Kalman Filter

Mar. 17, 2019

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Introduction

Visual odometry : 순서적인 형태의 짧은 궤적 정보만을 가지고 있음
-> 우리는 전체 운동 궤적이 오랫동안 최적 상태로 유지되기를 바람

Backend optimization

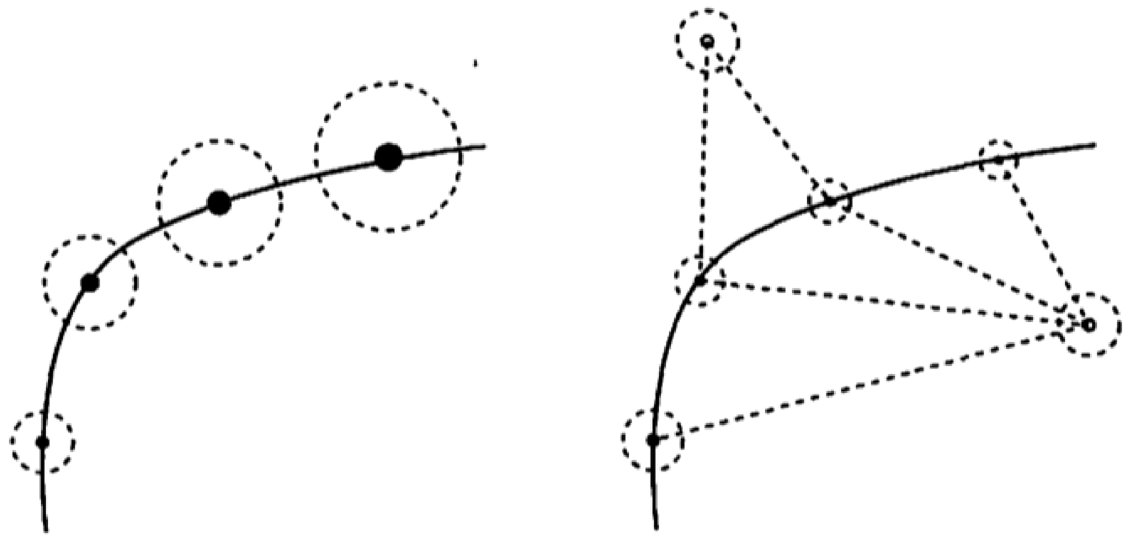
$$\begin{cases} \boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k) + \boldsymbol{w}_k \\ \boldsymbol{z}_{k,j} = h(\boldsymbol{y}_j, \boldsymbol{x}_k) + \boldsymbol{v}_{k,j} \end{cases} \quad k = 1, \ldots, N, \; j = 1, \ldots, M. \tag{10.1}$$

Batch process : 장기간 (또는 모든 시간) 상태 추정 문제를 고려하고 과거 정보를 사용하여 상태를 업데이트 할 뿐만 아니라 향후 정보로 업데이트
->X0, x1, x2, …. Xn 을 Z0, Z1, Z2, ….Zn 이 전부 주어질때 추정하는 방식

Incremental process : 현재 상태가 과거 순간 또는 바로 이전 순간에 의해서만 결정되는 경우
-> Xk 를 X0, X1, X2…. Xk-1 과 Z0, Z1, Z2, … Zk-1 로 추정하는 방식

$\boldsymbol{y}_j$는 항상 생성되지 않는다.
움직임을 측정할 장치가 없을 수도 있기 때문에 운동 방정식이 없을 수도 있다.
->몇 가지 관측 방정식으로 구성. SfM (Structure from Motion)

Introduction



운동방정식만을
고려한 경우

관측치를 기반으로
업데이트 해준경우

$$\begin{cases} \boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k) + \boldsymbol{w}_k \\ \boldsymbol{z}_{k,j} = h(\boldsymbol{y}_j, \boldsymbol{x}_k) + \boldsymbol{v}_{k,j} \end{cases} \quad k = 1, \ldots, N,\ j = 1, \ldots, M. \tag{10.1}$$

지속적으로 누적되는 error 를 제거하여 update 해준다.

