Factors and Polynomials

Remainder Theorum

If a polynomial $p(x)$ is divided by $x-a$, the remainder is $f(a)$.

Examples

a) x^3+2x^2-3x+5 is divided by $x-2$. Find the remainder

Divisor

$x-2=0$

$x=2$

$(2)^3+2(2)^2-3(2)+5$

$8+8-6+5$

Remainder $=15$

Factor Theorum

If $p(x)$ is divisible by $x-a$, than $p(a)=0$

$x-a$ is a factor

Solve/Factors a cubic Equations

Factorise $x^3+2x^2-3x-10$

First Factor: Trial and error

Factors of 10: $\pm1, \pm2, \pm5, \pm10$

$x=1 = 1^2+2-3-10$

$= -10 \neq 0$

$x=2$: $2^3+2(2)^2-3(2)-10$

$8+8-6-10$

$16-16=0$

$x-2$ is a factor

To find the remaining two possible factors:

Long Division

$$x-2 \;\overline{)\; x^3+2x^2-3x-10}$$

$$-(x^3-2x^2)$$

$$4x^2-3x$$

$$-(4x^2-8x)$$

$$5x-10$$

$$-(5x-10)$$

$$0$$

$$x^2+4x+5$$

$x^2+4x+5=0$ Solve as a quadratic

→ to check if you should be doing this

$$x^2+4x+5$$

$$b^2-4ac$$

$$16-4(1)(5)$$

$$16-20$$

$$=-4 \Rightarrow \text{No Real Roots}$$

$$x^3-2x^2-3x-10=(x-2)(x^2+4x+5)$$

Identities (comparing coeff)

Linear × (quadratic) = Cubic

$(x-2) \times (ax^2+bx+c) = x^3+2x^2-3x-10$

$ax^3+bx^2+cx-2ax^2-2bx-2c$

$ax^3+bx^2-2ax^2+cx-2bx-2c=x^3+2x^2-3x-10$

$ax^3+(b-2a)x^2+(c-2b)x-2c=x^3+2x^2-3x-10$

$a=1$

$b=b-2(1)=2$

$b=4$

$+2c=+10$

$c=5$

x^2+4x+5

28 $px = 10x^3 + ax^2 - 10x + b$ $2x+1=0 \Rightarrow x=-\frac{1}{2}$ $x=-2 \quad -24$

$10\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 - 10\left(-\frac{1}{2}\right) + b = 0$

$-\frac{5}{4} + \frac{a}{4} + 5 + b = 0$

$\frac{a}{4} + b = -\frac{15}{4}$

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$\times 4 \left(\frac{-24-b}{4} + b = -\frac{15}{4}\right)$

$= -24 - b + 4b = -15$

$= 3b = 9$

$b = 3$

$10(-1) + a(-1)^2 - 10(-1) + b = -24$

$-10 + a + 10 + b = -24$

$a + b = -24$

$a = -24 - 3$

$a = -24 - 3$

$a = -27$

(b) $10x^3 + ax^2 - 10x + b$

$= 10x^3 - 27x^2 - 10x + 3$

$$
\begin{array}{r|l}
 & 5x^2 - 16x + 3 \\
2x+1 & 10x^3 - 27x^2 - 10x + 3 \\
 & -(10x^3 + 5x^2) \\
 & \quad -32x^2 - 10x \\
 & \quad +(32x^2 + 16x) \\
 & \quad\quad 6x + 3 \\
 & \quad\quad -(6x + 3) \\
 & \quad\quad 0
\end{array}
$$

$(2x+1)(5x^2 - 16x + 3)$

$(2x+1)(5x^2 - 15x - x + 3)$

$(2x+1)(5x(x-3) - 1(x-3))$

$(2x+1)(5x-1)(x-3)$