Curling scientific

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Preface

During my days as an active Curler I spent quite a time thinking about what's happening when a rock slides over the ice or hits another one. And the longer I thought and the more people I asked the mystery did nothing but grow.

Some situations I could predict by experience rather fair. Some more skilled fellows could others. But some outcomes were completely miraculous.

So I started to gather scientific material to cover the topic from a theoretical base. Because once you got a model working properly for known situations, you can start to examine unknown ones and compare the model's predictions with reality. This way you get a fundamental understanding of what's going on.

At the beginning I considered this to be a kind of brain-jogging, but now I think the sport has developed so far yet that a team can't seriously practice competitive curling without at least noticing some theoretical knowledge e.g. about psychology but also physics.

Because to throw an *impossible* matchwinner you first have to recognise the option, then you need to dare it and surely, you need skill & luck to succeed.

Introduction

This paper wants to examine the theoretical basics of running rocks and hits at first and later apply these laws to real curling situations and get some useful hints & clues.

Chapter 1

Rock collission

Yet many brains boiled, there's poor secure knowledge still. Which directions take the involved rocks? How much momentum (or energy) is lost? How is the spin transferred?

Maybe this paper can show some ideas and suggest solutions.

Some things are going to be pretty much same with most of the following models. They are defined here and will be just referred in the latter. The variables' and constants' naming also is defined here.

The setup

Rock A hits rock B (fig. (1.1)), which equals A in all phys. properties. Both can be in motion, but usually only A is.

According to the geometric setup we change to a local coordinate-system with the y-axis pointing from A's center to B's. The x-axis is set perpendicular to get a right-handed system. See fig. (1.2).

1.1 Without spin, without loss of energy

This first model is the most primitive and simple one. But it's a proper approximation in most cases. Also it's a good basis for later refinement.

T	Time of contact
m	Mass
$\vec{v}$	Speed
$\vec{p}$	Momentum $\vec{p} = \mathrm{m} \cdot \vec{v}$
$\vec{F}$	Force $\vec{F} = \mathrm{m} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{p}$
J	Moment of inertia
ω	Angular speed, spin
$\vec{L}$	Angular momentum $\vec{L} = \mathrm{J} \cdot \vec{\omega}$
$\vec{M}$	Torque $\vec{M} = \mathrm{J} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\vec{\omega} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L} = \vec{R} \times \vec{F}$
$\vec{e}_{\parallel}$	unit vector in (parallel) direction of hit (see fig. (1.2))
$\vec{e}_{\perp}$	unit vector perpendicular to direction of hit (see fig. (1.2))
$F_{\parallel}$	$F_{\parallel} = \vec{F} \cdot \vec{e}_{\parallel}$ and $\vec{F} = F_{\perp} + F_{\parallel}$
$F_{\perp}$	$F_{\perp} = \vec{F} \cdot \vec{e}_{\perp}$ and $\vec{F} = F_{\perp} + F_{\parallel}$
$\vec{v}^{\mathrm{a}}$	$\vec{v}$ of rock A
$\vec{v}^{\mathrm{b}}$	$\vec{v}$ of rock B
$\vec{v}_1$	$\vec{v}$ before hit
$\vec{v}_2$	$\vec{v}$ after hit
Δx	$x_2 - x_1$ (difference in time)
${}^{a,b}\delta x$	$x^{\mathrm{b}} - x^{\mathrm{a}}$ (difference in space)

Table 1.1: Naming of variables (italic) and constants (roman)

1.1.1 General thoughts

Energy must be conserved:

$$E_1 = E_2 \tag{1.1}$$

$$E := \sum_{\text{all rocks}} E^i_{\mathrm{Kin}} + E^i_{\mathrm{Rot}} + E^i_{\mathrm{Fric}} \tag{1.2}$$

$$\text{here:} \quad 0 = E_{\mathrm{Rot}} = E_{\mathrm{Fric}} \tag{1.3}$$

1.1.2 Parallel component $p_{\parallel}$

If neglecting friction rock/ice and rock/rock only two forces appear:

$$F^{\mathrm{a}}_{\parallel}(t) = -F^{\mathrm{b}}_{\parallel}(t) \tag{1.4}$$

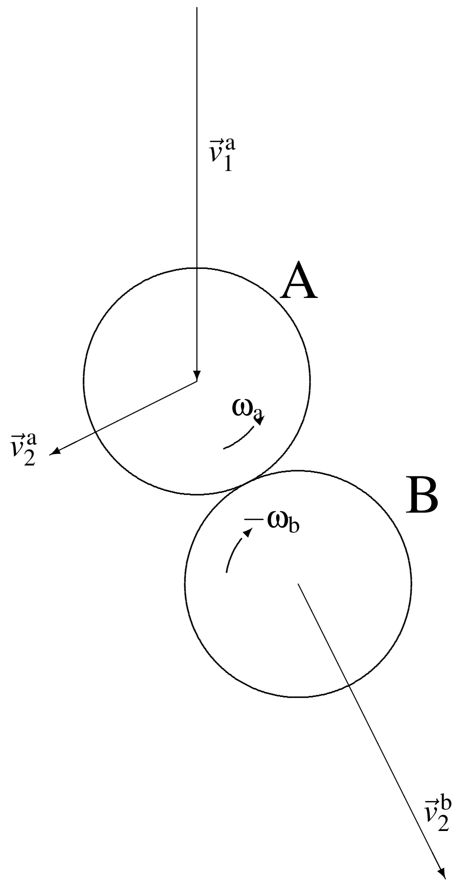

Figure 1.1: The setup: Rock A hits B. Both are *identical* in mass, size etc. After the hit i.g. both move.

$$\int_T F_{\parallel}^{\mathrm{a}}(t)\,\mathrm{d}t = -\int_T F_{\parallel}^{\mathrm{b}}(t)\,\mathrm{d}t \tag{1.5}$$

$$\Delta p_{\parallel}^{\mathrm{a}} = -\Delta p_{\parallel}^{\mathrm{b}} \tag{1.6}$$

With conservation of energy and assumption of equal masses we get:

$$v_{\parallel 2}^{\mathrm{a}} = v_{\parallel 1}^{\mathrm{b}} \tag{1.7}$$

$$v_{\parallel 2}^{\mathrm{b}} = v_{\parallel 1}^{\mathrm{a}} \tag{1.8}$$

I.e. the speed-components along the hitting direction just exchange.

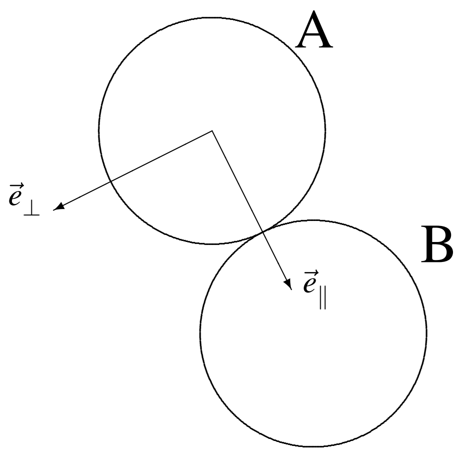

Figure 1.2: The coordinate-system. Positive turn is conterclockwise.

1.2 Without spin, with loss of energy

If we look at the rocks hitting each other, what can we see? I mean which equations do occur? First of all only the components along the hitting-direction interact at all, the rest remains unchanged. For these parallel components we state:

$$E_{Kin_1} = E_{Kin_2} + U \quad \text{with } U = \text{lost energy} \tag{1.9}$$

$$\vec{F}_a + \vec{F}_b + \vec{\xi} = 0 \tag{1.10}$$

If we assume the lost momentum to be small, e.g. because of a very short time of contact ($\ll 1$) and a *not* very huge friction rock/ice ($\not\gg 1$), we can assume $\xi \approx 0$ and solve this set of equations.

