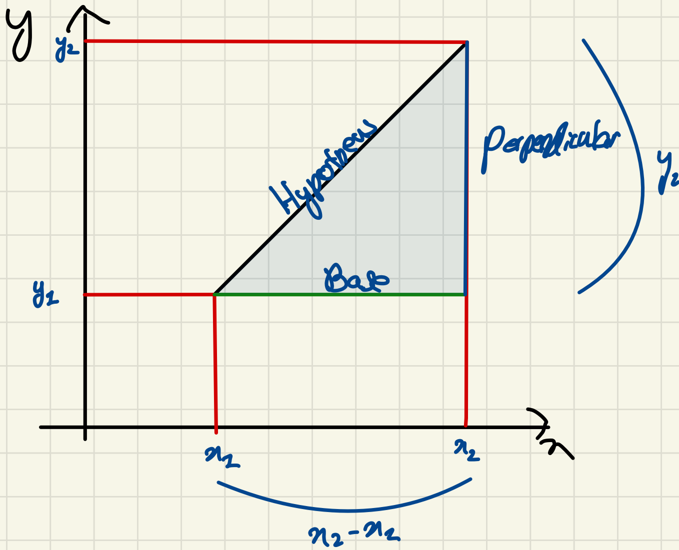
Coordinate Geometry

→ Distance b/w two points

$A = (x_1, y_1)$ $B = (x_2, y_2)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$Base^2 + Perpendicular^2 = Hyp^2$

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = Hyp^2$

$\sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$

Examples

1a: $A(3,5)$ $B(6,1)$

$AB = \sqrt{(6-3)^2 + (1-5)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= \boxed{5 \text{ units}}$

2) $A = (2,-5)$ $B(14,0)$

$AB = \sqrt{(14-2)^2 + (0 - -5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= \boxed{13 \text{ units}}$

2a) $A(a,5)$ $B(3,9)$. Given that $AB = 5$ units Find the values of a

$AB = \sqrt{(3-a)^2 + (9-5)^2} = 5$

$(3-a)^2 + 16 = 25$

$(3-a)^2 = 9$

$3 - a = \pm 3$

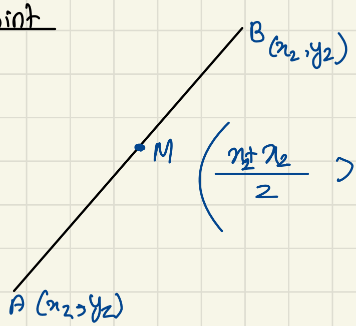
$\boxed{a = 0 \quad a = 6}$

(b) $P(p, -2)$ $Q(6, 13)$. Given that $PQ = 17$ units. Find the values of p.

$$\sqrt{(6-p)^2 + (13+2)^2} = 17$$
$$(6-p)^2 + 225 = 17^2$$
$$(6-p)^2 = 289 - 225$$
$$(6-p)^2 = 64$$
$$6 - p = \pm 8$$
$$-2 = p \quad , \quad 14 = p$$

$$\boxed{\begin{array}{l} p = -2 \\ p = 14 \end{array}}$$

→ Midpoint



M= midpoint

$$M\left(\frac{x_2 + x_2}{2}, \frac{y_2 + y_2}{2}\right)$$

Examples

1) A $(3, 9)$ B $(1, 5)$

$$M = \left(\frac{3+1}{2}, \frac{9+5}{2}\right)$$
$$= \boxed{2, 7}$$

(b) $P(-2, 0)$ $Q(4, 6)$

$$M\left(\frac{-2+4}{2}, \frac{0+6}{2}\right)$$
$$= \boxed{1, 3}$$

Qs2 $A(3, 5)$ $B(x, y)$. Given that midpoint of A & B is $M(4, 7)$

$$\left(\frac{3+x}{2}, \frac{5+y}{2}\right) = 4, 7$$

$$\frac{3+x}{2} = 4$$
$$3 + x = 8$$
$$\boxed{x = 5}$$

$$\frac{5+y}{2} = 7$$
$$5 + y = 14$$
$$\boxed{y = 9}$$

$$\boxed{B(5, 9)}$$

Qs3 $A(-1,0)$, $B(1,6)$ and $C(7,4)$

Show that triangle ABC is aright angle isosecles triangle

$$AB = \sqrt{(1+1)^2 + (6-0)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$

$$BC = \sqrt{(7-1)^2 + (4-6)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$