Introduction

운동방정식
관측방정식

$$\begin{cases} x_k = A_k x_{k-1} + u_k + w_k \\ z_k = C_k x_k + v_k \end{cases} \quad k = 1, \ldots, N. \tag{10.9}$$

Gaussian noise $$w_k \sim N(0, R). \quad v_k \sim N(0, Q). \tag{10.10}$$

Xk 의 상태를 예측하기 위해서

Prediction

$$P(x_k | x_0, u_{1:k}, z_{1:k-1}) = N\left(A_k \hat{x}_{k-1} + u_k, A_k \hat{P}_{k-1} A_k^T + R\right). \tag{10.11}$$

Error Covariance (between xk, xhat k)

$$\bar{x}_k = A_k \hat{x}_{k-1} + u_k, \quad \boxed{\bar{P}_k = A_k \hat{P}_{k-1} A_k^T + R.} \tag{10.12}$$

Markov Sequence

Markov Sequence

A random sequence $x(k), k = 0, 1, \cdots, N$ is said to be "Markov", if

$$P\left[x(k+1) / x(k), x(k-1), \cdots, x(0)\right] = P\left[x(k+1) / \underbrace{x(k)}_{1^{st}\ \text{order}}\right] \text{ for all } k$$

$$= P\left[x(k+1) / \underbrace{x(k), x(k-1)}_{2^{nd}\ \text{order}}\right] \text{ for all } k$$

ex) $x(k+1) = c(k)x(k) + w(k)$

where $c(k)$: deterministic, $w(k)$: random

→ We can predict tomorrow using the today's information without knowing all the past information!

Linear Gauss Markov Sequence

Linear Gauss Markov Sequence

$$\begin{cases} x(k+1) = f\left(x(k), x(k-1), \cdots, x(k-n+1)\right) : \textit{Nonlinear Markov Sequence} \\ P(k+1/k, k-1, \cdots, 0) = P\left(k+1/k, k-1, \cdots, k-(n-1)\right) \\ \qquad\qquad \rightarrow \text{n-th order Markov sequence} \end{cases}$$

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)w(k),\ x : n \times 1 \rightarrow \text{Linear 1'st order Markov seq.}$$

$$\begin{cases} \Phi(k), \Gamma(k):\ \text{deterministic} \\ x(0) \sim N(\bar{x}_0, X_0) \\ w(k) \sim N\left(\bar{w}(k), W(k)\right) \\ E\left[\{x(0) - \bar{x}(0)\}\{w(k) - \bar{w}(k)\}^T\right] = 0 \text{ for } k = 0, 1, \cdots \\ E\left[\{w(i) - \bar{w}(i)\}\{w(j) - \bar{w}(j)\}^T\right] = 0 \text{ for } i \neq j \end{cases}$$

Because $x(0)$ and $w(k)$ are Gaussian R.V. $\Rightarrow$ $x(k)$ is also Gaussian R.V.

Therefore, this system can be completely described by

1. mean-value sequence, $\bar{x}(k)$

2. covariance matrix sequence, $X(k) = E\left[\{x(k) - \bar{x}(k)\}\{x(k) - \bar{x}(k)\}^T\right]$

Linear Gauss Markov Sequence

$$\bar{x}(k+1) = E\left[x(k+1)\right]$$
$$= E\left[\Phi(k)x(k) + \Gamma(k)w(k)\right]$$
$$= \Phi(k)E\left[x(k)\right] + \Gamma(k)E\left[w(k)\right]$$
$$\therefore \bar{x}(k+1) = \Phi(k)\bar{x}(k) + \Gamma(k)\bar{w}(k) \text{ where } \bar{x}(0) : \text{initial condition}$$

$$X(k+1) = E\left[x'(k+1)x'(k+1)^T\right] = E\left[(x(k+1) - \bar{x}(k+1))(x(k+1) - \bar{x}(k+1))^T\right]$$

$$E\left[(\Phi(k)(x(k) - \bar{x}(k)) + \Gamma(k)w(k))(\Phi(k)(x(k) - \bar{x}(k)) + \Gamma(k)w(k))^T\right]$$

