

Indices and Standard form

$a^m \rightarrow$ power/index/exponent/order
 $a \rightarrow$ Base

$$\textcircled{1} a^m \times a^n = a^{m+n}$$

$$x^2 \times x^3 = x^{2+3} = x^5$$

$$2^3 \times 2^4 = 2^7$$

$$\textcircled{7} \left. \begin{array}{l} \text{(i)} \sqrt{a} = a^{1/2} \\ \text{(ii)} \sqrt[3]{a} = a^{1/3} \\ \text{(iii)} \sqrt[n]{a} = a^{1/n} \end{array} \right] \sqrt[n]{a} = a^{1/n}$$

$$\textcircled{2} a^m \div a^n = a^{m-n}$$

$$x^5 \div x^3 = x^{5-3} = x^2$$

$$5^4 \div 5^2 = 5^{4-2} = 5^2 \boxed{25}$$

$$\textcircled{8} \left. \begin{array}{l} \text{(i)} \sqrt[3]{a^2} = a^{2/3} \\ \text{(ii)} \sqrt{a^3} = a^{3/2} \\ \text{(iii)} \sqrt[2]{a^5} = a^{5/2} \end{array} \right] \sqrt[n]{a^m} = a^{m/n}$$

$$\textcircled{3} a^m \times b^m = (ab)^m$$

$$8^2 \times 5^2 = (15)^2 = 225$$

$$4^3 \times 25^3 = (100)^3 = 1000000$$

$$\textcircled{9} (a^m)^n = a^{mn} \text{ OR } a^{m \times n}$$

$$(x^2)^3 = x^6$$

$$(a^2b)^4 = a^8b^2$$

$$\sqrt{\frac{x^4y^8}{c}} = \left(\frac{x^4y^8}{c}\right)^{1/2} = \boxed{\frac{x^2y^4}{c^{1/2}}}$$

$$\textcircled{4} a^m \div b^m = \left(\frac{a}{b}\right)^m$$

$$10^2 \div 5^2 = \left(\frac{10}{5}\right)^2 = \boxed{8}$$

$$\textcircled{5} a^0 = 1$$

$$99878956^0 = 1$$

$$\textcircled{10} \left. \begin{array}{l} a^{-1} = \frac{1}{a} \\ a^{-2} = \frac{1}{a^2} \\ 8^{-2} = \frac{1}{9} \end{array} \right] a^{-m} = \frac{1}{a^m}$$

$$\textcircled{6} \frac{1^n}{1^{99878956}} = 1$$

$$\textcircled{11} a^m = a^n$$

$$m = n$$

Examples: Evaluate

$$\text{a) } 8^{3/2} \rightarrow \sqrt[3]{8^2} \begin{array}{l} \xrightarrow{\text{square}} \sqrt[3]{64} = 4 \\ \xrightarrow{\text{cube root}} 2^2 = 4 \\ \quad \downarrow \\ \quad \text{preferred} \end{array}$$

$$\text{b) } 125^{2/3} = \sqrt[3]{125^2} = 5^2 = 25$$

$$\text{c) } 16^{3/2} = \sqrt[2]{16^3} = 64$$

$$\text{d) } 4^{3/2} = \sqrt[2]{64} = 8$$

$$\text{e) } 1000^{2/3} = 10^2 = 100$$

$$\text{f) } 2^n = 8 \Rightarrow 2^n = 2^3 \Rightarrow \boxed{n=3}$$

$$\text{g) } 10^n = 0.01 \Rightarrow 10^n = \frac{1}{100} \Rightarrow 10^n = 10^{-2} \Rightarrow \boxed{n=-2}$$

$$\text{h) } 7^n = 1 \Rightarrow 7^n = 7^0 \Rightarrow \boxed{n=0}$$

→ Multiply & Divide

$$\text{a) } (3 \times 10^5) \times (6 \times 10^8) = \begin{array}{l} 18 \times 10^{13+1} \\ 1.8 \times 10^{14} \end{array}$$

$$\text{b) } (5 \times 10^{-11}) \times (4 \times 10^{-9}) = \begin{array}{l} 20 \times 10^{-20+1} \\ 2 \times 10^{-19} \end{array}$$

$$\text{c) } (9 \times 10^{-3})^2 = \begin{array}{l} 81 \times 10^{-6} \\ 8.1 \times 10^{-5} \end{array}$$

$$\text{d) } \left(\frac{5 \times 10^3}{4 \times 10^{-9}} \right) = \begin{array}{l} 1.25 \times 10^{3-(-9)} \\ 1.25 \times 10^{3+9} \\ 1.25 \times 10^{12} \end{array}$$

$$\text{e) } \left(\frac{2 \times 10^{-7}}{5 \times 10^3} \right)^{-} = \begin{array}{l} 0.4 \times 10^{-7-3} \\ 0.4 \times 10^{-10-1} \\ 4 \times 10^{-21} \end{array}$$

