Integration

Integration is the reverse process of differentiation. While differentiation is used to find the gradient function, integration is used to ~~fo~~ determine the equation of the curve and area under the curve. In order to integrate, the following models are applied:

<u>Model 1</u>

$$\int \frac{dy}{dx} = \int x^n \, dx$$

$$y = \frac{x^{n+1}}{n+1} + c$$

<u>Model 2</u>

$$\int \frac{dy}{dx} = \int ax^n \, dx$$

$$\frac{ax^{n+1}}{n+1} + c$$

<u>Model 3</u>

$$\int \frac{dy}{dx} = \int (ax+b)^n$$

(ax+b) must always be linear

$$y = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \frac{dy}{dx} = \int (4x+3)^{-1/2}$$

$$y = \frac{(4x+3)^{\frac{1}{2}}}{\frac{1}{2} \times 4^{2}} + c$$

$$y = \frac{\cancel{(4)} \sqrt{4x+3}}{2} + c$$

$$y = x^3 - 7x^2 + 5x - 8$$

$$\frac{dy}{dx} = 3x^2 - 14x + 5$$

$$\int \frac{dy}{dx} = \int 3x^2 - 14x + 5 \, dx$$

$$y = \frac{3x^3}{3} - \frac{14x^2}{2} + 5x + c$$

$$= x^3 - 7x^2 + 5x + \underline{c}$$

After performing integration we add +c to the result In order to ~~add~~ keep the place value of the constant which is dropped when the function is differentiated. This value of c can be found if the ~~function~~ point on the curve is substituted into the equation.

Model 4 Equation of a curve.

In order to find the equation of a curve, the following steps are applied.

Step1 Find the integral of given $\frac{dy}{dx}$ function

Step2 substitute the given point in the answer of step1.

Step3 Here find the value of c.

Step4 using the integral and the value of c found, form the equation of the curve.

Model 5 Definite Integrals / Integration within limits

When we are given definite integrals, there is no need to add c, because c will automatically cancel off.

$$\int_2^6 (4x+1)^{-\frac{1}{2}}$$

$$\frac{\sqrt{4u+1}}{2}\Bigg|_2^6$$

$$\frac{\sqrt{25}}{2} - \left(\frac{\sqrt{9}}{2}\right)$$

$$\frac{5}{2} - \frac{3}{2}$$

$$1$$

Model 6

If the limits of the integral are interchanged, the sign of the answer will change.

$$\int_a^b s = p \qquad \int_b^a s = -p$$

Model 7

$$\int_a^b f(x) + \int_b^c f(x) = \int_a^c f(x)$$

Model 8 Integ applied to exponential functions

$$\int e^x \, dx = e^x + c$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$$

$$\int e^{f(x)} \, dx = \frac{e^{f(x)}}{f'x} + c$$

Model 9 Integ applied to trig functions

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

~~$\int \tan x \, dx =$~~

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \tan x \, dx = -\ln|\cos x| + c$$

$$\int \cot x = \ln|\sin x| + c$$

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$$\int \sin ax\, dx = \frac{-\cos ax}{a} + C$$

$$\int \cos ax\, dx = \frac{\sin ax}{a} + C$$

$$\int \sec^2 ax\, dx = \frac{\tan ax}{a} + C$$

$$\int \tan ax\, dx = \frac{-\ln|\cos ax|}{a} + C$$

$$\int \cot ax = \frac{\ln|\sin ax|}{a} + C$$

$$\int \sin f(x)\, dx = \frac{-\cos f(x)}{f'(x)} \cancel{dx} + C$$

$$\int \cos f(x)\, dx = \frac{\sin f(x)}{f'(x)} + C$$

$$\int \sec^2 f(x)\, dx = \frac{\tan f(x)}{f'(x)} + C$$

$$\int \tan f(x)\, dx = \frac{-\ln|\cos f(x)|}{f'(x)} + C$$

$$\int \cot f(x)\, dx = \frac{\ln|\sin f(x)|}{f'(x)} + C$$

$$\int \cos(2x-\pi)\,dx = \frac{\sin(2x-\pi)}{2} + c$$

$$\int \sin(2\pi-3x)\,dx = \frac{+\cos(2\pi-3x)}{+3} + c$$

$$\int \sec^2 5x\,dx = \frac{\tan 5x}{5} + c$$

$$\int \tan(2x-\frac{\pi}{4}) = \frac{-\ln|\cos(2x-\frac{\pi}{4})|}{2}$$

$$\int \cot(\pi+3x) = \frac{\ln|\sin(\pi+3x)|}{3}$$

Model 10 integ using ln functions

If the differential of the denominator is ~~the~~ present ~~neum~~ in the neumerator or is the neumerator then, we will apply integ using ln function.