$$E\left[\Phi(k)(x(k) - \bar{x}(k))(x(k) - \bar{x}(k))^T\Phi(k)^T + \Gamma(k)w(k)w(k)^T\Gamma(k)^T\right]$$

$$\rightarrow \because E(x(k) \cdot w(k)) = 0$$

$$\boxed{\therefore X(k+1) = \Phi(k)X(k)\Phi(k)^T + \Gamma(k)W(k)\Gamma(k)^T}$$

Linear Gauss Markov Sequence

Linear Gauss Markov Sequence

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)w(k)$$

$$\begin{cases} \Phi(k), \Gamma(k): \text{ deterministic} \\ x(0) \sim N(\bar{x}_0, X_0) \\ w(k) \sim N(\bar{w}(k), W(k)) \\ E\left[\{x(0) - \bar{x}(0)\}\{w(k) - \bar{w}(k)\}^T\right] = 0 \text{ for } k = 0, 1, \cdots \\ E\left[\{w(i) - \bar{w}(i)\}\{w(j) - \bar{w}(j)\}^T\right] = 0 \text{ for } i \neq j \end{cases}$$

$$\bar{x}(k+1) = \Phi(k)\bar{x}(k) + \Gamma(k)\bar{w}(k)$$

$$\boxed{X(k+1) = \Phi(k)X(k)\Phi(k)^T + \Gamma(k)W(k)\Gamma(k)^T}$$

$x(0), X(0)$: initial condition

Estimation

< Criteria for Estimation >

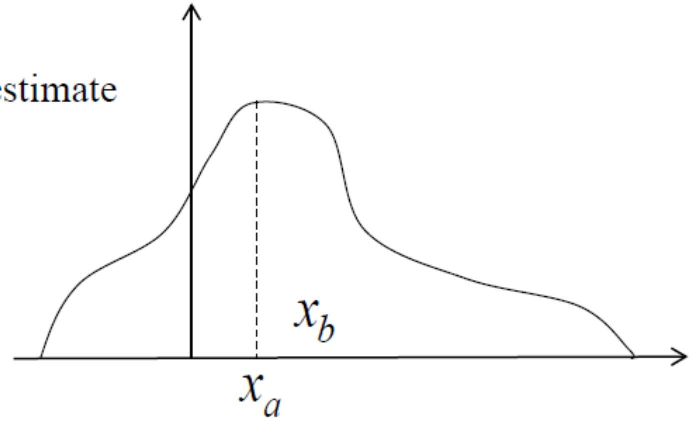
1. Maximize probability of x: most probable or likely estimate

$\rightarrow$ maximum likelyhood estimator

$$\hat{x} = \text{mode}(=\text{peak}) \text{ of } p(x \mid z) \Rightarrow \boxed{x_a}$$

2. Minumum variance: $\int (x-\hat{x})^2 \, p(x \mid z)\, dx$

$\rightarrow$ minimum variance estimator



$$J = \text{var}[(x \mid z)] = \frac{1}{2}\int_{-\infty}^{\infty} (x-\hat{x})^2 \, p(x \mid z)\, dx \Rightarrow \min_{\hat{x}} J$$

$$\frac{\partial J}{\partial \hat{x}} = 0 : \; -\int_{-\infty}^{\infty} (x-\hat{x})\, p(x \mid z)\, dx = 0$$

$$\hat{x}\int_{-\infty}^{\infty} p(x \mid z)\, dx = \int_{-\infty}^{\infty} x p(x \mid z)\, dx \quad \left(\because \int_{-\infty}^{\infty} p(x \mid z)\, dx = 1\right)$$

$$\hat{x} = \int_{-\infty}^{\infty} x p(x \mid z)\, dx = E\left[(x \mid z)\right] = (x \mid z)_m \text{ : mean value} \Rightarrow \boxed{x_b}$$

Estimation

3. Minimize max. of $|x - \hat{x}|$

$\rightarrow$ $\boxed{\text{minimum error estimator}}$

$$\hat{x} = \text{median of } p(x \mid z) \Rightarrow \boxed{x_c}$$

If $(x \mid z)$: Gaussian $\Rightarrow x_a = x_b = x_c$

$\therefore$ Kalman filter (based on Linear and Gaussian distribution) is $\begin{pmatrix} \text{Max. Likelyhood} \\ \text{Min. Variance} \\ \text{Min. Error} \end{pmatrix}$ Estimator!