If we assume equal masses, things end up pretty simple — quite as we like it! We get

$$p^{\mathrm{a}}_{\|2} = p^{\mathrm{a}}_{\|1} + p^{\mathrm{b}}_{\|1} - p^{\mathrm{b}}_{\|2} \tag{1.11}$$

$$p^{\mathrm{b}}_{\|2} = \frac{\mathrm{sgn}(p^{\mathrm{a}}_{\|1} - p^{\mathrm{b}}_{\|1})}{2}\sqrt{\left(p^{\mathrm{a}}_{\|1} - p^{\mathrm{b}}_{\|1}\right)^2 - 4U} + \frac{p^{\mathrm{a}}_{\|1} + p^{\mathrm{b}}_{\|1}}{2} \tag{1.12}$$

$$p^{\mathrm{a}}_{\|2} = \frac{\mathrm{sgn}(p^{\mathrm{b}}_{\|1} - p^{\mathrm{a}}_{\|1})}{2}\sqrt{\left(p^{\mathrm{a}}_{\|1} - p^{\mathrm{b}}_{\|1}\right)^2 - 4U} + \frac{p^{\mathrm{a}}_{\|1} + p^{\mathrm{b}}_{\|1}}{2} \tag{1.13}$$

1.2.1 The loss' amount

How big has the loss of energy U to be, to reduce the remaining path for a given distance Δs? It's got to be the friction's work along Δs. If assuming constant coulomb friction $|F| = \mu mg$ we get

$$|F| = \mu mg \tag{1.14}$$

$$W = F \cdot s \tag{1.15}$$

$$\Delta E = W \tag{1.16}$$

$$\Longrightarrow U = \Delta s \cdot \mu mg \tag{1.17}$$

1.2.2 Resumee

This model workes fine for rather full hits. But as the hits become extreme thin — the speed-component in hitting direction very small — we get weird results, e.g.:

The hitter changes it's direction, but the hitting rock remains unmoved! This tells me to include all $p_{\parallel}$, $p_{\perp}$ and L into the model, but how?

1.3 With spin, without loss (Try I)

The parallel component works like stated in 1.1.2. Much more interesting turns out to be the

1.3.1 Perpendicular component $p_\perp$ and spin ω

According to the friction rock/rock in the touching spot a tangential force appears.

Getting the equations

In the touching spot appear forces affecting A and B, speed and spin. The relations are (with fig. (A.1)):

$$F_\perp^{\mathrm{b}} = -F_\perp^{\mathrm{a}} \quad \text{(actio=reactio)} \tag{1.18}$$

$$M^{\mathrm{a}} = \mathrm{R} \cdot \vec{e}_\| \times \vec{F}^{\,\mathrm{a}} = -F_\perp^{\mathrm{a}} \cdot \mathrm{R} \tag{1.19}$$

$$M^{\mathrm{b}} = -\mathrm{R} \cdot \vec{e}_\| \times \vec{F}^{\,\mathrm{b}} = \mathrm{R} \cdot \vec{e}_\| \times \vec{F}^{\,\mathrm{a}} = -F_\perp^{\mathrm{a}} \cdot \mathrm{R} \tag{1.20}$$

Integration of the left sides:

$$F_\perp^{\mathrm{a}}(t) \rightarrow \int_{\mathrm{T}} F_\perp^{\mathrm{a}}(t)\,\mathrm{d}t = \Delta p_\perp^{\mathrm{a}} = \mathrm{m}\Delta v_\perp^{\mathrm{a}} \tag{1.21}$$

$$F_\perp^{\mathrm{b}}(t) \rightarrow \int_{\mathrm{T}} F_\perp^{\mathrm{b}}(t)\,\mathrm{d}t = \Delta p_\perp^{\mathrm{b}} = \mathrm{m}\Delta v_\perp^{\mathrm{b}} \tag{1.22}$$

$$M^{\mathrm{a}}(t) \rightarrow \int_{\mathrm{T}} M^{\mathrm{a}}(t)\,\mathrm{d}t = \Delta L^{\mathrm{a}} = \mathrm{J}\Delta\omega_{\mathrm{a}} \tag{1.23}$$

$$M^{\mathrm{b}}(t) \rightarrow \int_{\mathrm{T}} M^{\mathrm{b}}(t)\,\mathrm{d}t = \Delta L^{\mathrm{b}} = \mathrm{J}\Delta\omega_{\mathrm{b}} \tag{1.24}$$

Giving:

$$\Delta v_\perp^{\mathrm{a}} = +\frac{1}{\mathrm{m}} \int_{\mathrm{T}} F_\perp^{\mathrm{a}}(t)\,\mathrm{d}t \tag{1.25}$$

$$\Delta v_\perp^{\mathrm{b}} = -\frac{1}{\mathrm{m}} \int_{\mathrm{T}} F_\perp^{\mathrm{a}}(t)\,\mathrm{d}t \tag{1.26}$$

$$\Delta\omega_a = -\frac{R}{J}\int_T F_\perp^a(t)\,dt \tag{1.27}$$

$$\Delta\omega_b = -\frac{R}{J}\int_T F_\perp^a(t)\,dt \tag{1.28}$$

So much about how the transferred forces interact. Now let's say something about it's size. Because it's a friction, it exists only as long, as there is a difference of the rocks' surface-speeds. A point on a rotating circle's edge has the speed[1]:

$$\vec{v}^{\mathrm{eff}} = \vec{v} + \vec{\omega} \times \vec{R}(\alpha) \tag{1.29}$$

Or in our case:

$$v_\perp^{\mathrm{a\ eff}} = v_\perp^{\mathrm{a}} + \omega_a \times R\vec{e}_{\|} = v_\perp^{\mathrm{a}} - R\omega_a \tag{1.30}$$

$$v_\perp^{\mathrm{b\ eff}} = v_\perp^{\mathrm{b}} + \omega_b \times -R\vec{e}_{\|} = v_\perp^{\mathrm{a}} + R\omega_b \tag{1.31}$$

and

$${}^{a,b}\delta v_\perp^{\mathrm{eff}} = v_\perp^{\mathrm{b\ eff}} - v_\perp^{\mathrm{a\ eff}} \tag{1.32}$$

The friction stops when:

$${}^{a,b}\delta v_{\perp\,2}^{\mathrm{eff}} = 0 \tag{1.33}$$

$${}^{a,b}\delta v_{\perp\,2}^{\mathrm{eff}} - {}^{a,b}\delta v_{\perp\,1}^{\mathrm{eff}} = -{}^{a,b}\delta v_{\perp\,1}^{\mathrm{eff}} \tag{1.34}$$

$${}^{a,b}\delta\Delta v_\perp^{\mathrm{eff}} = \underbrace{-{}^{a,b}\delta v_{\perp\,1}^{\mathrm{eff}}}_{=:\,\mathrm{V_{eff}}} \tag{1.35}$$

$$\mathrm{V_{eff}} = {}^{a,b}\delta\Delta v_\perp^{\mathrm{eff}} = (\Delta v_\perp^{\mathrm{b}} + R\Delta\omega_b) - \Delta v_\perp^{\mathrm{a}} + R\Delta\omega_a \tag{1.36}$$

eq. (1.25) – eq. (1.28) and eq. (1.36) build a solvable set of equations.