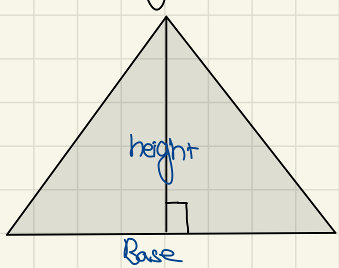
same values prove that they are isoseleces

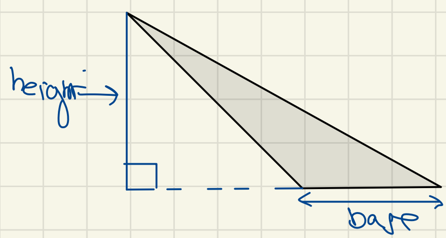
$$AC = \sqrt{(7+1)^2 + (4-0)^2}$$
$$= \sqrt{64+16}$$
$$= \sqrt{80}$$

$$AB^2 + BC^2 = AC^2$$
$$\sqrt{40}^2 + \sqrt{40}^2 = \sqrt{80}^2$$
$$40 + 40 = 80$$
$$80 = 80$$

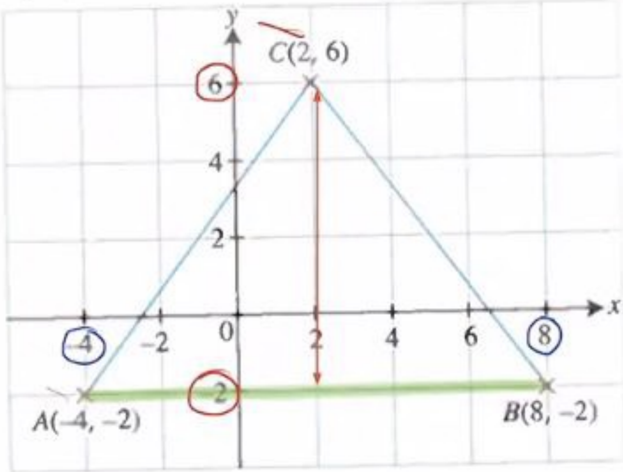
hence proved that they are right angled.

Area of Triangle





5. The vertices of ΔABC are $A(-4, -2)$, $B(8, -2)$ and $C(2, 6)$.



(i) Find the perimeter and the area of ΔABC.

(ii) Hence, find the length of the perpendicular from A to BC.

AB AC BC

$AB = 8 - -4$

$AB = 12$ units

$AC = \sqrt{(2--4)^2 + (6--2)^2}$

$AC = \sqrt{36+64}$

$AC = 10$ units

$BC = \sqrt{(2-8)^2 + (6--2)^2}$

$BC = \sqrt{36+64}$

$BC = 10$ units

Area $= \frac{1}{2} \times b \times h$

$h = 6-(-2)$

$h = 8$

$\frac{1}{2} \times 12 \times 8 = 48 \text{ unit}^2$

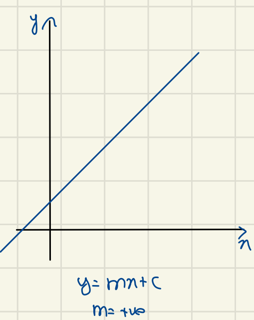
$P = 12+10+10$

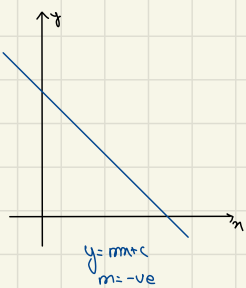
$P = 32$ units

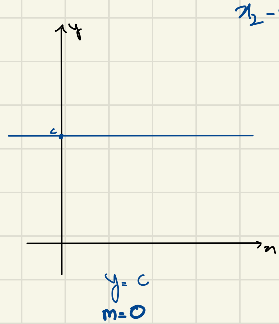
$\boxed{\text{Gradient}}$

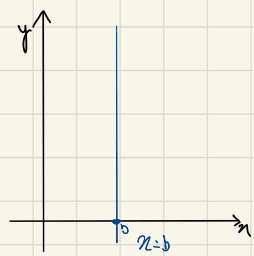
Measure of the steepness of a straight line