Estimation

구하고자 하는 것 likelihood

$$P\left(\boldsymbol{x}_k | \boldsymbol{x}_0, \boldsymbol{u}_{1:k}, \boldsymbol{z}_{1:k-1}\right) = N\left(\boldsymbol{A}_k \hat{\boldsymbol{x}}_{k-1} + \boldsymbol{u}_k, \boldsymbol{A}_k \hat{\boldsymbol{P}}_{k-1} \boldsymbol{A}_k^{\mathrm{T}} + \boldsymbol{R}\right). \tag{10.11}$$

Posterior

$$P\left(\boldsymbol{z}_k | \boldsymbol{x}_k\right) = N\left(\boldsymbol{C}_k \boldsymbol{x}_k, \boldsymbol{Q}\right). \tag{10.13}$$

Likelihood can expressed as

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{p(x)}.$$ [1]

The posterior probability can be written in the memorable form as

$$\text{Posterior probability} \propto \text{Likelihood} \times \text{Prior probability}.$$

$$N(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{P}}_k) = N\left(\boldsymbol{C}_k \boldsymbol{x}_k, \boldsymbol{Q}\right) \cdot N(\bar{\boldsymbol{x}}_k, \bar{\boldsymbol{P}}_k). \tag{10.14}$$

$$\left(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k\right)^{\mathrm{T}} \hat{\boldsymbol{P}}_k^{-1} \left(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k\right) = \left(\boldsymbol{z}_k - \boldsymbol{C}_k \boldsymbol{x}_k\right)^{\mathrm{T}} \boldsymbol{Q}^{-1} \left(\boldsymbol{z}_k - \boldsymbol{C}_k \boldsymbol{x}_k\right) + \left(\boldsymbol{x}_k - \bar{\boldsymbol{x}}_k\right)^{\mathrm{T}} \bar{\boldsymbol{P}}_k^{-1} \left(\boldsymbol{x}_k - \bar{\boldsymbol{x}}_k\right). \tag{10.15}$$

Maximum Likelihood Prob.

Given: $x \sim N(\bar{x}, M)$ *before* $z = Hx + v,\ v \sim N(0, V)$ *and* $E\left[(x - \bar{x})v^T\right] = 0$

Find: $x\ (= \hat{x}) \sim N\left(\bar{\hat{x}}, P\right)$ *after measurement*

$$p(x \mid z) = \frac{p(z \mid x)\, p(x)}{p(z)}$$

1) $x \sim N(\bar{x}, M)$

2) $(z \mid x) \sim N(Hx, V)$ $(x: \textit{must be treated as deterministic})$

$\quad$ *where* $\overline{(z \mid x)} = \overline{(Hx + v)} = Hx + \cancel{\bar{v}} = Hx$

$$E\left[(z - \bar{z})(z - \bar{z})^T\right] = E\left[(Hx + v - Hx)(Hx + v - Hx)^T\right] = E\left[v v^T\right] = V$$

3) $z \sim N\left(H\bar{x}, HMH^T + V\right)$

$\quad$ *where* $z = Hx + v,\ \bar{z} = H\bar{x}$ $(x: \textit{must be treated as stochastic})$

$$E\left[(z - \bar{z})(z - \bar{z})^T\right] = E\left[(Hx + v - H\bar{x})(Hx + v - H\bar{x})^T\right] = E\left[H(x - \bar{x})(x - \bar{x})^T H^T + v v^T\right]$$

$$\therefore E\left[(x - \bar{x}) \cdot v\right] = 0$$

Maximum Likelihood Prob.