But the extremal ${}^{a,b}\delta\Delta v_\perp^{\mathrm{eff}}$ isn't reached always. Both rocks might already separate while still rubbing. The maximum is limited by the pressure-force or $\Delta p_{\|}^{\mathrm{a}}$:[2]

[1] speed (along the path) + rotation

[2] This linear relation between $\Delta p_{\|}$ and the left side is only valid for pure Coulomb-friction:

$$F_\perp(t) = -\mathrm{sgn}(v)\cdot\mu\cdot|F_{\|}(t)| \tag{1.37}$$

$$\left|\int_T F_\perp^a(t)\,\mathrm{d}t\right| \leq \mu\cdot|\Delta p_\parallel^a| \tag{1.41}$$

$$\text{otherwise} \quad \int_T F_\perp^a(t)\,\mathrm{d}t = -\mathrm{sgn}(V_{\text{eff}})\cdot\mu\cdot|\Delta p_\parallel^a| \tag{1.42}$$

Solution

eq. (1.30), eq. (1.31) & eq. (1.32):

$$V_{\text{eff}} = -v^b_{\perp 1} - R\omega_{b1} + (v^a_{\perp 1} - R\omega_{a1}) \tag{1.43}$$

eq. (1.36):

$$V_{\text{eff}} = -\Delta v_\perp^a + R\Delta\omega_a + \Delta v_\perp^b + R\Delta\omega_b \tag{1.44}$$

Substituting eq. (1.25) through eq. (1.28):

$$V_{\text{eff}} = -\int_T F_\perp^a(t)\cdot\left(\frac{1}{m}+\frac{R^2}{J}+\frac{1}{m}+\frac{R^2}{J}\right)\mathrm{d}t \tag{1.45}$$

$$\underbrace{\int_T F_\perp^a(t)\,\mathrm{d}t}_{=:X} = \frac{-V_{\text{eff}}}{\frac{1}{m}+\frac{R^2}{J}+\frac{1}{m}+\frac{R^2}{J}} \tag{1.46}$$

$$\text{If} \quad |X| > \mu|\Delta p_\parallel^a| \tag{1.47}$$

$$\text{then} \quad X = -\mathrm{sgn}(V_{\text{eff}})\cdot\mu\cdot|\Delta p_\parallel^a| \tag{1.48}$$

$$\Delta v_\perp^a = +\frac{1}{m}\cdot X \tag{1.49}$$

$$\int_T F_\perp(t)\,\mathrm{d}t = -\mathrm{sgn}(v)\cdot\mu\cdot\int_T |F_\parallel(t)|\,\mathrm{d}t \tag{1.38}$$

no altering sign of $F_\parallel(t)$:

$$\int_T F_\perp(t)\,\mathrm{d}t = -\mathrm{sgn}(v)\cdot\mu\cdot\left|\int_T F_\parallel(t)\,\mathrm{d}t\right| \tag{1.39}$$

$$\int_T F_\perp(t)\,\mathrm{d}t = -\mathrm{sgn}(v)\cdot\mu\cdot|\Delta p_\parallel| \tag{1.40}$$

$$\Delta v_\perp^{\mathrm{b}} = -\frac{1}{\mathrm{m}} \cdot X \tag{1.50}$$

$$\Delta \omega_{\mathrm{a}} = -\frac{\mathrm{R}}{\mathrm{J}} \cdot X \tag{1.51}$$

$$\Delta \omega_{\mathrm{b}} = -\frac{\mathrm{R}}{\mathrm{J}} \cdot X \tag{1.52}$$

1.3.2 Check it out

O.k. so far, but what about the conservation of energy? This still should be ensured, so let's test it.

The energy consists of two parts, movement and spin and the motion part can be split into it's two components.

Beyond that, by using the form $z_2 = z_1 + \Delta z$ for all speeds and taking their squares we can collect the mixed parts into a remainder ξ

$$E_{Kin_1} + E_{Rot_1} \stackrel{!}{=} E_{Kin_2} + E_{Rot_2} \tag{1.53}$$

$$= E_{Kin_1} + E_{Rot_1} + \xi(x) \tag{1.54}$$

This remainder $\xi(x)$ must, for energy might be lost, but not gained, be less or equal 0.

To test this, we discuss the function $\xi(x)$.

$$\xi(x) = 2x(v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}\omega_{\mathrm{a}} - \mathrm{R}\omega_{\mathrm{b}}) + 2x^2 \frac{\mathrm{J} + \mathrm{mR}^2}{\mathrm{mJ}} \tag{1.55}$$

$$\xi''(x) = 4 \frac{\mathrm{J} + \mathrm{mR}^2}{\mathrm{mJ}} \tag{1.56}$$

$$\xi(0) = 0 \tag{1.57}$$

The symbols $v_\perp^{\mathrm{a}}$ through ω_{b}, strictly spoken, should be $v_{\perp 1}^{\mathrm{a}} \ldots \omega_{\mathrm{b}1}$. But for readability reasons in this section we skip the index.

Now let's check $\xi(X)$ with X from eq. (1.46):

$$X = \frac{-\mathrm{V_{eff}}}{\frac{1}{\mathrm{m}} + \frac{\mathrm{R}^2}{\mathrm{J}} + \frac{1}{\mathrm{m}} + \frac{\mathrm{R}^2}{\mathrm{J}}}$$

$$= -\frac{v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}(\omega_{\mathrm{a}} + \omega_{\mathrm{b}})}{2\frac{\mathrm{J+mR^2}}{\mathrm{mJ}}} \tag{1.58}$$

$$\xi(X) = 2X(v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}(\omega_{\mathrm{a}} + \omega_{\mathrm{b}})) + 2X^2 \frac{\mathrm{J} + \mathrm{mR}^2}{\mathrm{mJ}} \tag{1.59}$$

$$\begin{aligned}
&= -\frac{(v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}(\omega_{\mathrm{a}} + \omega_{\mathrm{b}}))^2}{\frac{\mathrm{J+mR^2}}{\mathrm{mJ}}} \\
&+ \frac{1}{2} \cdot \frac{(v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}(\omega_{\mathrm{a}} + \omega_{\mathrm{b}}))^2}{\frac{\mathrm{J+mR^2}}{\mathrm{mJ}}} && (1.60) \\
&= -\frac{1}{2} \cdot \frac{(v_\perp^{\mathrm{a}} - v_\perp^{\mathrm{b}} - \mathrm{R}(\omega_{\mathrm{a}} + \omega_{\mathrm{b}}))^2}{\frac{\mathrm{J+mR^2}}{\mathrm{mJ}}} < 0 && (1.61) \\
\Longrightarrow \xi(2 \cdot X) &= 0 && (1.62) \\
\Longrightarrow \min_{\forall x} \xi(x) &= \xi(X) && (1.63)
\end{aligned}$$

That means, as long as our values for x lie between 0 and X, we've got $\xi < 0$ and we're on the sunny side of life: We don't gain energy. Worst case we loose. I don't exactly understand where in the above equations this energy gets lost, but for the first we seem to have a plausible, more or less simple and handleable model for what happens with take-outs. The physical effect causing this loss surely is the friction rock/rock and ξ equals the energy lost here.

1.3.3 Resumee

The model seems to work rather fair and tells that both rocks don't separate under 90^0 in general. Spin has influence on direction of movement and vice versa.

But still there's a friction rock/ice that isn't included in this model, so it cannot be perfect in all situations.

Improved models are to be developed!

1.4 With spin and loss (Try I)

The approach 1.2 doesn't give proper results, so we try a new method. We assume the rock B not to experience acceleration until the initial friction F_H is overcome. This causes both, a loss of energy as well as as loss of momentum and spin. Besides we get a natural splitting of the loss of energy to the components $v_\parallel, v_\perp$ and ω.

For this purpose we need to know the forces *during* the time of contact T. To achieve this we need to make some assumptions about the rocks' elasticity mechanism. Here we assume the rocks to behave like springs and fulfilling Hook's law of elasticity. Also we assume the friction rock/rock to be constant $|F_\perp| = \mu \cdot |F_\parallel|$. This way we get to know the forces and everything's just fine.

In this section all letters (if not stated differently) mean rock A before the hit. E.g. x really is ${x^{\mathrm{a}}}_1$.

