Indices and surds

$\sqrt{a}$ surd form

$a^{1/2}$ Index form

Prime factorisation

$2\times2\times2$

2	8
2	2
2	2
	1

1) $\sqrt{8} = 2\sqrt{2}$

$2 \quad \sqrt{2\times2\times2}$

2) $4\sqrt{12} = 4\sqrt{2\times2\times3} \rightarrow 8\sqrt{3}$

3) $\sqrt{24} = 2\sqrt{6}$

4) $\sqrt{72} = 6\sqrt{2}$

5) $6\sqrt{27} = 18\sqrt{3}$

6) $3\sqrt{15} = 3\sqrt{15}$

Rationalisation

$$\frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

→ have to change this so multiply it by the same thing

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{5\sqrt{3}}{6\sqrt{2}} \Rightarrow \frac{5\sqrt{6}}{12}$$

$$\frac{3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \rightarrow \frac{3\sqrt{2}+3}{2+1} \rightarrow \frac{3(\sqrt{2}+1)}{3} \rightarrow \boxed{\sqrt{2}+1}$$

↳ use 3rd identity

Expand & Simplify

(i) $(2+4\sqrt{2})^2 = 2^2 + 2(2)(4\sqrt{2}) + (4\sqrt{2})^2$

$4 + 16\sqrt{2} + 32$

$\boxed{36+16\sqrt{2}}$

(ii) $(5-3\sqrt{2})^2 = 25 - 2(3\sqrt{2})(5) + (3\sqrt{2})^2$

$= 25 - 30\sqrt{2} + 18$

$= \boxed{43-30\sqrt{2}}$

(iii) $(3-4\sqrt{2})^2 = 9 - 24\sqrt{2} + 32$

$\boxed{41-24\sqrt{2}}$

(iv) $(1+2\sqrt{5})^2 = 1 + 4\sqrt{5} + 20$

$= \boxed{21+4\sqrt{5}}$

c) $(2^{2-x})(4^{2x+3}) = 2^3$

$(2^{2-x})(2^{4x+6}) = 2^3$

$2^{3x+8} = 2^3$

$3x+8 = 3$

$3x = -5$

$\boxed{x = -5/3}$

(4a) $2^{3x} \times 4^{x+1} = 64$

$2^{3x} \times 2^{2x+2} = 2^6$

$2^{5x+2} = 2^6$

$5x+2 = 6$

$5x = 4$

$x = 0.8$

(b) $2^{3x+1} \times 8^{x-1} = 128$

$2^{3x+1} \times 2^{3x-3} = 2^7$

$2^{6x-2} = 2^7$

$6x-2 = 7$

$6x = 5$

$x = 5/6$

(d) $3^{x+1} \times 3^{4-2x} = 3^{-3}$

$3^{-x+5} = 3^{-3}$

$-x+5 = -3$

$-x = -8$

$x = 8$

5a) $\frac{3^{6x}}{3^{5-x}} = \frac{3^{2x+1}}{3^{2x+6}}$

$3^{6x-(5-x)} = 3^{(2x+1)-(2x+6)}$

$3^{7x-5} = 3^{-5}$

$7x-\cancel{5} = \cancel{5}$

$x = 0$

$2(2^x)^2 - 7(2^x) - 4 = 0$ $\quad 2^x = y$

$2y^2 - 7y - 4 = 0$

$2y^2 - 8y + y - 4 = 0$

$2y(y-4) + 1(y-4) = 0$

$(y-4)(y+1) = 0$

$y = 4$ $\quad 2^x = 4$

$y = \frac{-1}{2}$ $\quad x = 2$

$4(3^{2x}) = 15 + 7(3^x)$ $\quad 3^x = a$

$4a^2 = 15 + 7a$

$4a^2 - 7a - 15 = 0$

$4a^2 - 12a + 5a - 15 = 0$

$4a(a-3) + 5(a-3) = 0$

$(a-3)(4a+5) = 0$

$a = 3$ $\quad 3^x = 3^1$ $\quad 3^x = \frac{-4}{5}$

$a = \frac{-4}{5}$ $\quad x = 1$ $\quad$ not possible