$$p(x)=\frac{1}{(2\pi)^{n/2}\,|M|^{1/2}}\exp\left[-\frac{1}{2}(x-\bar{x})^{T}M^{-1}(x-\bar{x})\right]$$

$$p(x\,|\,z)=\frac{p(z\,|\,x)\,p(x)}{p(z)}$$

$$\max_{x} p(x\,|\,z)=\max_{x}\left\{\exp\left[\begin{array}{ll}-\frac{1}{2}(z-Hx)^{T}V^{-1}(z-Hx) & \longrightarrow p(z\,|\,x)\\ -\frac{1}{2}(x-\bar{x})^{T}M^{-1}(x-\bar{x}) & \longrightarrow p(x)\\ +\frac{1}{2}(z-H\bar{x})\left(HMH^{T}+V\right)^{-1}(z-H\bar{x}) & \longrightarrow p(z):const.\end{array}\right]\right\}$$

$$\Rightarrow \min_{x} J=\frac{1}{2}(z-Hx)^{T}V^{-1}(z-Hx)+\frac{1}{2}(x-\bar{x})^{T}M^{-1}(x-\bar{x}) \qquad (1)$$

$$\rightarrow\ Solve\ x\ using\ \frac{\partial J}{\partial x}=0$$

Maximum Likelihood Prob.

$$\rightarrow \frac{\partial J}{\partial x} = 0$$

$$(z - Hx)^T V^{-1}(-H) + (x - \bar{x})^T M^{-1} = 0 \qquad \text{(2)}$$

$$\boxed{define \ (P')^{-1} \triangleq M^{-1} + H^T V^{-1} H} \quad (P': symmetric) \qquad \text{(3)}$$

$$(x - \bar{x})^T = (z - H\bar{x})^T V^{-1} H P'$$

$$(x - \bar{x}) = P' H^T V^{-1} (z - H\bar{x})$$

$$\therefore \hat{x} = x = \bar{x} + P' H^T V^{-1} (z - H\bar{x}) \qquad \text{(4)}$$

$\delta z = z - H\bar{x}$: residual (If 0, then the previous information is perfect!)

Maximum Likelihood Prob.

$$\hat{x} = \bar{x} + \left(M^{-1} + H^T V^{-1} H\right)^{-1} H^T V^{-1}\left(z - H\bar{x}\right)$$

If $M^{-1} = 0$ $(M \to \infty)$ $\left(\because y^T M^{-1} y \geq 0 \text{ for any } y\right)$

there is no previous information.

$$(2) \to \quad H^T V^{-1}(z - Hx) = 0$$

$$\left(H^T V^{-1} H\right)x = H^T V^{-1} z$$

$\boxed{\therefore\ \hat{x} = \left(H^T V^{-1} H\right)^{-1} H^T V^{-1} z}$: Weighted Least Square ——— (5)

If $V = \sigma^2 I$, $(5) \to \hat{x} = \left(H^T H\right)^{-1} H^T z$: Least Square

Error Update

$$define \quad e \triangleq \hat{x} - x : \text{ estimation error } \left(E\left(ee^T\right)?\right)$$

$$e = \bar{x} + P'H^T V^{-1}\left(z - H\bar{x}\right) - x$$

$$= \left(\bar{x} - x\right) + P'H^T V^{-1}\left(z - H\bar{x}\right) \qquad (z = Hx + v)$$

$$= \left(\bar{x} - x\right) + P'H^T V^{-1}\left[v - H\left(\bar{x} - x\right)\right] \quad \text{———— (6)}$$

$$\boxed{define \quad K \triangleq P'H^T V^{-1}}$$

$$(6) \rightarrow e = [I - KH]\left(\bar{x} - x\right) + Kv \quad \text{———— (7)}$$

$$P \triangleq E\left[ee^T\right] \quad = E\left[(I - KH)\left(\bar{x} - x\right)\left(\bar{x} - x\right)^T (I - KH)^T + Kvv^T K^T\right]$$