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int \frac{6x-5}{3x^2-5x+7}\,dx = \ln(3x^2-5x+7) + c$$

$$\int \frac{2e^{2x}}{e^{2x}+7}\,dx = \ln(e^{2x}+7) + c$$

$\int \tan x = -\int \frac{\sin x}{\cos x} = -\ln|\cos x| + C$

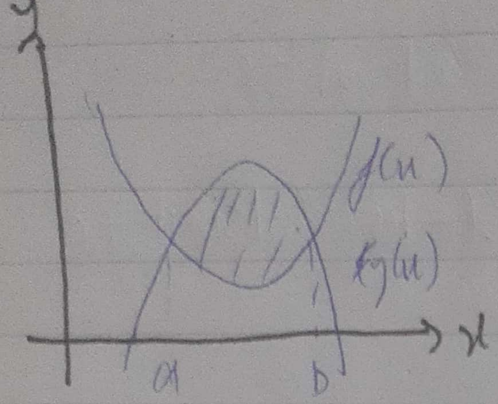
$\int \cot x = \int \frac{\cos x}{\sin x} = \cancel{\ln|\cos} \ln|\sin x| + C$

$\int \frac{3x-4}{3x^2-8x+5}$

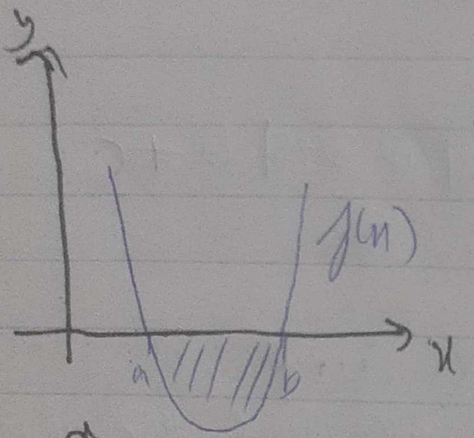
$\frac{1}{2}\int \frac{6x-8}{3x^2-8x+5} = \frac{1}{2}\ln|3x^2-8x+5| \, dx + C$

Model 11 Area under Curve

In order to find the area under the curve, we will ~~different~~ integrate the given function within the specified limit. Always remember area under the curve will always be positive became area are never negative. Thus, when finding area under the curve, we always use modulus sign, so that even if the results are negative they convert into positive values. Area under the curve means area b/w curve and x axis.

