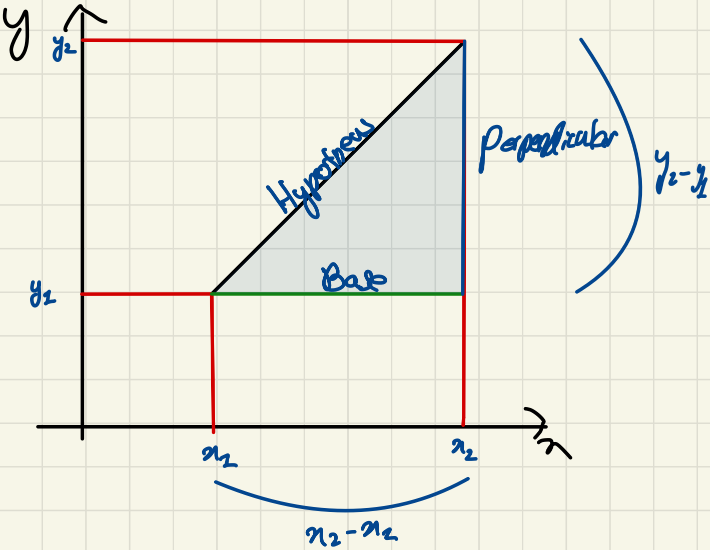
Coordinate Geometry.

→ Distance b/w two points

$A = (x_1, y_1)$ $B = (x_2, y_2)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$\text{Base}^2 + \text{Perpendicular}^2 = \text{Hyp}^2$

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = \text{Hyp}^2$

$\sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$

Examples

1) a: $A(3,5)$ $B(6,1)$

$AB = \sqrt{(6-3)^2 + (1-5)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= \boxed{5 \text{ units}}$

2) $A = (2,-5)$ $B(14,0)$

$AB = \sqrt{(14-2)^2 + (0--5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= \boxed{13 \text{ units}}$

2a) $A(a,5)$ $B(3,a)$. Given that $AB = 5$ units Find the values of a

$AB = \sqrt{(3-a)^2 + (a-5)^2} = 5$

$(3-a)^2 + 16 = 25$

$(3-a)^2 = 9$

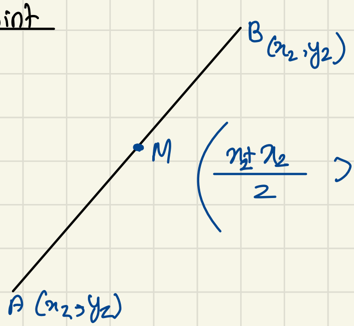
$3 - a = \pm 3$

$\boxed{a = 0 \quad a = 6}$

(b) $P(p, -2)$ $Q(6, 13)$. Given that $PQ = 17$ units. Find the values of p.

$$\sqrt{(6-p)^2 + (13+2)^2} = 17$$
$$(6-p)^2 + 225 = 17^2$$
$$(6-p)^2 = 289-225$$
$$(6-p)^2 = 64$$
$$6-p = \pm 8$$
$$-2 = p \quad , \quad 14 = p$$
$$\boxed{\begin{array}{l} p = -2 \\ p = 14 \end{array}}$$

→ <u>Midpoint</u>



M = midpoint

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_2 + y_1}{2}\right)$$

Examples

1) $A(3, 9)$ $B(1, 5)$

$$M = \left(\frac{3+1}{2}, \frac{9+5}{2}\right)$$
$$= \boxed{2, 7}$$

(b) $P(-2, 0)$ $Q(4, 6)$

$$M\left(\frac{-2+4}{2}, \frac{0+6}{2}\right)$$
$$= \boxed{1, 3}$$

Qs 2 $A(3, 5)$ $B(x, y)$. Given that midpoint of A & B is $M(4, 7)$

$$\left(\frac{3+x}{2}, \frac{5+y}{2}\right) = 4, 7$$

$$\frac{3+x}{2} = 4$$
$$3 + x = 8$$
$$\boxed{x = 5}$$

$$\frac{5+y}{2} = 7$$
$$5 + y = 14$$
$$\boxed{y = 9}$$

$$\boxed{B(5, 9)}$$

Qs3 $A(-1,0)$, $B(1,6)$ and $C(7,4)$

Show that triangle ABC is a right angle isosceles triangle

$$AB = \sqrt{(1+1)^2 + (6-0)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$

$$BC = \sqrt{(7-1)^2 + (4-6)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$