1.4.1 Contact behaviour

Hook says:

$$F = H \cdot x \tag{1.64}$$

$$E_{\mathrm{Pot}} = \frac{1}{2} H \cdot x^2 \tag{1.65}$$

$$H \cdot x = m \cdot a = m \frac{\mathrm{d}^2 x}{\mathrm{d}t} \tag{1.66}$$

$$\Longrightarrow \quad x(t) = A \cdot \sin \bar{\omega} t \tag{1.67}$$

$$\text{with} \quad \bar{\omega} = \sqrt{\frac{H}{m}} = \frac{\pi}{T} = \text{const!} \tag{1.68}$$

$$\text{and} \quad A = \sqrt{\frac{m}{H} {v_\parallel}^2} = \frac{-v_\parallel}{\bar{\omega}} = \frac{-v_\parallel T}{\pi} \tag{1.69}$$

$$x(t) = -\frac{v_\parallel}{\bar{\omega}} \cdot \sin \bar{\omega} t \tag{1.70}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} x(t) = -v_\parallel \cdot \cos \bar{\omega} t \tag{1.71}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) = v_\parallel \bar{\omega} \sin \bar{\omega} t \tag{1.72}$$

$$\Delta v_{\parallel} \quad = \quad -v_{\parallel}\left[\cos\bar{\omega}t'\right]_0^t = -v_{\parallel}\left(\cos\bar{\omega}t - 1\right) \tag{1.73}$$

O.k. so far. But what happens during such a hit? We split the whole contact $0 \leq t \leq T$ into three periods:

1. Overcome the initial friction $t < t_1$
2. Level the surface speeds $t < t_0$
3. Exchange the rest of the parallel momentum $t \leq T$

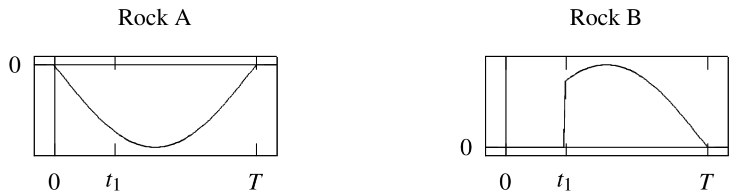

Figure 1.3: $F_{\parallel}(t)$. The parallel forces during the hit. As you see, B ain't accelerated until $t \geq t_1$.

1.4.2 Find t_0

t_0 is the time, when A's effective surface speed becomes 0 (=B's).

$$\vec{v}_2^{\text{eff}} \quad \stackrel{!}{=} \quad 0 \tag{1.74}$$

$$v_{\perp 2} - \mathrm{R}\omega_2 \quad = \quad 0 \tag{1.75}$$

$$v_{\perp} + \Delta v_{\perp} - \mathrm{R}(\omega + \Delta\omega) \quad = \quad 0 \tag{1.76}$$

$$\tag{1.77}$$

with eq. (1.25) & eq. (1.27):

$$\Delta\omega \quad = \quad \Delta v_{\perp}\frac{-\mathrm{mR}}{\mathrm{J}} \tag{1.78}$$

$$\Delta v_{\perp} \quad = \quad \frac{-\vec{v}^{\text{eff}}}{\frac{\mathrm{J+mR^2}}{\mathrm{J}}} \tag{1.79}$$

$$F_\perp = -\mathrm{sgn}(\vec{v}^{\mathrm{eff}})\cdot\mu\cdot|F_\|| \tag{1.80}$$

$$\Longrightarrow \quad \Delta v_\perp = -\mathrm{sgn}(\vec{v}^{\mathrm{eff}})\cdot\mu\cdot\left|-v_\|\cdot(\cos\bar{\omega}t_0-1)\right| \tag{1.81}$$

$$\Longrightarrow \quad \frac{-\vec{v}^{\mathrm{eff}}\frac{1}{\frac{\mathrm{J+mR^2}}{\mathrm{J}}}}{-\mathrm{sgn}(\vec{v}^{\mathrm{eff}})\mu|v_\||} = |\underbrace{\cos\bar{\omega}t_0-1}_{\leq 0\forall t_0}| \tag{1.82}$$

$$-\left|\frac{\frac{\vec{v}^{\mathrm{eff}}}{\frac{\mathrm{J+mR^2}}{\mathrm{J}}}}{v_\|\mu}\right|+1 = \cos\bar{\omega}t_0 \tag{1.83}$$

1.4.3 Find t_1

t_1 is the time, when the Hook-force equals the friction.

$$F_\|{}^2+F_\perp{}^2 = F_H^2 \tag{1.84}$$

$$F_\|{}^2+\mu^2F_\|{}^2 = F_H^2 \tag{1.85}$$

$$F_\| = \frac{F_H}{\sqrt{1+\mu^2}} \tag{1.86}$$

$$\mathrm{m}v_\|\bar{\omega}\sin\bar{\omega}t_1 = \frac{F_H}{\sqrt{1+\mu^2}} \tag{1.87}$$

$$\sin\bar{\omega}t_1 = \frac{F_H}{\mathrm{m}v_\|\bar{\omega}\sqrt{1+\mu^2}} \tag{1.88}$$

Result

The lost momentum is:

$$\Delta\vec{v} = -v_\|\left(\sqrt{1-\sin^2\bar{\omega}t_1}-1\right)\begin{pmatrix}-\mathrm{sgn}(\vec{v}^{\mathrm{eff}})\cdot\mu\\ -1\end{pmatrix} \tag{1.89}$$

If $\vec{v}^{\mathrm{eff}}$ becomes zero *before* the friction ends we need to find the friction parallel to $\vec{e}_\|$ only, but include the lost momentum in $\vec{e}_\perp$ direction as well.

$$\text{if} \quad t_0 < t_1 \iff \sin\bar{\omega}t_0 < \sin\bar{\omega}t_1 \tag{1.90}$$

$$\sin\bar{\omega}t_1 = \frac{F_H}{\mathrm{m}v_\|\bar{\omega}\cdot 1} \tag{1.91}$$

Lost momentum:

$$\perp \ : \ -\mathrm{sgn}(\vec{v}^{\text{eff}})\mu \cdot \left|-v_\| \cdot (\cos\bar{\omega}t_0 - 1)\right| \tag{1.92}$$

$$\| \ : \ v_\| \cdot (\sqrt{1-\sin^2\bar{\omega}t_1} - 1) \tag{1.93}$$

After applying the above loss of momentum to the *hitting* rock and a loss of spin of

$$\Delta\omega = -\Delta v_\perp \frac{\mathrm{mR}}{\mathrm{J}} \tag{1.94}$$

we can continue with a normal hit, e.g. like described in 1.3.

1.4.4 Loss of energy & F_H

The energy stored in a spring is

$$E = \frac{1}{2}Hx^2 \tag{1.95}$$

$$= \frac{1}{2}\bar{\omega}^2 \cdot \mathrm{m} \cdot x^2 \tag{1.96}$$

$$\text{with} \quad x = -v_\|/\bar{\omega} \cdot \sin\bar{\omega}t_H \tag{1.97}$$

$$\text{and} \quad F_H = \mathrm{m}v_\|\bar{\omega} \cdot \sin\bar{\omega}t_H \tag{1.98}$$

$$E = \frac{1}{2}\bar{\omega}^2 \cdot \mathrm{m} \cdot (-v_\|/\bar{\omega} \cdot \sin\bar{\omega}t_H)^2 \tag{1.99}$$

$$E = \frac{1}{2}\frac{F_H^2}{\mathrm{m}\bar{\omega}^2} \tag{1.100}$$

$$\implies F_H = \bar{\omega}\sqrt{2\mathrm{m}E} = \text{const!} \tag{1.101}$$

That tells us for a given loss of energy we'll get a constant initial friction. (Which, indeed, is a very nice result.)

1.4.5 Resumee

This model depends on Coulomb-friction rock/rock and a constant friction rock/ice for non-moving rocks. This friction can be very big, about 10^4 N!

The results presented by this model seem quite sensible. The only problem remaining is what happens if hitting a freeze? Do we have to overcome the friction two times? If yes this model works fine.

