Permutations and Combinations

Basic Counting Principle

T_1	T_2	T_3
Shirts (4)	Trousers (2)	Shoes
Blue	Brown	Black
Black	Blue	Brown
Pink		
Yellow		

Task 1 = m ways
Task 2 = n ways
Together = m×n ways

Together = 4×2 (T_1, T_2)
Together = 8 ways

Together = 4×2×2 (T_1, T_2, T_3)
= 16 ways.

→ Factorial notation

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

used to arrange n objects in a straight line without restrictions

Examples

1: Find the ways in which 5 people can be arranged in a straight line.

$$5! = \underset{1^{st}}{5} \quad \underset{second}{4} \quad \underset{3^{rd}}{3} \quad \underset{4^{th}}{2} \quad \underset{5^{th}}{1} = 5 \times 4 \times 3 \times 2 \times 1 = 5! \text{ or } {}^{5}P_5 = \boxed{120}$$

(Total: 5; Permute → Arrange)

2: In how many ways can you answer a test containing 5 True/False questions.

$$\underset{1}{2} \quad \underset{2}{2} \quad \underset{3}{2} \quad \underset{4}{2} \quad \underset{5}{2} = 2^5 = \boxed{32}$$

3. In how many ways can the letters of the word Ali be arranged?

$3! = 3 \times 2 \times 1 = \boxed{6}$

4. In how many ways can the letters of the word 'POP' be arranged?

$\frac{3!}{2} \rightarrow \frac{6}{2} = \boxed{3}$

Manually
1. POP
2. OPP
3. PPO

Note: When letters are repeated, we divide by the factorial of the number by the times each number is repeated.

Permutations: Used in arrangements. Applied when order/ment/rank is important

$3, 2 \neq 2\ 3$ Permutation
$A \& B = B \& A$ Combination.

$\underset{1^{st}}{A}\ \underset{2^{nd}}{B}\ \underset{3^{rd}}{C} \neq \underset{1^{st}}{B}\ \underset{2^{nd}}{A}\ \underset{3rd}{C}$ Permutation

$^{5}P_{3}$ → Total available; 5 → spaces available

(1) $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
$^{5}P_{5} = 120$

(2) $5 \times 4 \times 3$ OR $^{5}P_{3}$

(3) $6 \times 5 \times 4 \times 3 = {}^{6}P_{4} = 360$

A security code consists of three letters selected from A, B, C, D, E, F followed by 2 digits selected from 5, 6, 7, 8, 9.
Find the number of possible security codes if no letter or number can be repeated.

number of possibilities

$6 \times 5 \times 4 \times 5 \times 4 = 2400$

(Letters: $6 \times 5 \times 4$; Digits: 5×4)

OR

$^6P_3 \times {}^5P_2 = 2400$

31 **(a)** A 6-digit number is to be formed using the digits 1, 3, 5, 6, 8, 9. Each of these digits may be used only once in any 6-digit number. Find how many different 6-digit numbers can be formed if

(i) there are no restrictions, [1]

$^6P_6 = 720$

(ii) the number formed is even, [1]

_ _ _ _ _ 6, 8

$^5P_5 \times {}^2P_1 = 240$

(iii) the number formed is even and greater than 300 000. [3]

3 5, 8, 9 _ _ _ 6, 8

$^4P_1 \times {}^4P_4 \times {}^1P_1 = 96$

$^4P_1 \times {}^4P_4 \times {}^1P_1 = 96$

192

combination: Selection or Team or Choose

A & B = B & A (same team)

AB = BA ⇒ Combination.

3 How many different ways are there of selecting

a 3 photographs from 10 photographs $^{10}C_3 = 120$

b 5 books from 7 books $^{7}C_5 = 21$

c a team of 11 footballers from 14 footballers? $^{14}C_{11} = 364$

4 How many different combinations of 3 letters can be chosen from the letters P, Q, R, S, T? $^{5}C_3 = 10$