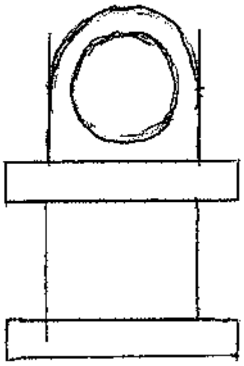
Target weight

$m_T = ?$

$V_T = ?$



$\rho_s = 7800 \frac{kg}{m^3}$

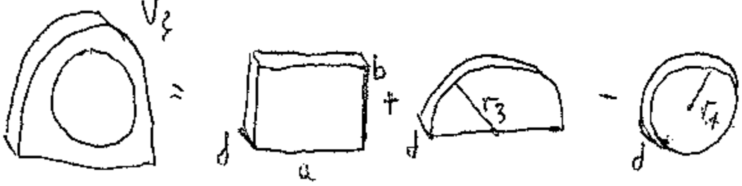
V_1

2x  $r_1 = 30mm$ $h_1 = 10mm$

V_2

1x  $r_2 = 20mm$ $h_2 = 30mm$

V_3

1x 

$d = 10mm$ $a = 40mm$ $b = 20mm$ $r_3 = 20mm$ $r_4 = 15mm$

$$V_1 = \pi r_1^2 h_1 \cdot 2$$

$$V_2 = \pi r_2^2 h_2$$

$$V_3 = a \cdot b \cdot d + \frac{1}{2} \pi r_3^2 d - \pi r_4^2 d$$

$$V_T = (V_1 + V_2 + V_3) \cdot \rho_s = \pi r_1^2 h_1 \cdot 2 + \pi r_2^2 h_2 + abd + \frac{1}{2} \pi r_3^2 d - \pi r_4^2 d =$$

$$= 2\pi (30mm)^2 (10mm) + \pi (20mm)^2 (30mm) + (10mm) \cdot (40mm) \cdot (20mm) + \frac{1}{2}\pi (20mm)^2 (10mm) - \pi (15mm)^2 (10mm) =$$

$$\approx 56548.6 mm^3 + 37699.1 mm^3 + 8000 mm^3 + 6283.2 mm^3 - 7068.6 mm^3$$

$$V_T \approx 101462.3 mm^3 = 0.0001014623 m^3$$

$m_T = V_T \cdot \rho_s = 0.79141 kg$ $\quad M_T \approx 0.8 kg$ (round up)

The target weighs 791 g which for the sake of calculations I will round it up to 800g. This will also add extra margin.

Robot weight and carry capacity

1 CPU : 0.5kg

2 Batteries $2 \cdot 1kg = 2kg$

2 Motors $2 \cdot 0.7kg = 1.4kg$

2 Base / Platform mass $2 \cdot 0.2kg = 0.4kg$

2 Wheels $2 \cdot 0.2kg = 0.4kg$

2 castor wheels

$r = 15mm$

$m = \rho_A \cdot V_w = 2700 \frac{kg}{m^3} \cdot \pi (15mm)^2 \cdot 10mm =$

$\approx 0.02kg$

I will estimate the castor wheel chassis to be 0.03kg.

so 1 wheel will be 0.05kg

$2 \cdot 0.05kg = 0.1kg$

4 Steel rods mass

$r = 5mm$

$h: 190mm$

$m = \rho_s \cdot V_r \cdot 4 = 7800 \frac{kg}{m^3} \cdot \pi (5mm)^2 \cdot 190mm =$

$\approx 0.085kg \cdot 4 \approx 0.34\ kg$

$M = 0.5kg + 2kg + 1.4kg + 0.4kg + 0.4kg + 0.1kg + 0.34kg = \underline{5.14kg}$

$M \approx 5.2\ kg$

I round it up because its better to overestimate its weight than to underestimate it.

Motor torque $= 3Nm$

Wheel radius $= 75mm = 0.075m$

$T = r \cdot F_m$

motor $F_m = \frac{T}{r}$

$F_m = \frac{3Nm}{0.075m} = 40N$

because we have 2 wheels

total $F_T = 80N$

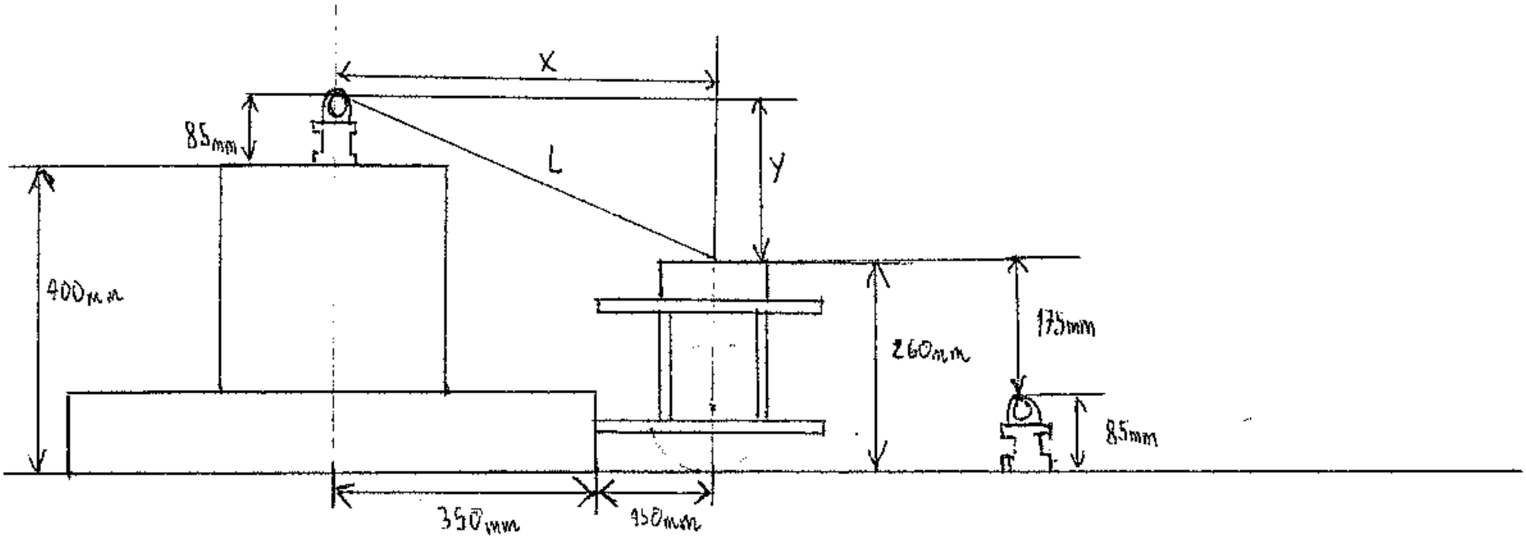
$F = m \cdot a$, $F = m \cdot g$ $m = \frac{F}{g}$

