AP Calculus
3.3 Worksheet Day 2

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the product rule?

$uv' + v'u$

2. What is the quotient rule?

$\frac{vu' - uv'}{v^2}$

3. Let $f(x) = (3x^3 + 4x^2)(2x^4 - 5x)$.

a) Find $f'(x)$ without using the product rule

$6x^7 - 15x^4 + 8x^6 - 20x^3$

$42x^6 - 60x^3 + 48x^5 - 60x^2$

b) Find $f'(x)$ using the product rule.

$(3x^3 + 4x^2)(8x^3 - 5) + (2x^4 - 5x)(9x^2 + 8x)$

$24x^6 - 15x^3 + 32x^5 - 20x^2 + 18x^6 + 16x^5 - 45x^3 - 40x^2$

$42x^6 - 60x^3 + 48x^5 - 60x^2$

4. Let $f(x) = \frac{x^2 + 4}{x}$.

a) Find $f'(x)$ without using the quotient rule

$\frac{x^2}{x} + \frac{4}{x}$

$x + \frac{4}{x} \Rightarrow 1 - \frac{4}{x^2}$

b) Find $f'(x)$ using the quotient rule.

$\frac{x(2x) - (x^2+4)(1)}{x^2}$

$\frac{2x^2 - x^2 - 4}{x^2} \Rightarrow \frac{x^2 - 4}{x^2}$

5. Find $\frac{dy}{dx}$ for each of the following functions.

a) $y = \frac{2x - 5}{3x + 2}$

$\ln y = \ln(2x-5) - \ln(3x+2)$

$\frac{y'}{y} = \frac{2}{2x-5} - \frac{3}{3x+2}$

$y' = \frac{\cancel{2x-5}}{3x+2}\left(\frac{\cancel{6x}+4 - \cancel{6x}+15}{\cancel{(2x-5)}(3x+2)}\right)$

$y' = \frac{19}{(3x+2)^2}$

b) $y = (3 - x)(2 + x^2)^{-1}$

$\frac{-1}{3-x} - \frac{2x}{2+x^2}$

$\ln y = \ln(3-x) - \ln(2+x^2)$

$y' = \frac{\cancel{(3-x)}}{(2+x^2)}\left(\frac{-2 - x^2 - 6x + 2x^2}{\cancel{(3-x)}(2+x^2)}\right)$

$y' = \frac{x^2 - 6x - 2}{(2+x^2)}$

$y =$

c) $y = \frac{x^3}{8 - x^2}$

6. For $a - d$, write a expression for $f'(x)$ and then use it to find $f'(2)$ given the following information:

$$g(2) = 3 \quad g'(2) = -2$$
$$h(2) = -1 \quad h'(2) = 4$$

a) $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2(-2) + 4$

$f'(2) = 0$

b) $f(x) = 4 - h(x)$

$f'(x) = 0 - h'(x)$

$f'(2) = -4$

c) $f(x) = g(x)h(x)$

$f'(x) = g'(x)h'(x)$

$f'(2) = -2(4)$

$f'(2) = -8$

d) $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{g'(x)}{h'(x)}$

$f'(2) = \dfrac{-2}{4} = -1/2$

7. Suppose u and v are differentiable functions of $x = 3$ and that $u(3) = 4$, $\left.\dfrac{du}{dx}\right|_{x=3} = -3$, $v(3) = 2$, and $\left.\dfrac{dv}{dx}\right|_{x=3} = 3$. Find the values of the following derivatives at $x = 3$.

a) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$

$\dfrac{vu' - uv'}{v^2} = \dfrac{2(-3) - 4(3)}{2^2}$

$= \dfrac{-12 - 6}{4} = 4/2$

b) $\dfrac{d}{dx}(uv)$

$vu' + uv'$

$2(-3) + 4(3)$

$-6 + 12$

$= 6$

c) $\dfrac{d}{dx}(5u - 2v + 4uv)$

$5u' - 2v' + 4(6)$

$5(-3) - 2(3) + 4(6)$

$-15 - 6 + 10 = -11$

d) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$

$\dfrac{uv' - vu'}{u^2}$

$\dfrac{4(3) - 2(-3)}{4^2} = \dfrac{18}{16} = 9/8$

8. Solve for a and b in order for $f(x)$ to be both continuous and differentiable at $x = 1$. (be sure to use the definition of continuity)

$$f(x) = \begin{cases} x^2 + 2 & ;x \leq 1 \\ a\left(x - \frac{1}{x}\right) + b & ;x > 1 \end{cases}$$

9. For each of the following, find the equation of the tangent line to the given function at the indicated point.

a) $f(x) = (x^3 - 3x + 1)(x + 2)$ at the point $(1, -3)$.

b) $y = \dfrac{8}{4 + x^2}$ at the point $(-2, 1)$.

10. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

A) $\left(\frac{1}{2}, \frac{1}{2}\right)$
B) $\left(\frac{1}{2}, \frac{1}{8}\right)$
C) $\left(1, -\frac{1}{4}\right)$
D) $\left(1, \frac{1}{2}\right)$
E) $(2, 2)$

11. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

A) 0.4
B) 0.5
C) 2.6
D) 3.4
E) 5.5

12. An equation of the line tangent to the graph of $y = \dfrac{2x + 3}{3x - 2}$ at the point $(1, 5)$ is

A) $13x - y = 8$
B) $13x + y = 18$
C) $x - 13y = 64$
D) $x + 13y = 66$
E) $-2x + 3y = 13$

13. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

A) -2
B) $\frac{1}{6}$
C) $\frac{1}{2}$
D) 2
E) 6

14. If u, v, and w are nonzero differentiable functions of x, then the $\dfrac{d}{dx}\left(\dfrac{uv}{w}\right)$ is

A) $\dfrac{uv' + u'v}{w'}$

B) $\dfrac{u'v'w - uvw'}{w^2}$

C) $\dfrac{uvw' - uv'w - u'vw}{w^2}$

D) $\dfrac{u'vw + uv'w + uvw'}{w^2}$

E) $\dfrac{uv'w + u'vw - uvw'}{w^2}$

15. When an object is thrown off a 100 foot cliff with an initial velocity of 40 feet/second, the height h, in feet, of the object can be modeled as a function of time t, in seconds, using the function

$$h(t) = -16t^2 + 45t + 100.$$

a) Find $\dfrac{dh}{dt}$ ... What is the unit of measurement for this equation?

b) Find $\dfrac{d^2h}{dt^2}$ ... What is the unit of measurement for this equation?

16. Let $g(x) = x - \dfrac{1}{x}$. Find the following:

a) $g'(x)$

b) $g''(x)$

c) The tangent line equation when $x = 2$