Chapter 2

Rock trajectory

2.1 Applying the 'Denny' model

Mark Denny published a model in [Den98], see B.1. To apply it for simulation we need to

- compute the ice properties from the draw-to-tee time and curl.
- transform from $t_0 = 0$ to $t_0 > 0$
- transform the equations from rock-coordinates to world-coordinates
- compute the initial speed of a rock from the hog-to-hog time and y_0

2.1.1 Basic equations

After substituting eq. (B.5) the basic equations are:

$$x(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\mu t^3\,(3g\mu t - 4v_0)}{48\varepsilon R} \tag{2.1}$$

$$x'(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\mu t^2\,(g\mu t - v_0)}{4\varepsilon R} \tag{2.2}$$

$$x''(t) = -\mathrm{sgn}(\omega_0)\frac{b\,g\mu t\,(3g\mu t - 2v_0)}{4\varepsilon R} \tag{2.3}$$

$$y(t) = -\frac{t\,(g\mu t - 2v_0)}{2} \tag{2.4}$$

$$y'(t) = -(g\mu t - v_0) \tag{2.5}$$

$$y''(t) = -g\mu \tag{2.6}$$

$$\alpha(t) = \frac{\omega_0 \varepsilon \left(g\mu t \left(-\frac{g\mu t - v_0}{v_0}\right)^{\frac{1}{\varepsilon}} - v_0 \left(-\frac{g\mu t - v_0}{v_0}\right)^{\frac{1}{\varepsilon}} + v_0 \right)}{(\varepsilon + 1)\, g\mu} \tag{2.7}$$

$$\alpha'(t) = \omega_0 \left(-\frac{g\mu t - v_0}{v_0}\right)^{\frac{1}{\varepsilon}} \tag{2.8}$$

$$\alpha''(t) = \frac{\omega_0\, g\mu \left(-\frac{g\mu t - v_0}{v_0}\right)^{\frac{1}{\varepsilon}}}{\varepsilon\,(g\mu t - v_0)} \tag{2.9}$$

2.1.2 The ice-properties μ and b

To calculate μ and b from the time T and curl B of a draw-to-tee we set up the equations

$$x(T) = B \quad \text{(curl)} \tag{2.10}$$

$$x'(T) = 0 \tag{2.11}$$

$$y(T) = D \quad \text{(distance hog to tee)} \tag{2.12}$$

$$y'(T) = 0 \tag{2.13}$$

Solving the set of this 4 equations leads to

$$b = -\frac{12 B \varepsilon R}{D^2} \tag{2.14}$$

$$\mu = \frac{2D}{gT^2} \tag{2.15}$$

$$v_0 = \frac{2D}{T} \tag{2.16}$$

2.1.3 Some initial speeds

How hard do we have to throw a rock, that will take 12 seconds hog to hog?

To calculate v_0 at the far hog we don't use the time hog-to-tee for not every rock reaches the tee-line. Here the time hog-to-hog (T_H) is better.

$$y(T_H) = H \quad \text{(dist. hog-to-hog)} \tag{2.17}$$

$$\Rightarrow \quad v_0 = \frac{g\mu T_H^2 + 2H}{2T_H} \tag{2.18}$$

If we don't want v_1 at the far hog but at any given distance y_1, the following equations apply:

$$y(t) = y_1 \tag{2.19}$$

$$y'(t) = v_1 \tag{2.20}$$

$$v_1 = -\frac{g\mu T^2 - 2y_1}{2T} \tag{2.21}$$

$$v_0 = \frac{g\mu T^2 + 2y_1}{2T} \tag{2.22}$$

eq. (**??**) + eq. (**??**):

$$H = v_0 T_H - T_H \frac{\mu g}{2} + T_H t_0 \mu g \tag{2.23}$$

$$\Longrightarrow \quad t_0 = \frac{H - v_0 T_H + T_H^2 \frac{\mu g}{2}}{T_H \mu g} \tag{2.24}$$

Substituting this into eq. (**??**) and solving for v_0 gives

$$\Longrightarrow v_0 = \frac{1}{2} \cdot \sqrt{\left(\mu g T_H + 2\frac{H}{T_H}\right)^2 + 4\mu g H - 8\mu g y_0} \tag{2.25}$$

If you prefer using Y_0 measured from the tee, use

$$\Longrightarrow v_0 = \frac{1}{2} \cdot \sqrt{\left(\mu g T_H + 2\frac{H}{T_H}\right)^2 + 4\mu g H - 8\mu g(\text{far-hog-to-tee} - Y_0)} \tag{2.26}$$

2.1.4 Coordinate transformation

Because of Denny assumes the rock to start at $(0,0)^T$ with v_0 pointing along the $\hat{y}$-axis, we need a rotation and shift to get the general equations.

$$\begin{pmatrix} x \\ y \end{pmatrix} := \frac{\vec{v}_{\text{real}}}{|\vec{v}_{\text{real}}|} \tag{2.27}$$

$$\text{The required transformation is:} \quad \begin{pmatrix} y & x \\ x & -y \end{pmatrix} \tag{2.28}$$

Applying this trafo to e.g. $\binom{a}{b}$ results in

$$\begin{pmatrix} y & x \\ x & -y \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ay+bx \\ ax-by \end{pmatrix} \tag{2.29}$$

where a and b are polynomes of max. fourth degree:

$$a =: At^4 + Bt^3 + Ct^2 + Dt + E \tag{2.30}$$

$$b =: \alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi \tag{2.31}$$

This leads to a x-component of

$$\begin{aligned} & y(At^4 + Bt^3 + Ct^2 + Dt + E) + \\ & x(\alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi) && (2.32) \\ = \; & (Ay + \alpha x)t^4 + \ldots + (Ey + \phi x) && (2.33) \end{aligned}$$

and a y-component of

$$\begin{aligned} & x(At^4 + Bt^3 + Ct^2 + Dt + E) + \\ & -y(\alpha t^4 + \beta t^3 + \gamma t^2 + \delta t + \phi) && (2.34) \\ = \; & (Ax - \alpha y)t^4 + \ldots + (Ex - \phi y) && (2.35) \end{aligned}$$

Now just the shift is missing.

Appendix A

Some physical/math. Basics

A.1 Rotation equations using cross-products

$$\begin{aligned} \text{velocity:} \quad \vec{v} &= \vec{\omega} \times \vec{R} & \text{(A.1)} \\ \text{torque:} \quad \vec{M} &= \vec{R} \times \vec{F} & \text{(A.2)} \\ \text{angular mom.:} \quad \vec{L} &= \vec{R} \times \vec{p} = J\vec{\omega} & \text{(A.3)} \\ \text{energy:} \quad E_{Rot} &= \frac{1}{2}\vec{L} \cdot \vec{\omega} = \frac{|\vec{L}|^2}{2J} & \text{(A.4)} \end{aligned}$$

A.2 Force-splitting

See fig. (A.1).

A.3 Time-distance of two spheres on straight paths

Having one object's center at $\vec{a}$ with speed $\vec{v}_a$, the other at $\vec{b}$ with speed $\vec{v}_b$, we introduce

$$\vec{x} := \vec{b} - \vec{a} \quad \text{(A.5)}$$

$$\vec{v} := \vec{v}_b - \vec{a}_a \quad \text{(A.6)}$$

Now we need the time until the centers' distance equals the two radii:

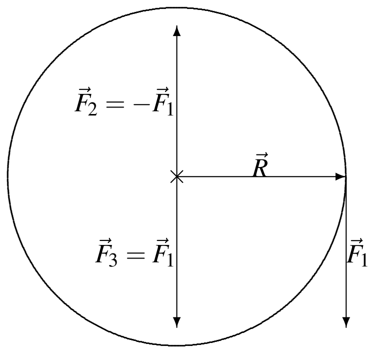

Figure A.1: Force-splitting: The force at the edge $\vec{F}_1$ can, after adding $\vec{F}_{2,3}$ be split into a central force $\vec{F}_3$ and a torque-causing force-pair $\vec{F}_{1,2}$. The result is a force $\vec{F}_2$ in the center of mass and a torque $\vec{F}_1 \times \vec{R}$. See [Gre85, P. 142f], [GHS95, P. 34fff].

$$|\vec{x} + t \cdot \vec{v}| = (r + r) \tag{A.7}$$

$$(\vec{x} + t \cdot \vec{v}) \cdot (\vec{x} + t \cdot \vec{v}) = (r + r)^2 \tag{A.8}$$

$$\vec{x}\vec{x} + 2t\vec{v}\vec{x} + t^2\vec{v}\vec{v} - (r + r)^2 = 0 \tag{A.9}$$

$$t_{1,2} = \frac{-\vec{v}\vec{x} \pm \sqrt{(\vec{v}\vec{x})^2 - |\vec{v}|^2 \cdot (|\vec{x}|^2 - (r + r)^2)}}{|\vec{v}|^2} \tag{A.10}$$

In our case the solution is the smaller (earlier) result:

$$t = \frac{-\vec{v}\vec{x} - \sqrt{(\vec{v}\vec{x})^2 - |\vec{v}|^2 \cdot (|\vec{x}|^2 - (r + r)^2)}}{|\vec{v}|^2} \tag{A.11}$$

If $\vec{v}\vec{x} = 0$ the rocks pass each other (or don't move at all), for $\vec{v}\vec{x} > 0$ they separate.

