Circular Measure

→ Introduction to radian measure

Radian: One radian is the measure of a central angle when the arc length is equal to the radius

$360 = 2\pi$ rad

→ Degree to Radian

$\times \frac{\pi}{180}$

$\text{Degree} \times \frac{\pi}{180} = \text{Radian}$

① $\cancel{30} \times \frac{\pi}{\cancel{180}_{6}} = \text{Radian}$

$\boxed{\frac{\pi}{6} \text{ rad}}$

② $\cancel{300}^{5} \times \frac{\pi}{\cancel{180}_{3}}$

$\boxed{\frac{5}{3}\pi \text{ rad}}$

③ $\cancel{135}^{3} \times \frac{\pi}{\cancel{180}_{4}} = \boxed{\frac{3}{4}\pi \text{ rad}}$

→ Radian to degree

$\text{Rad} \times \frac{180}{\pi} = \text{Degree}$

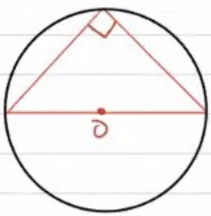
① $\frac{5}{7}\pi \times \frac{180}{\pi} = 128.6^{\circ}$

② $2.5 \text{rad} \times \frac{180}{\pi} = \boxed{143.2^{\circ}}$

Formula	Degree	Radian
Area of circle	πr^2	πr^2
Circumference	$2\pi r$	$2\pi r$
Area of a sector	$\frac{\theta}{360} \times \pi r^2$	$\frac{1}{2} r^2 \theta$ OR $\frac{1}{2} r s$
Arc length	$\frac{\theta}{360} \times 2\pi r$	$\frac{\theta}{2\pi} \times 2\pi r = r\theta$
Perimeter of a sector	$P = 2r + \frac{\theta}{360} \times 2\pi r$	$P = 2r \times r\theta$
Area of Triangle	$\frac{1}{2} \times a \times b \times \sin c$	$\frac{1}{2} \times a \times b \times \sin c$
Sin rule		
Cosine rul		

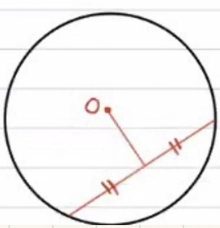
Circle Properties to Remember

1)



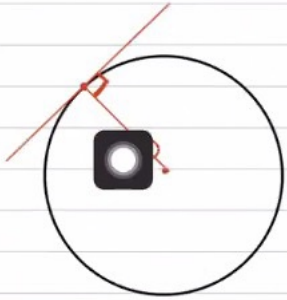
Angle opposite the diameter is always 90°

2)



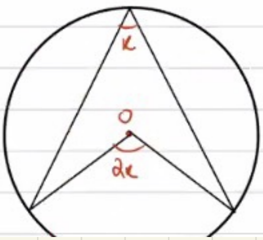
The perpendicular from the centre of a circle to a chord bisects the chord

3)



Radius & tangent meet at 90°.
Radius is always perpendicular to tangent

4)

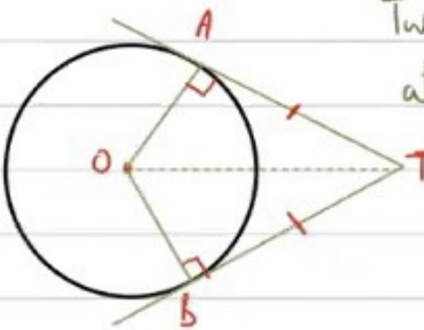


Angle at the centre is twice the angle at the circumference

5)

Two tangents me
at an external

5)



Two tangents meeting at an external point are equal.

Find the arc length of a sector of

a radius 8 cm and angle 1.2 radians **b** radius 2.5 cm and angle 0.8 radians.

a) $S = r\theta$

$S = 8 \times 1.2$

$\boxed{S = 9.6\text{cm}}$

(b) $S = r\theta$

$S = 2.5 \times 0.8$

$\boxed{S = 2\text{cm}}$

Find, in radians, the angle of a sector of

a radius 4 cm and arc length 5 cm **b** radius 9 cm and arc length 13.5 cm.

a) $S = r\theta$

$5 = 4\theta$

$\boxed{\theta = 1.25\text{rad}}$

(b) $S = r\theta$

$13.5 = 9\theta$

$\boxed{\theta = 1.5\text{rad}}$

Find the area

a radius 6 cm and angle $\frac{\pi}{3}$ **b** radius 15 cm and angle $\frac{3\pi}{5}$

c radius 10 cm and angle $\frac{7\pi}{10}$ **d** radius 9 cm and angle $\frac{5\pi}{6}$.

