Matrices

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1. Order of a matrix : # of Rows x # of Columns

$$A = \begin{array}{cc} C_1 & \\ \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{array}{l} \to R_2 \\ \to R_1 \end{array} \end{array} \qquad B = \begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_2 \\ R_1 \end{array} & \begin{bmatrix} 3 & 2 & 5 \\ 7 & 1 & -4 \end{bmatrix} \end{array} \qquad C = \begin{bmatrix} 5 \end{bmatrix}$$

2×1 $\qquad$ 2×3 $\qquad$ 1×1

Column Matrix

$$D = \begin{bmatrix} 3 & -1 \\ 4 & 8 \end{bmatrix}$$

2×2

2. Addition & Subtraction : Two or more matrices can only be added or subtracted if & only if the order is the same.

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 9 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -2 \\ -7 & 0 \end{bmatrix}$$

$A + B$ = Not Possible

$$A + C = \begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -7 & 0 \end{bmatrix} = \begin{bmatrix} 3+3 & 5+-2 \\ 1+-7 & 8+0 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -6 & 8 \end{bmatrix}$$

3. Scalar Multiplication

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

① $A = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$, find $2A$

$$2\begin{bmatrix} 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \end{bmatrix}$$

② $B = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$, find $3B$

$$3\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 12 & 6 \end{bmatrix}$$

③ $A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$ $C = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$

(a) $2A$

$$2\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

(b) $A - 2B$

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 10 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

(c) $2A + 3B - 3C$

$$\begin{bmatrix} 6 \\ -4 \end{bmatrix} + \begin{bmatrix} 15 \\ -24 \end{bmatrix} - \begin{bmatrix} 0 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} 21 \\ -49 \end{bmatrix}$$

4. Matrix multiplication : Two matrices can only be multiplied if the number of columns of the first matrix is equal to the number of rows of the second matrix.

Note: $AB \neq BA$

① $A = \begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, find AB

$2 \times \underbrace{2 \quad \quad 2}_{\text{possible}} \times 1$

2×1: Order of the product

Row

Column

$$\begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{array}{c c} & C_1 \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} (3 \times 1) + (5 \times -3) \\ (-2 \times 1) + (1 \times -3) \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 3 - 15 \\ -2 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} -12 \\ -5 \end{bmatrix}$$

② $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \begin{bmatrix} 5 \\ -7 \end{bmatrix}$ = Not Possible

2×1 $\quad 2 \times 1$

(3)

$$\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = \begin{array}{c} \\ R_1 \\ R_2 \end{array} \begin{array}{c} C_1 \quad\quad\quad\quad\quad\quad C_2 \\ \begin{bmatrix} 3\times0 + -2\times3 & (3\times1)+(-2\times2) \\ (1\times0)+(5\times3) & (1\times1)+(5\times2) \end{bmatrix} \end{array}$$

$$\underbrace{2 \times \underbrace{2 \quad 2}_{=} \times 2}_{2\times2}$$

$$\begin{bmatrix} -6 & -1 \\ 15 & 11 \end{bmatrix}$$

(4)

$$\underset{2\times2}{\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}} \underset{2\times3}{\begin{bmatrix} 1 & 5 & 3 \\ 0 & -2 & -4 \end{bmatrix}} = \begin{bmatrix} 4+0 & 20+-6 & 12+-12 \\ 2+0 & 10+-2 & 6+-4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 14 & 0 \\ 2 & 8 & 2 \end{bmatrix}$$

Calculate $\begin{pmatrix} 3 & 7 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 4 & 2 \end{pmatrix}$.

$$\begin{bmatrix} 22 & 17 \\ 18 & 7 \end{bmatrix}$$

5. Identity Matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (One of matrices)
1

$5 \times 1 = 5$
$10 \times 1 = 10$
$-3 \times 1 = -3$

Note: Any matrix multiplied by the identity matrix will remain unchanged

$AI = A$

$IB = B$

① $A = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$, find AI

$$\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3+0 & 0+-2 \\ 2+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$$

② $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3+0 \\ 0-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$\sin x = 0.5$
$x = \sin^{-1}(0.5)$

$AI = A$

$I = A^{-1}A$ Any matrix multiplied by its inverse will give the identity matrix.

$$\begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{3}{2} \\ -3 & \frac{5}{2} \end{bmatrix} = \begin{bmatrix} 10+-9 & -\frac{15}{2}+\frac{15}{2} \\ 12+-12 & \frac{-18}{2}+\frac{20}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$AA^{-1} = I$

6. Inverse of a matrix (2×2)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \det(A) = ad-bc$$

Examples

① $\begin{bmatrix} 3 & 1 \\ -4 & 5 \end{bmatrix}^{-1}$

$$\frac{1}{15-(-4)}\begin{bmatrix} 5 & -1 \\ 4 & 3 \end{bmatrix}$$

$$\frac{1}{19}\begin{bmatrix} 5 & -1 \\ 4 & 3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \frac{5}{19} & \frac{-1}{19} \\ \frac{4}{19} & \frac{3}{19} \end{bmatrix}$$

② $B = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$, Find B^{-1}

$$\frac{1}{10-(-3)}\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$$

$$\frac{1}{13}\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$$

Calculate the inverse of $\begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}$.

$$\frac{1}{2}\begin{bmatrix} 4 & -3 \\ -6 & 5 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} 2 & -\frac{3}{2} \\ -3 & \frac{5}{2} \end{bmatrix}$$

Note:

$$AX = B \qquad\qquad XA = B$$

$$X = A^{-1}B \qquad\qquad X = BA^{-1}$$