$$\text{f) } \sqrt{4 \times 10^{-10}} = (4 \times 10^{-10})^{1/2} = 2 \times 10^{-5}$$

$$h) \sqrt{8.1 \times 10^{13}} \rightarrow \left(\begin{array}{l} 8.1 \times 10^{13} \\ \underline{81 \times 10^{12}} \\ 9 \times 10^6 \end{array} \right)^{1/2}$$

$$(j) \sqrt[3]{2.7 \times 10^{16}} \Rightarrow \begin{aligned} & (2.7 \times 10^{16})^{1/3} \\ & = (27 \times 10^{15})^{1/3} \\ & = 9 \times 10^5 \end{aligned}$$

$$(i) \sqrt{2.5 \times 10^{-9}} \Rightarrow \left(\begin{array}{l} 2.5 \times 10^{-9-1} \\ \underline{25 \times 10^{-10}} \\ 5 \times 10^{-5} \end{array} \right)^{1/2}$$

→ Addition & subtraction

- ① $5x^2 + 3x^2 = 8x^2$
- ② $10x^3 - 4x^3 = 6x^3$
- ③ $5x^2 - 4x^3 =$ Not possible to simplify.

Examples

$$(a) \begin{array}{c} + \\ 3 \times 10^5 + 4 \times 10^5 = 7 \times 10^5 \end{array}$$

$$(b) 9 \times 10^{-6} - 4 \times 10^{-6} = 5 \times 10^{-6}$$

$$(c) (3.5 \times 10^8) + (4.2 \times 10^9) \rightarrow \left(\begin{array}{l} 3.5 \times 10^8 \\ + 42 \times 10^8 \\ \hline 45.5 \times 10^8 \\ = 4.55 \times 10^9 \end{array} \right) \left. \begin{array}{l} \text{Preference} \\ \text{10}^8 \text{ has to be changed} \end{array} \right\}$$

OR

$$\begin{array}{l} (3.5 \times 10^{8+1}) + (4.2 \times 10^9) \\ (0.35 \times 10^9) + (4.2 \times 10^9) \\ \hline 4.55 \times 10^9 \end{array}$$

$$d) \quad 7.85 \times 10^{-6} + 3.5 \times 10^{-5}$$

$$7.85 \times 10^{-6} + 35 \times 10^{-6}$$

$$42.85 \times 10^{-6+1}$$

$$\boxed{4.285 \times 10^{-5}}$$

$$e) \quad 8.3 \times 10^{12} - 6.5 \times 10^{14} = 830 \times 10^{14} - 6.5 \times 10^{14}$$

$$823.5 \times 10^{14}$$

$$\boxed{8.235 \times 10^{12}}$$

$$f) \quad 5.25 \times 10^4 + 3.75 \times 10^5 \Rightarrow 5.25 \times 10^4 + 37.5 \times 10^4$$

$$42.75 \times 10^4$$

$$\boxed{4.275 \times 10^5}$$

Past paper

- 1 The table shows information about the oil production, in barrels per day, of five countries during one year.

Country	Oil production (barrels per day)
India	8.97×10^5
Brazil	2.63×10^6
United States	8.4×10^6
Russia	1.09×10^7
Saudi Arabia	9.9×10^6

- (a) Which country had the highest oil production?

Russia

(1)

- (b) Calculate the difference between the oil production of Brazil and the oil production of India. Give your answer in standard form.

$$\begin{aligned}
 & b) \quad 2.63 \times 10^6 - 8.97 \times 10^5 \\
 & \quad 26.3 \times 10^5 - 8.97 \times 10^5 \\
 & \quad 17.33 \times 10^5 \Rightarrow \boxed{1.733 \times 10^6}
 \end{aligned}$$

During the same year, the oil production of California was 6.3×10^5 barrels per day.

(c) Work out the oil production of California as a proportion of the oil production of the United States.

$$\begin{aligned}
 & c) \quad \frac{6.3 \times 10^5}{8.4 \times 10^6} \rightarrow \frac{63 \times 10^{-1}}{84} \rightarrow 0.75 \times 10^{-1} \\
 & \quad \quad \quad \boxed{0.075}
 \end{aligned}$$

3 The table gives the surface areas, in square kilometres, of five seas.

Sea	Surface area in square kilometres
Mediterranean Sea	2.97×10^6
East China Sea	1.25×10^6
Baltic Sea	4.22×10^5
Red Sea	4.38×10^5
Okhotsk Sea	1.59×10^6

(a) Write 1.59×10^6 as an ordinary number.

1590000
(1)

(b) Work out the difference, in square kilometres, between the largest surface area and the smallest surface area for these five seas.
Give your answer in standard form.

$$\begin{aligned}
 & (2.97 \times 10^6) - (4.22 \times 10^5) \\
 & 29.7 \times 10^5 - 4.22 \times 10^5 \\
 & 35.48 \times 10^5 \\
 & \boxed{3.548 \times 10^6}
 \end{aligned}$$