$m = \frac{80N}{9.81 \frac{m}{s^2}} = 8.15\ kg$

$8.15\ kg - 5.2kg = \underline{2.95\ kg}$

The robot can carry additional 2.95 kg

Assuming target weight of 800g we have $\underline{2.15\ kg}$ to build the arm



$Y = 400mm + 85mm - 260mm = 225mm$

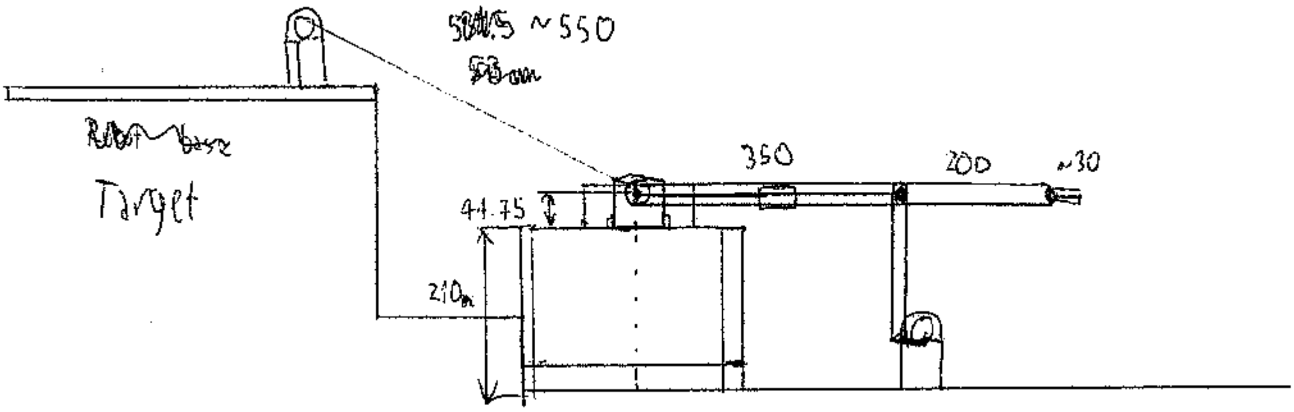
$X = 350mm + 150mm = 500mm$

$L = \sqrt{X^2 + Y^2} = \sqrt{(500mm)^2 + (225mm)^2} = \sqrt{300625 \, mm^2} \approx 548.3mm$

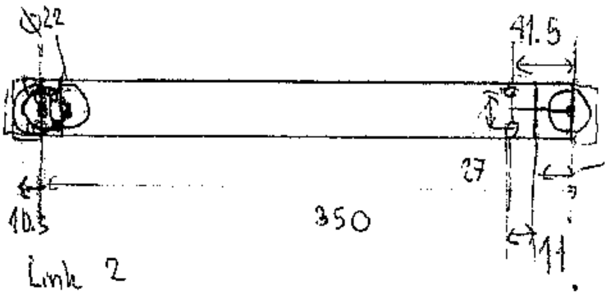
The arm should be able to place the target on top of the destination with a length of 55cm if it was attached directly on top of the CPU and in a straight line. This means that depending on the mounting system the arm can be even shorter.

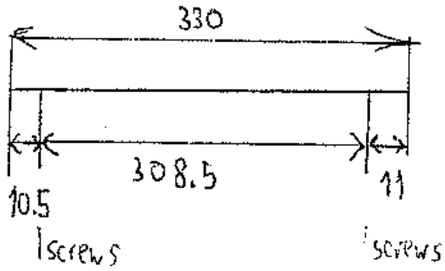
If link first link is horizontal the second link has to be 175mm long to reach the target on the floor

Design of arm (mm)

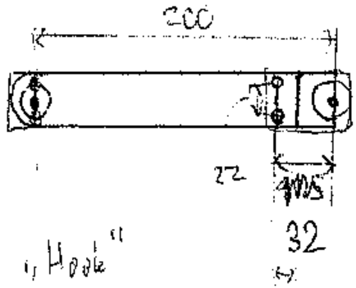


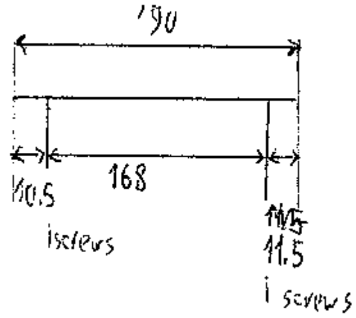
Link 1 2 Beams on either side of the servo



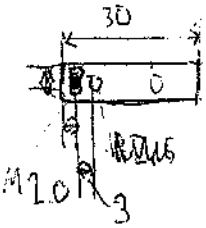


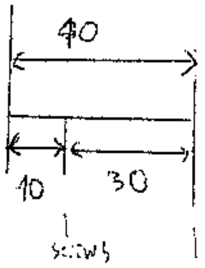
Link 2



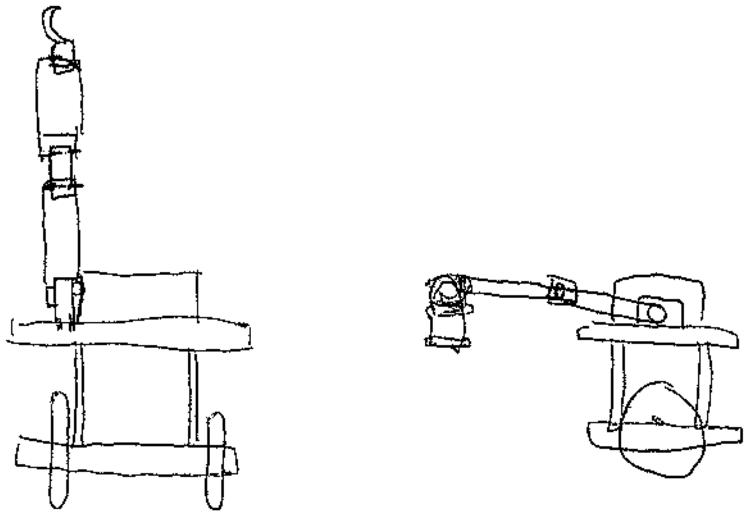


„Hook"





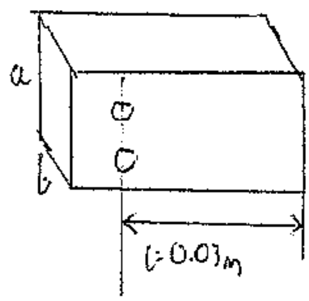
The design idea



Limitations : - Robot carry weight (2.45kg)

- Arm might put robot off balance
- The robot has to reach the ~~floor and the~~ target on the floor and on the destination
- The arm is 2 link with a controlled "hook" at the end, so the motors have to be power-full enough to lift 800g on a 55cm rod and support their weight