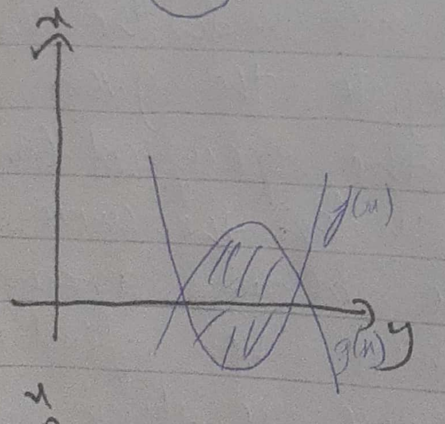


$$\int_a^b g(x) - \int_a^b f(x)$$

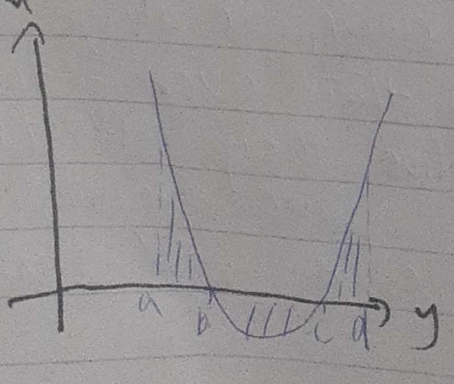


$$-\int_a^b f(x)$$

or modulus

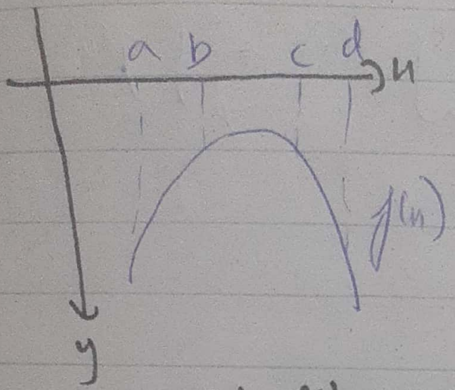


$$-\int_a^b f(x) + \int_a^b g(x)$$



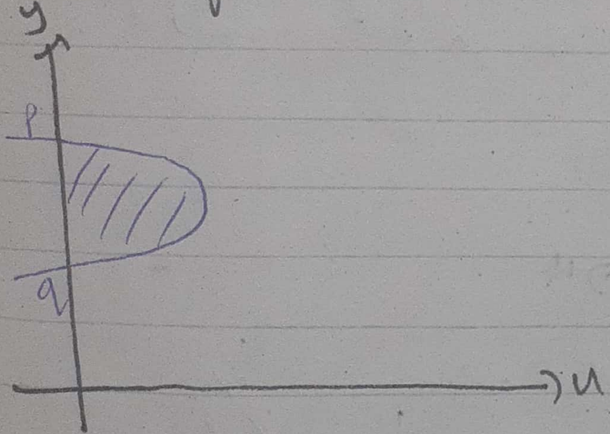
$$\int_a^b f(x) + \left|\int_b^c f(x)\right| + \int_c^d f(x)$$

$$\neq \int_a^d f(x)\,dx$$

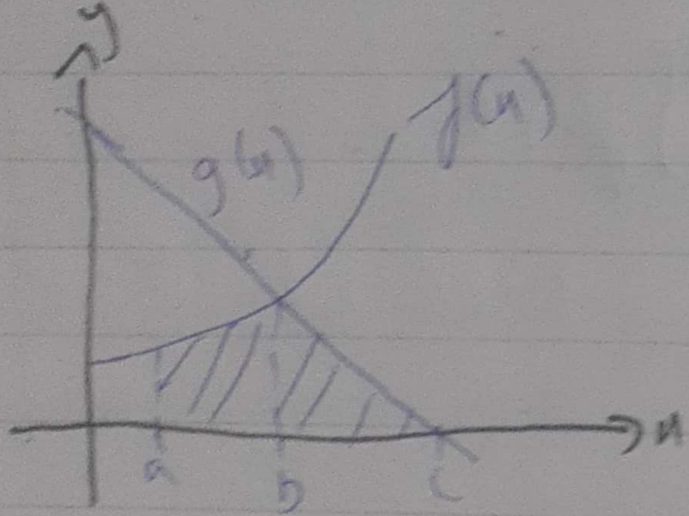


$$\int_a^q f(x)\,dx$$

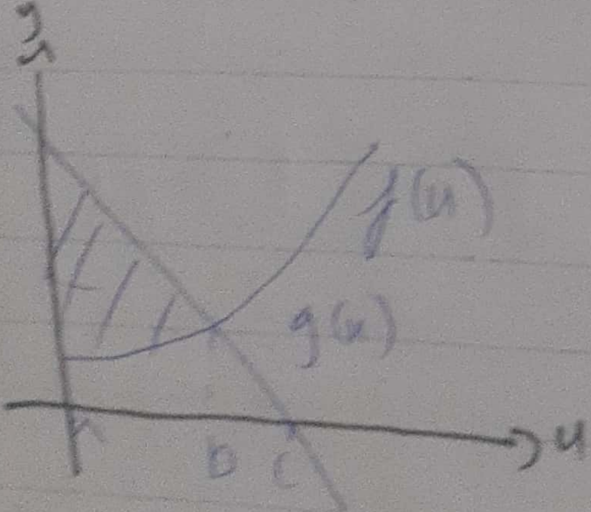
If like this:



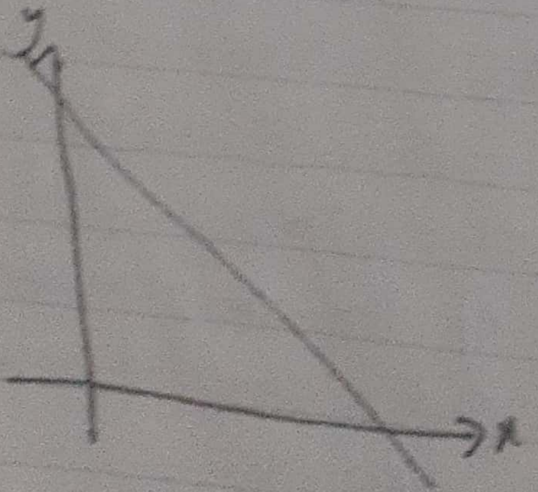
Step 1:
Make x the subject of the formula
Step 2:
Apply limits on y axis
Step 3:
$$\int_p^q f(y)$$