Figure A.2: When do 2 objects hit? The setup.

Appendix B

Ice models

B.1 The 'Denny' model

Mark Denny published a model in [Den98]. It's quite simple and provides polynomes of fourth degree to describe the rock's motion along the sheet.

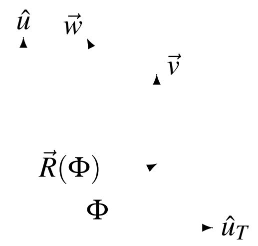

Figure B.1: The *running band* of radius R is the surface of contact between rock and ice. Here the velocity components, relative to the ice, of a point on the running band are shown. v is the CM velocity, and w is the angular component at radius R. The unit vectors (u_T, u) coincide with (u_x, u_y) at time $t = 0$. In this case the rock is curling in a counterclockwise sense (so angular velocity unit vector $= u_z$).

$$x(t) \approx -\frac{bv_0^2}{4\varepsilon R\tau}\left(\frac{t^3}{3} - \frac{t^4}{4\tau}\right) \tag{B.1}$$

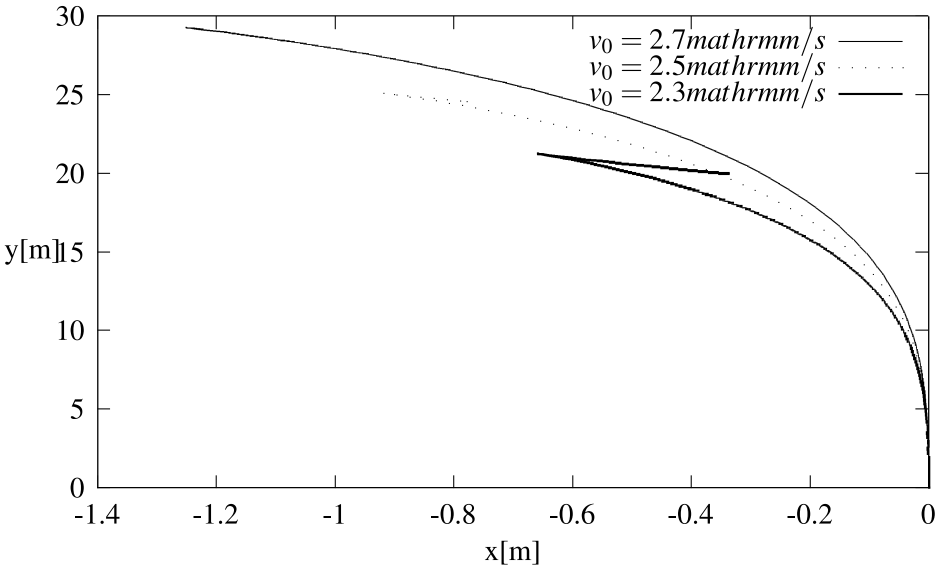

Figure B.2: Approximate curling rock trajectories, calculated from eq. (B.1), eq. (B.2) and eq. (B.4) for three values of initial velocity v_0. The other parameters are: $\mu = 0.0127$, $b_{\mathrm{LR}} = 0.003$, $\varepsilon = 2.63$, $R = 0.065$.

$$y(t) \approx v_0 \left(t - \frac{t^2}{2\tau}\right) \tag{B.2}$$

$$\omega(t) \approx \omega_0 \left(1 - \frac{t}{\tau}\right)^{1/\varepsilon} \tag{B.3}$$

$$\alpha(t) \approx \int_0^t \omega(t)\,\mathrm{d}t = -\frac{\omega_0\,\varepsilon \left(\tau\,\left(\frac{\tau-t}{\tau}\right)^{\frac{1}{\varepsilon}} - t\,\left(\frac{\tau-t}{\tau}\right)^{\frac{1}{\varepsilon}} - \tau\right)}{\varepsilon+1} \tag{B.4}$$

t	[s]	time
x	[m]	curl
y	[m]	distance along the track
b	[1]	parameter for the curl's magnitude
ε	[1]	parameter $\frac{I}{mR^2}$
τ	[s]	total time until $v = 0$
R	[m]	radius of the touching area rock/ice $\approx$ 6.3e-2
μ	[1]	friction coefficient rock/ice
I		moment of inertia (z-direction)
g	[N/kg]	9.81 gravitation

The final properties are:

$$\tau = \frac{v_0}{\mu g} \tag{B.5}$$

$$x(\tau) \approx -\frac{b_{\mathrm{LR}}}{12\varepsilon}\frac{y^2(\tau)}{R} \tag{B.6}$$

$$y(\tau) \approx \frac{v_0^2}{2\mu g} \tag{B.7}$$

B.2 The 'Shegelski et al.' model

This model is much more complicated. A sketch is published in [SNW96]:

...

B.2.1 Equations of motion of a curling rock

We next give the equations that determine the motion of the curling rock down the pebbled ice surface. This is a straightforward exercise in determining the net force and torque exerted by the ice on the rock.

We choose the coordinate axes as follows. The $+y$ axis is in the direction of the *initial* velocity of the rock, and the $+x$ axis is perpendicular to the initial velocity and in the direction in which the rock is expected to curl. In obtaining the results given in the next section, we also used a $+y'$ axis, which is in the direction of the instantaneous velocity of the rock, as well as a $+x'$ axis.

We assume that both wet and dry frictional forces exerted on a small section of the annulus of the rock are in the direction opposite to the velocity of the point relative to the ice. The magnitude of the dry friction is

$$\Delta F^d = \mu M g \left(\frac{\Delta\Theta}{2\pi} \right) \tag{B.8}$$

where μ is the coefficient of kinetic friction.[1] We take the magnitude of the wet friction increase with velocity, in analogy with increased drag on an object moving in a fluid, and write

$$\Delta F^w = k \left[u(\Theta) \right]^\Phi \tag{B.9}$$

where k and Φ are parameters to be discussed below, and $u(\Theta)$ is the net speed, relative to the ice, of the portion of the contact annulus at angle Θ, as shown in fig. (B.3).

To minimize the numerical analysis, instead of integrating over the entire area of the annulus, we break each semicircle into an outer and inner semicircle. By considering each quadrant of the rock seperately, the following equations for the x and y components of the force on the rock are readily obtained. We first give the equations, then complete our definitions of symbols used.

The x-component of dry friction is:

[1] In the case of a curling rock, the nonuniformity in the normal force around the rock is neglegible in our model; the nonuniformity is a consequence of the acceleration due to friction.