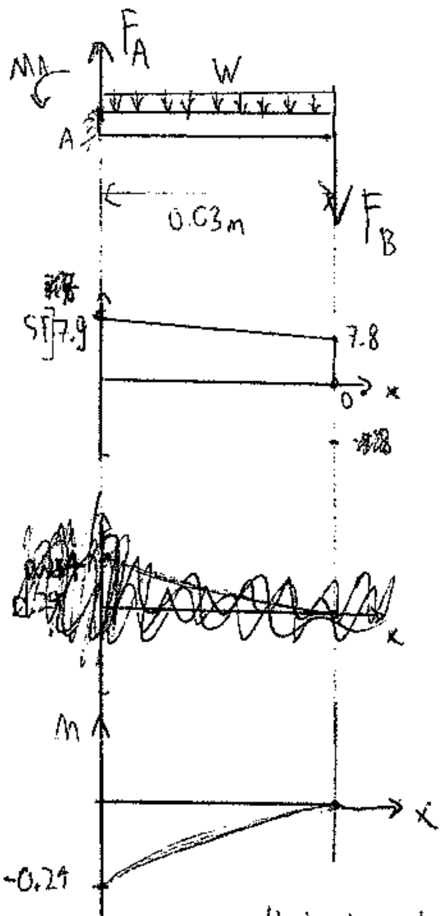
„Hook"



a = beam height , b = beam width , L = beam length
ρ = beam material density (ABS) $\rho_{ABS} = 1040 \frac{kg}{m^3}$

$a = 0.015\,m$ $\quad b = 0.015\,m$

The right side of the screws will not experience any significant loads.
The right side will have most of the loads. The screw position will be a fixed end, since there is 2 screws. The load will be at the end to account for the worst case (0.8kg target)



$F_B = 0.8\,kg \cdot 9.81 \frac{m}{s^2} = \underline{7.8\,N}$

$W = \frac{m \cdot g}{l} = \frac{a \cdot b \cdot l \cdot \rho \cdot g}{L} = ab\rho g$ $\quad W = 0.015 \cdot 0.015 \cdot 1040 \cdot 9.81 =$

$W = 2.3 (\frac{N}{m})$

$F_A + Wl + F_B = 0$

$F_A = 7.8N + 0.069\,N \approx 7.9N \approx \underline{7.9\,N}$

$M_A - W\frac{1}{2}l^2 - F_B l = 0 \quad M_A = 0.0345 + 0.234$

$M_A \approx 0.24\,Nm$

SF between A and B at B $F = F_A - 2.3 \cdot 0.03 = 0.069 \approx 0.1$

M at A $\quad M_A = 7.8N \cdot 0.03\,m + 2.3\frac{N}{m} \cdot 0.03\,m \cdot 0.015\,m = 0.234 + 0.001035\,Nm$

$M_A \approx 0.24\,Nm$ $\quad M_B = 0$

Highest bending moment is at the screw location $M = 0.24\,Nm$

$\underline{M = 0.24\,Nm}$

Stress in „Hook"

$$\frac{M}{J} = \frac{\sigma}{y} = \frac{E}{R} \qquad \sigma = \frac{My}{J}$$

$$y = \frac{1}{2}a \,, \quad J = \frac{ba^3}{12} \qquad M = 0.24\,Nm \qquad a = b = 0.015\,m$$

for ABS:
$$\sigma_y = 43\,MPa = 43000000\,Pa$$

$$\sigma = \cancel{\frac{0.234\,Nm \cdot 0.0075\,m}{(0.015)^4 \cdot \frac{1}{12}}} = \frac{M \cdot \frac{1}{2}a}{\frac{b \cdot a^3}{12}} = \frac{M \cdot 6}{a^3} = \frac{0.24\,Nm \cdot 6}{(0.015\,m)^3} =$$

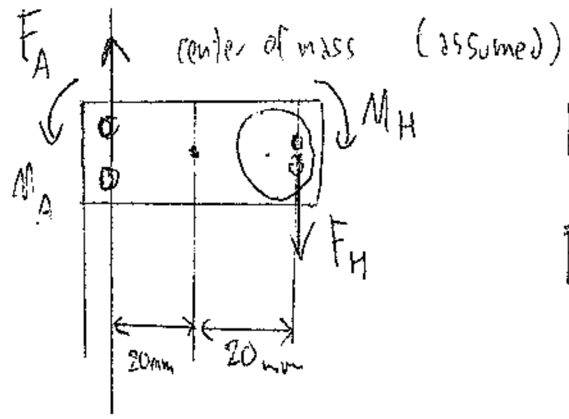
$$= 426\,666\,\frac{N}{m^2} = \underline{426\,666\,Pa} \qquad \tau = \frac{F}{A} = \frac{7.9\,N}{(0.015\,m)^2} \approx \underline{35\,111\,Pa}$$

$$\frac{\sigma + \tau}{\sigma_y} = \frac{426\,666\,Pa + 35111\,Pa}{43000000\,Pa} \approx 0.01 \qquad \frac{\sigma_y}{\sigma + \tau} = 93$$

~~This~~ The ~~yield~~ stress to yield stress ratio is ~~about~~ ~~100~~ ~90 which means the hook should withstand the stress

Assuming the screw holes

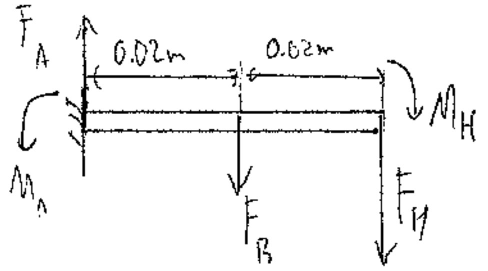
Forces in Hook to second limb servo



$m = 82g$

F_H and M_H - force and moment from the hook

F_A and M_A : ~~forces~~ o Reaction forces on the second limb



F_B : force from servo mass (assumed at the centre

$F_B = 0.082\,kg \cdot 9.81\,\frac{m}{s^2} = 0.8N$

$F_H = 7.9N$ $\quad M_H = 0.24\,Nm$

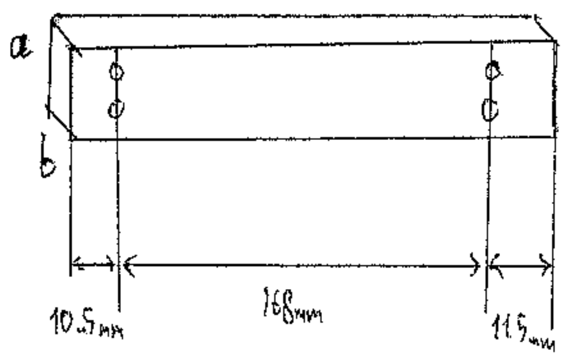
$F_A = F_B + F_H$ $\quad F_A = 8.7N$

$M_A + M_H + F_B \cdot 0.02 + F_H \cdot 0.04 = 0$

$M_A = 0.6\,Nm$

The servo is a dynamixel XM 430-W350 servo with 4.1 Nm stall torque. Since $M_H < 4.1\,Nm$ the servo should work fine.