$m = \frac{\text{Rise}}{\text{Run}} \to \frac{(\text{difference in } y)}{(\text{difference in } x)}$ so the formula would be $= \frac{y_2 - y_1}{x_2 - x_1}$









Examples

(1) $A(1,3)$ $B(4,7)$

$x_1\ y_1$ $x_2\ y_2$

$$m = \frac{7-3}{4-1} = \boxed{\frac{4}{3}}$$

(2) $P(-1,2)$ $Q(0,-3)$

$x_1\ y_1$ $x_2\ y_2$

$$m = \frac{-3-2}{0+1} = \frac{-5}{1} = \boxed{-5}$$

3. If the gradient of the line joining the points $(-3,-7)$ and $(4,p)$ is $\frac{3}{5}$, find the value of p

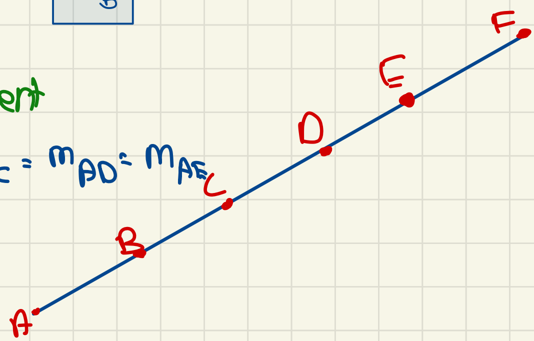
$(-3,-7)$ $(4,p)$

$$\frac{p--7}{4+3} = \frac{7+p}{7} = \frac{3}{5} = 35+5p=21 \rightarrow 5p=-14 \rightarrow \boxed{p = \frac{-14}{5}}$$

Collinear Points have the same gradient

Points lying on the same line

$$m_{AB} = m_{BC} = m_{AD} = m_{AE}$$



Example

1) $A(3,5)$, $B(4,8)$, $C(-1,c)$

All lie on the same straight line

$$m_{AB} = \frac{8-5}{4-3} = 3$$

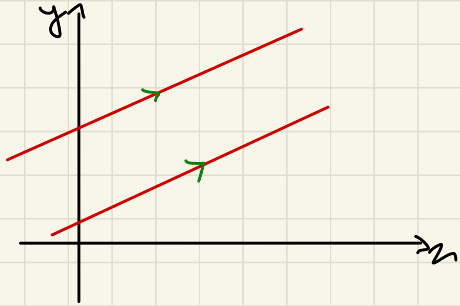
$$m_{AB} = m_{BC}$$
$$3 = \frac{c-8}{-1-4}$$
$$-15 = c-8$$
$$\boxed{-7 = c}$$

2) The points $P(2,-3)$, $Q(3,-2)$ and $R(8,z)$ are collinear. Find the value of z.

$$m_{PQ} = \frac{-2+3}{3-2} = \frac{1}{1} = 1$$

$$m_{PQ} = m_{QR}$$
$$1 = \frac{z+2}{8-3}$$
$$5 = z+2$$
$$\boxed{3 = z}$$

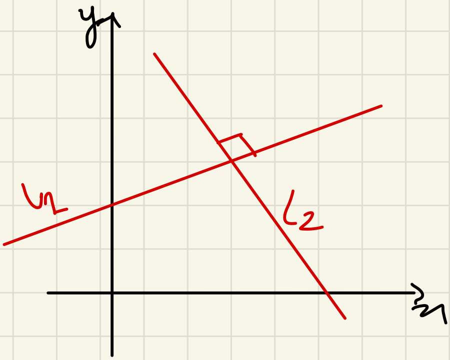
Parallel lines have the same gradient



$L_1 // L_2$

$m_1 = m_2$

Product of the gradient perpendicular line is -1



$m_1 \times m_2 = -1$

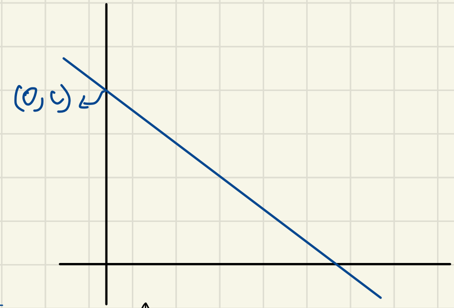
$m_1 = \frac{-1}{m_2}$

→ Equation Of Straight line

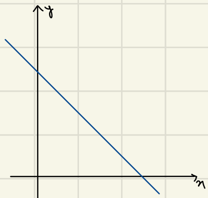
$y = mx + c$

m → gradient

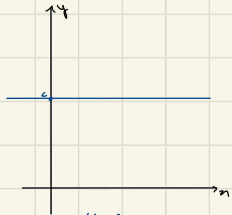
c → y-intercept where the point is $(0, c)$