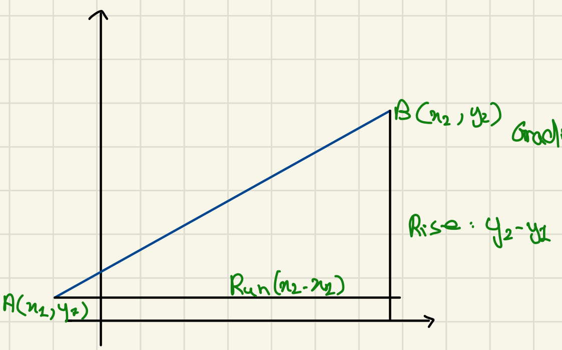
same values prove that they are isoseleces

$$AC = \sqrt{(7+1)^2 + (4-0)^2}$$
$$= \sqrt{64+16}$$
$$= \sqrt{80}$$

$$AB^2 + BC^2 = AC^2$$
$$\sqrt{40}^2 + \sqrt{40}^2 = \sqrt{80}^2$$
$$40 + 40 = 80$$
$$80 = 80$$

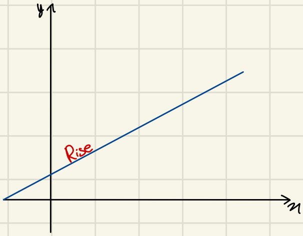
hence proved that they are right angled.

Gradient/slope : Measure of steepness of a line

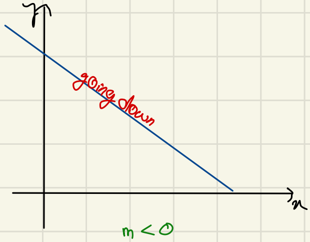


Gradient

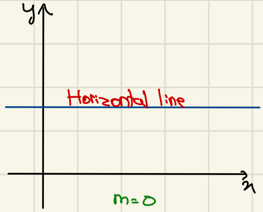
$$m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



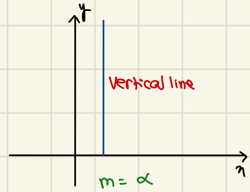
$m > 0$



$m < 0$



$m = 0$



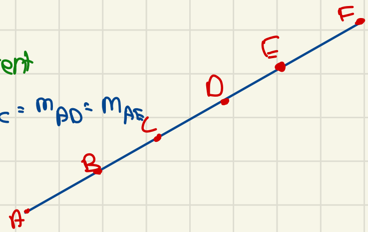
$m = \propto$

Key Point

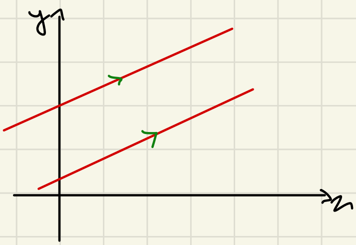
① Collinear Points have the same gradient

Points lying on the same line

$m_{AB} = m_{BC} = m_{AD} = m_{AE}$



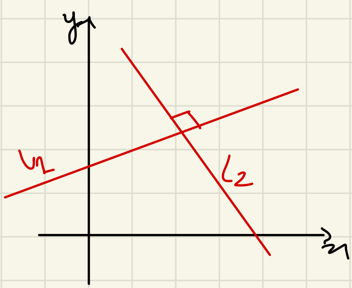
② Parallel lines have the same gradient



$L_1 \parallel L_2$

$m_1 = m_2$

③ Product of the gradient perpendicular line is -1

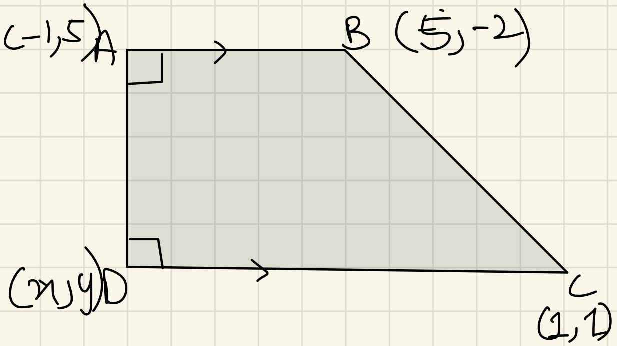


$m_1 \times m_2 = -1$

$m_1 = \frac{-1}{m_2}$

4) A $(-1,-5)$, B $(5,-2)$ and $C(1,1)$

ABCD is a trapezium
AB is parallel to DC and angle BAD is 90°
Find the cordinates of D.



