Vectors

There are two types of quantities in maths:- the scalar quantity and the vector quantity. When we refer to scalar quantities, our concern is only the magnitude, irrespective of the direction. However, when we refer to vector quantities, both magnitude as well as direction are important.

① Representation of Vectors

A vector is ~~repre~~ represented in three ways: coordinate form, column vector form, cartesian form.

(x, y) $\quad$ $\begin{pmatrix} x \\ y \end{pmatrix}$ $\quad$ $xi + yj$

where i denotes movement across x axis and j denotes movement along y axis.

A vector quantity is differentiable from a scalar quantity by the fact that a vector quantity will always have an arrow above it, distinguishing it from a scalar quantity, which does not have an arrow.

② Position Vector

Position vector is a vector with respect to origin. In a vector quantity, coordinate of any point is its position vector.

③ Magnitude:

Magnitude is the length of a vector. In order calculate magnitude, we use the formula $\sqrt{(x\ component)^2 + (y\ comp)^2}$.

$\vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$|\vec{AB}| = \sqrt{3^2 + 4^2} = 5$.

④ Zero Vector

Vectors are considered to be zero vectors if their magnitude is 0. It is represented as 0. Only the origin is a zero vector.

⑤ Equal Vectors

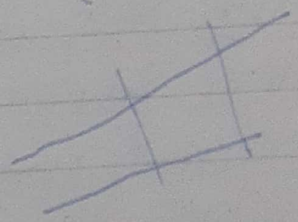
Vectors are considered to be equal vectors if they have the same magnitude and are in the same direction.

⑥ Negative Vectors

vectors are considered to be negative vectors if they have the same magnitude but are in opposite directions.

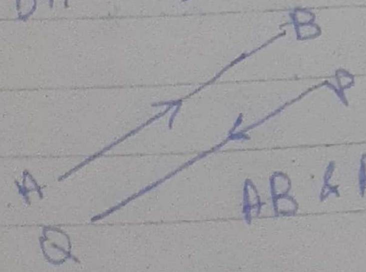
$\overrightarrow{OB} = a$ $\qquad$ $\overrightarrow{PQ} = \begin{pmatrix}2\\3\end{pmatrix}$

$\overrightarrow{BO} = -a$ $\qquad$ $\overrightarrow{QP} = \begin{pmatrix}-2\\-3\end{pmatrix}$



$\overrightarrow{AB} = a-b$

$\overrightarrow{BA} = b-a$

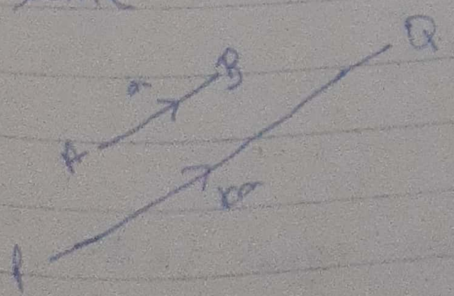


AB & PQ are negative vectors.

⑦ Parallel Vectors

Vectors are considered to be parallel vectors if they are in the same direction but have different magnitude which is in the same ratio

$\overrightarrow{PQ} = k\overrightarrow{AB}$



$\overrightarrow{XY} = \begin{pmatrix}1\\3\end{pmatrix}$

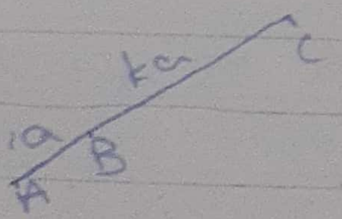
$\overrightarrow{LM} = \begin{pmatrix}4\\12\end{pmatrix}$

$\overrightarrow{LM} = 4\begin{pmatrix}1\\3\end{pmatrix}$

$\overrightarrow{LM} = 4\overrightarrow{XY}$

(8) Collinear Vectors

Vectors are considered to be collinear vectors if they lie on the same straight line. Thus, having the same direction but a different magnitude which is in the same ratio.