Second limb



$a = 33.5mm = 0.0335m$

$b = 15mm = 0.015m$

$l = 168 + 11.5\ (mm) = 179.5mm = 0.1795m$

$\rho_{ABS} = 1040\ \frac{kg}{m^3}$

F_B - force from the hook including the target and servo

M_B - Moment from hook with target and servo

~~F_B = (hook mass + servo mass + limit mass)·g~~

~~F_B = (1040·0.015·0.03·0.90) + 0.082 + 0.8) kg · 9.81 m/s² =~~

~~= 8.7N~~

~~M_B = hook mass · distance to centre + target mass · distance + servo mass · distance to centre~~

~~M_B = (0.015·0.015·0.05·1040) g~~

$F_B = 8.7N \qquad M_B = 0.6\ Nm$

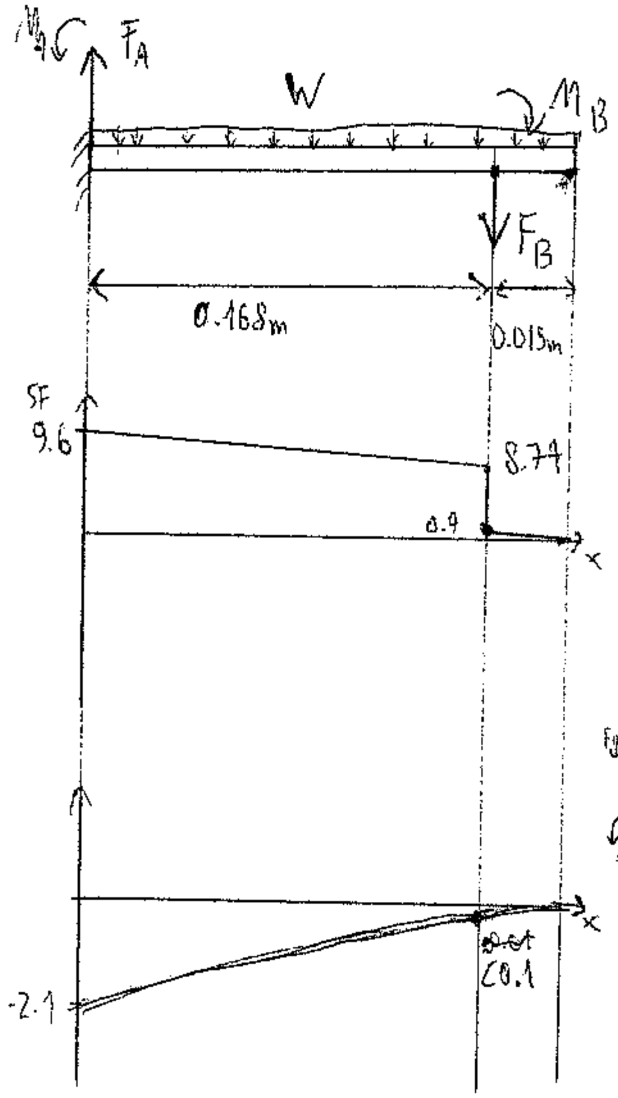
$W = ab\rho g = 0.0335 \cdot 0.015 \cdot 1040 \cdot 9.81 = \underline{5.1\ \left(\frac{N}{m}\right)}$

$\Sigma F_y \quad F_A + F_B + Wl = 0 \qquad -F_A = 8.7 + 5.1 \cdot 0.1795 = \underline{9.6N}$

$\Sigma M_A \quad M_A + W\frac{1}{2}l^2 + F_B \cdot 0.168 + M_B = 0$

$M_A = 5.1 \cdot \frac{1}{2} \cdot (0.1795)^2 + 8.7 \cdot 0.168 + 0.6 = \underline{2.1\ Nm}$

$\underline{M_{max} = 2.1\ Nm}$



Second limit stress

$$\sigma = \frac{My}{J} \qquad y = \frac{1}{2}a \qquad J = \frac{ba^3}{12} \qquad M = 2.1\,Nm$$

$$a = 0.0335\,m \qquad b = 0.015\,m \qquad \sigma_y = 43000000\,Pa$$

$$\sigma = \frac{M \cdot \frac{1}{2}a}{\frac{b \cdot a^3}{12}} = \frac{6M}{a^2 b} = \frac{6 \cdot 2.1\,Nm}{(0.0335\,m)^2 \cdot 0.015\,m} = \underline{748\,496\,Pa}$$

$$\tau = \frac{F_a}{ab} = \frac{96\,N}{0.0335\,m \cdot 0.015\,m} = \underline{19104.5\,Pa}$$

$$\frac{\sigma_y}{\sigma + \tau} = \frac{43000000\,Pa}{748\,496\,Pa + 19104.5\,Pa} = 56$$

The yield stress to stress ratio is 56, so the stress needs to be 56 times larger to break the material. This leaves enough room to add screw holes without compromising the integrity.

Second limb deflection

$L = 168mm$

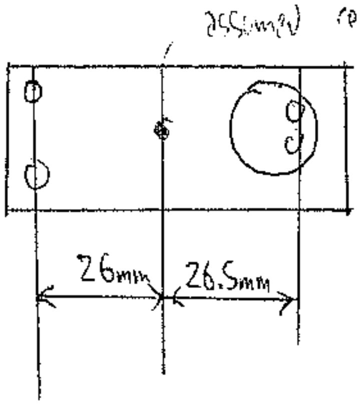
$$y_{max} = \frac{FL^3}{3EJ} + \frac{wL^4}{8EJ} + \frac{ML^2}{2EJ}$$

$$y_{max} = \frac{8FL^3 + 3wL^4 + 12ML^2}{24EJ} = \frac{L^2(8F_BL + 3wL^2 + 12M_B)}{24E\cdot\frac{ba^3}{12}} =$$

$$= \frac{0.168^2(8\cdot 8.7\cdot 0.168 + 3\cdot 5.1\cdot 0.168^2 + 12\cdot 0.6)}{24\cdot 2.3\cdot 10^9\cdot\frac{0.0335^3\cdot 0.015}{12}} = \frac{0.545}{2594.1} =$$

$$= 0.0002m \sim 0.2mm$$

Forces in Second limb to first limb servo

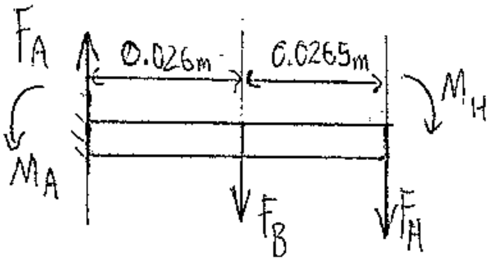


$m = 165g$

$F_B = 0.165 \cdot 9.81 = 1.6N$ mass

$F_H = 9.6N \qquad M_H = 2.1Nm$

$F_\Sigma \quad F_A + F_B + F_H = 0 \qquad F_A = 1.6N + 9.6N = \underline{11.2N}$



