Logs

(1) $a^b = c$ → Index form $\log_a c = b$ → Argument (c), → Base (a)

Examples

Index form	log form
$2^3 = 8$ →	$\log_2 8 = 3$
$3^3 = 27$	$\log_3 27 = 3$
$5^2 = 25$	$\log_5 25 = 2$
$1^x = 7$ not possible	$\log_1 7 =$ Not possible
$7^0 = 1$	$\log_7 1 = 0$
$5^1 = 5$	$\log_5 5 = 1$

(2) log to be defined $\sqrt{-9}$ → not possible

$\log_b a$

(1) $a > 0$

(2) $b > 0$ & $b \neq 1$

③ $\log_a 1 = 0$ $\qquad a^0 = 1$

④ $\log_a a = 1$

⑤ $\log_a b^n = n \log_a b$

→ Solving exponential equations using logs (lg and ln)

① log with base 10 is called lg

$\log_{10} 5 = \lg 5$

$\log_{10} 7 = \lg 7$

→ $\lg 10 = 1$

Example 1

$2^n = 8$

$n = 3$

Example 2

$2^n = 9$

$\lg 2^n = \lg 9$

$n \lg 2 = \lg 9$

$n = \frac{\lg 9}{\lg 2}$

$\boxed{n = 3.17}$

Example 3

$3^n = 15$

$\lg 3^n = \lg 15$

$n = \frac{\lg 15}{\lg 3}$

$n = 2.464$

$\boxed{n = 2.46}$

Example 4

$5^n - 2 = 1$

$5^n = 3$

$\lg 5^n = \lg 3$

$n \lg 5 = \lg 3$

$n = \frac{\lg 3}{\lg 5}$

$\boxed{n = 0.683}$

Example 5

$$8^{x-1} = 11$$

$$\lg 8^{x-1} = \lg 11$$

$$(x-1)\lg 8 = \lg 11$$

$$x-1 = \frac{\lg 11}{\lg 8}$$

$$x = \frac{\lg 11}{\lg 8} + 1$$

$$\boxed{x = 2.15}$$

Example 6

$$5^{x+1} = 3^{x+2}$$

$$5^x \times 5 = 3^x \times 3^2$$

$$\left(\frac{5}{3}\right)^x = \frac{9}{5}$$

$$\lg\left(\frac{5}{3}\right)^x = \lg\frac{9}{5}$$

$$x = \frac{\lg 9/5}{\lg 5/3}$$

$$\boxed{x = 1.15}$$

② log with base 'e' is called ln

$\rightarrow \ln e = 1$

(a) $e^x = 7$

$$\ln e^x = \ln 7$$

$$x \ln e = \ln 7$$

$$x = \ln 7$$

$$x = 1.95$$

(b) $\ln x = 3$

$$e^3 = x$$

$$\boxed{x = 20.1}$$

(c) $\ln(x+1) = 7$

$$e^7 = x+1$$

$$x = e^7 - 1$$

$$x = 1095.63$$

$$\boxed{x = 1100}$$

⑦ Product Law

$$\log_b a + \log_b c = \log_b(ac)$$

$$\log_2 4 + \log_2 8 = \log_2(4 \times 8)$$

$$= \log_2 32$$

$$2 + 3 = 5$$

⑧ Quotient Rule

$$\log_a b - \log_a c = \log_a \frac{b}{c}$$

$$\log_2 8 - \log_2 4 = \log_2 2$$

$$3 - 2 = 1$$

Write as a simple logarithm

(a) $\lg 5 - 2$

$= \lg 5 - \lg 10^2$

$= \lg 5 - \lg 100$

$= \lg 5/100$

$= \boxed{\lg 1/20}$

(b) $3 - \log_4 10$

$\log_4 4^3 - \log_4 10$

$\log_4 64 - \log_4 10$

$\boxed{\log_4 6.4}$

→ Change of Base Law

$$\log_4 64 = \frac{\log_2 64}{\log_2 4} = \frac{6}{2} = \boxed{3}$$

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

eg

$$\log_9 81 = \frac{\log_3 81}{\log_3 9} = \frac{4}{2} = 2$$

2

same

→ Reciprocal Law

$$\boxed{\log_b a = \frac{1}{\log_a b}}$$

$$\log_2 8 = \frac{1}{\log_8 2}$$

$$\log_3 5 = \frac{1}{\log_5 3}$$

change of base law

$$\frac{\log_5 5}{\log_5 3} \rightarrow \frac{1}{\log_5 3}$$

Past Papers

53 a) Solve the equation $\log_6(2x-3) = \frac{1}{2}$ Give your answer in exact form