$$\boxed{= (I - KH) M (I - KH)^T + KVK^T} \quad \text{———— (8)}$$

$$P' \times (3) : \qquad I = P'M^{-1} + P'H^T V^{-1} H \quad \text{———— (9)}$$

$$(9) \times M : \qquad M = P' + P'H^T V^{-1} HM$$

$$\text{where } K = P'H^T V^{-1}$$

$$\boxed{P' = (I - KH) M} \quad \text{———— (10)}$$

Error Update

$$(8): \quad P = P'(I-KH)^T + P'H^TV^{-1}VK^T$$

$$\therefore P = P'$$

$$\boxed{P^{-1} = M^{-1} + H^TV^{-1}H} \qquad \text{———————} \quad (11)$$

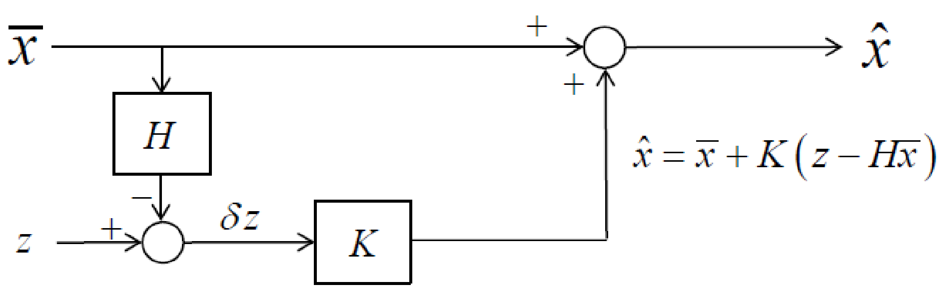
$$(7): \quad E[e] = 0 \ \leftarrow \ \textit{unbiased}$$

from (3), since $H^TV^{-1}H$ is positive-semidefinite $\left(y^T\left(H^TV^{-1}H\right)y \geq 0 \text{ for any } y\right)$

$$P^{-1} \geq M^{-1}$$

$$\Rightarrow P \leq M \quad \left(i.e.\ \ y^TPy \leq y^TMy \text{ for any } y\right)$$

$\therefore$ Uncertainty of x never increase after measurement update!



Discrete Kalman filter

Discrete Kalman Filter

1. Measurement update

$$\hat{x}(k)=\bar{x}(k)+K_d(k)\left[z(k)-H(k)\bar{x}(k)\right] \quad \left(\delta z(k)=z(k)-H(k)\bar{x}(k)\right)$$

where $K_d(k)=P(k)H(k)^T V^{-1}(k)$

$$P(k)^{-1}=M(k)^{-1}+H(k)^T V(k)^{-1} H(k)$$

2. Time update

$$\bar{x}(k+1)=\Phi(k)\hat{x}(k)$$

$$M(k+1)=\Phi(k)P(k)\Phi(k)^T+\Gamma_d(k)W_d(k)\Gamma_d(k)^T$$

$$x(k+1)=\Phi_d x(k)+\Gamma_d w_d(k)$$

$$z(k)=Hx(k)+v(k)$$

$$where\ w_d(k)\sim \mathrm{N}(0,W_d),\ v(k)\sim \mathrm{N}(0,V)$$

$$E\left[\left\{x(0)-\bar{x}(0)\right\} w_d(k)^T\right]=0$$

$$E\left[\left\{x(k)-\bar{x}(k)\right\} v^T(k)\right]=0$$

Estimation, Error Update

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{A}_k \boldsymbol{x}_{k-1} + \boldsymbol{u}_k + \boldsymbol{w}_k \\ \boldsymbol{z}_k = \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{v}_k \end{cases} \quad k = 1, \ldots, N. \tag{10.9}$$

$$\boldsymbol{w}_k \sim N(\boldsymbol{0}, \boldsymbol{R}). \quad \boldsymbol{v}_k \sim N(\boldsymbol{0}, \boldsymbol{Q}). \tag{10.10}$$

1. 예측

$$\bar{\boldsymbol{x}}_k = \boldsymbol{A}_k \hat{\boldsymbol{x}}_{k-1} + \boldsymbol{u}_k, \quad \bar{\boldsymbol{P}}_k = \boldsymbol{A}_k \hat{\boldsymbol{P}}_{k-1} \boldsymbol{A}_k^{\mathrm{T}} + \boldsymbol{R}. \tag{10.24}$$