$$m_{AB} = \frac{-2+5}{5--1} = \frac{3}{6}$$

$$m_{AB} = m_{DC}$$

$$\frac{1-y}{1-x} = \frac{3}{6} \rightarrow$$

$$6-6y = 3-3x$$

$$3+3x = 6y$$

$$\frac{1+x}{2} = y$$

$$y = \frac{1+1}{2}$$

$$y = 1$$

$$m_{AD} = -\frac{1}{m_{AB}}$$

$$m_{AD} = -\frac{6}{3}$$

$$\frac{5-y}{-1-x} = -\frac{6}{3}$$

$$5-y = +2+2x$$

$$3-2x = y$$

$$2(3-2x) = 1+x$$

$$6-4x = 1+x$$

$$5 = 5x$$

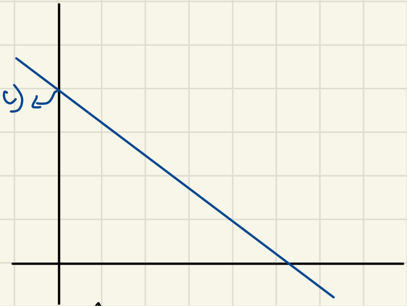
$$x = 1$$

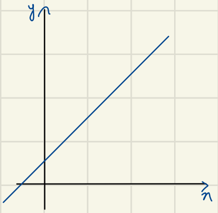
→ Equation Of Straight line

$y = mx + c$

m → gradient

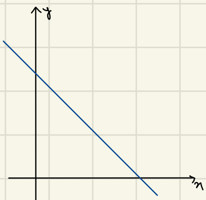
c → y-intercept where the point is $(0, c)$





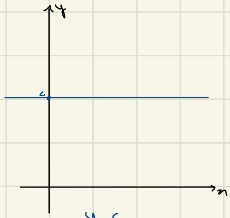
$y = mx + c$

$m = +ve$



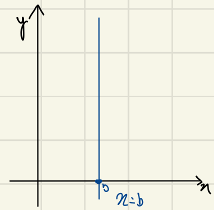
$y = mx + c$

$m = -ve$



$y = c$

$m = 0$



$x = b$

$m = \infty$

How to find the equation of a straight line

1. Gradient & y intercept. — $y = mx + c$
2. Gradient & a point on the line — $y - y_1 = m(x - x_1)$
3. Two points on the line

m = gradient

(x_1, y_1) = Point on the line

1. Gradient & y intercept.

$m = 3$ $c = 2$

$y = mx + c$

$\boxed{y = 3x + 2}$

2. Gradient & a point on the line

$y = mx + c$ $P(2, 3)$

$m = 4$

$y = 4x + c$

$3 = 4(2) + c$

$\boxed{-5 = c}$

$\boxed{y = 4x - 5}$

New way

$y - y_1 = m(x - x_1)$

$m = 4$ $P(2, 3)$

$y - 3 = 4(x - 2)$

$y - 3 = 4x - 8$

$\boxed{y = 4x - 5}$

Two points

$P(1,-3)$ $Q(1,7)$

$y-y_2 = m(x-x_1)$

$y-7 = 2(x-1)$

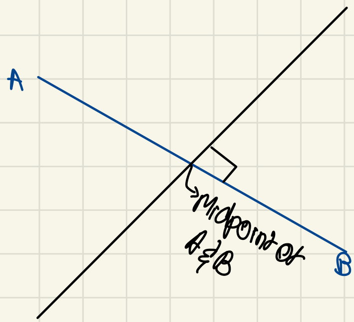
$y-7 = 2x-2$

$\boxed{y = 2x+5}$

$m = \frac{7-3}{1--1} = \frac{4}{2} = 2$

Perpendicular Bisector

Equation of perpendicular bisector of line AB



1. Find the midpoint of A & B
2. (i) Find the gradient of line AB (m_1)
   (ii) Take the negative reciprocal $m_2 = \frac{-1}{m_1}$
3. Form an equation using $y-y_1 = m(x-x_1)$

Example

Find the equation of the perpendicular bisector of

(a) $A(-1,2)$ $B(1,6)$

$M = \left(\frac{-1+1}{2}, \frac{2+6}{2}\right)$

$= \boxed{(0, 4)}$

$m_{AB} = \frac{6-2}{1--1}$

$m_{AB} = 2$

$m_{pb} = \frac{-1}{2}$

$y-4 = \frac{-1}{2}(x-0)$

$\boxed{y = \frac{-x}{2} + 4}$