$$\log_6(2x-3) = \frac{1}{2}$$

$$6^{1/2} = 2x-3$$

$$\sqrt{6}+3 = 2x$$

$$\boxed{\frac{\sqrt{6}+3}{2} = x}$$

OR

$$\log_6(2x-3) = \frac{1}{2}\log_6 6$$

$$\cancel{\log_6}(2x-3) = \cancel{\log_6}\sqrt{6}$$

$$2x-3 = \sqrt{6}$$

$$2x = \sqrt{6}+3$$

$$\boxed{x = \frac{\sqrt{6}+3}{2}}$$

(b) Solve the equation $\ln 2u - \ln(u-4) = 1$ Give your answer in exact form.

$$\ln\frac{2u}{u-4} = 1$$

$$e^1 = \frac{2u}{u-4}$$

$$2u = ue - 4e$$

$$2u - ue = -4$$

$$u(2-e) = -4$$

$$\boxed{u = \frac{-4}{2-e} \text{ OR } \frac{4}{e-2}}$$

Write $3 + 2\lg a - \lg b$ as a logarithm to base 10

$3 + 2\lg a - \lg b$

$3\lg 10 + \lg a^2 - \lg b$

$\lg 1000 + \lg a^2 - \lg b$

$\boxed{\lg\left(\frac{1000a^2}{b^4}\right)}$

b) Solve the equation $3\log_a 4 + 2\log_4 a = 7$

Substituition

$\log_4 a = y$

$3\log_a 4 + 2\log_4 a = 7$

$\frac{3}{\log_4 a} + 2y = 7$

$\frac{3}{y} + 2y = 7$

$3 + 2y^2 = 7y$

$2y^2 - 7y + 3$

$2y^2 - 6y - y + 3$

$2y(y-3) - (y-3)$

$(y-3)(2y-1)$

$y = 6$

$y = 1/2$

$\log_4 a = 3$

$a = 4^3$

$\boxed{a = 64}$

$\log_4 a = \frac{1}{2}$

$a = 4^{1/2}$

$\boxed{a = 2}$

Solve the equation $4\log_y 2 + \log_2 y = 4$

$\log_2 y = x$

$\frac{4}{\log_2 y} + x = 4$

reciprocal law

$\frac{4}{x} + x = 4$

$4 + x^2 = 4x$

$x^2 - 4x + 4 = 0$

$x^2 - 2x - 2x + 4 = 0$

$x(x-2) - 2(x-2)$

$x = 2$

$\log_2 y = 2$

$2^2 = y$

$\boxed{4 = y}$

Given that $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$ find the value of p

$\cancel{\log_a} \frac{5p}{4} = \cancel{\log_a} 20$

$\frac{5p}{4} = 20$

$5p = 80$

$\boxed{p = 16}$

No Calc

$\log_2(y+1) = 3 - 2\log_2 x$

$\log_2(x+2) = 2 + \log_2 y$

(a) Show that $x^3 + 6x^2 - 32 = 0$

$\log_2(y+1) = 3\log_2 2 - \log_2 x^2$

$\log_2(y+1) = \log_2 8 - \log_2 x^2$

$\cancel{\log_2}(y+1) = \cancel{\log_2} \frac{8}{x^2}$

$\boxed{y+1 = \frac{8}{x^2}} \rightarrow 1$

$\log_2(x+2) = \log_2 4 + \log_2 y$

$\cancel{\log_2}(x+2) = \cancel{\log_2} 4y$

$x + 2 = 4y$

$2 \rightarrow \boxed{x = 4y - 2}$

$$y = \frac{8}{x^2} - 1 \rightarrow x + 2 = 4y$$

$$x + 2 = 4\left(\frac{8}{x^2} - 1\right)$$

$$x + 2 = \frac{32}{x^2} - 4$$

$$x + 6 = \frac{32}{x^2}$$

$$x^3 + 6x^2 = 32$$

$$\boxed{x^3 + 6x^2 - 32 = 0}$$