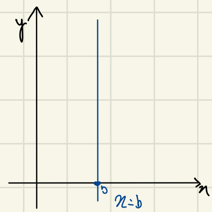
$y = mx + c$
$m = +ve$



$y = mx + c$
$m = -ve$



$y = c$
$m = 0$



$x = b$
$m = \infty$

How to find the equation of a straight line

1. Gradient & y intercept. — $y = mx + c$
2. Gradient & a point on the line — $y - y_1 = m(x - x_1)$
3. Two points on the line

m = gradient

(x_1, y_1) = Point on the line

① Gradient & y intercept.

$m = 3 \qquad c = 2$

$y = mx + c$

$\boxed{y = 3x + 2}$

② Gradient & a point on the line

$y = mx + c \qquad P(2, 3)$

$m = 4$

$y = 4x + c$

$3 = 4(2) + c$

$\boxed{-5 = c}$

$\boxed{y = 4x - 5}$

③ Two points on the line

$P(-1, -3) \qquad Q(1, 7)$

$m = \frac{7+3}{1+1} = m = \frac{10}{2} = 5$

$y = 5x + c$

$7 = 5(1) + c$

$7 - 5 = c \rightarrow c = 2$

$c = 2$

$y = mx + c$

$\boxed{y = 5x + 2}$

Find the equation of the line which passes through both points in each case

(a) A (2, 3) and B (4, 11)

$m = \frac{11-3}{4-2} = \frac{8}{2} = 4$

$y = 4x + c$

$11 = 16 + c$

$-5 = c$

$\boxed{y = 4x - 5}$

(b) A (3, -5) B (7, 12)

$m = \frac{12+5}{7-3} = \frac{17}{4}$

$y = \frac{17}{4}x + c$

$-5 = \frac{17}{4}(3) + c$

$-5 - \frac{51}{4} = c$

$\frac{-20-51}{4} = c$

$c = \frac{-71}{4}$

$c = -17\frac{3}{4}$

$\boxed{y = \frac{17}{4}x - \frac{71}{4}}$

Find the equation of the line

1. Passes through (0, 7) at a gradient 3

$y = mx + c$

$y = 3x + c$

$7 = 0 + c$

$7 = c$

$\boxed{y = 3x + 7}$

2. Passes through (2, 3) at a gradient of 2

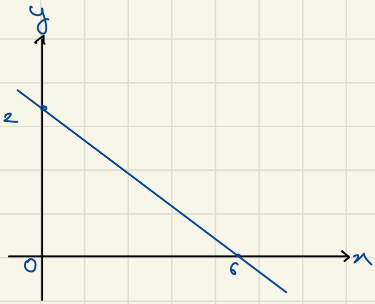
$y = 2x + c$

$3 = 2(2) + c$

$-1 = c$

$\boxed{y = 2x - 1}$

Qs



(0, 2) (6, 0)

$y = mx + c$ $c = 2$

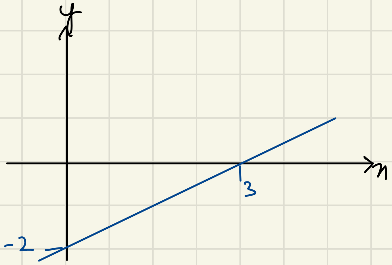
$y = mx + 2$

$\boxed{y = -3x + 2}$

$m = \frac{0-2}{6-0}$

$= \frac{-2}{6}$

$= -3$



$$y = mx + c$$

$$y = mx - 2$$

$$m = \frac{3-0}{0+2}$$

$$= \frac{3}{2}$$

$$\boxed{y = \frac{3}{2}x - 2}$$