$\circlearrowright_A \quad M_A + M_H + F_H \cdot (0.0265 + 0.026)_m + F_B \cdot 0.026_m = 0$

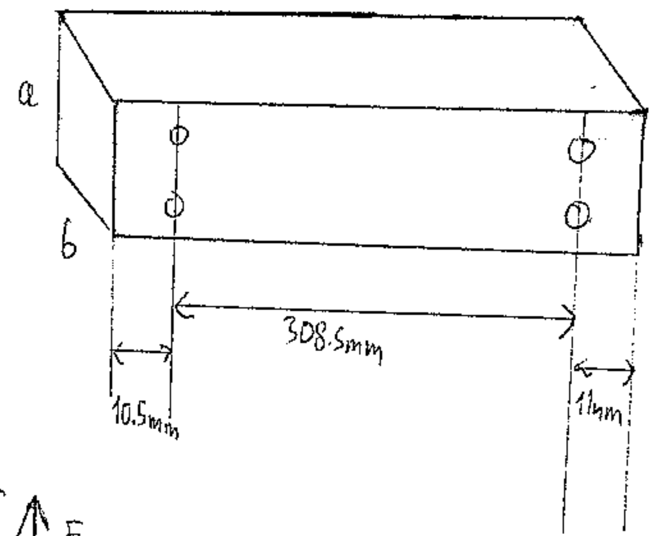
$M_A = \underline{2.6\,Nm}$

The servo is a dynamixel XM 540 - W270 - T/R

It has a stall torque of 10.6 Nm, so it should have no issues operating.

First limb

This limb will be made out of 2 equal beams, so the forces will be half.



$a = 33.5mm = 0.0335m$ $\rho_{ABS} = 1090 \frac{kg}{m^3}$

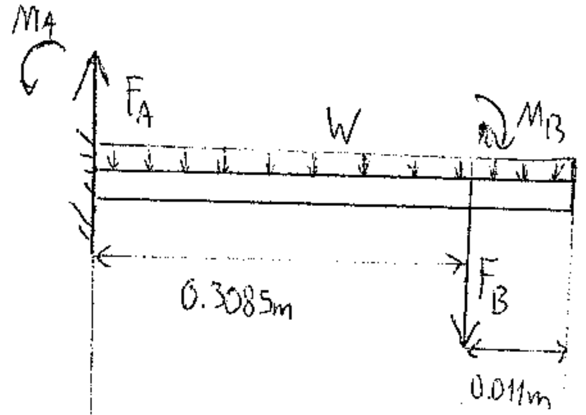
$b = \cancel{20}mm =$ ~~[illegible]~~ $0.02m$

$l = 319.5mm = 0.3195m$

$F_B = \frac{1}{2}$ previous force $= \frac{1}{2} \cdot 11.2N = 5.6N$

$M_B = \frac{1}{2}$ previous moment $= \frac{1}{2} \cdot 2.6Nm = 1.3Nm$

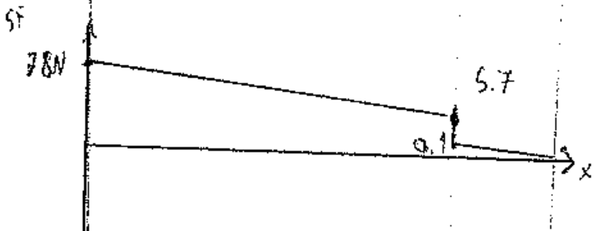
$W = a b \rho g = 6.8 \frac{N}{m}$



F_y $F_A + Wl - F_B = 0$ $F_A = Wl - F_B = \underline{7.8N}$

$M_A + W\frac{1}{2}l^2 + F_B 0.3085m + M_B = 0$

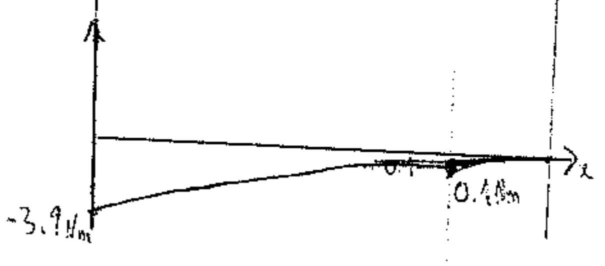
$\underline{M_A = 3.4Nm}$



$0.3085m \cdot 6.8\frac{N}{m} = 2.1N$ $7.8 - 2.1 = 5.7$

$0.3085^2 \cdot 6.8 \cdot \frac{1}{2} = 0.3\ (Nm)$ $0.3085 \cdot 5.6 = 1.7$

$3.4 - 0.3 - 1.3 = 1.7 =$



First limb stress

$$\sigma = \frac{My}{J} \quad y = \frac{1}{2}a \quad J = \frac{ba^3}{12} \quad M = 3.9\ Nm$$

$a = 0.0335m \quad b = 0.02m \quad \rho_{ABS} = 1040\ \frac{kg}{m^3} \quad \sigma_y = 93000000\ Pa$

$$\sigma = \frac{M \cdot \frac{1}{2}a}{\frac{ba^3}{12}} = \frac{6M}{a^2 b} = \frac{6 \cdot 3.4\ Nm}{(0.0335m)^2 \cdot 0.02m} = \underline{908888\ Pa}$$

$$\tau = \frac{F}{ab} = \underline{11642\ Pa}$$

$$\frac{\sigma_y}{\sigma + \tau} = \frac{93000000 Pa}{908888\ Pa + 11642 Pa} = 47$$

The yield stress to stress ratio is 47. ~~Even~~ Remembering that there is two ~~...~~ ~~this~~ ~~should~~ screw holes, this extra should be enough.

Limb 1 deflection

$L = 308.5mm$

$$y_{max} = \frac{FL^3}{3EJ} + \frac{WL^4}{8EJ} + \frac{ML^2}{2EJ}$$

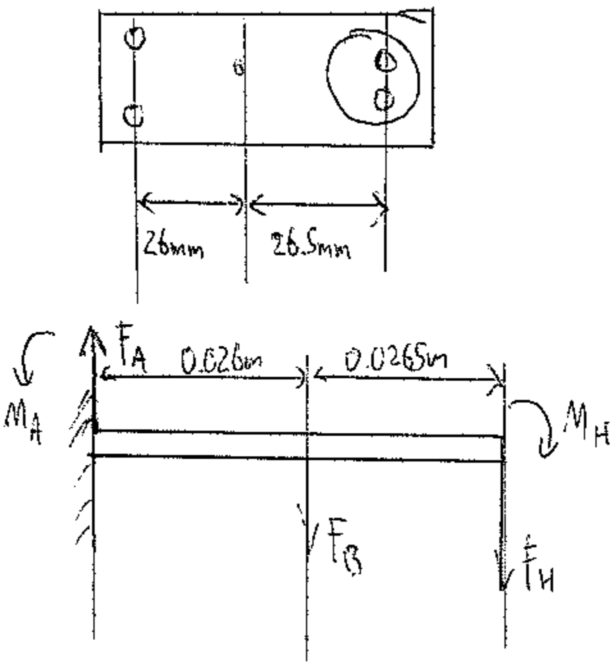
$$\cancel{y_{max} = \frac{8FL^3 + 3WL^4 + 12ML^2}{24 \; EJ}} \; 8 +$$

