Binomial Theorum

Power	no. of terms
1	2
2	3
3	4
4	5
5	6
n	n+1

→ Introduction

Expand the following expression

$(1+x)^1 = 1+x$

$(1+x)^2 = 1+2x+x^2$

$(1+x)^3 = (1+x)^2(1+x) = 1+3x+3x^2+x^3$

$(1+x)^4 = (1+x)^2(1+x)^2 =$

$(1+x)^9 \rightarrow$ cant do this in the way we know

$$^nC_r (a)^{n-r} b^r$$

Expansion Examples

1) $(1+x)^4$ $\qquad (a+b)^n = {}^nC_r (a)^{n-r} b^r$

$r=0 \quad {}^4C_0 (1)^4 (x)^0 = 1(1)(1) = 1$

$r=1 \quad {}^4C_1 (1)^3 (x)^1 = 4(1)(x) = 4x$

$r=2 \quad {}^4C_2 (1)^2 (x)^2 = 6(1)(x)^2 = 6x^2$

$r=3 \quad {}^4C_3 (1)^1 (x)^3 = 4(1)(x)^3 = 4x^3$

$r=4 \quad {}^4C_4 (1)^0 (x)^4 = 1(1)(x)^4 = x^4$

$$\boxed{(1+x)^4 = 1+4x+6x^2+4x^3+x^4}$$

$2 = (1-2x)^5$

$r=0 \quad {}^5C_0 (1)^5 (-2x)^0 \rightarrow 1(1)(1) = 1$

$r=1 \quad {}^5C_1 (1)^4 (-2x)^1 \rightarrow 5(1)(-2x) = -10x$

$r=2 \quad {}^5C_2 (1)^3 (-2x)^2 \rightarrow 10(1)(4x^2) = 40x^2$

$r=3 \quad {}^5C_3 (1)^2 (-2x)^3 \rightarrow 10(1)(-8x^3) = -80x^3$

${}^5C_4 (1)^1 (-2x)^4 \rightarrow 5(1)(16x^4) = 80x^4$

${}^5C_5 (1)^0 (-2x)^5 \rightarrow 1(1)(-32x^5) = -32x^5$

$$(1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

→ Finding the coefficient of x if:

(a) x is the second term

Q. Find the coefficient of x^2 & x^5 in the expansion of $(1-2x)^5$

$(1-2x)^5$

$r=0 \quad {}^5C_0 (1)^5 (-2x)^0 \rightarrow 1(1)(1) = 1$

$r=1 \quad {}^5C_1 (1)^4 (-2x)^1 \rightarrow 5(1)(-2x) = -10x$

$r=2 \quad {}^5C_2 (1)^3 (-2x)^2 \rightarrow 10(1)(4x^2) = 40x^2$

$r=3 \quad {}^5C_3 (1)^2 (-2x)^3 \rightarrow 10(1)(-8x^3) = -80x^3$

${}^5C_4 (1)^1 (-2x)^4 \rightarrow 5(1)(16x^4) = 80x^4$

${}^5C_5 (1)^0 (-2x)^5 \rightarrow 1(1)(-32x^5) = -32x^5$

$$(1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

$x^2 : r = 2 \quad \& \quad x^5 : r = 5$

$r=2 \quad {}^5C_2 (1)^3 (-2x)^2 = 40x^2 \rightarrow \boxed{40}$

$r=5 \quad {}^5C_5 (1)^0 (-2x)^5 = -32x^5 \rightarrow \boxed{-32}$

Note: When x is the second term, the value of r is equal to the required power of x

(ii) $(3-2x)^5$

$r=2 \Rightarrow x^2$ $\quad$ $r=5 \rightarrow x^5$

$^5C_2(3)^3(-2x)^2 \rightarrow 1080x^2 \rightarrow 1080$

$^5C_5(-2x)^5 \rightarrow -32x \rightarrow -32$

(iv) $(3-\frac{1}{2}x)^9$ $\quad$ x^2 $\quad$ x^5

$^9C_2(3)^7(-\frac{1}{2}x)^2 \rightarrow 19683x^2 \rightarrow \boxed{19683}$

$^9C_5(3)^4(-\frac{1}{2}x)^5 \rightarrow -318.93x^5 \rightarrow \boxed{-318.9\cancel{35}}$

→ Finding the coefficient of x if:

x is the first term

Q Find the coefficient of x^2 & x^5 in the expansion of

(i) $(x-2)^5$

$r=0$ $\quad$ $^5C_0(x)^5(-2)^0 \rightarrow x^5 \longrightarrow x^5 = \boxed{1}$

$^5C_1(x)^4(-2)^1 \rightarrow -10x^4$

$^5C_2(x)^3(-2)^2 \rightarrow 40x^3$

$^5C_3(x)^2(-2)^3 \rightarrow -80x^2 \longrightarrow -80x^2 = \boxed{-80}$

$^5C_4(x)(-2)^4 \rightarrow 80x$

$^5C_5(x)^0(-2)^5 \rightarrow -32$

Expansion of ${}^{n}C_r$ without a calculator

$^{5}C_0 = 1$ regardless the value of n if $r = 0$ the value will be 1

$^{10}C_0 = 1$

$^{15}C_0 = 1$

$^{n}C_0 = 1$

$^{5}C_1 = 5$

$^{10}C_1 = 10$ If $r = 1$ than the answer is the value of n

$^{15}C_1 = 15$

$^{n}C_1 = n$

$^{5}C_5 = 1$ If $n = r$ than the value is 1

$^{10}C_{10} = 1$

$^{15}C_{15} = 1$

$^{n}C_n = 1$

Factorial (!)

$3! = 3 \times 2 \times 1 = 6$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362\ 880$

$n! = n-1 \times n-2 \times n-3 \ldots\ldots 3 \times 2 \times 1$

$^{7}C_2 = \frac{7 \times 6}{2 \times 1}$ $^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$

$^nC_0 = 1$

$^nC_1 = n$

$^nC_2 = \frac{n \times (n-1)}{2 \times 1}$

$^nC_3 = \frac{n(n-1)(n-2)}{3 \times 2 \times 1}$

$^nC_4 = \frac{n(n-1)(n-2)(n-3)}{4 \times 3 \times 2 \times 1}$

$^nC_5 = \frac{n(n-1)(n-2)(n-3)(n-4)}{5 \times 4 \times 3 \times 2 \times 1}$

Eg 1

When $\left(1 - \frac{x}{3}\right)^n$ is expanded in ascending powers of x, the coefficient of x^2 is 4. Given that n is the positive integer, find the value of n.

$r = 2$

$^nC_2 (1)^{n-2} \left(\frac{-x}{3}\right)^2$

$\frac{n(n-1)}{2 \times 1} \times 1 \times \frac{\cancel{x}^2}{9}$

$\frac{n^2 - n}{2} \times \frac{x^2}{9}$ → dont need this

$\frac{n^2 - n}{18} = 4$

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$\boxed{n = 9}$, $n = -8$

→ Not possible