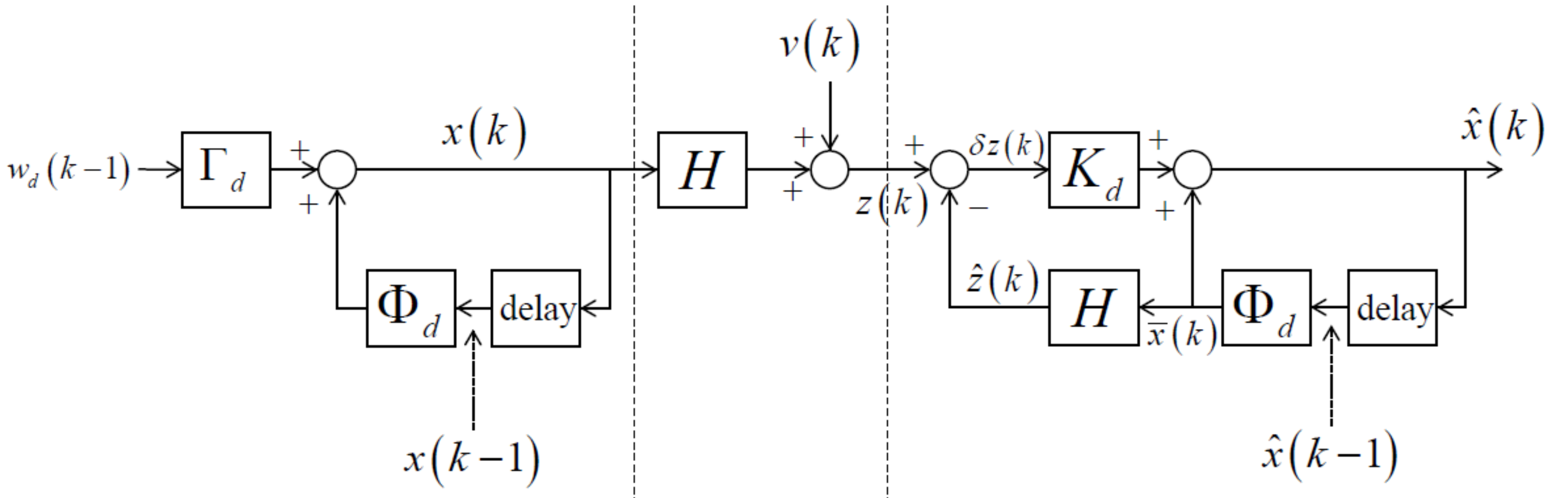
2. 업데이트 : 칼만 게인이라고도 하는 K를 먼저 계산합니다.

$$\boldsymbol{K} = \bar{\boldsymbol{P}}_k \boldsymbol{C}_k^{\mathrm{T}} \left(\boldsymbol{C}_k \bar{\boldsymbol{P}}_k \boldsymbol{C}_k^{\mathrm{T}} + \boldsymbol{Q}_k \right)^{-1}. \tag{10.25}$$

그런 다음 사후 확률 분포를 계산하십시오.

$$\begin{aligned} \hat{\boldsymbol{x}}_k &= \bar{\boldsymbol{x}}_k + \boldsymbol{K} \left(\boldsymbol{z}_k - \boldsymbol{C}_k \bar{\boldsymbol{x}}_k \right) \\ \hat{\boldsymbol{P}}_k &= \left(\boldsymbol{I} - \boldsymbol{K} \boldsymbol{C}_k \right) \bar{\boldsymbol{P}}_k. \end{aligned} \tag{10.26}$$

