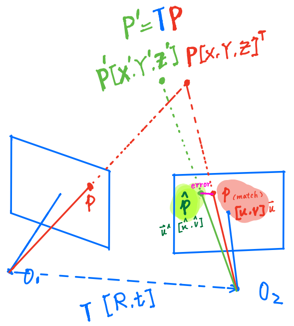
最小化重投影误差求解PnP



$s\hat{u} = KTP$

$s\vec{u} = KP'$

$\vec{u}_i = [u_i, v_i]$

$$T^* = \underset{T}{\arg\min} \frac{1}{2}\sum_{i=1}^{n} \left\| \vec{u}_i - \frac{1}{s_i}\underbrace{KTP}_{P'} \right\|_2^2$$

$s_i \vec{u}_i = KTP$

尺度因子　像素坐标　相机内参矩阵　前一帧坐标系转换到当前帧坐标系（相机运动）　前一帧坐标系

(error)

残差 residual $= \vec{u} - \frac{1}{s}KTP$

G-N . L-M 优化.

线性化残差，迭代求解Δx更新x，待优化变量.

$e(x+\Delta x) \approx e(x) + J_{(x)}^T \Delta x$

其中J为残差函数对待优化变量x的导数.

待优化变量为T，然而不能对其求导，将其转换为对T对应的李代数求导. $TP=\exp(\ ^\wedge)\ \xi$

$$\frac{\partial(\vec{u} - \vec{u}^\wedge)}{\partial(TP)} \rightarrow -\frac{\partial(u^\wedge)}{\partial(TP)}$$

$$s\begin{bmatrix}u^\wedge\\ v^\wedge\\ 1\end{bmatrix} = \begin{bmatrix}f_x & 0 & c_x\\ 0 & f_y & c_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}$$

$\vec{u}^\wedge$

$u^\wedge = f_x\frac{X'}{Z'} + c_x$

$v^\wedge = f_y \cdot \frac{Y'}{Z'} + c_y$

优化变换T，求Jacobi:

$$\frac{\partial(\vec{u} - \frac{1}{s}KTP)}{\partial \xi}$$

$$= \frac{\partial(\vec{u} - \frac{1}{s}KTP)}{\partial(TP)}\frac{\partial(TP)}{\partial\xi}$$

$$= -\frac{\partial\begin{bmatrix} f_x\frac{X'}{Z'} + c_x\\ f_y\frac{Y'}{Z'} + c_y\end{bmatrix}}{\partial\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}}(TP)^\odot$$

$$= -\underbrace{\begin{bmatrix} f_x\frac{1}{Z'} & 0 & -f_x\frac{X'}{Z'^2}\\ 0 & f_y\frac{1}{Z'} & -f_y\frac{Y'}{Z'^2}\end{bmatrix}}_{2\times 3}\underbrace{[I, -P'^\wedge]}_{3\times 6}$$

3×3　3×3

$$= -\begin{bmatrix} f_x\frac{1}{Z'} & 0\\ 0 & f_y\frac{1}{Z'}\\ -f_x\frac{X'}{Z'^2} & -f_y\frac{Y'}{Z'^2}\\ -f_x\frac{X'Y'}{Z'^2} & -f_y - f_y\frac{Y'^2}{Z'^2}\\ f_x + f_x\frac{X'^2}{Z'^2} & f_y\frac{X'Y'}{Z'^2}\\ -f_x\frac{Y'}{Z'} & -f_y\frac{X'}{Z'}\end{bmatrix}^T$$

$$se(3) = \left\{ \xi = \begin{bmatrix}\rho\\ \phi\end{bmatrix} \in \mathbb{R}^6, \rho \in \mathbb{R}^3, \phi \in so(3), \xi^\wedge = \begin{bmatrix}\phi^\wedge & \rho\\ 0^T & 0\end{bmatrix} \right\}$$

translation（ρ）　rotation（ϕ）

$$\xi = \begin{bmatrix}x\\ y\\ z\\ a\\ b\\ c\end{bmatrix} \qquad \xi^\wedge = \begin{bmatrix}0 & -c & b & x\\ c & 0 & -a & y\\ -b & a & 0 & z\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$\frac{\partial(TP)}{\partial\xi}$ SE(3)李代数求导推导

$$= \lim_{\delta\xi\to 0}\frac{\exp(\delta\xi^\wedge)\exp(\xi^\wedge)P - \exp(\xi^\wedge)P}{\delta\xi}$$

$$= \lim_{\delta\xi\to 0}\frac{(I+\delta\xi^\wedge)\exp(\xi^\wedge)P - \exp(\xi^\wedge)P}{\delta\xi}$$

$$= \lim_{\delta\xi\to 0}\frac{\delta\xi^\wedge\exp(\xi^\wedge)P}{\delta\xi}$$

$$= \lim_{\delta\xi\to 0}\frac{\begin{bmatrix}\delta\phi^\wedge & \delta\rho\\ 0^T & 0\end{bmatrix}\begin{bmatrix}RP+t\\ 1\end{bmatrix}}{\begin{bmatrix}\delta\rho\\ \delta\phi\end{bmatrix}}$$

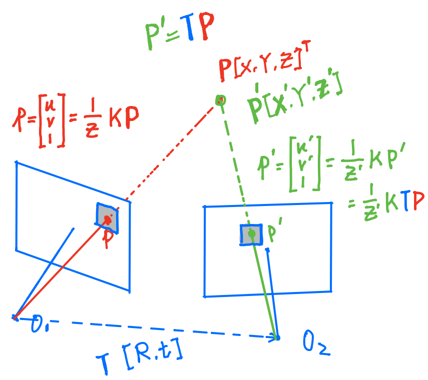
$$= \lim_{\delta\xi\to 0}\frac{\begin{bmatrix}\delta\phi^\wedge(RP+t)+\delta\rho\\ 0^T\end{bmatrix}}{\begin{bmatrix}\delta\rho\\ \delta\phi\end{bmatrix}}$$

$a^\wedge b$
$= -b^\wedge a$

$$= \lim_{\delta\xi\to 0}\left[\frac{\delta\phi^\wedge(RP+t)+\delta\rho}{\delta\rho}\quad \frac{\delta\phi^\wedge(RP+t)+\delta\rho}{\delta\phi}\right]$$

$$= [I, \underbrace{-(RP+t)^\wedge}_{-P'^\wedge}] \overset{\text{def}}{=} (TP)^\odot$$

直接法



$$T^* = \underset{T}{\arg\min}\frac{1}{2}\sum_{i=1}^{n}\| I_1(p_i) - I_2(p_i') \|$$

$e = I_1(p) - I_2(p')$

$$J = \frac{\partial(I_1(p) - I_2(p'))}{\partial\xi}$$

$$= -\frac{\partial I_2(\frac{1}{Z'}KTP)}{\partial\xi}$$

$$= -\frac{\partial I_2}{\partial(\frac{1}{Z'}KTP)}\cdot\frac{\partial(\frac{1}{Z'}KTP)}{\partial TP}\cdot\frac{\partial TP}{\partial\xi}$$

$$= -\frac{\partial I_2}{\partial p'}\cdot\frac{\partial\begin{bmatrix}f_x\frac{X'}{Z'}+c_x\\ f_y\frac{Y'}{Z'}+c_y\end{bmatrix}}{\partial\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}}(TP)^\odot$$

像素灰度梯度.