

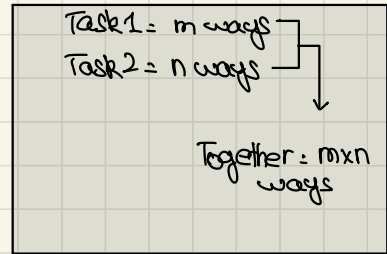
# Permutations and Combinations

## Basic Counting Principle

$T_1$   
Shirts (4)  
Blue  
Black  
Pink  
Yellow

$T_2$   
Trousers (2)  
Brown  
Blue

$T_3$   
Shoes  
Black  
Brown



Together =  $4 \times 2$  ( $T_1, T_2$ )  
Together = 8 ways

Together =  $4 \times 2 \times 2$  ( $T_1, T_2, T_3$ )  
= 16 ways.

## → Factorial notation

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

used to arrange  $n$  objects in a straight line without restrictions

## Examples

1. Find the ways in which 5 people can be arranged in a straight line.

$$5! = \frac{5}{1^{st}} \frac{4}{2^{nd}} \frac{3}{3^{rd}} \frac{2}{4^{th}} \frac{1}{5^{th}} = 5 \times 4 \times 3 \times 2 \times 1 = 5! \text{ or } {}^5P_5 = \boxed{120}$$

Total  $\nwarrow$   
Permute  $\swarrow$   
Arrange  $\rightarrow$

2. In how many ways can you answer a test containing 5 True/False questions.

$$\frac{2}{1} \frac{2}{2} \frac{2}{3} \frac{2}{4} \frac{2}{5} = 2^5 = \boxed{32}$$

3. In how many ways can the letters of the word Ali be arranged?

$$3! = 3 \times 2 \times 1 = \boxed{6}$$

4. In how many ways can the letters of the word 'POP' be arranged?

$$\frac{3!}{2} \rightarrow \frac{6}{2} = \boxed{3}$$

Manually  
 ① POP  
 ② OPP  
 ③ PPO

Note: When letters are repeated, we divide by the factorial of the number by the times each number is repeated.

Permutations: Used in arrangements. Applied when order/rank is important

3, 2  $\neq$  2, 3      Permutation  
 A & B = B & A      Combination.

$\frac{A}{1^{st}} \frac{B}{2^{nd}} \frac{C}{3^{rd}} \neq \frac{B}{1^{st}} \frac{A}{2^{nd}} \frac{C}{3^{rd}}$       Permutation

①  $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$   
 ${}_5P_5 = 120$

②  $5 \times 4 \times 3$  or

$\rightarrow$  spaces available  
 ${}_5P_3 \rightarrow$  Total available

③  $6 \times 5 \times 4 \times 3 = {}_6P_4 = 360$

A security code consists of three letters selected from A, B, C, D, E, F followed by 2 digits selected from 5, 6, 7, 8, 9.

Find the number of possible security codes if no letter or number can be repeated.

→ number of possibilities

$$\underbrace{6 \times 5 \times 4}_{\text{Letters}} \times \underbrace{5 \times 4}_{\text{Digits}} = 2400$$

OR

$${}^6P_3 \times {}^5P_2 = 2400$$

- 31 (a) A 6-digit number is to be formed using the digits 1, 3, 5, 6, 8, 9. Each of these digits may be used only once in any 6-digit number. Find how many different 6-digit numbers can be formed if

- (i) there are no restrictions,

[1]

$${}^6P_6 = 720$$

- (ii) the number formed is even,

[1]

— — — — — 6, 8

$${}^5P_5 \times {}^2P_2 = 240$$

- (iii) the number formed is even and greater than 300 000.

[3]

3, 5, 6, 8, 9 — — — 6, 8

$${}^4P_2 \times {}^4P_4 \times {}^2P_2 = 96$$

$${}^4P_2 \times {}^4P_4 \times {}^2P_2 = 96$$

$$192$$

combination: Selection or Team or Choice

$$A \& B = B \& A \text{ (same team)}$$

$$AB = BA \Rightarrow \text{Combination.}$$

3 How many different ways are there of selecting

a 3 photographs from 10 photographs  $^{10}C_3 = 120$

b 5 books from 7 books  $^7C_5 = 21$

c a team of 11 footballers from 14 footballers?  $^{14}C_{11} = 364$

4 How many different combinations of 3 letters can be chosen from the letters P, Q, R, S, T?  $^5C_3 = 10$