$$y_{max} = \frac{8FL^3 + 3WL^4 + 12ML^2}{24} = \frac{L^2\,(8F_B L + 3WL^2 + 12M_B)}{24 \cdot EJ} =$$

$$= \frac{0.3085^2\,(8 \cdot 7.8 \cdot 0.3085 + 3 \cdot 6.8 \cdot 0.3085^2 + 12 \cdot 1.3)}{24 \cdot 2.3 \cdot 10^9 \cdot \frac{0.02 \cdot 0.0335^3}{12}} = \frac{3.502}{3458.77} =$$

$$= 0.00101m \approx 1mm$$

First limb to mounting bracket forces



$m = 165g$

$F_B = 0.165 \cdot 9.81 = 1.6 (N)$ - mass

$F_H = 2 \cdot$ first limb force $= 2 \cdot 7.8 N = 15.6 N$

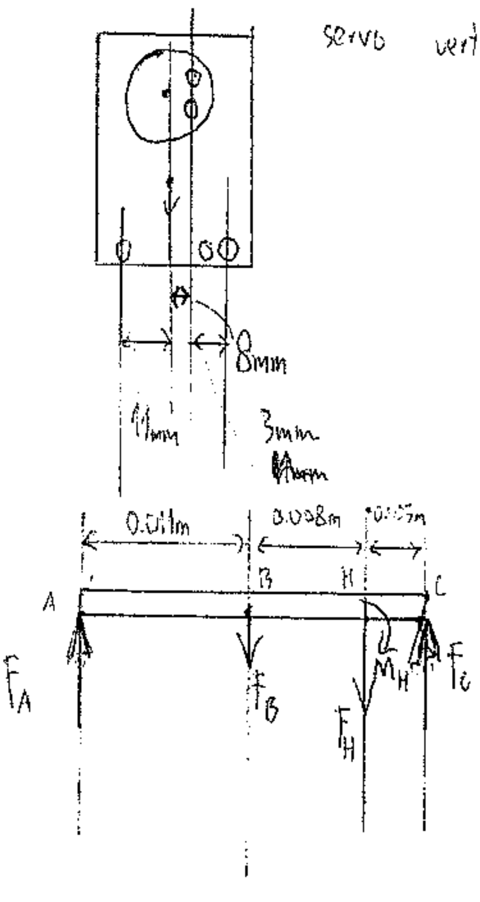
$M_H = 2 \cdot$ first limb moment $= 2 \cdot 3.4 Nm = 6.8 Nm$

$\uparrow \quad F_A + F_B + F_H = 0 \quad F_A = 1.6N + 15.6N = \underline{17.2 N}$

$\circlearrowleft \quad M_A + F_B \cdot 0.026m + F_H (0.0265 + 0.026m) + M_H = 0$

$\underline{M_A = 7.7 Nm}$

servo vertically

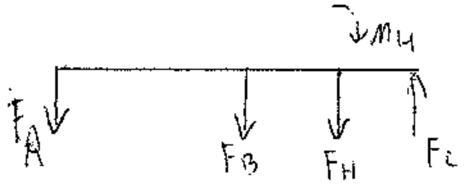


$\circlearrowleft_C \quad M_H - F_H \cdot 0.003 - F_B \cdot 0.011 + F_A \cdot 0.022 = 0$

$F_A = \dfrac{F_H \cdot 0.003 + F_B \cdot 0.011 - M_H}{0.022} = \underline{-306.1} \ (N)$

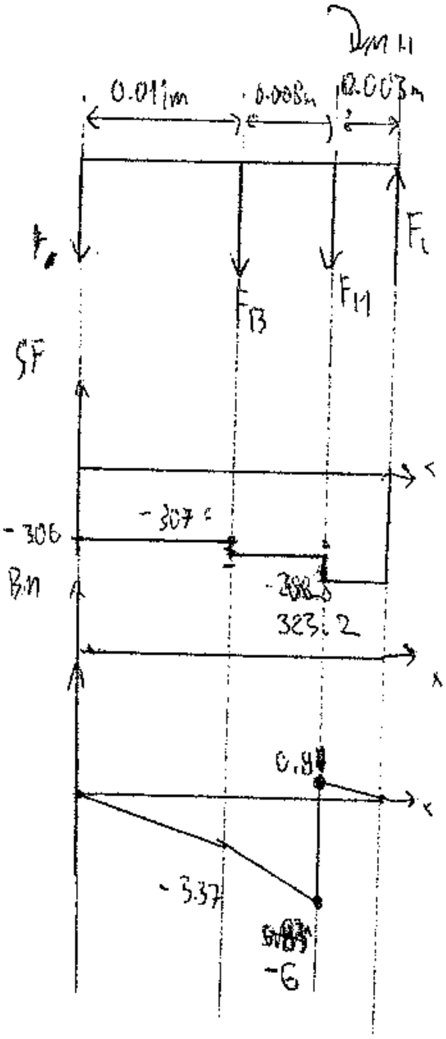
$\uparrow \quad F_A - F_B - F_H + F_C = 0 \qquad F_C = F_A - F_B - F_H$

$F_C = \underline{323.2} \ (N)$



- actual direction of forces

$F_C = 323.2 N \quad , \quad F_A = 306 N$



$F_A = 306\ N$
$F_B = 1.6N$
$F_H = 15.6N$
$F_C = 323.2\ N$

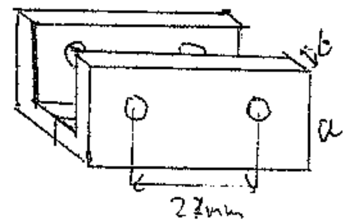
$-306 \cdot 0.011 = -3.37\ (Nm)$
$-323.2 \cdot 0.008 = -2.59\ (Nm)$
$-3.37 - 2.59 = -5.96\ (Nm) \approx 6\ Nm$
$-6 + 6.8 = 0.8\ Nm$