a) $\frac{1}{2}r^2\theta$

$\frac{1}{2} \times 6^2 \times \frac{\pi}{3}$

$\boxed{A = 6\pi\text{cm}^2}$

(b) $\frac{1}{2}r^2\theta$

$\frac{1}{2} \times 15^2 \times \frac{3\pi}{5}$

$\boxed{A = 67.5\pi\text{cm}^2}$

(c) $\frac{1}{2}r^2\theta$

$\frac{1}{2} \times 10^2 \times \frac{7\pi}{10}$

$\boxed{35\pi\text{cm}^2}$

(d) $\frac{1}{2}r^2\theta$

$\frac{1}{2} \times 9^2 \times \frac{5\pi}{6} \rightarrow \boxed{\frac{135}{4}\pi\text{cm}^2}$

POQ is the sector of a circle, centre *O*, radius 10 cm.

The length of arc *PQ* is 8 cm. Find

a angle *POQ*, in radians **b** the area of the sector *POQ*.

(a) $S = r\theta$

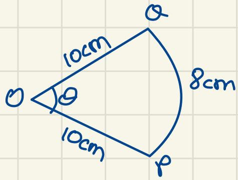
$8 = 10(\theta)$

$\boxed{\theta = 0.8\text{rad}}$

(b) $\frac{1}{2} rs$

$\frac{1}{2} \times 10 \times 8$

$\boxed{40\text{cm}^2}$



7.

The circle has radius 12 cm and centre *O*.

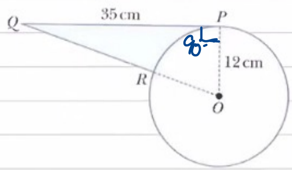
PQ is a tangent to the circle at the point *P*.

QRO is a straight line. Find

a angle *POQ*, in radians

b the area of sector *POR*

c the area of the shaded region.



(a)

(a) $\angle OPQ = 90^\circ$ or $\frac{\pi}{2}$

$\tan POQ = \frac{35}{12}$

$POQ = \tan^{-1}\left(\frac{35}{12}\right)$ → do this in radians

$\boxed{POQ = 1.24 \text{ rad}}$

(b) $\frac{1}{2} r^2\theta$

$\frac{1}{2} \times 12^2 \times 1.24$

$\boxed{89.3 = \text{Area}}$

(c) $\frac{1}{2} \times 35 \times 12$

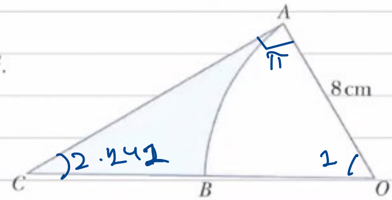
$210 - 89.3$

$\boxed{120.7\text{ cm}^2 = AQSR}$

8.

AOB is the sector of a circle, centre *O*, radius 8 cm.
AC is a tangent to the circle at the point *A*.
CBO is a straight line and the area of sector *AOB* is 32 cm^2.
Find

a angle *AOB*, in radians
b the area of triangle *AOC*
c the area of the shaded region.



(a) $A = \frac{1}{2}r^2\theta$

$32 = \frac{1}{2} \times 8^2 \times \theta$

$64 = 64 \times \theta$

$\boxed{\theta = 1}$

b) $\tan(1) = \frac{x}{8}$

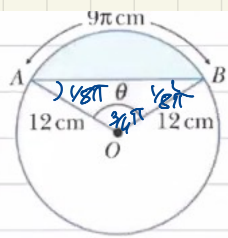
$x = 12.46\text{cm}$

$\frac{1}{2} \times 12.46 \times 8 \Rightarrow \boxed{49.8\text{cm}^2}$

c) $49.8 - 32 \Rightarrow \boxed{17.8\text{cm}^2}$

The diagram shows a circle, centre *O*, radius 12 cm.
Angle $AOB = \theta$ radians.
Arc $AB = 9\pi\text{ cm}$.

a Show that $\theta = \frac{3\pi}{4}$.
b Find the area of the shaded region.



(a) $r\theta = S$

$S = r\theta$

$9\pi = 12\theta$

$\frac{9\pi}{12} = \theta \longrightarrow \boxed{\frac{3}{4}\pi = \theta}$ hence shown

(b) $\frac{1}{2}rs$

$\frac{1}{2} \times 12 \times 9\pi$

$54\pi - 36\sqrt{2}$

$\boxed{119\text{ cm}^2}$

$\frac{1}{2} \times 12 \times 12 \times \sin(\frac{3}{4}\pi)$

$\frac{72}{2} \times \frac{\sqrt{2}}{2}$

$36\sqrt{2}$