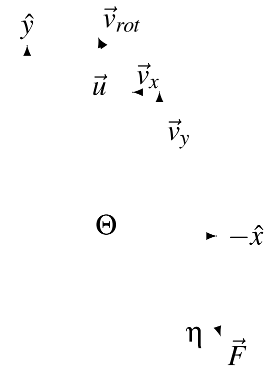

Figure B.3: The contributions $\vec{v}_y$, $\vec{v}_x$ and $\vec{v}_{rot}$ (with $\vec{v}_{rot} = \vec{r}\omega$) to the net velocity $\vec{u} \equiv \vec{u}(\Theta)$, relative to the ice, of a portion of the contact annulus located at the angle Θ relative to the x-axis. The figure shown is for a portion located in the first quadrant; similar figures are readily constructed for the other three quadrants. The angle Θ in each case is from the x-axis toward the y-axis. Note that the force $\vec{F}$ exerted on the portion is in the opposite direction to $\vec{u}$. The angle η appears in the equations explicitely as the arctan of the ratio of the x and y components of $\vec{u}$.

$$
\begin{aligned}
F_x^d = & -f_L^d \frac{\mu M g}{2\pi} \int_0^{\pi/2} d\Theta \left\{ \sin\left(\tan^{-1}\left[\frac{r_2 \omega \sin\Theta + v_x}{v_y + r_2 \omega \cos\Theta} \right] \right) + \sin\left(\tan^{-1}\left[\frac{r_2 \omega \sin\Theta + v_x}{v_y - r_2 \omega \cos\Theta} \right] \right) \right\} \\
& + f_T^d \frac{\mu M g}{2\pi} \int_0^{\pi/2} d\Theta \left\{ \sin\left(\tan^{-1}\left[\frac{r_1 \omega \sin\Theta - v_x}{v_y + r_1 \omega \cos\Theta} \right] \right) + \sin\left(\tan^{-1}\left[\frac{r_1 \omega \sin\Theta - v_x}{v_y - r_1 \omega \cos\Theta} \right] \right) \right\}
\end{aligned} \tag{B.10}
$$

The x-component of wet friction is:

$$
\begin{aligned}
F_x^w = & -k f_L^w \int_0^{\pi/2} d\Theta \left[(r_1 \omega \sin\Theta + v_x)^2 + (v_y + r_1 \omega \cos\Theta)^2 \right]^{\Phi/2} \sin\left(\tan^{-1}\left[\frac{r_1 \omega \sin\Theta + v_x}{v_y + r_1 \omega \cos\Theta} \right] \right) \\
& -k f_L^w \int_0^{\pi/2} d\Theta \left[(r_1 \omega \sin\Theta + v_x)^2 + (v_y - r_1 \omega \cos\Theta)^2 \right]^{\Phi/2} \sin\left(\tan^{-1}\left[\frac{r_1 \omega \sin\Theta + v_x}{v_y - r_1 \omega \cos\Theta} \right] \right) \\
& +k f_T^w \int_0^{\pi/2} d\Theta \left[(r_2 \omega \sin\Theta - v_x)^2 + (v_y + r_2 \omega \cos\Theta)^2 \right]^{\Phi/2} \sin\left(\tan^{-1}\left[\frac{r_2 \omega \sin\Theta - v_x}{v_y + r_2 \omega \cos\Theta} \right] \right)
\end{aligned}
$$

$$+kf_T^w \int_0^{\pi/2} d\Theta \left[(r_2\omega\sin\Theta - v_x)^2 + (v_y - r_2\omega\cos\Theta)^2\right]^{\Phi/2} \sin\left(\tan^{-1}\left[\frac{r_2\omega\sin\Theta - v_x}{v_y - r_2\omega\cos\Theta}\right]\right) \tag{B.11}$$

Similar equations hold for the y-component of the force.

The following equations give the torque due to dry and wet friction:

$$\begin{aligned}
\tau^d \quad = \quad & -r_2\frac{\mu Mg}{2\pi}\int_0^{\pi/2} d\Theta f_L^d \sin\left(\Theta + \frac{\pi}{2} - \tan^{-1}\left[\frac{r_2\omega\sin\Theta + v_x}{v_y + r_2\omega\cos\Theta}\right]\right) \\
& +r_2\frac{\mu Mg}{2\pi}\int_0^{\pi/2} d\Theta f_L^d \sin\left(\Theta + \frac{\pi}{2} + \tan^{-1}\left[\frac{r_2\omega\sin\Theta + v_x}{v_y - r_2\omega\cos\Theta}\right]\right) \\
& -r_1\frac{\mu Mg}{2\pi}\int_0^{\pi/2} d\Theta f_T^d \sin\left(\frac{\pi}{2} - \Theta + \tan^{-1}\left[\frac{r_1\omega\sin\Theta - v_x}{v_y + r_1\omega\cos\Theta}\right]\right) \\
& +r_1\frac{\mu Mg}{2\pi}\int_0^{\pi/2} d\Theta f_T^d \sin\left(\frac{\pi}{2} - \Theta - \tan^{-1}\left[\frac{r_1\omega\sin\Theta - v_x}{v_y - r_1\omega\cos\Theta}\right]\right)
\end{aligned} \tag{B.12}$$

$$\begin{aligned}
\tau^w \quad = \quad & -r_1kf_L^w \int_0^{\pi/2} d\Theta \left[(r_1\omega\sin\Theta + v_x)^2 + (v_y + r_1\omega\cos\Theta)^2\right]^{\Phi/2} \sin\left(\Theta + \frac{\pi}{2} - \tan^{-1}\left[\frac{r_1\omega\sin\Theta + v_x}{v_y + r_1\omega\cos\Theta}\right]\right) \\
& +r_1kf_L^w \int_0^{\pi/2} d\Theta \left[(r_1\omega\sin\Theta + v_x)^2 + (v_y - r_1\omega\cos\Theta)^2\right]^{\Phi/2} \sin\left(\Theta + \frac{\pi}{2} + \tan^{-1}\left[\frac{r_1\omega\sin\Theta + v_x}{v_y - r_1\omega\cos\Theta}\right]\right) \\
& -r_2kf_T^w \int_0^{\pi/2} d\Theta \left[(r_2\omega\sin\Theta - v_x)^2 + (v_y + r_2\omega\cos\Theta)^2\right]^{\Phi/2} \sin\left(\frac{\pi}{2} - \Theta + \tan^{-1}\left[\frac{r_2\omega\sin\Theta - v_x}{v_y + r_2\omega\cos\Theta}\right]\right) \\
& +r_2kf_T^w \int_0^{\pi/2} d\Theta \left[(r_2\omega\sin\Theta - v_x)^2 + (v_y - r_2\omega\cos\Theta)^2\right]^{\Phi/2} \sin\left(\frac{\pi}{2} - \Theta - \tan^{-1}\left[\frac{r_2\omega\sin\Theta - v_x}{v_y - r_2\omega\cos\Theta}\right]\right)
\end{aligned} \tag{B.13}$$

In these equations f_L^d is the effective fraction of the leading semicircle of the rock which experiences dry friction, and similarly for f_T^d, etc., r_1 and r_2 are the effective values of the radii of the inner and outer portions of the semicircle, respectively; v_x and v_y are the components of the velocity of the centre of mass of the rock; Θ in each quadrant is measured from the x-axis toward the y-axis, as shown in fig. (B.3); and ω is the instantaneous angular speed of the rock.

Fig. (B.3) is presented to assist in understanding and appreciating the significance of the above equations. The figure shows the contributions $\vec{v}_y$, $\vec{v}_x$ and $\vec{v}_{rot}$

(with $\vec{v}_{rot} = \vec{r}\omega$) to the net velocity $\vec{u} \equiv \vec{u}(\Theta)$, relative to the ice, of a portion of the contact annulus located at the angle Θ; note that $\vec{u} = \vec{v}_y + \vec{v}_x + \vec{v}_{rot}$, and recall that the velocity of the centre of mass of the rock relative to the ice is $\vec{v} = \vec{v}_y + \vec{v}_x$. Figure fig. (B.3) also shows that the force $\vec{F}$ exerted on that portion the direction opposite to $\vec{u}$. Similar figures are readily structured for the other three quadrants of the rock. We p??? the equations above with each of the four quadrants exp??? displayed so that the reader may easily recognize the ???-bution from the four quadrants and further appreciate asymmetries that arise, which are responsible for the curl of the rock.