These forces apply to the bracket

$M_{Max} = -6\ Nm$
$F_{max} = 323.2\ N$

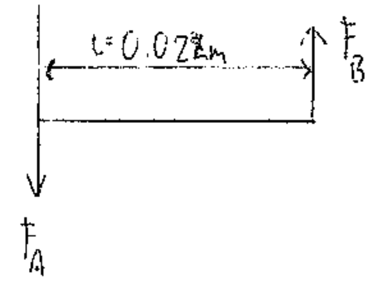
The servo is also a dynamixel XM540-W270-T/R, so it should operate with no issues.
since $10.6\ Nm > 6.8\ Nm$

bracket holding first servo attached to robot



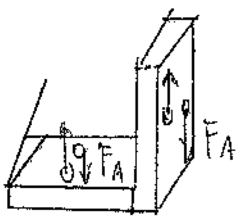
two identical brackets on either side i
each will carry half the load

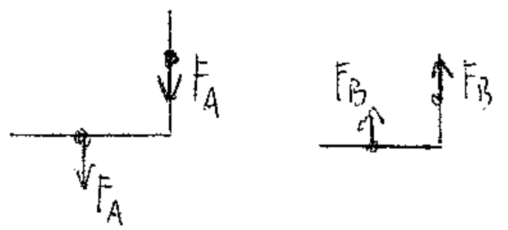
$$F_A = \frac{1}{2}\cdot 323.2\,N = 161.6\,N$$

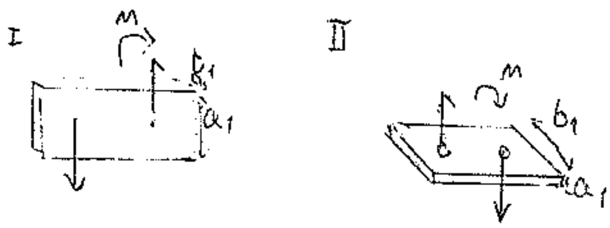


$$F_B = \frac{1}{2}\cdot 306\,N = 153\,N$$

For I will assume the forces on the side of the bracket will be identical as those on the bottom of the bracket acting on the screws







$\sigma_b = \frac{My}{J}$ $\quad y = \frac{1}{2}a$ $\quad J = \frac{ba^3}{12}$ $\quad M = \frac{1}{2}\cdot 6\,Nm = 3\,Nm$

$a = 1mm$ ~~[illegible]~~ $\quad b_1 = 2mm \quad a_1 = 10mm \quad a_2 = 1mm \quad b_2 = 10mm$

$$\sigma_1 = \frac{6M}{a_1^2 b_1} = \frac{6\cdot 3Nm}{(0.01m)^2\cdot 0.002m} = 90\,MPa \qquad \sigma_{ys} = 250\,MPa$$ – too much stress

$a_1 = 20mm \quad b_1 = 6mm \qquad a_2 = 6mm \quad b_2 = 20mm$

$\sigma_1 = 7.5\,MPa \qquad \sigma_2 = \frac{6M}{a_2^2 b_2^2} = 50\,MPa$ - too high

$a_2 = 10mm \quad b_2 = 20mm$

$$\sigma_2 = \frac{6M}{a_2^2\cdot b_2} = 9\,MPa$$

$F_{max} = 323.2\cdot\frac{1}{2} = 161.6 \qquad \tau_1 = \frac{F_{max}}{a_1 b_1} = 1.3\,MPa$

$$\tau_2 = \frac{F_{max}}{a_2 b_2} = 0.8\,MPa$$

Bracket stress

$\sigma_1 = 7.5MPa$ $\tau_1 = 1.3MPa$ $\sigma_{ys} = 250MPa$

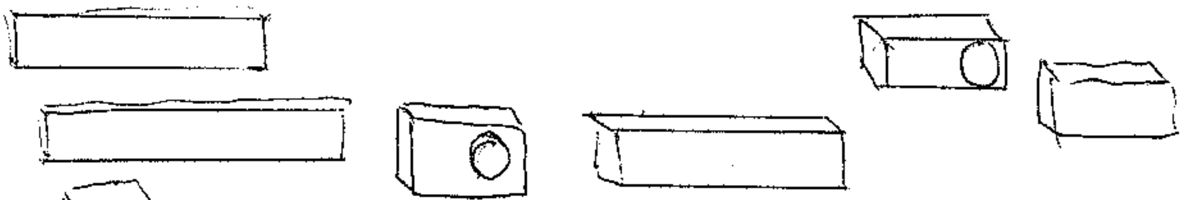
$$\frac{\sigma_y}{\sigma_1 + \tau_1} = \frac{250\,MPa}{7.5MPa + 1.3MPa} = 28.4$$

$\sigma_2 = 9MPa$ $\tau_2 = 0.8MPa$

$$\frac{\sigma_y}{\sigma_2 + \tau_2} = \frac{250MPa}{9MPa + 0.8MPa} = 25.5 \approx 26$$

yield stress to stress ratio in the bracket sections of 26 and 28 should be sufficient.

Arm mass



Weight of screws is negligible

Brackets $m_B = 2\cdot(0.02\cdot 0.006\cdot 0.022\cdot 7800 + 0.01\cdot 0.02\cdot 0.022\cdot 7800) = 0.11\ kg$

Servo 1 = 165 g = 0.165 kg

Limb 1 $m_{L1} = 2(0.0335\cdot 0.02\cdot 0.3195\cdot 1090) = 0.95\ kg$

Servo 2 = 165 g = 0.165 kg

Limb 2 $m_{L2} = 0.0335\cdot 0.015\cdot 0.1795\cdot 1090 = 0.1\ kg$

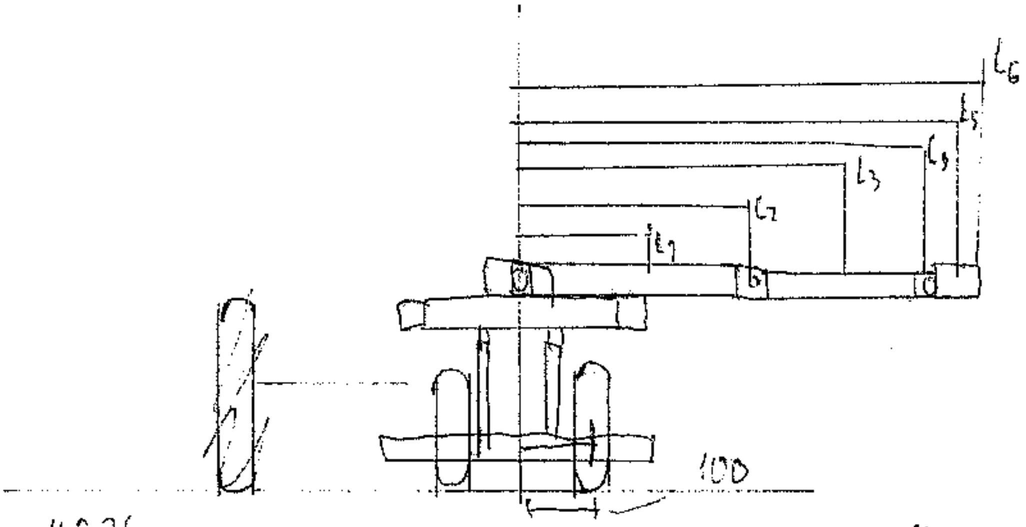
Servo 3 = 82 g = 0.082 kg

Limb 3 (hook) $m_H = 0.015\cdot 0.015\cdot 0.04\cdot 1090 = 0.01\ kg$

$$\Sigma M = 0.11 + 0.165 + 0.95 + 0.165 + 0.1 + 0.082 + 0.01 = \underline{1.1\ kg}$$

