varepsilon\,R} \tag{1.1}$$

$$x'(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\,\mu\,t^2\,(g\,\mu\,t - v_0)}{4\,\varepsilon\,R} \tag{1.2}$$

$$x''(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\,\mu\,t\,(3\,g\,\mu\,t - 2\,v_0)}{4\,\varepsilon\,R} \tag{1.3}$$

$$y(t) = -\frac{t\,(g\,\mu\,t - 2\,v_0)}{2} \tag{1.4}$$

$$y'(t) = -\,(g\,\mu\,t - v_0) \tag{1.5}$$

$$y''(t) = -g\,\mu \tag{1.6}$$

$$\alpha(t) = \frac{\omega_0\,\varepsilon\left(g\,\mu\,t\left(-\frac{g\,\mu\,t-v_0}{v_0}\right)^{\frac{1}{\varepsilon}} - v_0\left(-\frac{g\,\mu\,t-v_0}{v_0}\right)^{\frac{1}{\varepsilon}} + v_0\right)}{(\varepsilon+1)\,g\,\mu} \tag{1.7}$$

$$\alpha'(t) = \omega_0\left(-\frac{g\,\mu\,t-v_0}{v_0}\right)^{\frac{1}{\varepsilon}} \tag{1.8}$$

$$\alpha''(t) = \frac{\omega_0\,g\,\mu\left(-\frac{g\,\mu\,t-v_0}{v_0}\right)^{\frac{1}{\varepsilon}}}{\varepsilon\,(g\,\mu\,t-v_0)} \tag{1.9}$$

1.1.2 The ice-properties μ and b

To calculate μ and b from the time T and curl B of a draw-to-tee we set up the equations

$$x(T) = B \quad \text{(curl)} \tag{1.10}$$

$$x'(T) = 0 \tag{1.11}$$

$$y(T) = D \quad \text{(distance hog to tee)} \tag{1.12}$$

$$y'(T) = 0 \tag{1.13}$$

Solving the set of this 4 equations leads to

$$b = -\frac{12\,B\,\varepsilon\,R}{D^2} \tag{1.14}$$

$$\mu = \frac{2\,D}{g\,T^2} \tag{1.15}$$

$$v_0 = \frac{2\,D}{T} \tag{1.16}$$

1.1.3 Some initial speeds

How hard do we have to throw a rock, that will take 12 seconds hog to hog?

To calculate v_0 at the far hog we don't use the time hog-to-tee for not every rock reaches the tee-line. Here the time hog-to-hog (T_H) is better.

$$y(T_H) = H \quad \text{(dist. hog-to-hog)} \tag{1.17}$$

$$\Rightarrow \quad v_0 = \frac{g\,\mu\,T_H^2 + 2\,H}{2\,T_H} \tag{1.18}$$

If we don't want v_1 at the far hog but at any given distance y_1, the following equations apply:

$$y(t) = y_1 \tag{1.19}$$

$$y'(t) = v_1 \tag{1.20}$$

$$v_1 = -\frac{g\,\mu\,T^2 - 2\,y_1}{2\,T} \tag{1.21}$$

$$v_0 = \frac{g\,\mu\,T^2 + 2\,y_1}{2\,T} \tag{1.22}$$

eq. (**??**) + eq. (**??**):

$$H = v_0 T_H - T_H \frac{\mu g}{2} + T_H t_0 \mu g \tag{1.23}$$

$$\Longrightarrow \quad t_0 = \frac{H - v_0 T_H + T_H^2 \frac{\mu g}{2}}{T_H \mu g} \tag{1.24}$$

Substituting this into eq. (**??**) and solving for v_0 gives

$$\Longrightarrow v_0 = \frac{1}{2} \cdot \sqrt{\left(\mu g T_H + 2\frac{H}{T_H}\right)^2 + 4\mu g H - 8\mu g y_0} \tag{1.25}$$

If you prefer using Y_0 measured from the tee, use

$$\Longrightarrow v_0 = \frac{1}{2} \cdot \sqrt{\left(\mu g T_H + 2\frac{H}{T_H}\right)^2 + 4\mu g H - 8\mu g(\text{far-hog-to-tee} - Y_0)} \tag{1.26}$$

1.1.4 Coordinate transformation

Because of Denny assumes the rock to start at $(0,0)^T$ with v_0 pointing along the $\hat{y}$-axis, we need a rotation and shift to get the general equations.

$$\begin{pmatrix} x \\ y \end{pmatrix} := \frac{\vec{v}_{\text{real}}}{|\vec{v}_{\text{real}}|} \tag{1.27}$$

$$\text{The required transformation is:} \quad \begin{pmatrix} y & x \\ x & -y \end{pmatrix} \tag{1.28}$$

Applying this trafo to e.g. $\binom{a}{b}$ results in

$$\begin{pmatrix} y & x \\ x & -y \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ay + bx \\ ax - by \end{pmatrix} \tag{1.29}$$

where a and b are polynomes of max. fourth degree:

$$a =: At^4 + Bt^3 + Ct^2 + Dt + E \tag{1.30}$$

$$b =: \alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi \tag{1.31}$$

This leads to a x-component of

$$\begin{aligned} & y(At^4 + Bt^3 + Ct^2 + Dt + E) + \\ & x(\alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi) \end{aligned} \tag{1.32}$$

$$= (Ay + \alpha x)t^4 + \ldots + (Ey + \phi x) \tag{1.33}$$

and a y-component of

$$\begin{aligned} & x(At^4 + Bt^3 + Ct^2 + Dt + E) + \\ & -y(\alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi) \end{aligned} \tag{1.34}$$

$$= (Ax - \alpha y)t^4 + \ldots + (Ex - \phi y) \tag{1.35}$$

Now just the shift is missing.