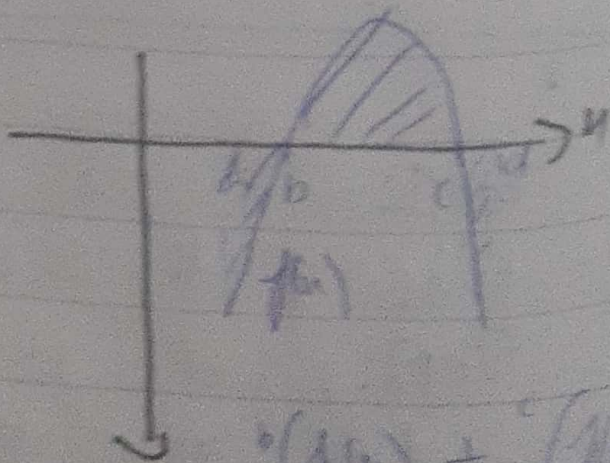


$\int_a^b f(x) + \int_b^c g(x)$



$\int_a^b g(x) - \int_a^b f(x)$





$\int_a^b f(x) + \int_b^c f(x) + \int$

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Model 12 Reverse Integration

Reverse Integration is a process in which in the first part of the question you have to ~~different~~ differentiate the given equation and then using the results, we will integrate ~~it~~ the answer hence obtained. Usually, there are those functions whose integration is other wise not possible.

Given that $y = \frac{x+5}{\sqrt{2x-1}}$.

i) Find $\frac{dy}{dx}$

ii) Hence find $\int \frac{x-6}{\sqrt{(2x-1)^3}}$

i) $u = x+5$ $\quad$ $v = \sqrt{2x-1}$

$u' = 1$ $\quad$ $v' = (2x-1)^{\frac{-1}{2}}(2)$

$= \frac{2}{\sqrt{2x-1}}$

$$\frac{\sqrt{2x-1} - \frac{x+5}{\sqrt{2x-1}}}{2x-1}$$

$$\frac{2x-1-x-5}{(2x-1)\sqrt{2x-1}}$$

$$\frac{x-6}{(2x-1)^{\frac{3}{2}}}$$

$$\cancel{\frac{x-6}{\sqrt{(2x-1)^3}}} \cancel{\frac{d\left(\frac{x+5}{\sqrt{2x-1}}\right)}{dx}} = \int \frac{x-6}{\sqrt{(2x-1)^3}}$$

$$\cancel{y=} \quad y = \frac{x+5}{\sqrt{2x-1}} + c$$

1 a) Find $\frac{dy}{dx}$ of $y = x\sqrt{x^2-4}$

b) Hence find $\int \frac{x^2-2}{\sqrt{x^2-4}}\,dx$

2 a) $y = 3(x+1)\sqrt{x-5}$; show:

a) $\frac{dy}{dx} = \frac{9(x-3)}{2\sqrt{x-5}}$

b) find $\int \frac{x-3}{\sqrt{x-5}}\,dx$

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1 a)

$u = x$ $\quad v = \sqrt{x^2-4}$

$u' = 1$ $\quad v' = \frac{1}{2}(x^2-4)^{-\frac{1}{2}}(2x)$

$$\sqrt{x^2-4} + \frac{x^2}{\sqrt{x^2-4}}$$

$$\frac{\sqrt{x^2-4}\sqrt{x^2-4}+x^2}{\sqrt{x^2-4}} \quad \frac{2x^2-4}{\sqrt{x^2-4}}$$

$$\frac{2(x^2-2)}{\sqrt{x^2-4}}$$

$$\int \frac{d}{dx}\left(x\sqrt{x^2-4}\right)dx = \int \frac{2(x^2-2)}{\sqrt{x^2-4}}$$

$$\frac{x\sqrt{x^2-4}}{2} + c = \int 2x \frac{x^2-2}{\sqrt{x^2-4}}$$

2) $u = x+1$ $\qquad v = \sqrt{x-5}$

$u' = 1$ $\qquad v' = \frac{1}{2\sqrt{x-5}}$

$$\frac{x+1}{2\sqrt{x-5}} + \sqrt{x-5}$$

$$\frac{x+1+2x-10}{2\sqrt{x-5}}$$

$$3\left(\frac{3x-9}{2\sqrt{x-5}}\right)$$

$$\frac{9x-27}{2\sqrt{x-5}}$$

$$\int \frac{9(x-3)}{2\sqrt{x-5}}dx = \frac{d}{dx}\left(3(x+1)\sqrt{x-5}\right)$$

$$\frac{{}^{2}\cancel{6}(x+1)\sqrt{x-5}}{\cancel{9}\,3}$$

$$\frac{2(x+1)\sqrt{x-5}}{3} + c$$