which is less than the maximum 2.15 kg meaning the robot can still move

Robot centre of mass



$L_1 = 162.25$

$L_2 = 392.5$

$L_3 = 455$

$L_4 = 557$

$L_5 = 592$

$L_6 = 607$

$M_1 = 0.95$

$M_2 = 0.165$

$M_3 = 0.1$

$M_4 = 0.082$

$M_5 = 0.09$

$M_6 = 0.8$

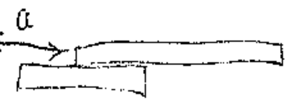
$M_R = 5.2$

$L_R = 0$

tipping point $x = \frac{\Sigma (m \cdot L)}{\Sigma m} = \frac{700}{6.8} = \underline{103 \text{ mm}}$

which is past the wheel, so the robot will fall. to prevent this I will add a counterweight

$$\frac{700 + (M \cdot a)}{6.8 + M} < 100 \quad \text{where } a < 0$$



$$700 - 100 \cdot 6.8 < (100 - a) M$$

$$20 < (100 - a) M$$

if $a = -130$, $M = 0.5$

$$x = \frac{700 + 0.5 \cdot (-130)}{7.3} = \underline{87 \text{ mm}}$$

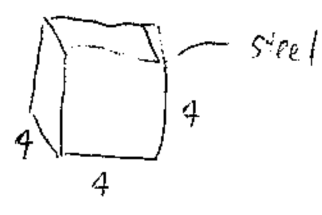
which is within the robot tipping point.

The counterweight will be just a steel block (cube) glued to the robot 130mm from the centre.
It shouldn't experience any forces and thus stay in place

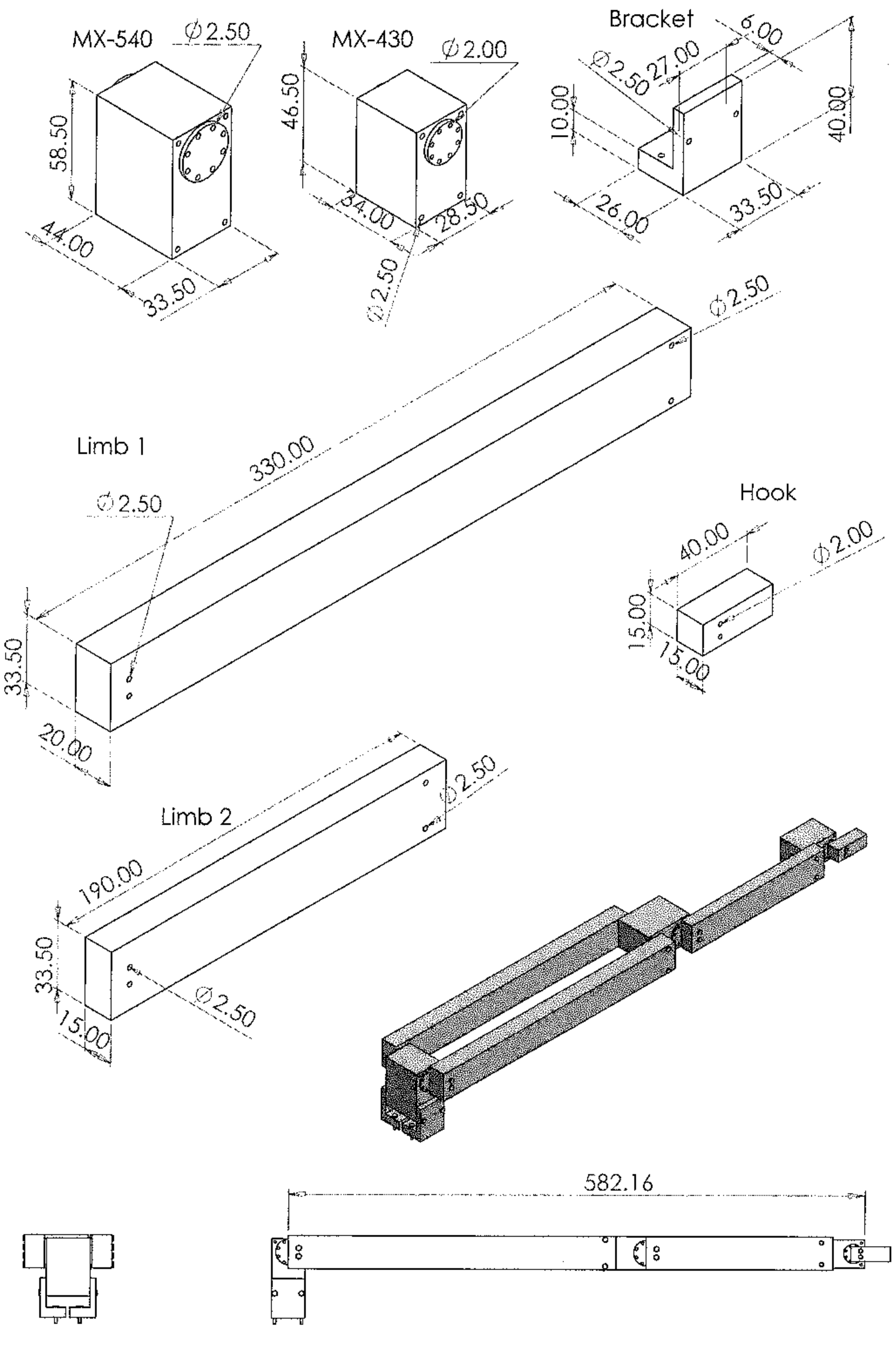
Cube dimentions:

$m = V \cdot \rho_s \qquad V = \frac{m}{\rho_s} \qquad a^3 = \frac{m}{\rho_s} \qquad a = \sqrt[3]{\frac{m}{\rho_s}}$

$$a = \sqrt[3]{\frac{0.5}{7800}} = 0.04\text{m} = 4\text{cm}$$



The counterweight will ~~bring~~ increase the robot weight by 0.5kg, leaving the extra carry weight to be 0.65 kg, so the robot should still be able to fully move.


MX-540
Ø2.50
58.50
44.00
33.50
MX-430
Ø2.00
46.50
34.00
28.50
Ø2.50
Bracket
6.00
Ø2.50
27.00
10.00
40.00
26.00
33.50
Ø2.50
Limb 1
330.00
Ø2.50
33.50
20.00
Hook
40.00
Ø2.00
15.00
15.00
Ø2.50
Limb 2
190.00
33.50
15.00
Ø2.50
582.16