The x and y components of the velocity of the rock at any time t are readily obtained numerically from the ??? equations along with $\vec{F} = Md\vec{v}/dt$, and initial conditions. The location of the rock is then obtained by using

$$x(t) = x_0 + \int_0^t dt' v_x(t') \tag{B.14}$$

and

$$y(t) = y_0 + \int_0^t dt' v_y(t') \tag{B.15}$$

The angular speed ω is given by

$$\omega(t) = \omega_0 + \int_0^t dt' \alpha(t') \tag{B.16}$$

where $\tau = MR^2\alpha/2$ determines the time development of the angular acceleration of the rock.

In the first phase of the rock's motion, we put $f_L^d = f_T^w = 1$, $f_T^d = f_L^w = 0$ and $r_1 = r_2 = r$.

In the middle phase of the rock's motion, we put $f_L^d = f_T^w = f_T^d = f_L^w = 1/2$ and $r_1 = r - \Delta r/2$, and $r_2 = r + \Delta r/2$.

In the final phase, we have $r_1 = r_2 = r$, $f^d = 0$, and $f^w = 1$ for $0 \le \Theta' \le \Theta_0'$, and $f^d = 1$, and $f^w = 0$ for $\Theta_0' \le \Theta' \le 2\pi$, where Θ' is measured counterclockwise from the $-x'$ direction, and Θ_0' gives the point on the rock where the velocity relative to the ice is radially outwarded from the centre of the rock.

We do not have a first-priciples derivation of the function that should be used to describe the transition between the first and middle, and middle and last phases of motion. Instead we use a smooth function for the transition in f_L^d that has the following features: $f_L^d \approx 1$ for $v \geq 1.5\,\mathrm{m\,s^{-1}}$; $f_L^d \approx 1/2$ for $1.5\,\mathrm{m\,s^{-1}} \geq v \geq 1.0\,\mathrm{m\,s^{-1}}$; and $f_L^d \approx 0$ for $1.0\,\mathrm{m\,s^{-1}} \geq v \geq 0\,\mathrm{m\,s^{-1}}$. The transition from one value to the next is smooth and occurs within a full width of about $0.2\,\mathrm{m\,s^{-1}}$. Small variations in the functional form of f_L^d give only very slight changes in the results presented in the next section.

...

B.3 My own considerations

B.3.1 Introduction

The rocks are not running straight in general: Besides slowing down they *curl* to one side. So the force $\vec{F}$ caused by friction rock/ice is not parallel to the speed $\vec{v}$. Neihter it is constant. The most general form is:

$$\vec{F} = \vec{F}(\vec{v}, \omega, \vec{x}, \alpha) \tag{B.17}$$

If ice and rock are homogenous and free of debri it simplifies a little bit:

$$\vec{F} = \vec{F}(\vec{v}, \omega) \tag{B.18}$$

The components of $\vec{F}$ are:

$$\begin{array}{ll} F_y \parallel \vec{v} & \text{force parallel running direction} \\ F_x \perp \vec{v} & \text{force perpendicular to running direction} \end{array} \tag{B.19}$$

B.3.2 Parameters to describe the friction

Statements:

$$\begin{array}{ll} F_x \approx 0 \text{ for } \omega \gg \omega_0 (\approx \frac{2.5\pi}{25s}) & \text{Straight running spinners} \\ F_x \approx 0 \text{ for } \omega \approx 0 & \text{Straight if without handle} \\ F_x \max \text{ for } \omega = \omega_0 & \text{max. curl for a smooth handle} \\ F_y \ll \text{ for } \omega \gg \omega_0 & \text{few friction for spinners} \\ F_y \gg \text{ for } \omega \approx 0 & \text{much friction if without handle} \end{array} \tag{B.20}$$

Practicable parameters to describe the ice might be:

- time hog to tee for a draw-to-tee,
- curl for a draw-to-tee,
- curl for a take-out of defined speed (e.g. 9s hog-to-tee),
- do the rocks slow down quick once they started to or don't they.

B.3.3 Base for $\vec{F}$

The area of contact rock/ice is a fine ring on the rock's bottom. $\vec{F}$ is the sum of the forces affecting this ring. For a given spot $(x,y)^T$ on this ring this force is in general:

$$\vec{f} = \vec{f}(\vec{v}, \omega, x, y) \tag{B.21}$$

Then $\vec{F}$ is:

$$\vec{F} = \int_{R}^{R+\Delta R} \int_{0}^{2\pi} \vec{f}(\vec{v}, \omega, x, y)\, \mathrm{d}\varphi\, \mathrm{d}r \tag{B.22}$$

Bibliography

[BOW76] K. J. Bathe, J. T. Oden, and W. Wunderlich, Editor. *A Finite Element Method For Large Dispacement Contact and Impact Problems, Formulation and Computational Algorithms in Finite Element Analysis*. US-German Symposium on Finite Element Method, 1976.

[Bra92] Brach. *Mechanical impact dynamics: rigid body collisions*. 1992. TuM-Sig: 1/92 A 2875.

[Dan86] J. M. Daniels. *American Journal of Physics*, 54:777, 1986.

[Den98] Mark Denny. Curling rock dynamics. *Canadian Journal of Physics*, 76(4):295–304, April 1998.

[FLS89] Richard Feynman, Robert B. Leighton, and Matthew L. Sands. *Mechanics, Radiation and Heat*, Volume I of *Lectures on Physics*. Addison Wesley, Reading, Massachusetts, commemorative issue edition, 1989.

[GHS95] Dietmar Gross, Werner Hauger, and Walter Schnell. *Technische Mechanik*, Volume 1. Statik. Springer Verlag, fünfte edition, 1995.

[GR] Gammert and Reckling. *Mechanik*.

[Gre85] Walter Greiner. *Mechanik II*, Volume 2 of *Theoretische Physik*. 1985.

[Her95] Heinrich Hertz. Über die Berührung fester elastischer Körper, 1881; über die Berührung fester elastischer Körper und über die Härte, 1882. In Ph. Lenard, Editor, *Gesammelte Werke von Heinrich Hertz*, Volume 1. Leipzig, 1895.

[HNS93] Hills, Novell, and Sackfield. *Mechanics of elastic contacts*. 1993. TuM-Sig: 1/93 A 846.

[HTS+95] T. J. R. Hughes, R. L. Taylor, L. Sackmann, A. Curnier, and W. Kanoknukulchai. A finite element method for a class of contact-impact problems. *Comp. Meth. Appl. Mech. Engng.*, 8:249–276, 1995.

[Joh81] G. W. Johnston. *Can. Aeronaut. Space Journal*, 27:144, 1981.

[Leh85] Theodor Lehmann. *Kinetik*, Volume III of *Elemente der Mechanik*. 1985.

[MV93] Meyberg and Vachenauer. *Höhere Mathematik*, Volume 1. Springer, Berlin, Heidelberg, zweite edition, 1993.

[SNW96] Mark R. A. Shegelski, Ross Niebergall, and Mark A. Walton. The motion of a curling rock. *Canadian Journal of Physics*, 74(9 and 10):663–670, September and October 1996.

[Sut88] Uli Sutor. Curling. Diplomarbeit, Physik Department der TU-München, 1988.

[Van89] T. Vu Van. *Zur Behandlung von Stoßkontaktproblemen mit Reibung unter Verwendung der Finite-Element-Methode*. PHD Thesis, Fachbereich Bauingenieur- und Vermessungswesen der Universität Hannover, 1989. TuM-Sig: 1/90 A 1966.

[VE85] K. Voyenli and E. Eriksen. *American Journal of Physics*, 53:1149, 1985.

[VE86] K. Voyenli and E. Eriksen. *American Journal of Physics*, 54:778, 1986.