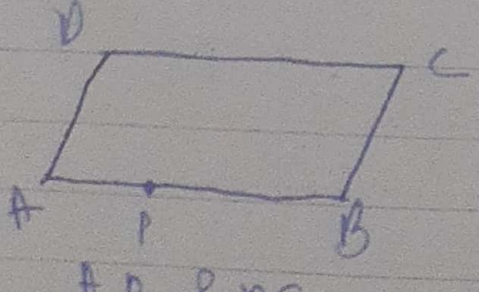


$\overrightarrow{AB} = a$

$\overrightarrow{BC} = ka$

→ can distinguish b/w parallel and collinear vectors via common point

$\overrightarrow{AB}$ $\overrightarrow{BA}$



AB & DC are equal vectors

AD & BC " " "

AP & AB " parallel "

~~APB~~

① Unit Vectors

Unit vectors are vectors w/ the magnitude of 1. In order to achieve unit vectors, we divide the vector with its magnitude. Unit vector is denoted with a cap over the vector.

$$\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

If you are told to find the unit vector parallel to a given vector, ~~y~~ then, ~~you~~ we are required to find the unit vector of the given vector.

Find the unit vector of $PQ = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

$$|\vec{PQ}| = \sqrt{9^2 + 12^2}$$

$$\hat{PQ} = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$$

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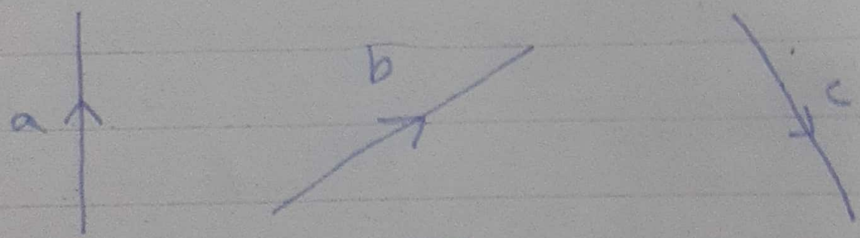
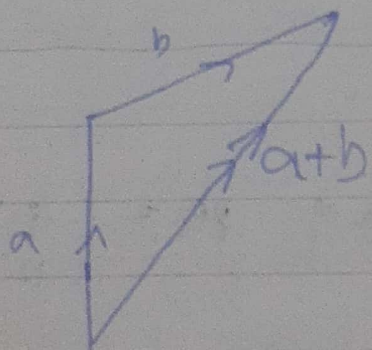
⑩ Scalar Multiple of Vectors

If a constant is multiplied to a vector, then it must be multiplied to all components of the vector.

⑪ Head to Tail Rule

Head to tail rule indicates that the point where one vector ends, the other vector begins





⑫ Addition of Vectors

In order to add vectors, we apply the head to tail rule

$\vec{OA} = a$ $\vec{OB} = b$ $\vec{OC} = c$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -a + b$

$= b - a$

$\cancel{\overrightarrow{AO} =}\ \overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -a + c$

$= \cancel{-c} \ c - a$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= -b + c$

$= c - b$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -b \cancel{+a} + c \cancel{-a}$

$= c - b$

$\boxed{3a + 6b}$

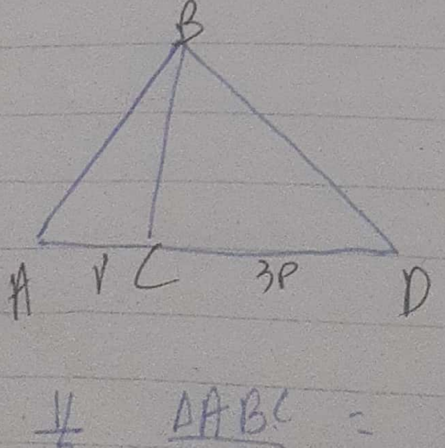
Note: Inorder to find the ratio of areas, in questions involving vectors, we will apply the following

① Observe if the triangles are similar, & if they are, then we will use the formula

$$\left(\frac{l_1}{l_2}\right)^2 = \frac{A_1}{A_2}$$

② If triangles are not similar, then, we will observe if there is a common side along with either parallel lines or common base. If they exist, we will use the formula

$$\frac{\cancel{\frac{1}{2}} \times b \times \cancel{h}}{\cancel{\frac{1}{2}} \times B \times \cancel{h}} = \frac{b}{B}$$



$$\cancel{\frac{1}{2}} \; \frac{\Delta ABC}{\Delta BCD} = \frac{P\,1}{3}$$