Discrete Kalman filter



system | measurement | kalman filter

$$x(k+1) = \Phi_d x(k) + \Gamma_d w_d(k)$$

$$z(k) = Hx(k) + v(k)$$

$$\hat{x}(k) = \bar{x}(k) + K_d(k)\left[z(k) - H(k)\bar{x}(k)\right]$$

$$\begin{bmatrix} K_d(k) = P(k)H(k)^T V^{-1}(k) \\ P(k)^{-1} = M(k)^{-1} + H(k)^T V(k)^{-1} H(k) \\ \bar{x}(k+1) = \Phi_d(k)\hat{x}(k) \\ M(k+1) = \Phi_d(k)P(k)\Phi_d(k)^T + \Gamma_d(k)W_d(k)\Gamma_d(k)^T \end{bmatrix}$$

Extended Kalman filter

$$\begin{cases} \dot{x}_N = f(x_N, u_N) + \Gamma w \\ z_N = h(x_N) + v \end{cases}$$

for nominal (or, trim, equilibrium) condition x_{N0}, u_{N0}

$$\dot{x}_N = \left\{ f(x_{N0}, u_{N0}) + \left[\left. \frac{\partial f}{\partial x} \right|_{x_{N0}, u_{N0}} \right] \cdot \delta x + \left[\left. \frac{\partial f}{\partial u} \right|_{x_{N0}, u_{N0}} \right] \cdot \delta u + h.o.t \right\} + \Gamma w \qquad \text{———— (1)}$$

$$where \quad \dot{x}_{N0} = f(x_{N0}, u_{N0})$$

$$\begin{cases} x_N = x_{N0} + \delta x \\ u_N = u_{N0} + \delta u \end{cases}$$

$$\dot{x}_N = \dot{x}_{N0} + \delta \dot{x} \qquad \text{———— (2)}$$

from $(1),(2)$:

$$\delta \dot{x} = F \delta x + G \delta u + \Gamma w \qquad \text{———— (3)}$$

$$\left(where\ F = \left[\left. \frac{\partial f}{\partial x} \right|_{x_{N0}, u_{N0}} \right], G = \left[\left. \frac{\partial f}{\partial u} \right|_{x_{N0}, u_{N0}} \right] \right)$$

Extended Kalman filter

$$\text{define } x \triangleq \delta x \text{ and } u \triangleq \delta u \text{ in } (3)$$

$$(3) \Rightarrow \therefore \boxed{\dot{x} = Fx + Gu + \Gamma w} \quad \text{———} \quad (4)$$

$$\text{likewise, } z_N = \left\{ h(x_{N0}) + \left[\frac{\partial h}{\partial x}\bigg|_{x_{N0}} \right] \cdot \delta x \right\} + v \quad \text{———} \quad (5)$$

$$where \begin{pmatrix} z_N = z_{N0} + \delta z \quad \text{———} \quad (6) \\ z_{N0} = h(x_{N0}) \end{pmatrix}$$

$$\therefore \delta z = Hx + v \qquad \left(where\ H \equiv \left[\frac{\partial h}{\partial x}\bigg|_{x_{N0}} \right] \right)$$

$$define\ z \triangleq \delta z$$

$$\therefore \boxed{z = Hx + v}$$

EKF 의 한계

- **K와 K-1 사이의 관계만을 고려하는 Incremental 방식**

-> 현재 상태가 실제로 오래 전의 데이터 (예: 루프 폐쇄)와 관련되어 있으면 필터를 처리하기가 어려울 수 있습니다. 반면에 비선형 최적화 기반 방법은 모든 히스토리 데이터를 사용합니다.

- **선형화로 인한 오차**

->EKF 필터는 한 번만 선형화 한 다음 선형화 결과를 기반으로 사후 확률을 직접 계산합니다. 강한 비선형성을 가지면 선형 근사는 매우 작은 범위에서만 유효하며 큰 거리에서 선형성으로 근사화 된 것으로 간주 할 수 없습니다.

- **Large scale 에 부적합**

-> 프로그램 구현 관점에서 볼 때, EKF는 상태 파라미터들 (로봇의 포즈)의 평균과 분산을 저장하고 이를 유지하고 업데이트 해야 합니다. 랜드마크도 상태로 놓으면 시각적 SLAM의 랜드마크가 많기 때문에 저장량은 상당히 크며 상태 파라미터의 수에 제곱에 해당됩니다.

Thank you

Q&A