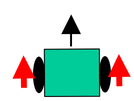
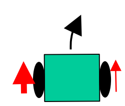
Kinematics of Differential Steering with an Aside about Tricycle Steering

Differential Steering

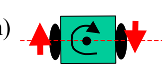
- 2 wheel configuration
- Steer by differential power applied to each wheel
  - Equal power causes movement along a straight line



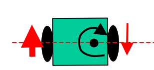
  - Unequal power of same polarity causes a turn



  - Equal power of opposite polarity causes a pivot (spin) around the center point between the wheels



  - Unequal power of opposite polarity causes a pivot about a point along the axle, but not the center (for the boundary case of one wheel with 0 power, the pivot point is the 0 powered wheel)

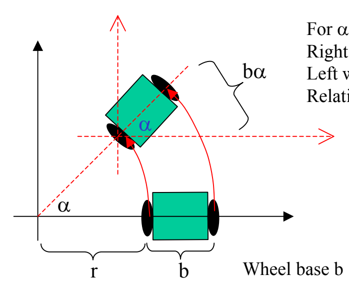


Forward Kinematics

- Kinematics describes the mapping between a model's parameters and the model's configuration (shape, position, or orientation)
- Inverse kinematics is the determination of parameters from model configuration
  - Robot training typically employs this technique so that actions may be repeated
- Forward kinematics is determining the configuration from parameters
  - For robot arms, forward kinematics refers to finding the end position given joint angles for an arm
- Determining the path that will be followed when using differential steering is a forward kinematics problem
  - The parameters are the wheel speed of each wheel
    - Speed is a function of power, but power does not predict wheel speed
    - Simplifying assumptions to reduce complexity
      - Wheel speed is constant (ignore acceleration to reach speed)
      - No wheel slippage or friction effects
      - Constantly applied power reaches constant speed in negligible time

Predictive Model

- Since the robot is a rigid body, the travel of one wheel relative to the other is the same regardless of location of coordinate reference:



For α in radians:
Right wheel distance = $(r + b)\ \alpha$
Left wheel distance = $r\ \alpha$
Relative distance = $b\ \alpha$

- This indicates that the path followed by the axle midpoint will be circular (each wheel having a constant speed)
  - the limiting case of a turn with the left wheel velocity = 0 is obviously circular

Predictive Model(2)

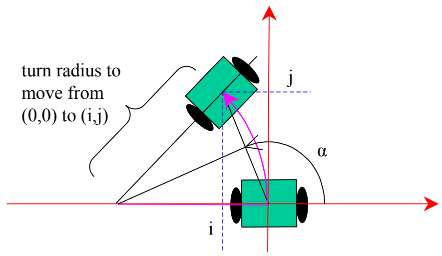
- Let the two wheel velocities be v_1, v_2 , where $v_1 > v_2$
  - Taking α as a function of time
    - Relative distance $b\alpha(t) = (v_1 - v_2)t$
  - $\alpha(t) = \alpha_0 + (v_1 - v_2)t/b$ where $\alpha_0 = \alpha(0)$
- Considering the polar coordinates $(v(t), \alpha(t))$ of the axle center
  - $(dx/dt, dy/dt) = (v(t)\cos(\alpha(t)), v(t)\sin(\alpha(t)))$
  - For constant velocity, velocity of center is $(v_1+v_2)/2$
  - $dx/dt = ((v_1+v_2)/2)\cos(\alpha(t))$
  - $dy/dt = ((v_1+v_2)/2)\sin(\alpha(t))$

Predictive Model(2)

- Integrate
  - → $dx = ((v_1+v_2)/2)\cos(\alpha_0 + (v_1 - v_2)t/b)dt$
  - → $dy = ((v_1+v_2)/2)\sin(\alpha_0 + (v_1 - v_2)t/b)dt$
  - → $x(t) = ((v_1+v_2)/2)(b/(v_1 - v_2))\sin(\alpha_0 + (v_1 - v_2)t/b) + C_x$
  - → $y(t) = (-(v_1+v_2)/2)(b/(v_1 - v_2))\cos(\alpha_0 + (v_1 - v_2)t/b) + C_y$
  - → Assuming $x(0) = x_0$ and $y(0) = y_0$ then
    - $x(t) = x_0 + (b(v_1+v_2)/2(v_1 - v_2))[\sin(\alpha_0 + (v_1 - v_2)t/b) - \sin(\alpha_0)]$
    - $y(t) = y_0 + (-b(v_1+v_2)/2(v_1 - v_2))[\cos(\alpha_0 + (v_1 - v_2)t/b) - \cos(\alpha_0)]$
- Remark
  - → If $v_1 \approx v_2$ the above equations approach a line in the direction of the initial orientation (L'Hospital's rule applies).
    - Case of the robot going straight is the limiting case

Predictive Model (3)

- $b(v_1+v_2)/2(v_1 - v_2))$ gives the turn radius to the center of the robot axle
- Navigate to point (i,j) relative to the axle center (0,0), $\alpha_0 = 0$



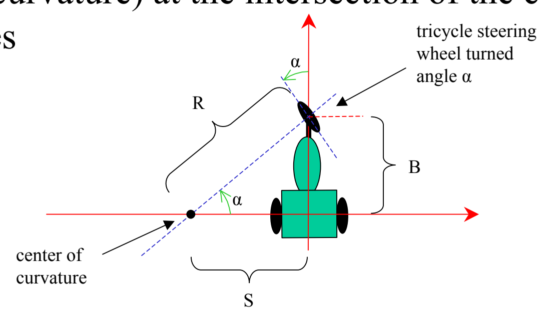
- Fix one of t, v_1 and v_2 to get 2 (non-linear) equations in 2 variables (i, j are knowns, $x_0 = y_0 = \alpha_0 = 0$)
  - $i = (b(v_1+v_2)/2(v_1 - v_2))[\sin((v_1 – v_2)t/b)]$
  - $j = (-b(v_1+v_2)/2(v_1 - v_2))[\cos((v_1 – v_2)t/b) - 1]$

Deliberative Planning

- Simplifying Assumptions
  - Acceleration is not considered
  - Velocity of each wheel is constant
- Wheel axis gives orientation
  - assign coordinates for (i, j) using distance r (sonar) and angle α to the wheel base (direction of sonar relative to the axle).
  - (i, j) = (rcos α, rsin α)
- Planning can dynamically update wheel speeds based on current world view

Tricycle Steering Kinematics

- Tricycle steering kinematics are simpler than differential steering kinematics
  - All 3 wheels to rotate about a point (the center of curvature) at the intersection of the common axes



Tricycle Steering Kinematics (2)

- $B = R \sin \alpha$; $R = B / \sin \alpha$
- $S = R \cos \alpha = B \cos \alpha / \sin \alpha = B \cot \alpha$

  $\rightarrow S = B \tan (\pi/2 - \alpha)$
- For turn angle α
  - the steering wheel follows a circle of radius R
  - center of axle follows a circle of radius S
- Assumption is that powering a wheel to provide velocity does not change the kinematics