Appendix A

Ice models

A.1 The 'Denny' model

Mark Denny published a model in [Den98]. It's quite simple and provides polynomes of fourth degree to describe the rock's motion along the sheet.

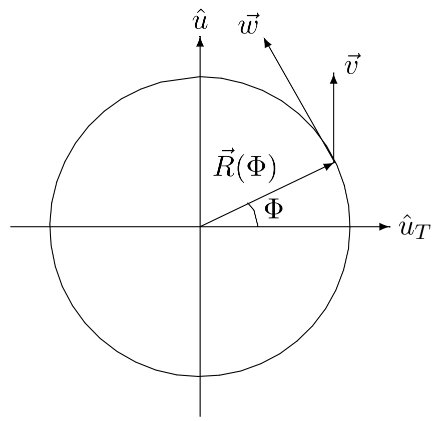

Figure A.1: The *running band* of radius R is the surface of contact between rock and ice. Here the velocity components, relative to the ice, of a point on the running band are shown. v is the CM velocity, and w is the angular component at radius R. The unit vectors (u_T, u) coincide with (u_x, u_y) at time $t = 0$. In this case the rock is curling in a counterclockwise sense (so angular velocity unit vector $= u_z$).

$$x(t) \approx -\frac{bv_0^2}{4\epsilon R\tau}\left(\frac{t^3}{3} - \frac{t^4}{4\tau}\right) \tag{A.1}$$

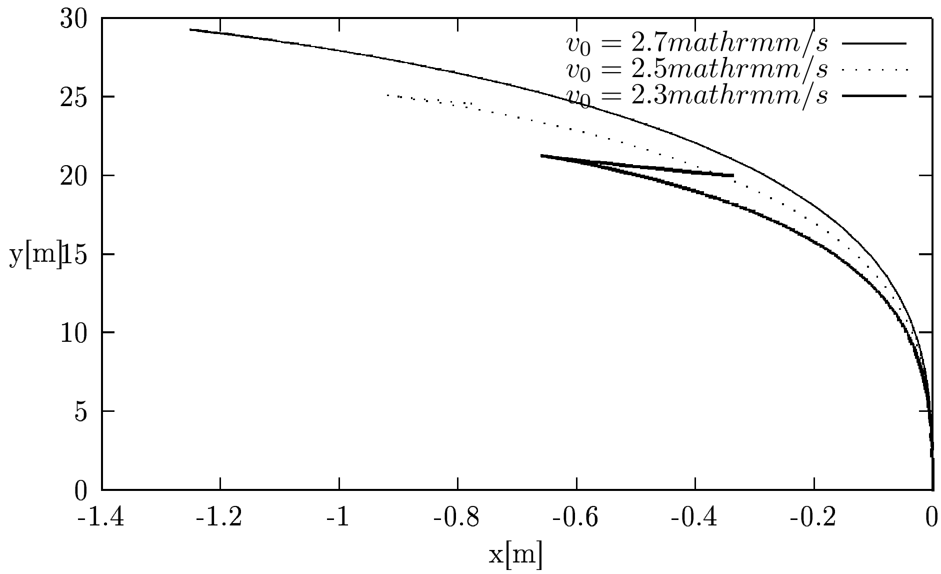

Figure A.2: Approximate curling rock trajectories, calculated from eq. (A.1), eq. (A.2) and eq. (A.4) for three values of initial velocity v_0. The other parameters are: $\mu = 0.0127$, $b_{\mathrm{LR}} = 0.003$, $\epsilon = 2.63$, $R = 0.065$.

$$y(t) \approx v_0 \left(t - \frac{t^2}{2\tau} \right) \tag{A.2}$$

$$\omega(t) \approx \omega_0 \left(1 - \frac{t}{\tau} \right)^{1/\epsilon} \tag{A.3}$$

$$\alpha(t) \approx \int_0^t \omega(t)\,\mathrm{d}t = -\frac{\omega_0\,\varepsilon \left(\tau\ \left(\frac{\tau - t}{\tau}\right)^{\frac{1}{\varepsilon}} - t\ \left(\frac{\tau - t}{\tau}\right)^{\frac{1}{\varepsilon}} - \tau \right)}{\varepsilon + 1} \tag{A.4}$$

t	[s]	time
x	[m]	curl
y	[m]	distance along the track
b	[1]	parameter for the curl's magnitude
ϵ	[1]	parameter $\frac{I}{mR^2}$
τ	[s]	total time until $v = 0$
R	[m]	radius of the touching area rock/ice $\approx$ 6.3e-2
μ	[1]	friction coefficient rock/ice
I		moment of inertia (z-direction)
g	[N/kg]	9.81 gravitation

The final properties are:

$$\tau = \frac{v_0}{\mu g} \tag{A.5}$$

$$x(\tau) \approx -\frac{b_{\mathrm{LR}}}{12\epsilon}\frac{y^2(\tau)}{R} \tag{A.6}$$

$$y(\tau) \approx \frac{v_0^2}{2\mu g} \tag{A.7}$$

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